

Effective Tolman temperature induced by trace anomaly on the four-dimensional Schwarzschild background

Tolman temperature

which is a local temperature detected
by the free fall (proper) observer at the finite distance
from the black hole in the thermal equilibrium state.

Tolman temperature

A diagram consisting of a large black rectangular box. Inside this box, on the left side, is a smaller orange rounded rectangle containing the text 'Tolman temperature'. A horizontal orange line extends from the right side of this rounded rectangle, then turns 90 degrees upwards and ends in an orange arrowhead pointing towards the top-right corner of the large black box.

which is a local temperature detected
by the free fall (proper) observer at the finite distance
from the black hole in the thermal equilibrium state.

Effective Tolman temperature induced by trace anomaly on the four-dimensional Schwarzschild background

Yongwan Gim
(Sogang University, Korea)

collaborated with Wontae Kim and Myungseok Eune
and based on arXiv : 1511.09135

which is a local temperature detected
by the free fall (proper) observer at the finite distance
from the black hole in the thermal equilibrium state.

Effective Tolman temperature induced by trace anomaly on the four-dimensional Schwarzschild background

Yongwan Gim
(Sogang University, Korea)

collaborated with Wontae Kim and Myungseok Eune
and based on arXiv: 1511.09135

	<div>The ordinary Tolman Temperature in 4D black hole</div> <div>[R.C.Tolman,Phys.Rev.35,904 (1930)]</div>
Conservation law	
The fluid eq.	
Trace	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	<div>The ordinary Tolman Temperature in 4D black hole</div> <div>[R.C.Tolman,Phys.Rev.35,904 (1930)]</div>
Conservation law	
The fluid eq.	
Trace	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The ordinary Tolman Temperature in 4D black hole [R.C.Tolman,Phys.Rev.35,904 (1930)]
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	
Trace	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The ordinary Tolman Temperature in 4D black hole [R.C.Tolman,Phys.Rev.35,904 (1930)]
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$
Trace	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The ordinary Tolman Temperature in 4D black hole [R.C.Tolman,Phys.Rev.35,904 (1930)]
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$T^{\mu}_{\mu} = 0$
The first law	
Stefan-Boltzmann law	
Proper temperature	

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The ordinary Tolman Temperature in 4D black hole [R.C.Tolman,Phys.Rev.35,904 (1930)]
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ ρ : proper energy density, p : proper pressure
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	
Proper temperature	

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}}$ <p>where $f = -g_{00}(r)$,</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}}$ <p>where $f = -g_{00}(r)$,</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}}$ <p>where $f = -g_{00}(r)$, $\frac{C_0}{\sqrt{\gamma}}$: Hawking temperature (1974)</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}} = \frac{T_H}{\sqrt{f(r)}}$ <p>where $f = -g_{00}(r)$, $\frac{C_0}{\sqrt{\gamma}}$: Hawking temperature (1974)</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}} = \frac{T_H}{\sqrt{f(r)}} \sim \text{Trace anomaly}$ <p>where $f = -g_{00}(r)$, $\frac{C_0}{\sqrt{\gamma}}$: Hawking temperature (1974)</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0 \quad \text{Traceless?}$
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}} = \frac{T_H}{\sqrt{f(r)}} \quad \sim \text{Trace anomaly}$ <p>where $f = -g_{00}(r)$, $\frac{C_0}{\sqrt{\gamma}}$: Hawking temperature (1974)</p>

	<p>The ordinary Tolman Temperature in 4D black hole</p> <p>[R.C.Tolman,Phys.Rev.35,904 (1930)]</p>
Conservation law	$\nabla_{\mu} T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$ <p>ρ : proper energy density, p : proper pressure</p> <p>Isotropic?</p>
Trace	$p = \frac{1}{3}\rho \Leftrightarrow T^{\mu}_{\mu} = 0$ <p>Traceless?</p>
The first law	$dU = TdS - pdV$
Stefan-Boltzmann law	$\rho = 3\gamma T^4 \qquad p = \gamma T^4$ <p>where γ is the Stefan-Boltzmann constant.</p>
Proper temperature	$T = \frac{C_0}{\sqrt{\gamma f(r)}} = \frac{T_H}{\sqrt{f(r)}}$ <p>where $f = -g_{00}(r)$, $\frac{C_0}{\sqrt{\gamma}}$: Hawking temperature (1974)</p> <p>~Trace anomaly</p>

	The trace anomaly-induced Tolman Temperature
Conservation law	
The fluid eq.	
Trace anomaly	
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole	

	The trace anomaly-induced Tolman Temperature
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$
The fluid eq.	
Trace anomaly	
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole	

	The trace anomaly-induced Tolman Temperature
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu + p_t g^{\mu\nu} + (p_r - p_t)n_{(r)}^\mu n_{(r)}^\nu$
	p_t : the tangential pressure, p_r : the radial pressure, ρ : proper energy density, u^μ : the proper velocity $n_{(r)}^\mu$: the unit spacelike vector in the radial direction, $n_{(\theta)}^\mu$ and $n_{(\phi)}^\mu$: the unit normal vectors which are orthogonal to $n_{(r)}^\mu$
Trace anomaly	
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole	

	The trace anomaly-induced Tolman Temperature	
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$	
The fluid eq.	$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu + p_t g^{\mu\nu} + (p_r - p_t)n_{(r)}^\mu n_{(r)}^\nu$: anisotropic	
	<p>p_t : the tangential pressure, p_r : the radial pressure, ρ : proper energy density, u^μ : the proper velocity $n_{(r)}^\mu$: the unit spacelike vector in the radial direction, $n_{(\theta)}^\mu$ and $n_{(\phi)}^\mu$: the unit normal vectors which are orthogonal to $n_{(r)}^\mu$</p>	
Trace anomaly		
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole		

	The trace anomaly-induced Tolman Temperature	
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$	
The fluid eq.	$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu + p_t g^{\mu\nu} + (p_r - p_t)n_{(r)}^\mu n_{(r)}^\nu$: anisotropic	
	<p>p_t : the tangential pressure, p_r : the radial pressure, ρ : proper energy density, u^μ : the proper velocity $n_{(r)}^\mu$: the unit spacelike vector in the radial direction, $n_{(\theta)}^\mu$ and $n_{(\phi)}^\mu$: the unit normal vectors which are orthogonal to $n_{(r)}^\mu$</p>	
Trace anomaly	$T^\mu_\mu = \alpha(\mathcal{F} + (2/3)\square R) + \beta\mathcal{G} \neq 0$	
	<p>where $\mathcal{F} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 2R^{\mu\nu} R_{\mu\nu} + R^2/3$, and $\mathcal{G} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$.</p>	
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole		

	The trace anomaly-induced Tolman Temperature	
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$	
The fluid eq.	$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu + p_t g^{\mu\nu} + (p_r - p_t)n_{(r)}^\mu n_{(r)}^\nu$: anisotropic	
	p_t : the tangential pressure, p_r : the radial pressure, ρ : proper energy density, u^μ : the proper velocity $n_{(r)}^\mu$: the unit spacelike vector in the radial direction, $n_{(\theta)}^\mu$ and $n_{(\phi)}^\mu$: the unit normal vectors which are orthogonal to $n_{(r)}^\mu$	
Trace anomaly	$T^\mu_\mu = \alpha(\mathcal{F} + (2/3)\square R) + \beta\mathcal{G} \neq 0$	
	where $\mathcal{F} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 2R^{\mu\nu} R_{\mu\nu} + R^2/3$, and $\mathcal{G} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$. : curvature	
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole		

The trace anomaly-induced Tolman Temperature	
Conservation law	$\nabla_\mu T^{\mu\nu} = 0$
The fluid eq.	$T^{\mu\nu} = (\rho + p_t)u^\mu u^\nu + p_t g^{\mu\nu} + (p_r - p_t)n_{(r)}^\mu n_{(r)}^\nu$: anisotropic
	<p> p_t : the tangential pressure, p_r : the radial pressure, ρ : proper energy density, u^μ : the proper velocity $n_{(r)}^\mu$: the unit spacelike vector in the radial direction, $n_{(\theta)}^\mu$ and $n_{(\phi)}^\mu$: the unit normal vectors which are orthogonal to $n_{(r)}^\mu$ </p>
Trace anomaly	$T^\mu_\mu = \alpha(\mathcal{F} + (2/3)\square R) + \beta\mathcal{G} \neq 0$
	<p> where $\mathcal{F} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 2R^{\mu\nu} R_{\mu\nu} + R^2/3$, and $\mathcal{G} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2$. </p> : curvature
The stress tensor in thermal equilibrium with conformal anomaly for 4D Schwarzschild black hole	
$T^\mu_\nu = \frac{\pi^2}{90} \left(\frac{1}{8\pi M} \right)^4 \left[\frac{1 - (4 - \frac{6M}{r})^2 (\frac{2M}{r})^6}{(1 - \frac{2M}{r})^2} (\delta^\mu_\nu - 4\delta^\mu_0 \delta^0_\nu) + 24 \left(\frac{2M}{r} \right)^6 (3\delta^\mu_0 \delta^0_\nu + \delta^\mu_1 \delta^1_\nu) \right]$	

	The trace anomaly-induced Tolman Temperature
Proper energy density	
The radial pressure	
The tangential pressure	
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The trace anomaly-induced Tolman Temperature
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$
The first law	
Stefan-Boltzmann law	
Proper temperature	

	The trace anomaly-induced Tolman Temperature	
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	negative and finite near the horizon
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The first law		
Stefan-Boltzmann law		
Proper temperature		

	The trace anomaly-induced Tolman Temperature	
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> <div>$\rho = 3\gamma T^4$</div>
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The first law		
Stefan-Boltzmann law		
Proper temperature		

	The trace anomaly-induced Tolman Temperature	
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{ ?!}$
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The first law		
Stefan-Boltzmann law		
Proper temperature		

	The trace anomaly-induced Tolman Temperature	
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{ ?!}$
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	
The first law	$dU = TdS - p_r dV$	
Stefan-Boltzmann law		
Proper temperature		

	The trace anomaly-induced Tolman Temperature		
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^{\textcolor{orange}{4}} ?!$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The first law	$dU = TdS - p_r dV$		
Stefan-Boltzmann law	$\rho = 3\gamma T^4 - \frac{3}{8} T_\mu^\mu$	$p_r = \gamma T^4 + \frac{3}{8} T_\mu^\mu$	$p_t = \gamma T^4 + \frac{1}{8} T_\mu^\mu$
Proper temperature			

	The trace anomaly-induced Tolman Temperature		
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{?!}$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The first law	$dU = TdS - p_r dV$		
Stefan-Boltzmann law	$\rho = 3\gamma T^4 - \frac{3}{8} T_\mu^\mu$	$p_r = \gamma T^4 + \frac{3}{8} T_\mu^\mu$	$p_t = \gamma T^4 + \frac{1}{8} T_\mu^\mu$
Proper temperature			

	The trace anomaly-induced Tolman Temperature		
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{?!}$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The first law	$dU = TdS - p_r dV$		
Stefan-Boltzmann law	$\rho = 3\gamma T^4 - \frac{3}{8} T_\mu^\mu$	$p_r = \gamma T^4 + \frac{3}{8} T_\mu^\mu$	$p_t = \gamma T^4 + \frac{1}{8} T_\mu^\mu$
Proper temperature	$T = \frac{1}{\gamma^{1/4} \sqrt{f}} \left(C_0 - \frac{3}{8} f^2 T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)^{1/4}$		
	where $C_0 = \gamma T_H^4$, γ is the Stefan-Boltzmann constant, and $f = -g_{00}(r)$.		

	The trace anomaly-induced Tolman Temperature		
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{?!}$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The first law	$dU = TdS - p_r dV$		
Stefan-Boltzmann law	$\rho = 3\gamma T^4 - \frac{3}{8} T_\mu^\mu$	$p_r = \gamma T^4 + \frac{3}{8} T_\mu^\mu$	$p_t = \gamma T^4 + \frac{1}{8} T_\mu^\mu$
Proper temperature	$T = \frac{1}{\gamma^{1/4} \sqrt{f}} \left(C_0 - \frac{3}{8} f^2 T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)^{1/4}$		
	where $C_0 = \gamma T_H^4$, γ is the Stefan-Boltzmann constant, and $f = -g_{00}(r)$.		

	The trace anomaly-induced Tolman Temperature		
Proper energy density	$\rho = \frac{3}{f^2} \left(C_0 - \frac{f^2}{2} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The radial pressure	$p_r = \frac{1}{f^2} \left(C_0 + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$	<div>negative and finite near the horizon</div> $\rho = 3\gamma T^4 \text{ ?!}$	
The tangential pressure	$p_t = \frac{1}{f^2} \left(C_0 - \frac{f^2}{4} T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)$		
The first law	$dU = TdS - p_r dV$		
Stefan-Boltzmann law	$\rho = 3\gamma T^4 - \frac{3}{8} T_\mu^\mu$	$p_r = \gamma T^4 + \frac{3}{8} T_\mu^\mu$	$p_t = \gamma T^4 + \frac{1}{8} T_\mu^\mu$
Proper temperature	$T = \frac{1}{\gamma^{1/4} \sqrt{f}} \left(C_0 - \frac{3}{8} f^2 T_\mu^\mu + \int^r \frac{f}{4r} (-2f + 3r \partial_r f) T_\mu^\mu dr \right)^{1/4}$		
	where $C_0 = \gamma T_H^4$, γ is the Stefan-Boltzmann constant, and $f = -g_{00}(r)$.		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature		
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph	<p style="text-align: center;">T</p> <p style="text-align: center;">ordinary Tolman temperature</p> <p style="text-align: center;">Diverge at the horizon</p> <p style="text-align: center;">Firewall-like object in the thermal equilibrium...??</p> <p style="text-align: center;">T_{max}</p> <p style="text-align: center;">T_H</p> <p style="text-align: center;">2M r_c r</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	
Graph	<p>Diverge at the horizon</p> <p>Firewall-like object in the thermal equilibrium...??</p> <p>Violation of the equivalence principle...??</p> <p>ordinary Tolman temperature</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph	<p>The graph shows the temperature T as a function of the radial coordinate r. The vertical axis T has marked levels T_{max} and T_H. The horizontal axis r has marked positions 2M and r_c. A blue dashed curve represents the temperature profile, which decreases from a high value near the horizon towards the Hawking temperature T_H at large r. A label 'ordinary Tolman temperature' with an arrow points to this curve.</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph		

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph	<p>The graph illustrates the relationship between temperature T and radial coordinate r for a 4D Schwarzschild black hole. The x-axis represents r, with key points $2M$ (the event horizon) and r_c (the location of maximum temperature for the trace anomaly-induced case). The y-axis represents temperature T, with T_H being the asymptotic temperature at large r. The dashed purple curve represents the ordinary Tolman temperature, which decreases from infinity at $r=2M$. The solid blue curve represents the trace anomaly-induced Tolman temperature, which peaks at T_{\max} at $r=r_c$ before decreasing towards T_H.</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph	<p>ordinary Tolman temperature</p> <p>The trace anomaly-induced Tolman temperature</p> <p>vanish at the horizon</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph	<p>ordinary Tolman temperature</p> <p>The trace anomaly-induced Tolman temperature</p> <p>vanish at the horizon</p> <p>Nothing at the horizon</p>	

	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$	
Proper temperature	$T = \frac{1}{8\pi G M \sqrt{1 - \frac{2M}{r}}}$	$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$
Graph		

vanish at the horizon

Nothing at the horizon

Equivalence principle is recovered.

Conclusions and Discussions

The anomaly-induced
Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_{\mu}^{\mu} \quad p_r = \gamma T^4 + \frac{3}{8}T_{\mu}^{\mu} \quad p_t = \gamma T^4 + \frac{1}{8}T_{\mu}^{\mu}$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann law.

ex) Hawking radiation, Cosmology etc...

The anomaly-induced
Tolman
temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

Conclusions and Discussions

The anomaly-induced Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_\mu^\mu \quad p_r = \gamma T^4 + \frac{3}{8}T_\mu^\mu \quad p_t = \gamma T^4 + \frac{1}{8}T_\mu^\mu$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann law.

ex) Hawking radiation, Cosmology etc...

The anomaly-induced Tolman temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

Conclusions and Discussions

The anomaly-induced
Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_\mu^\mu \quad p_r = \gamma T^4 + \frac{3}{8}T_\mu^\mu \quad p_t = \gamma T^4 + \frac{1}{8}T_\mu^\mu$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann law.

ex) Hawking radiation, Cosmology etc...

The anomaly-induced
Tolman
temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

Conclusions and Discussions

The anomaly-induced Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_{\mu}^{\mu} \quad p_r = \gamma T^4 + \frac{3}{8}T_{\mu}^{\mu} \quad p_t = \gamma T^4 + \frac{1}{8}T_{\mu}^{\mu}$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann law.

ex) Hawking radiation, Cosmology etc...

The anomaly-induced Tolman temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

In addition, 'the general formalism of the trace-anomaly induced Tolman T' for 2D black hole is in [Eur.Phys.J.C 75 (2015) 549].

Conclusions and Discussions

The anomaly-induced Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_{\mu}^{\mu} \quad p_r = \gamma T^4 + \frac{3}{8}T_{\mu}^{\mu} \quad p_t = \gamma T^4 + \frac{1}{8}T_{\mu}^{\mu}$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann law.

ex) Hawking radiation, Cosmology etc...

The anomaly-induced Tolman temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right]^{1/4}$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

In addition, 'the general formalism of the trace-anomaly induced Tolman T' for 2D black hole is in [Eur.Phys.J.C 75 (2015) 549]

Conclusions and Discussions

The anomaly-induced
Stefan-Boltzmann law

$$\rho = 3\gamma T^4 - \frac{3}{8}T_\mu^\mu \quad p_r = \gamma T^4 + \frac{3}{8}T_\mu^\mu \quad p_t = \gamma T^4 + \frac{1}{8}T_\mu^\mu$$

Stefan-Boltzmann law should be modified with the trace of the energy-momentum tensor. So, when we calculate the temperature, we should use this modified Stefan-Boltzmann

Thank you for your attention!!

The anomaly-induced
Tolman temperature

$$T = \frac{1}{8\pi M} \left[\left(1 - \frac{2M}{r} \right) \sum_{n=1}^6 \frac{n(n+1)}{2} \left(\frac{2M}{r} \right)^{n-1} \right],$$

Thanks to this formula, the temperature vanishes when the observer arrives at the horizon. Therefore, there does not exist firewall-like object in the thermal equilibrium of the blackhole system, and the equivalence principle is recovered at the horizon.

In addition, 'the general formalism of the trace-anomaly induced Tolman T' for 2D black hole is in [Eur.Phys.J.C 75 (2015) 549]