Caustic-singularity-free scalar field theory with shift symmetry

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Based on

"Is the DBI scalar field as fragile as other *k*-essence fields?"

with S. Mukohyama and R. Namba, [1605.06418](1605.06418), PRD **94**, 023514 (2016)

Outline

- Singularity in nonlinear wave solution

  where $\partial^2\phi$ diverge,
  effective description breaks down,
  theory should be replaced

  (: essentially review) Babichev 1602.00735

- Theory without singularity in the solution

  Mukohyama, Namba, YW 1605.06418

Scalar theories

- Simplest models of Inflation: scalar theories

e.g. $$I = \int d^n x \sqrt{-g}\, \mathcal{L}(\phi, X) \qquad X = -\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$\ni$ Dirac-Born-Infeld theory $\xleftarrow{\text{low energy}}$ brane

$$\mathcal{L}(\phi, X)_{\mathrm{DBI}} = -f(\phi)^{-1}\sqrt{1-2Xf(\phi)} - V(\phi)$$

- Consider $I = \int d^n x\, \mathcal{L}(X)$: shift sym. $\phi \to \phi + \text{const.}$

on a fixed Minkowski background (for simplicity)

Equation of Motion (EoM)

EoM

$$(-\mathcal{L}_X g^{\mu\nu} + \mathcal{L}_{XX}\partial^\mu\phi\partial^\nu\phi)\partial_\mu\partial_\nu\phi = 0 \qquad \leftarrow \quad I = \int d^n x\, \mathcal{L}(X)$$

(1+1 dim.) planar sym. $\phi = \phi(t, x)$

$\tau \equiv \dot{\phi}\,, \qquad \chi \equiv \phi'$

$$\rightarrow \quad \begin{cases} A\dot{\tau} + 2B\tau' + C\chi' = 0 \\ \tau' - \dot{\chi} = 0 \end{cases}$$

A, B, C: functions of $\tau,\ \chi$

$B^2 - AC > 0$

$\leftrightarrow$ (sound speed)$^2 > 0$

Take linear combination to align differential directions

$$\leftrightarrow \qquad (\partial_t + \xi_\pm\partial_x)\tau + \xi_\mp(\partial_t + \xi_\pm\partial_x)\chi = 0 \qquad \xi_\pm = \xi_\pm(\tau, \chi)$$

$(t, x) \rightarrow (\sigma_+, \sigma_-)$ s.t. $\partial_{\sigma_\pm} \propto (\partial_t + \xi_\pm\partial_x)$: characteristic directions

Interpretation of characteristics: $\partial_{\sigma_\pm} \propto (\partial_t + \xi_\pm \partial_x)$

$\xi_\pm = \pm 1 \leftarrow \mathcal{L}(X) = X$

- Suppose we can consider background & perturbation:

$$\phi = \bar{\phi} + \delta\phi$$

i.e. time & length scales of $\delta\phi$ are shorter than those of $\bar{\phi}$

Linear EoM $\quad \bar{A}\delta\ddot{\phi} + 2\bar{B}\delta\dot{\phi}' + \bar{C}\delta\phi'' \simeq 0$

$\rightarrow \quad \bar{A}\omega^2 - 2\bar{B}\omega k + \bar{C}k^2 \simeq 0$

$\rightarrow$ $\omega = \xi_\pm k$

$\xi_\pm$: phase velocity (→ sound cone) of perturbation

- In fact, $\xi_\pm$: relativistic additions of background-fluid velocity $v = \partial^x\phi/\partial^t\phi$ & sound speed of perturbation $\pm c_s$

Back to nonlinear analysis

EoM $\quad \rightarrow \quad \partial_{\sigma_\pm}\tau + \xi_\mp \partial_{\sigma_\pm}\chi = 0$

Analytically integrable along $\partial_{\sigma_\pm}$

$$\rightarrow \quad \int \frac{dX}{X c_s(X)} \pm \ln\frac{1+v}{1-v} = \Gamma_\pm(\sigma_\mp)$$

Integral curves along $\partial_{\sigma_\pm}$ in τ-χ plain

e.g. for $\mathcal{L}(X) = X + \frac{1}{2}X^2 \quad \rightarrow$

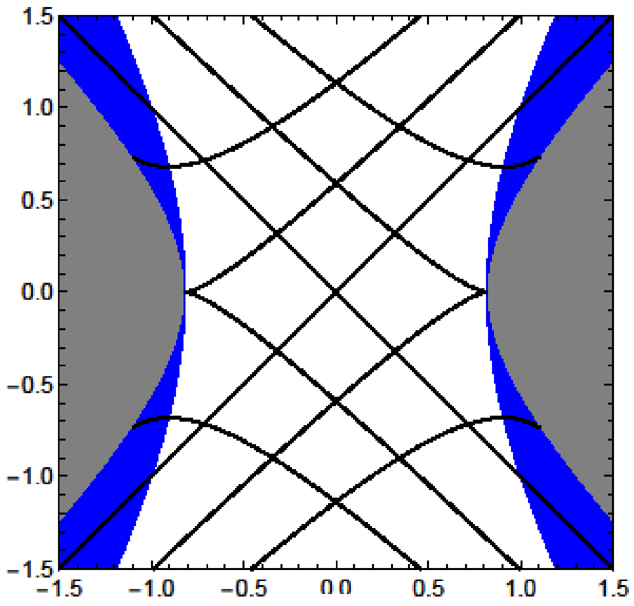
: independent of solutions in t-x plain

$$X = \frac{1}{2}(\tau^2 - \chi^2)$$

$$v \equiv -\chi/\tau$$

$$c_s \equiv \left(\frac{\mathcal{L}_X}{\mathcal{L}_X + 2X\mathcal{L}_{XX}}\right)^{1/2}$$

$\Gamma_\pm$: integration constants

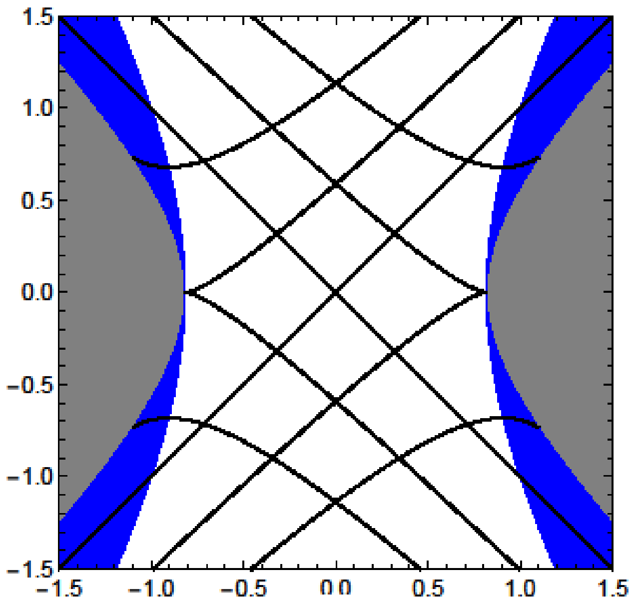


τ

χ

Solutions in t-x plain

Courant & Friedrichs (1948)



τ

χ

1. Steady state:

Both τ & χ are constant (trivial)

2. Simple wave:

Whose image in τ-χ plain lies entirely on one of the integration curves

i.e. τ & χ depends on only one of $\sigma_\pm$

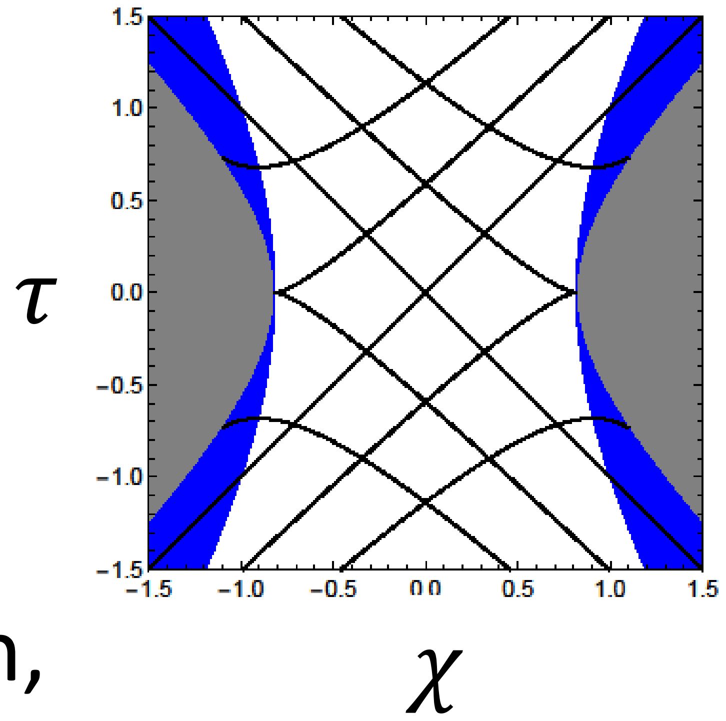
3. General wave

Whose image in τ-χ plain does not entirely lie on the integration curves

Simple wave solution (easily constructed)

- τ & χ change only along (e.g.) ∂_{σ_-},

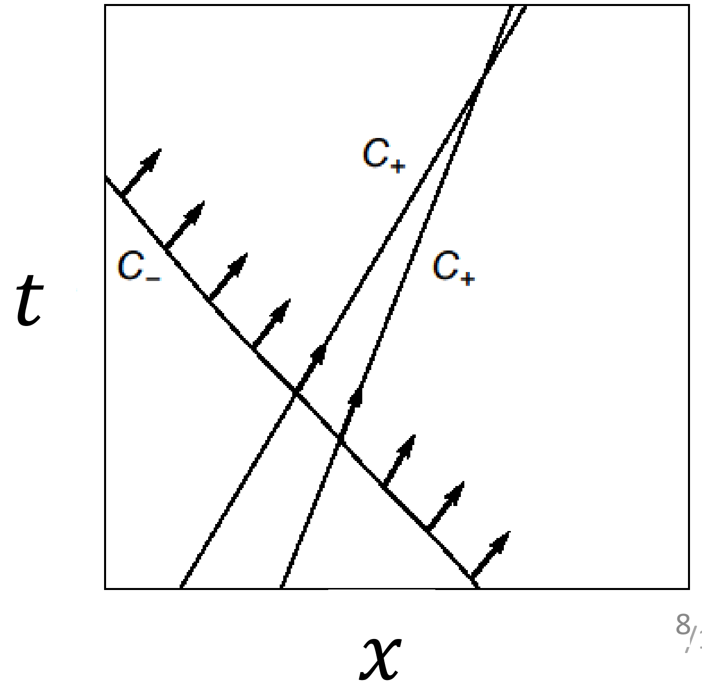
  constant along ∂_{σ_+}



- Choose an integration curve in τ-χ plain, define σ_- dependence on the curve of τ or χ

  $\rightarrow \quad \xi_\pm = \xi_\pm(\sigma_-)$ is determined

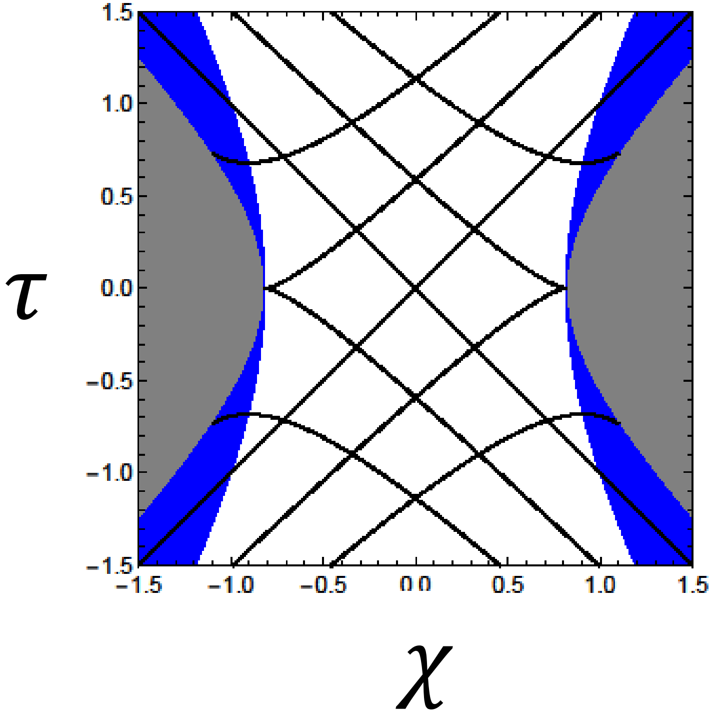
- Integrating $\partial_{\sigma_\pm} \propto (\partial_t + \xi_\pm \partial_x)$ gives a solution in t-x plain ($\simeq$ right moving)



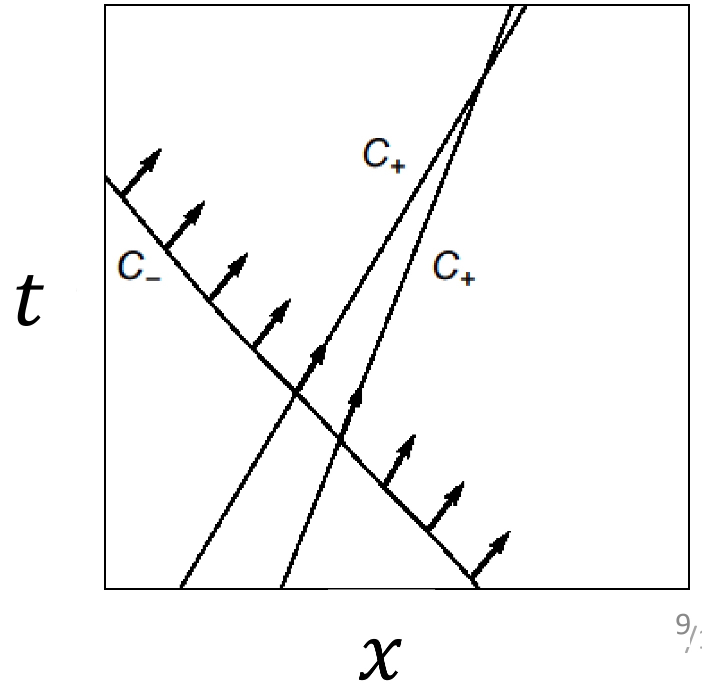
Divergence of $\partial^2\phi$

Now $\xi_\pm = \xi_\pm(\sigma_-)$, two right-going curves in t-x plain intersect.

Each curve carry different constant values of $\tau, \chi \simeq \partial\phi$.



→ At the intersection, $\partial^2\phi$ diverge and ϕ isn't single valued. ∴Effective description by the theory breaks down, the theory should be replaced to describe motions after that.



This divergence is generic   Babichev [1602.00735](https://arxiv.org/abs/1602.00735)

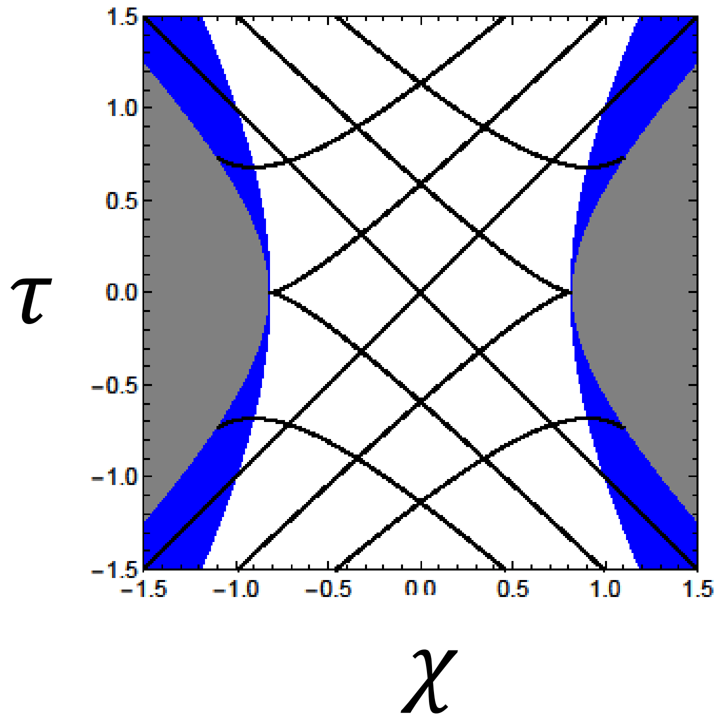
- The divergence does not happen only if $\partial_{\sigma_-}\xi_+ = 0$.

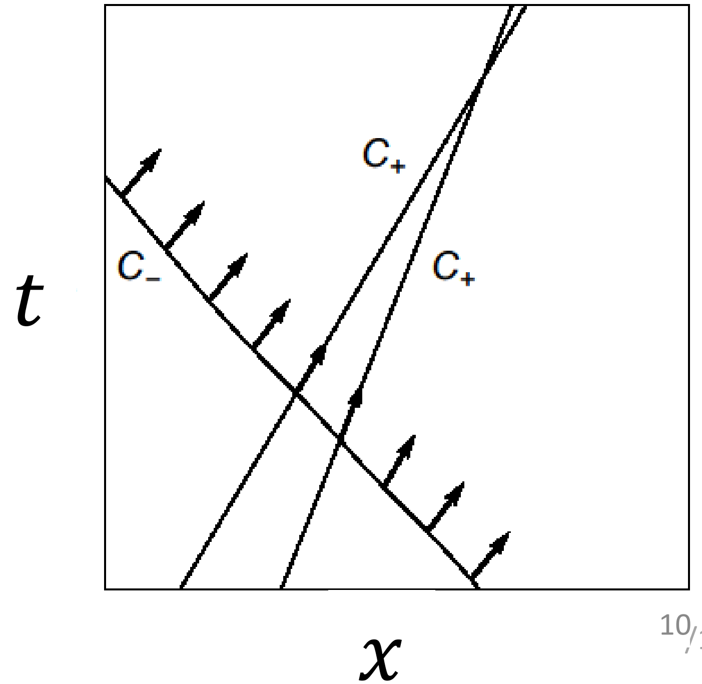
  ⇕ EoM: $\partial_{\sigma_\pm}\tau + \xi_\mp \partial_{\sigma_\pm}\chi = 0$

  Choose an integration curve in τ-χ plain which is linear, fine-tuning initial condition as $(\partial\phi)^2 = 0$

- The divergence is not cured by higher-dim. (Horndeski) terms yielding 2nd-order EoM

- Not diverge only $\mathcal{L}(X) = X$? $(\xi_\pm = \pm 1)$





All theories without the singularity

Mukohyama, Namba, YW 1605.06418

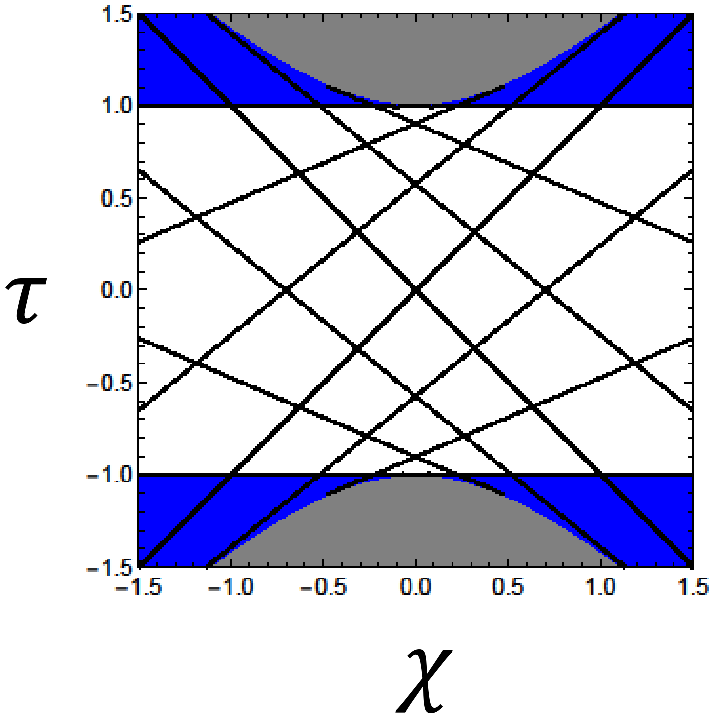
$\partial_{\sigma_\mp}\xi_\pm = 0 \leftrightarrow$ There exists a (const.) s.t. the integration curves in τ-χ plain along characteristics, $d\tau + \xi_\pm d\chi = 0$ be linear: $\tau = a\chi + b$ for any b (const.).

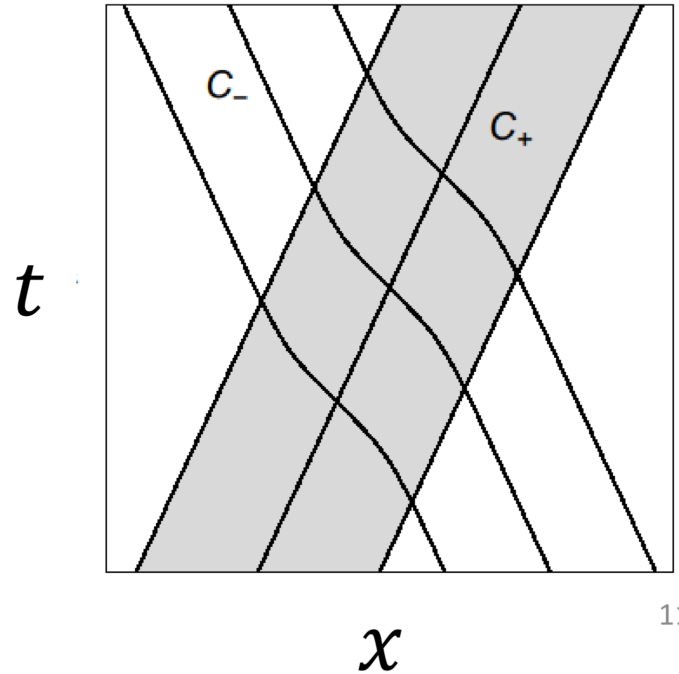
$\rightarrow \xi_\pm \rightarrow c_s(X)$ is determined

$$(\ln \mathcal{L}_X)_X = \frac{1}{2X}\left(\frac{1}{c_s^2(X)} - 1\right)$$

$\mathcal{L}(X) = X$ or $-(\text{const.})\sqrt{1-2X}$

up to const. and rescaling of ϕ





Conclusion

- We can solve $\mathcal{L} = \mathcal{L}((\partial\phi)^2)$ on a fixed Minkowski background by (1+1)-dim. nonlinear wave solutions where $\partial\phi$ is constant along a family of characteristics

- Without fine-tuning initial conditions, characteristics intersect $\rightarrow \partial^2\phi$ diverge and ϕ isn't single valued. Effective description by the theory breaks down, the theory should be replaced.

  This behavior does not change by adding higher-dim. (Horndeski) terms yielding 2^{nd}-order EoM.

- We've found all the theories without the singularity in the solution: canonical scalar and Dirac-Born-Infeld theory