



# **Systematic NR fits to GW Peak Luminosity and Final States of BBH Mergers**

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**Universitat de les Illes Balears**  
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**Session B2**



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# Motivation and goals

- \* Relevant **astrophysical information** can be obtained from **BBH merger and final states**.

\*\* Spin distribution, maximal spin systems, radiated energy, luminosity...

- \* **Fits are used to calibrate phenomenological models.** Hannam+2014 , Husa+ 2015, Khan+2015

\*\* Quasinormal ringdown frequency directly computed from final mass and spin.

- \* **Fits can be applied to PE posterior distributions.**

- \* We aim to build a **consistent hierarchical method** to get **non-precessing fits** (luminosity, final spin, energy radiated) based on:

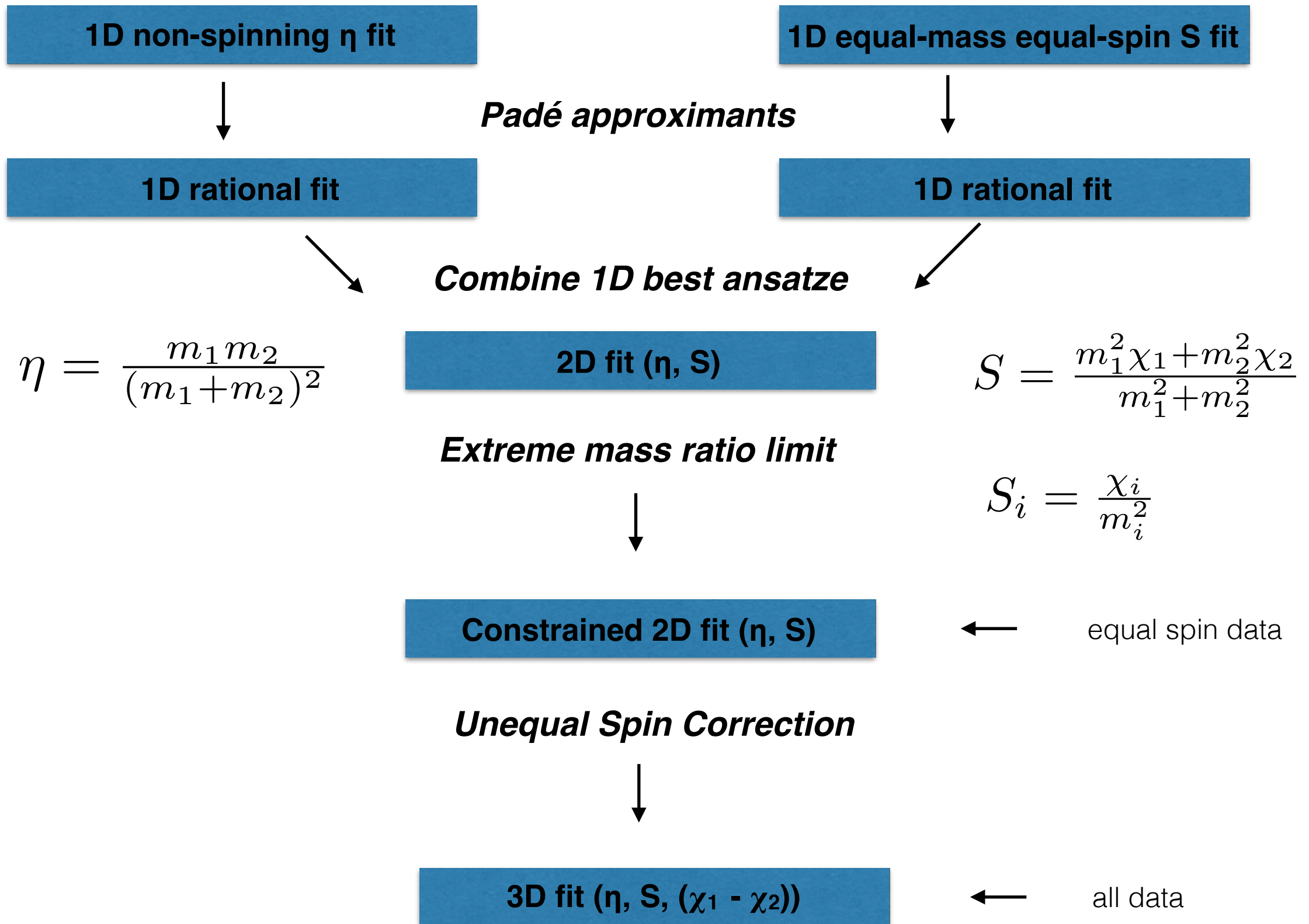
\*\* NR Data quality studies.

\*\* Fits better constrained in the best covered 1D parameter regions ( $q=1$ ,  $S=0$ ).

\*\* Constrained to extreme mass-ratio limit.

\*\* Capture unequal-spin ( $\chi_1$ - $\chi_2$ ) effects as a perturbation of the 2D fits ( $\eta, S$ ).

# Final states. Build up the model.



# Final Spin. Parameter coverage

Data : 259 Cases

Outliers : 12 Cases

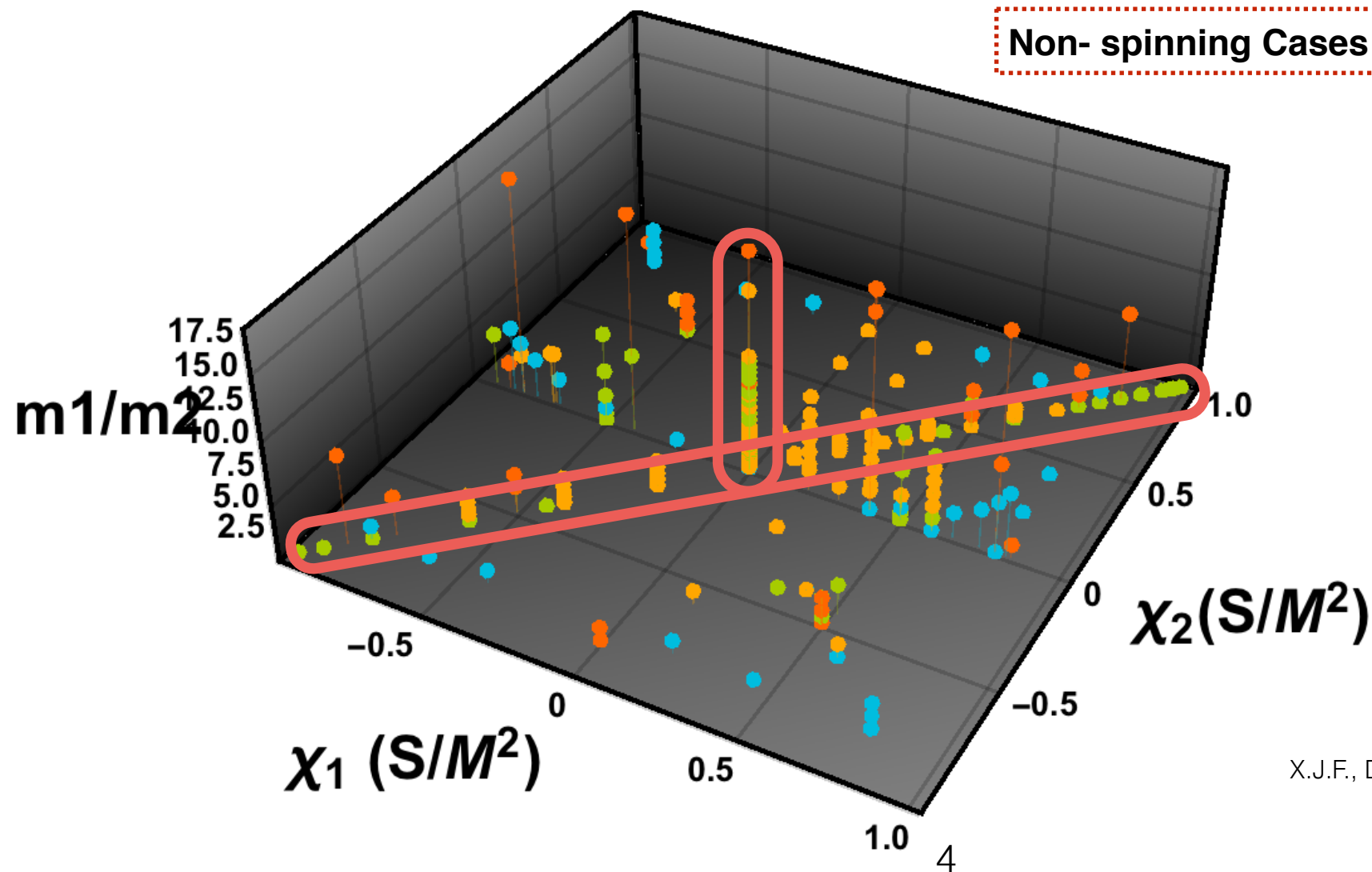
Total : 247 cases used for the fit

Unequal Spin Cases : 105

Equal Spin Cases : 142

Non-spinning Cases : 75

$q=1$  Eq. S. Cases: 67



- **BAM** UIB+Cardiff \*
- **SXS** Mroue+ 2013. [www.black-holes.org/](http://www.black-holes.org/)
- **RIT** Healy+ 2014.
- **GaTech** Jani+ 2016.

# Final states. Build up the model.

- Combine the best 1D fits.

**Non-Spinning fit :**

$$\frac{4.73a_0\eta^2 + 0.98a_2\eta^3 + 2\sqrt{3}\eta}{2.7a_1\eta + 1}$$

**3 free parameters**

**Equal-mass equal-spin fit :**

$$\frac{0.27b_0S^2 + 0.0064b_1S^3 - 0.69b_3S}{1 - 0.43b_2S}$$

**4 free parameters**

***Combine + Solve 1D constraints***

**Final 2D Ansatz :**

**16 free parameters**

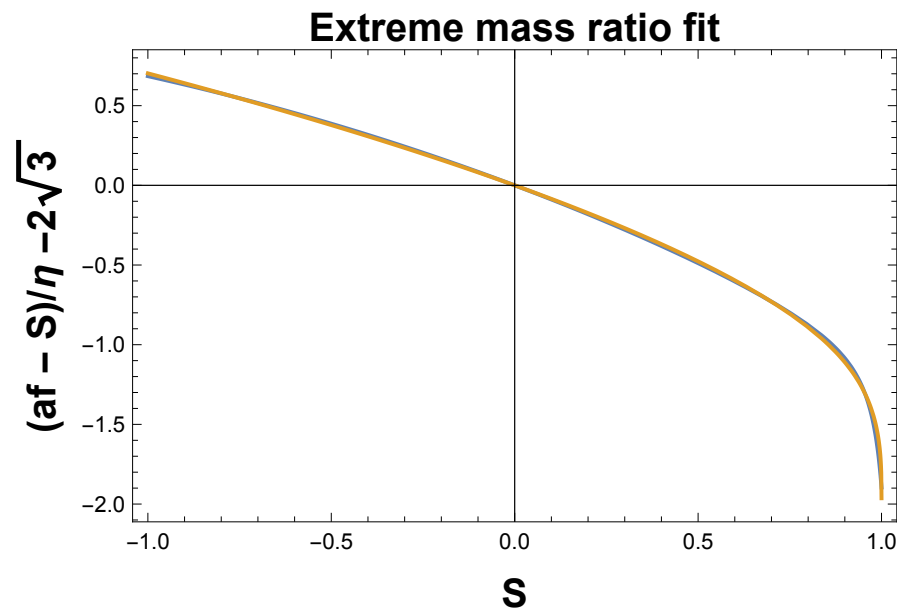
$$\frac{-10.1177\eta^3 + 16.7998\eta^2 + 2\sqrt{3}\eta}{6.24531\eta + 1} + \frac{(0.075788S^2(f_{00} + f_{01}\eta + f_{02}\eta^2 + f_{03}\eta^3) + 0.007475S^3(f_{10} + f_{11}\eta + f_{12}\eta^2 + f_{13}\eta^3) - 0.194118S(f_{20} + f_{21}\eta + f_{22}\eta^2 + f_{23}\eta^3))}{1 - 0.529026S(f_{30} + f_{31}\eta + f_{32}\eta^2 + f_{33}\eta^3)}$$

**\* We know the final states in the extreme mass ratio limit case from unperturbed Kerr solution + the Kerr geodesic equation (Bardeen+ 1972).**

**\* This provides an additional constraint equation.**

# Final Spin. Build up the model.

## - Extreme Mass Ratio Limit.



$$\frac{S((-0.307346 S - 1.21955) S + 1.63799)}{1.83167 S - 1.89027}$$

Extreme Limit data

$$\lim_{\eta \rightarrow 0} \frac{\text{fit2D}(\eta, S)}{\eta} \approx \frac{S(S(-0.143 f_{01} - 0.0141 f_{11} S) + 0.367 f_{21})}{f_{30} S - 1.890}$$

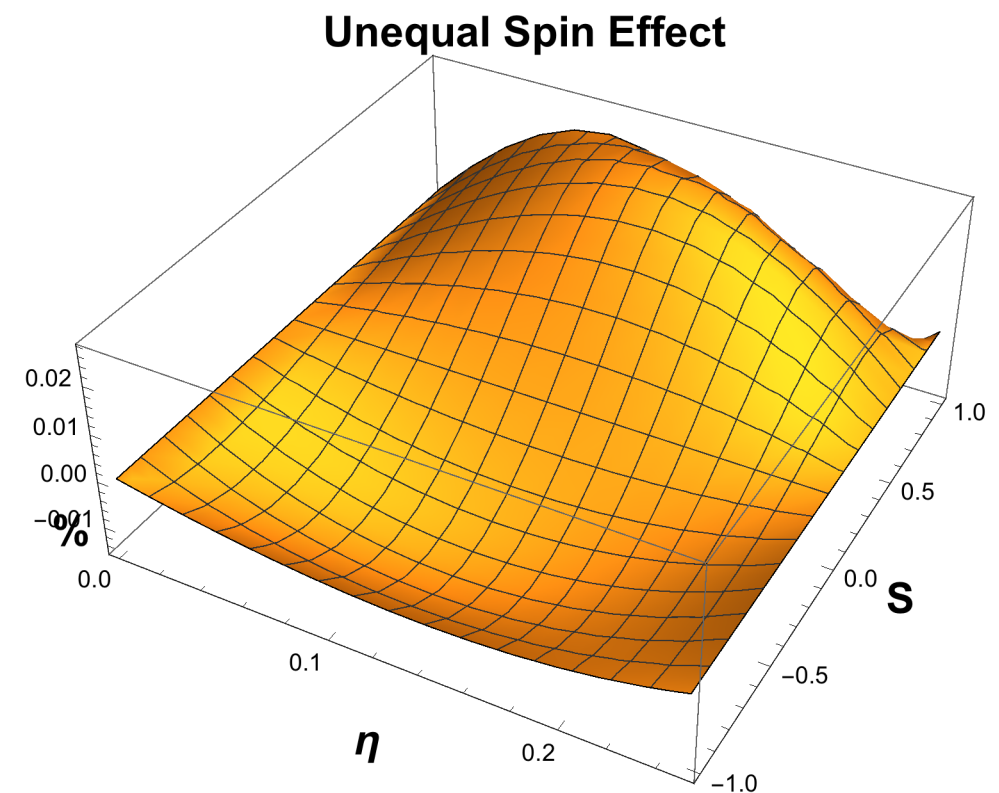
Number of Free parameters after applying the constrains for the 2D fit → 3

## - Unequal Spin effect.

\* Subdominant effect ~ 5%,  
treat as perturbation of 2D fit.

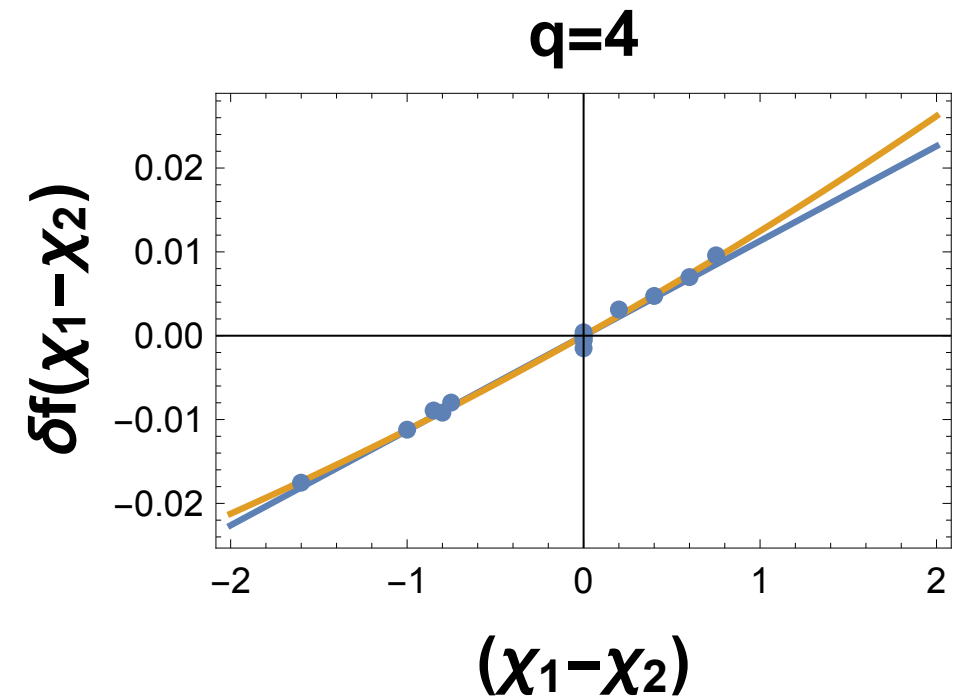
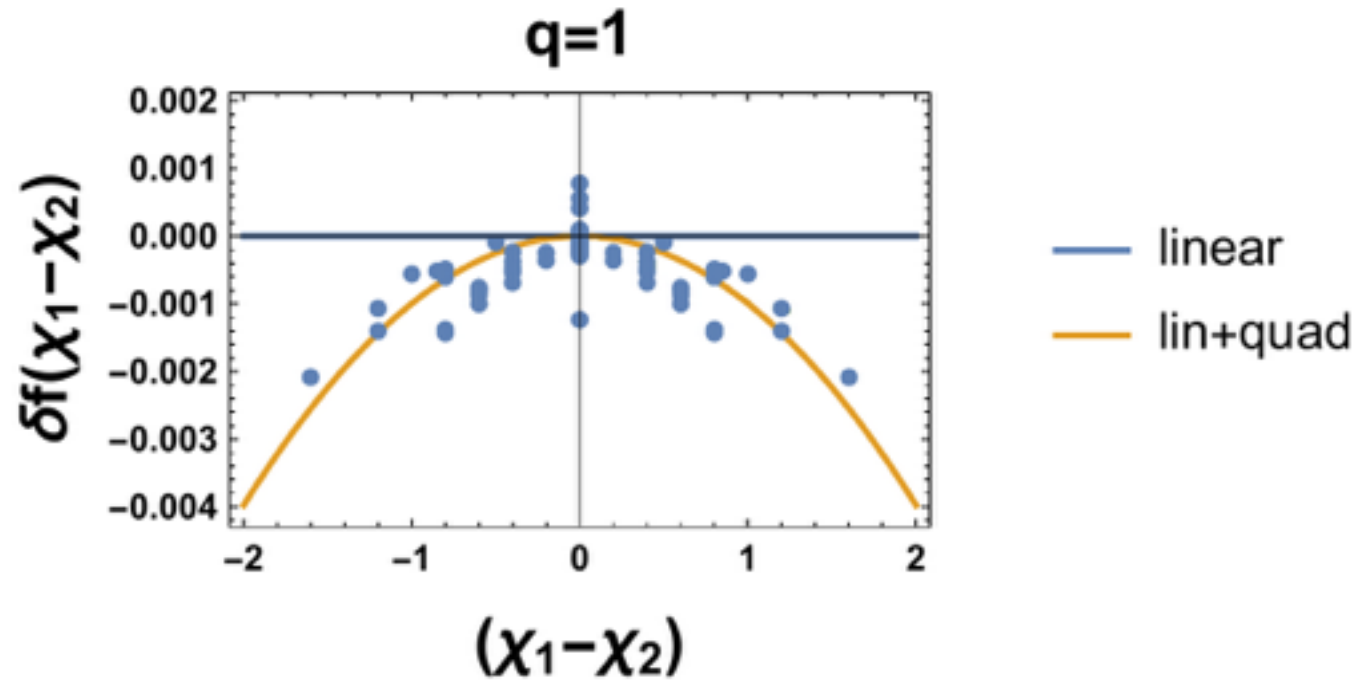
\* rms 2D fit. 0.00517 : rms 3D fit. 0.00062

$$\text{fit2D}(\eta, S) + f_a(\eta)(\chi_1 - \chi_2) + f_b(\eta)(\chi_1 - \chi_2)^2$$

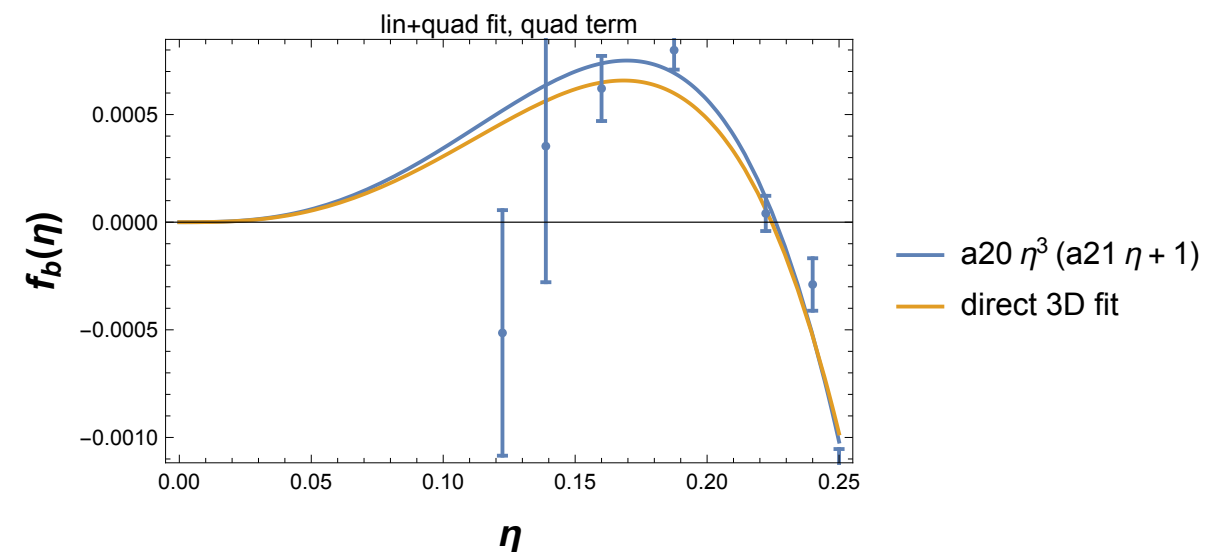
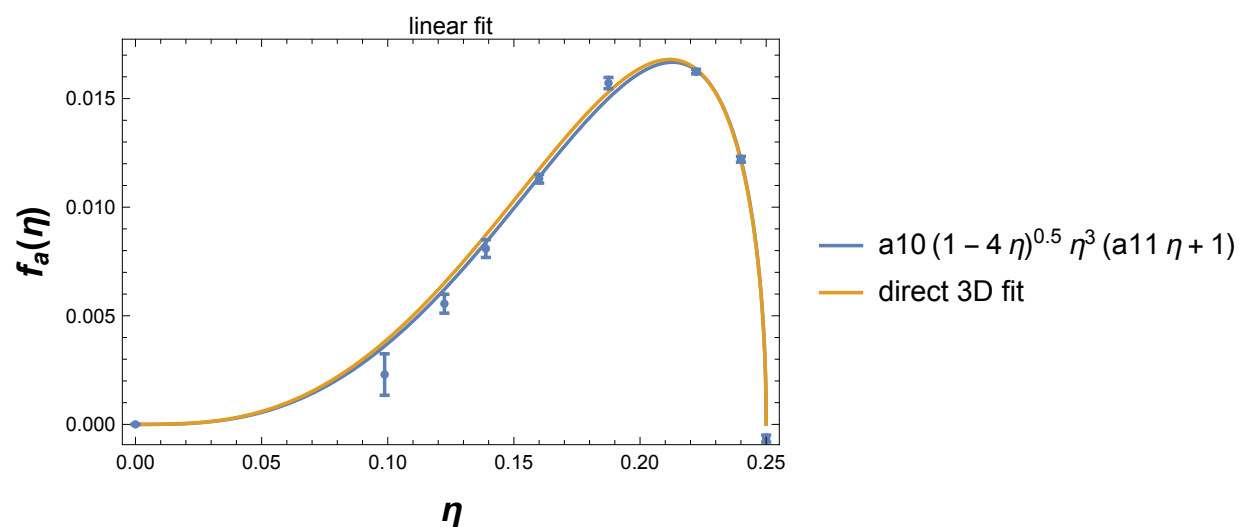


# Final Spin. Build up the model.

- Unequal Spin effect.

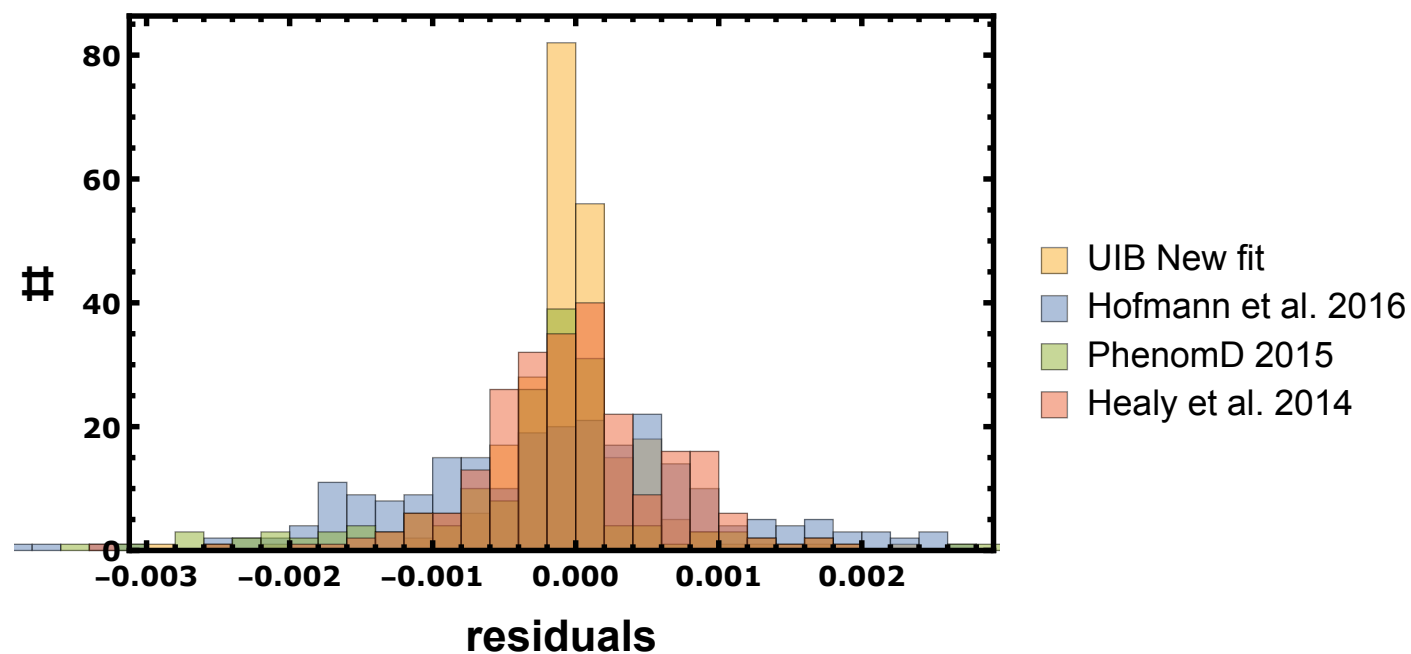


Doing an analysis per mass-ratio one finds evidence of both linear and quadratic dependence.



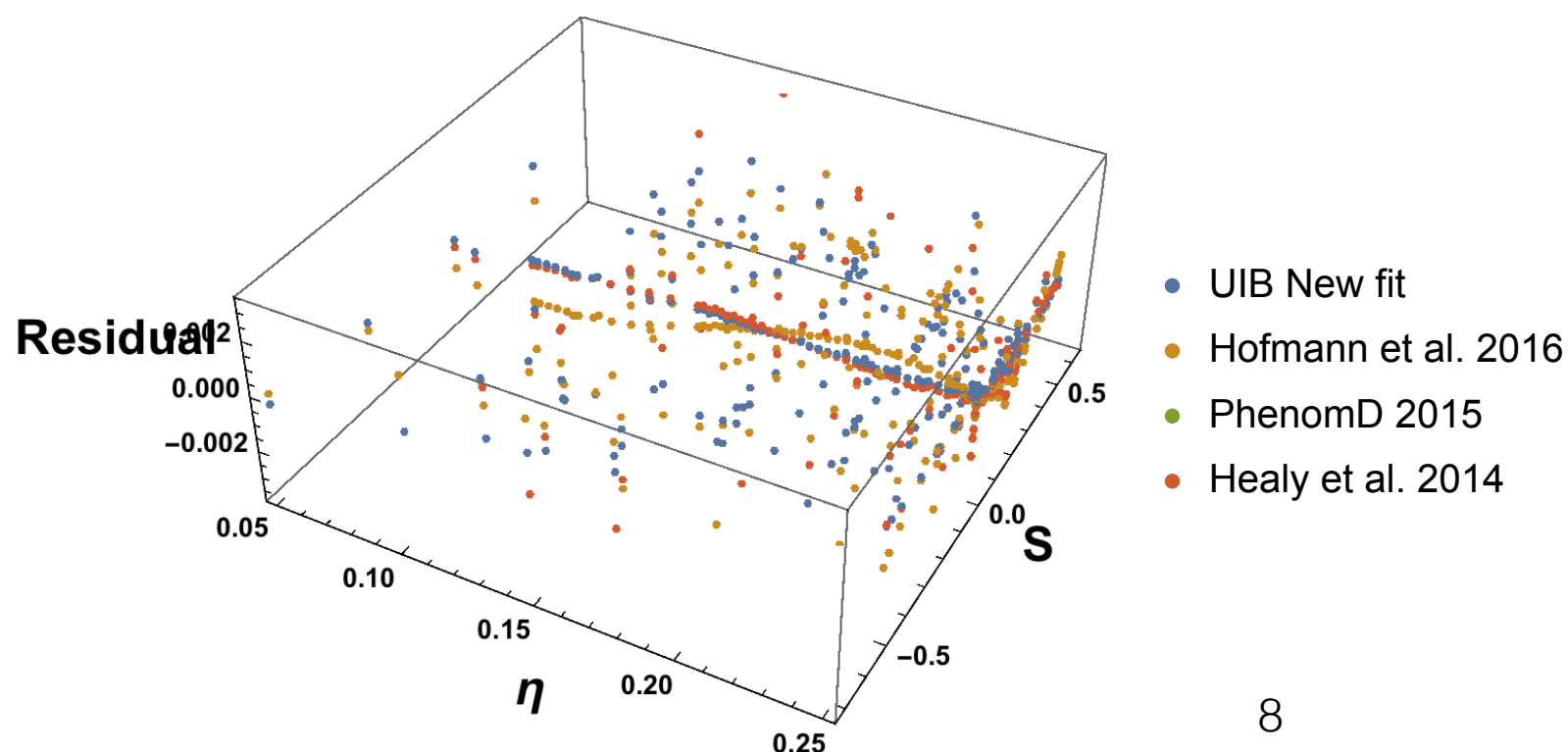
# Final Spin. Test the model.

- Comparison with previous fits.



Fits		$\mu$	$\sigma$
3D Fit	Hofmann et al. 2016	-0.000132	0.00116
3D Fit	UIB New fit	-0.000023	0.00051
2D Fit	PhenD fit 2015	-0.000200	0.00566
3D Fit	Healy et al. 2014	0.000017	0.00079

**\*\* mean of residuals and root mean squared residuals**

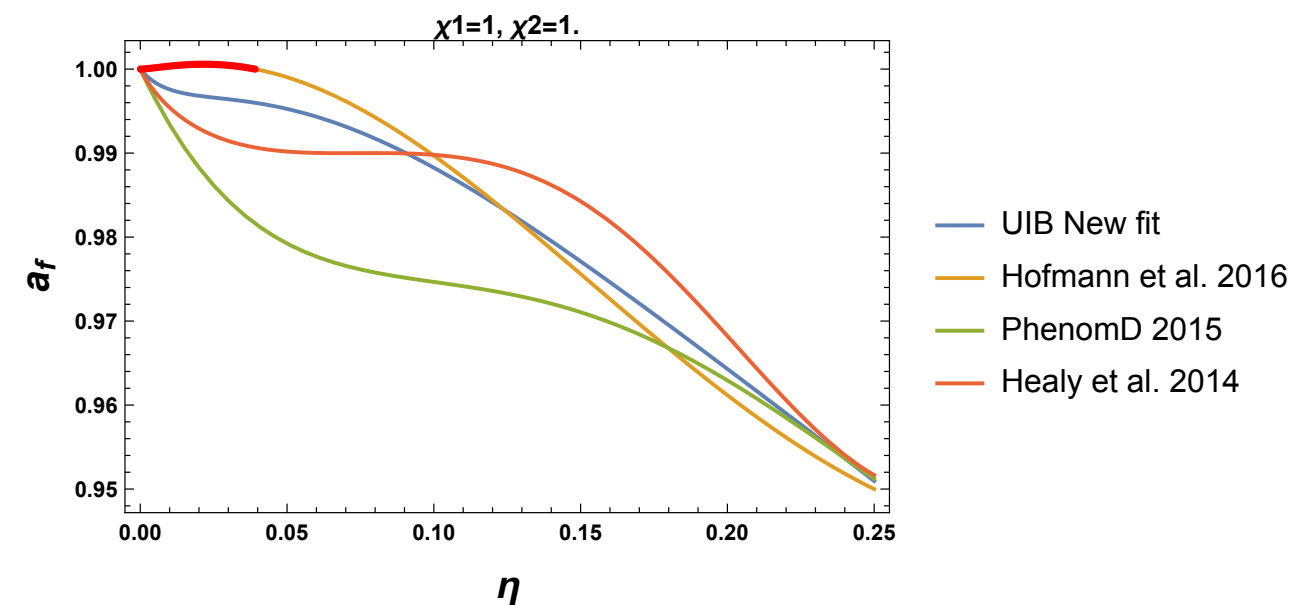
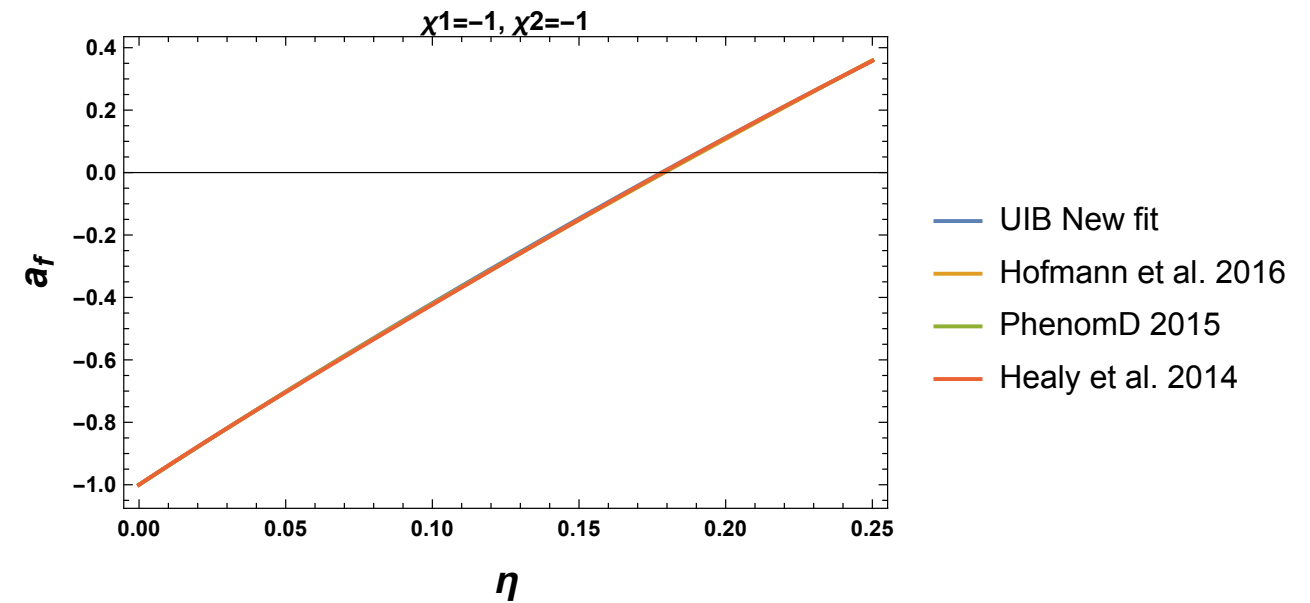
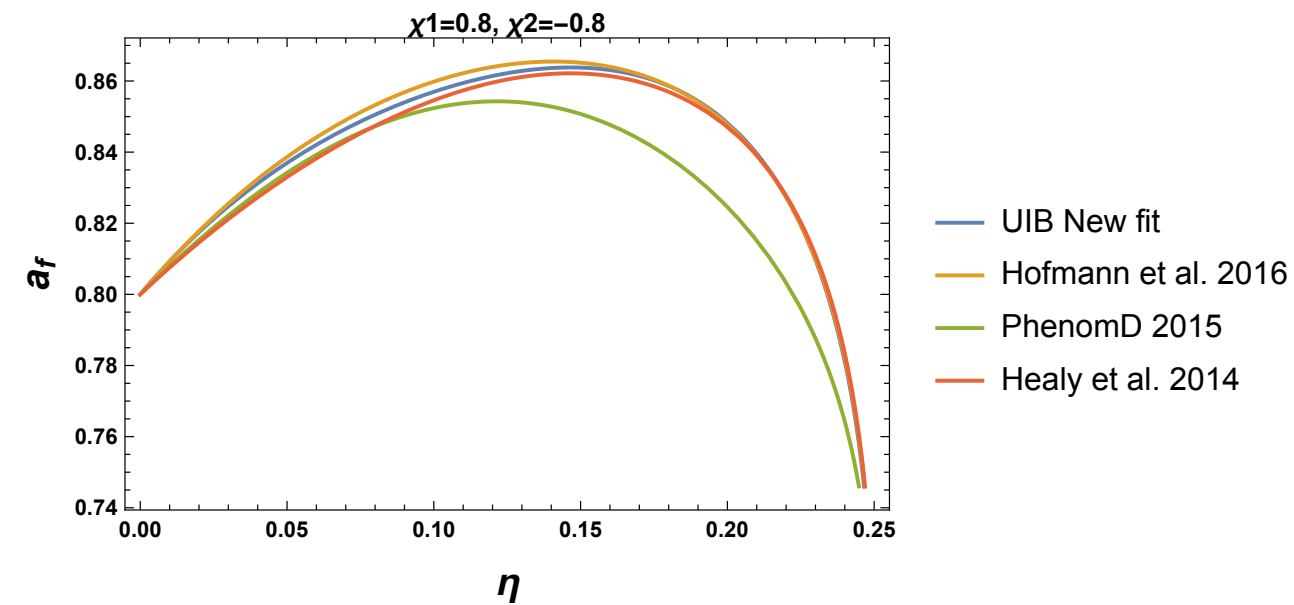


- \* no bias between various fits
- \* compatible distributions of residuals
- \* However naive comparisons might be misleading since different fits have been calibrated to different data sets.



# Final states. Test the model.

- Comparison with previous fits. Some special cases.



\* The approach to the extreme limit is different for different fits.

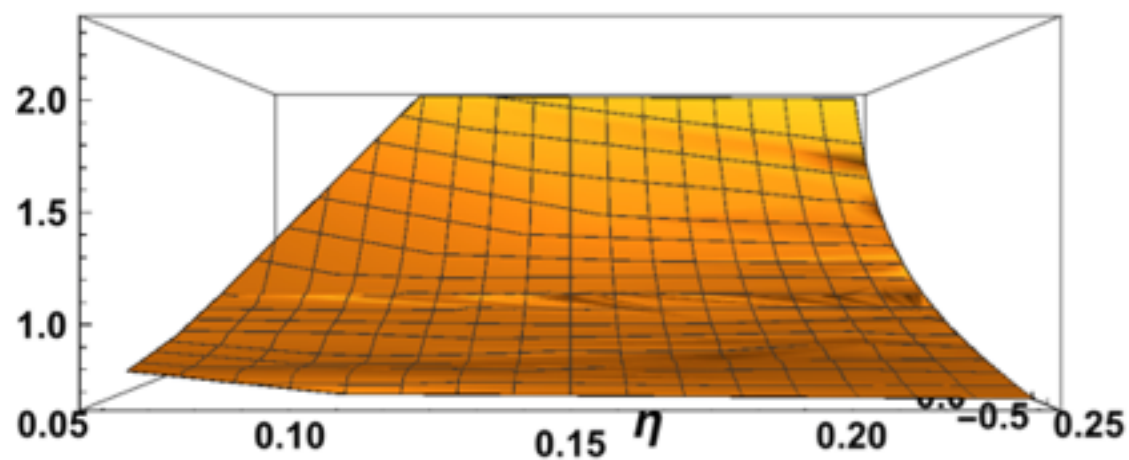
\* The actual extreme mass ratio limit is constrained.

\* But no NR data below  $q=18$ . Differences are due to extrapolation.

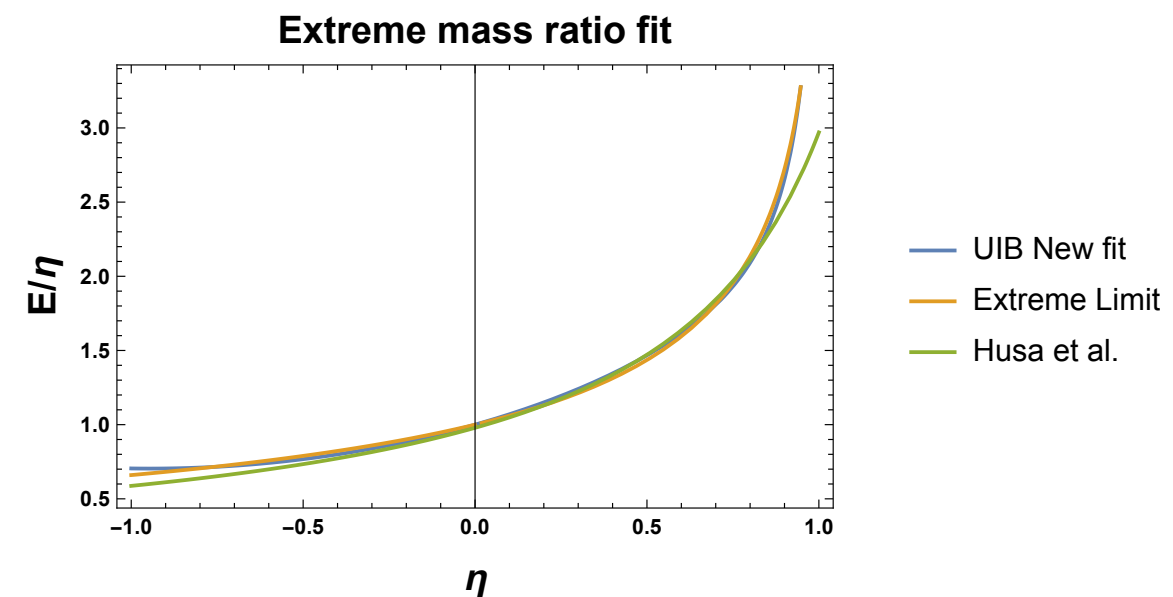
# Final Mass

*Preliminary!*

- Same hierarchical procedure as for final spin.
- 2D ansatz  $[\eta, S]$  is now a product (instead of sum).

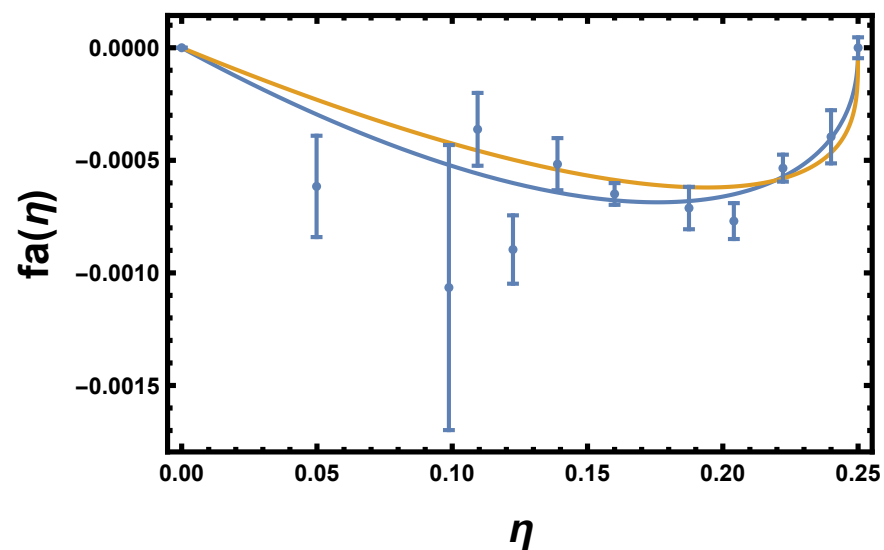


data/fit( $\eta$ )



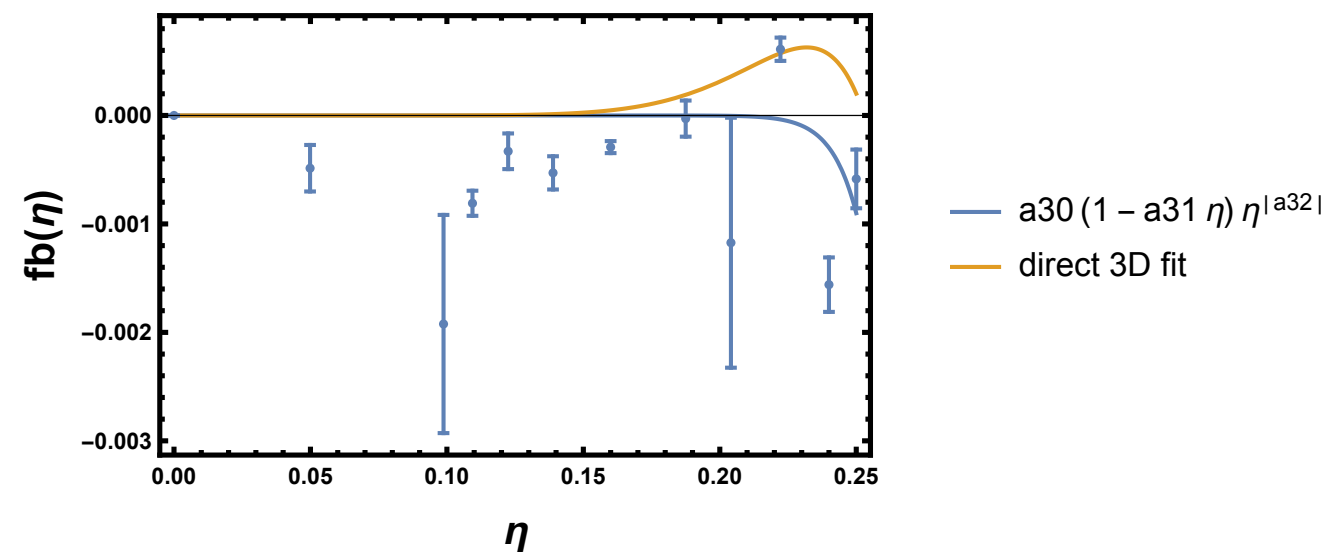
- Here we can currently only pull out the linear ( $\chi_1$ - $\chi_2$ ) term.

linear fit



$a_{10} \eta (1 - 4 \eta)^{\frac{|a_{12}|}{2}}$   
direct 3D fit

quad. term



$a_{30} (1 - a_{31} \eta) \eta^{|a_{32}|}$   
direct 3D fit

# Luminosity

- An early version of this procedure has been already used for the peak luminosity.

<https://dcc.ligo.org/LIGO-T1600018/public>

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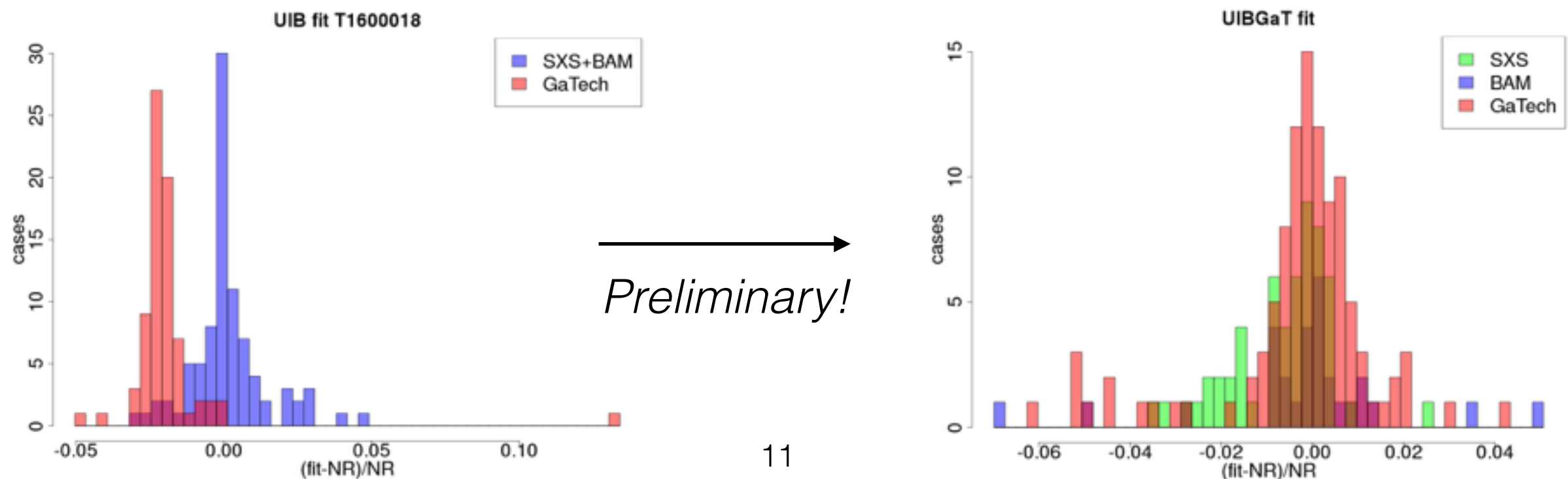
- Used in LVC publication on GW150914 and GW151226:

*PRL* **116**, 241103 (2016),

*PRL* **116**, 061102 (2016),

arXiv:1606.04856

- Previously fitted to SXS and BAM data (89 cases).
- New version in preparation with modes up to  $\ell = 6$  and including GaTech data (arXiv:1605.03204 - processed for fit by L. London).



# Conclusions

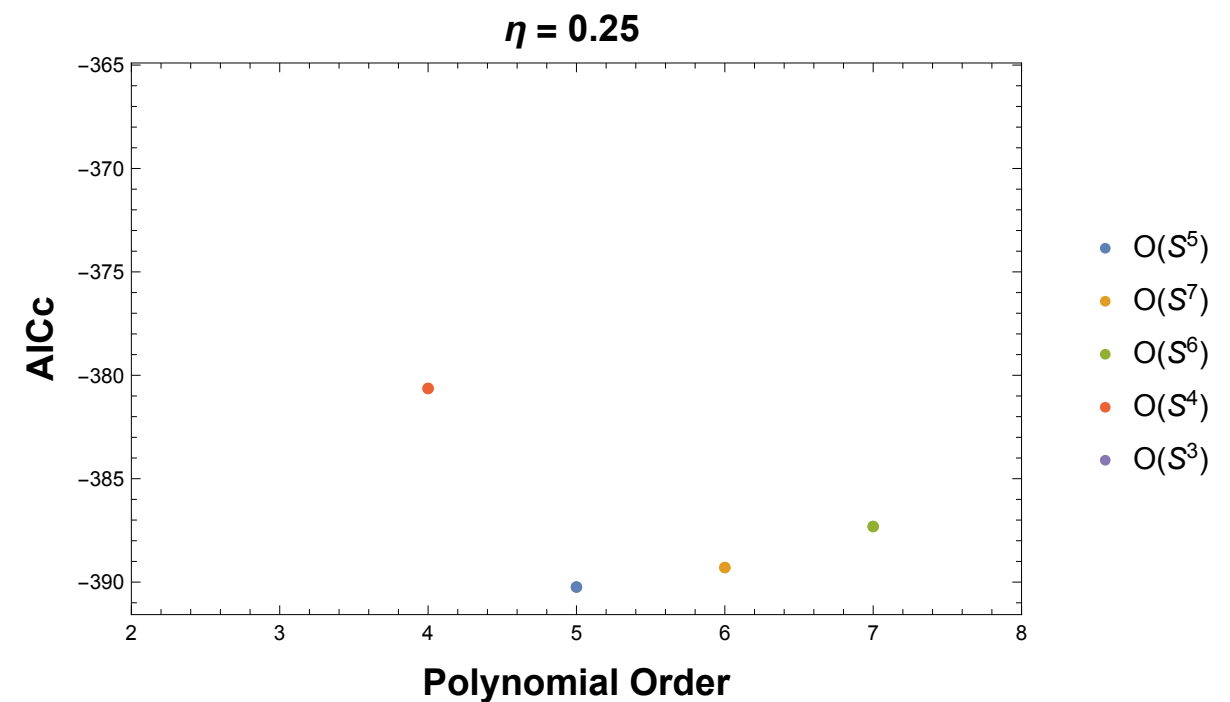
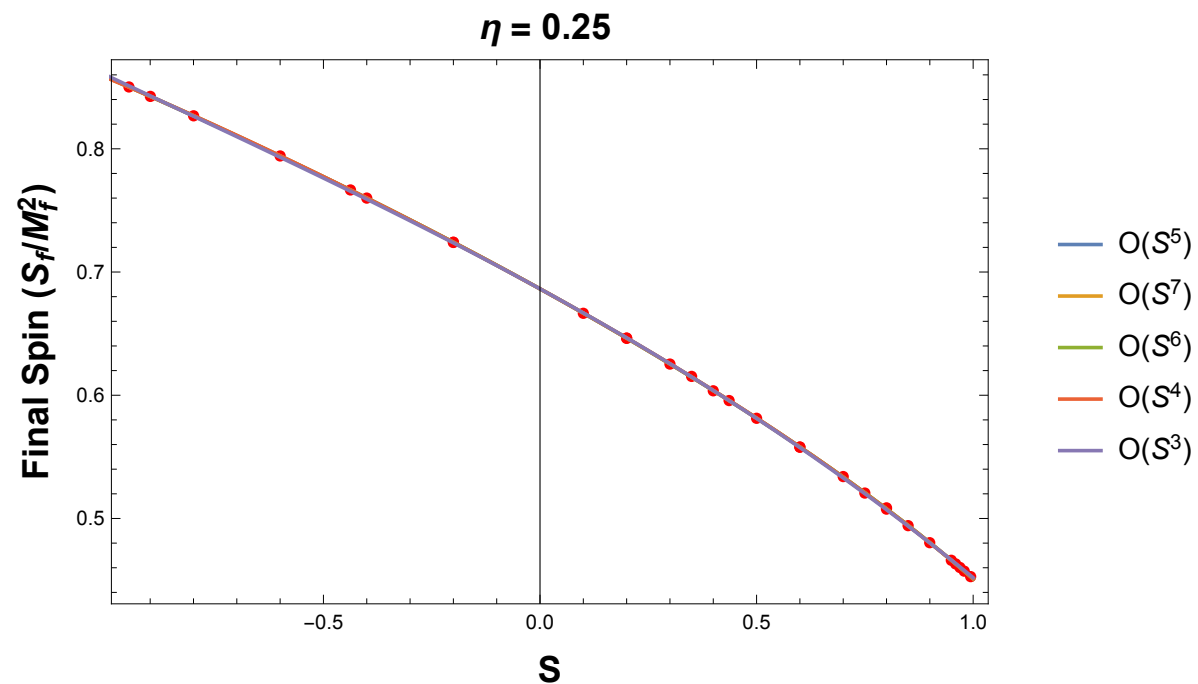
- \* We aim to build a **consistent hierarchical method** to get **non-precessing fits** (luminosity, final spin, energy radiated) based on:
- \* Performing this **hierarchical study** provides a **better understanding** of the **3D parameter space** structure and control of **extrapolation behaviour**.
- \* **Improved final spin fit. Residuals compatible** with Healy et al., Hofmann et al.
- \* Same **approach** for **final mass** showing **similar results**, although **higher spin difference** effects **difficult to constrain**.
- \* An early **version** of this procedure has **been** already **used for the peak luminosity**. New **version** in **preparation**.
- \* When **combining NR results from several codes/catalogues**, it is important *to **carefully study** their **data quality***.

See also talk by Sascha Husa, Wednesday, Session C2.

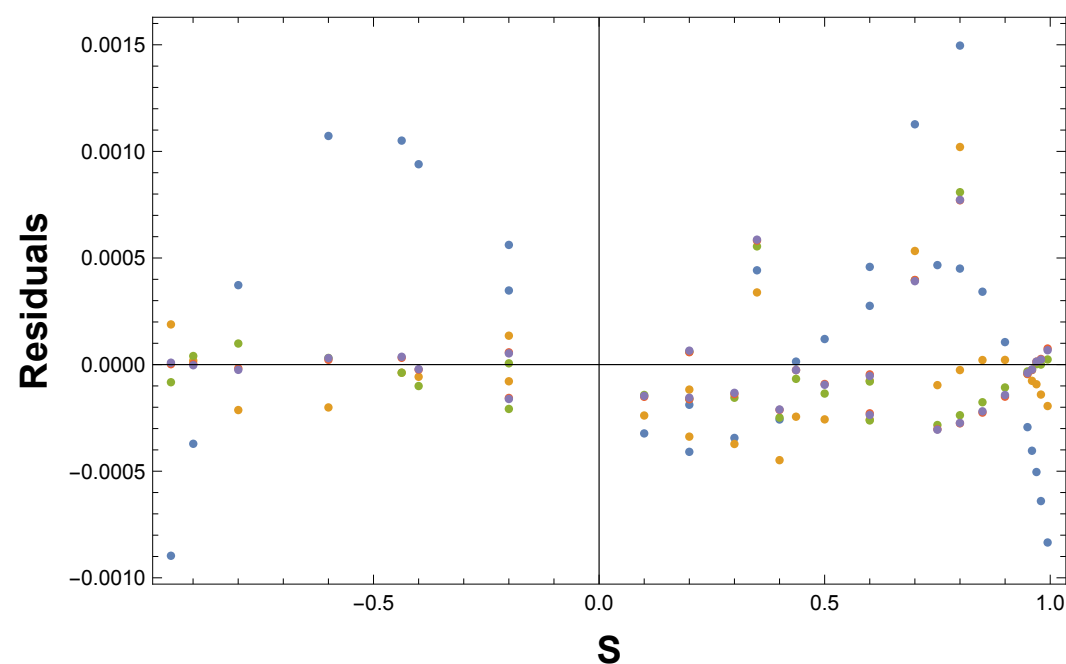
# Thanks!

# Final Spin. Build up the model.

- Get 1D fits from the most populated and NR most accurate regions:  $f(\eta=0.25, S)$  and  $f(\eta, 0)$ .



We base our decision on Akaike Information Criterion (AICc), residuals and the t-statistics of the free parameters.

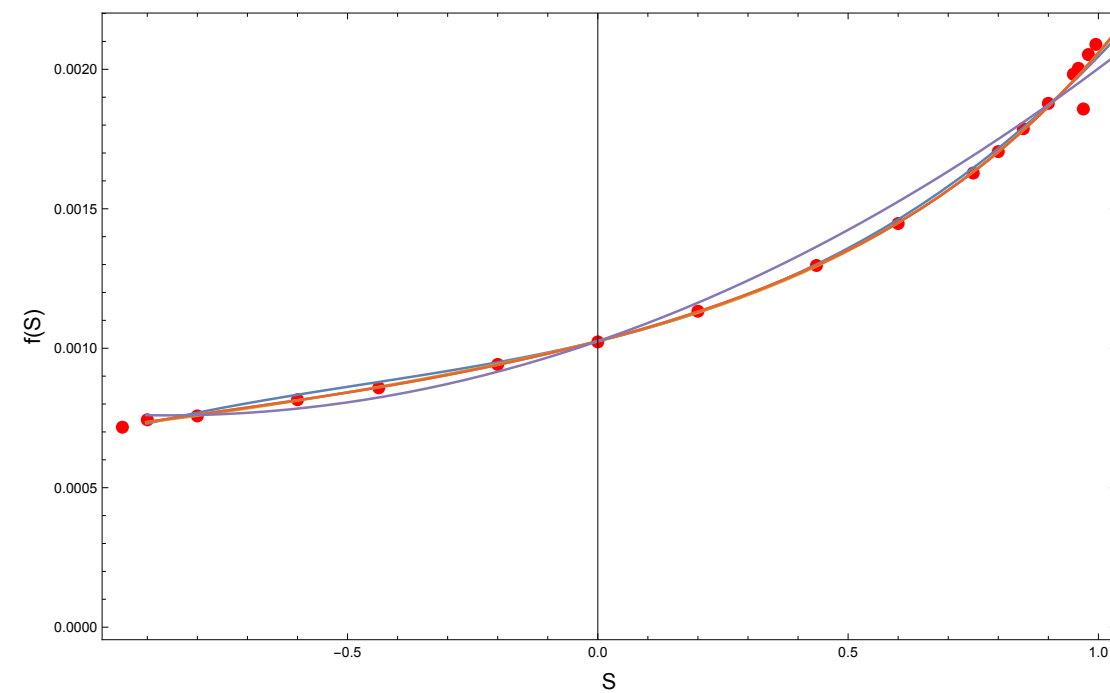
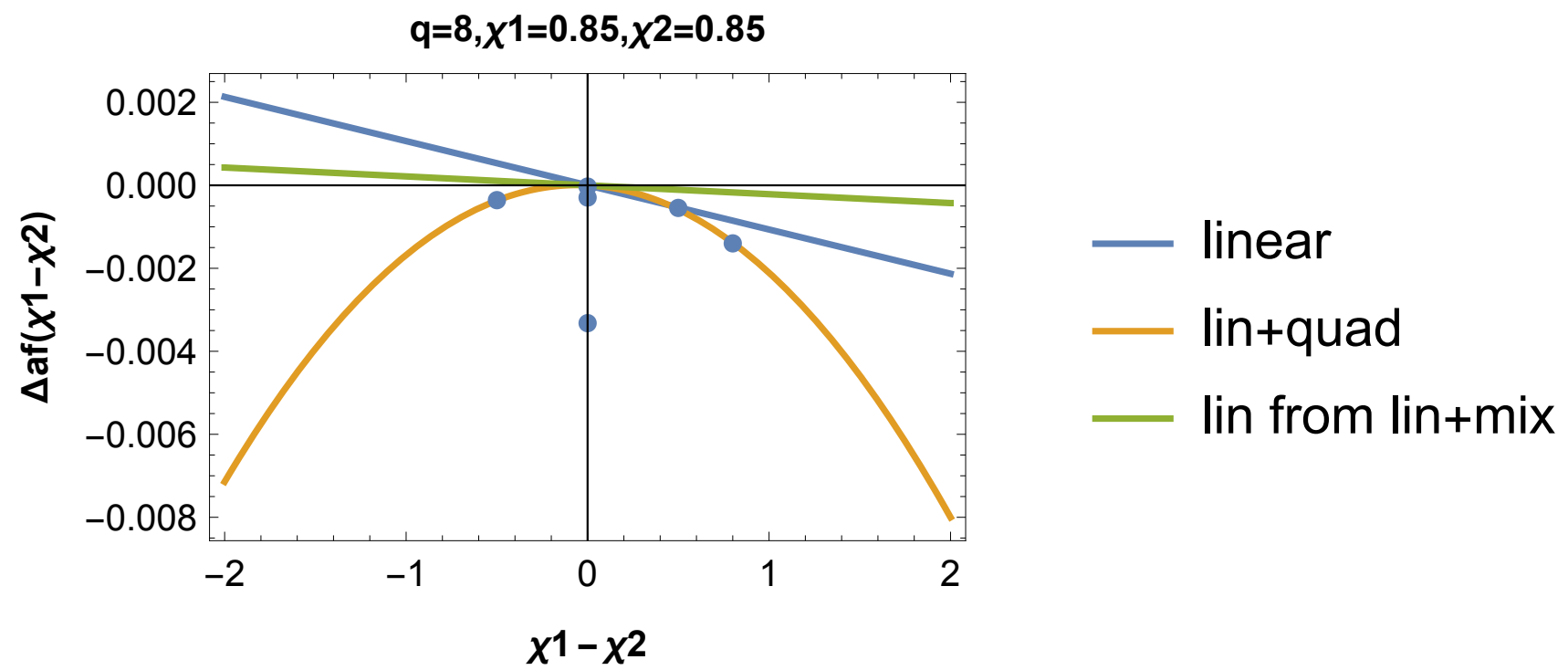


$$0.686355 + b_1 S + b_2 S^2 + b_3 S^3 + b_4 S^4 + b_5 S^5$$

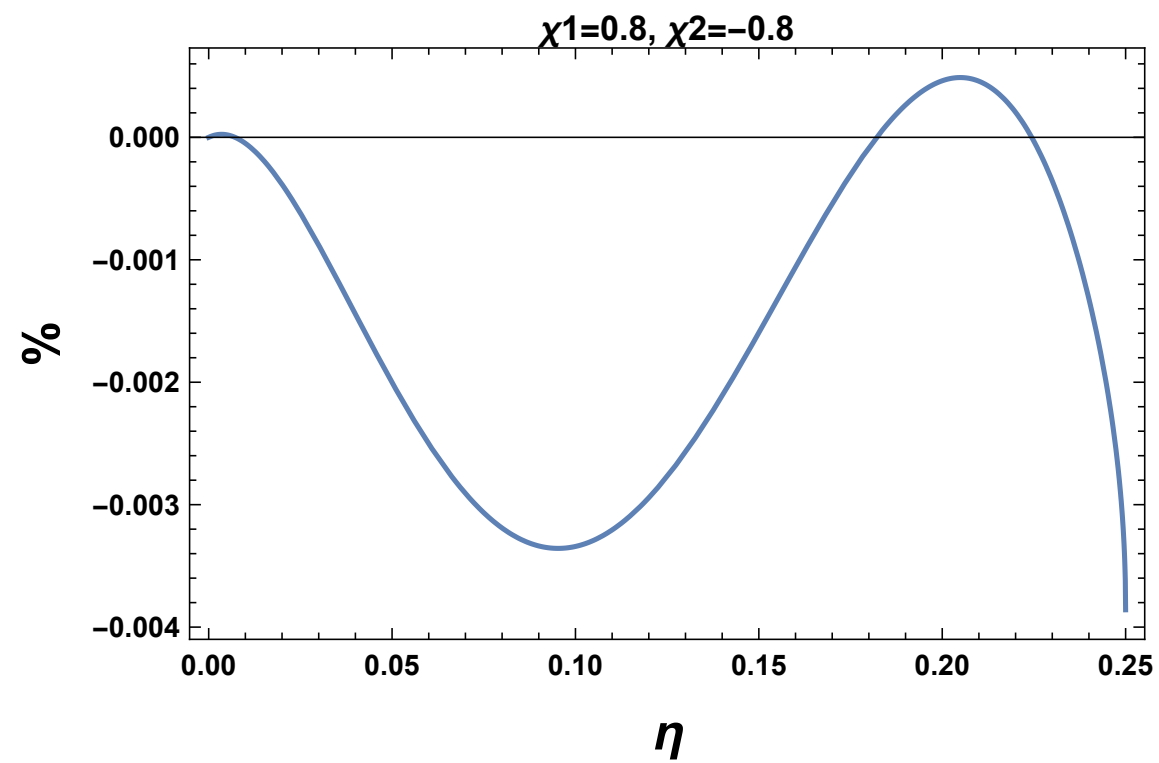
	Estimate	Standard Error	t-Statistic	P-Value
b1	-0.194412	0.000363411	-534.964	$1.94821 \times 10^{-50}$
b2	-0.0265515	0.000455693	-58.2663	$2.31492 \times 10^{-27}$
b3	-0.00496448	0.00140788	-3.52621	0.00172663
b4	-0.00515274	0.000577171	-8.92758	$4.29214 \times 10^{-9}$
b5	-0.00423716	0.00123315	-3.43605	0.00215744

**\*\* First step before using Padé approximants.**

# Bonus slide



# Bonus slide



— 1-Hofmann et al. 2016/UIB New fit