The Hartle-Hawking Wave Function in Causal Set Quantum Gravity

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Outline

- CST 101
- Continuum Inspired Dynamics for CST : a review of results in 2d CST
- The Hartle-Hawking Wave Function in 2d CST
- The Large N limit.

L. Glaser and S. Surya, Class.Quant.Grav. (2016)

L. Glaser, D. O'Connor and S. Surya, In Preparation

- Spacetime continuum is replaced by a locally finite countable partially ordered set.

$$\boxed{\text{Order} + \text{Number} \sim \text{Spacetime geometry}}$$

- Continuum approximation: via a Poisson Process:

$$P_V(n) \equiv \frac{1}{n!}\exp^{-\rho V}(\rho V)^n, \quad < N >= \rho V$$

- Local Lorentz invariance: there are no preferred directions

– L.Bombelli, J.Henson, R. Sorkin, Mod.Phys.Lett. 2009

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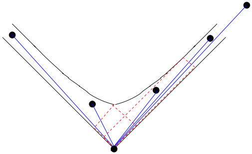

A Continuum Inspired Dynamics For CST

- First principles: Quantum sequential growth using the *quantum measure formulation.*
- Continuum Inspired Dynamics:

$$Z_\Omega = \sum_{c \in \Omega} \exp^{\frac{i}{\hbar} S(c)}$$

  - $S(C)$ is the *Benincasa-Dowker action* which is the analog of the Einstein-Hilbert action in CST.

    –D. Benincasa and F. Dowker, Phys.Rev.Lett. (2010)

    –F. Dowker and L. Glaser, CQG (2013)

  - Ω is a sample space of causal sets (e.g.: the set of all past-finite causal sets)

Analytic Continuation

$$Z_\Omega = \sum_{c\in\Omega} \exp^{\frac{i\beta}{\hbar}S(c)} \longrightarrow Z_\Omega = \sum_{c\in\Omega} \exp^{-\frac{\beta}{\hbar}S(c)}$$

- Space of Configurations Ω is unchanged: There are no Euclidean causal sets!
- Analytic continuation of parameter: $i\beta \to -\beta$
- MCMC for Ω_N:
- Covariant/label invariant observables $\langle O \rangle$

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- Analytic continuation of parameter: $i\beta \to -\beta$
- MCMC for Ω_N:
    - $\beta = 0$:
        - In the $N \to \infty$ limit the *Kleitman-Rothschild* posets dominate.
        - The onset of the asymptotic regime occurs for $N > 80$

– J. Henson, D. Rideout, R. Sorkin and S.Surya, 2016

    - Studies have begun on $\beta \neq 0$ – a challenge!
- Covariant/label invariant observables $\langle O \rangle$

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- Covariant/label invariant observables $\langle O \rangle$
  - Ordering Fraction: $R/\binom{N}{2}$
  - Myrheim-Myer Dimension
  - Homology
  - Height
  - Abundance of Causal Diamonds of a given volume

2d CST

$$Z_{2d} = \sum_{c \in \Omega_{2d}(N)} \exp^{-\frac{1}{\hbar}\beta S_{2d}}$$

–G. Brightwell, J. Henson, S.Surya, Class.Quant.Grav. 25, 2008

–S. Surya, Class.Quant.Grav. 29, 2012

–L. Glaser, D. O'Connor and S. Surya, in preparation

- $\Omega_{2d}(N) \subset \Omega(N)$: N-element 2d-orders: $U \cap V$, where U and V are total orders:
- 2d Benincasa-Dowker Action: $\frac{1}{\hbar} S(\epsilon) = 4\epsilon \left(N - 2\epsilon \sum_{n=0}^{N-2} N_n f(n, \epsilon) \right)$

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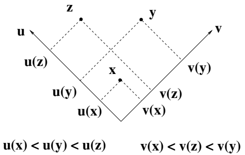
- $\Omega_{2d}(N) \subset \Omega(N)$: N-element 2d-orders: $U \cap V$, where U and V are total orders:
  - $x \prec y \quad \Leftrightarrow \quad u(x) < u(y)$ and $v(x) < v(y)$



  - Includes all continuum-like causal sets $\sim$ topologically trivial causal diamond in 2d.
  - Includes posets with no continuum approximation.
- 2d Benincasa-Dowker Action: $\frac{1}{\hbar}S(\epsilon) = 4\epsilon\left(N - 2\epsilon\sum_{n=0}^{N-2} N_n f(n,\epsilon)\right)$

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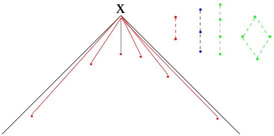
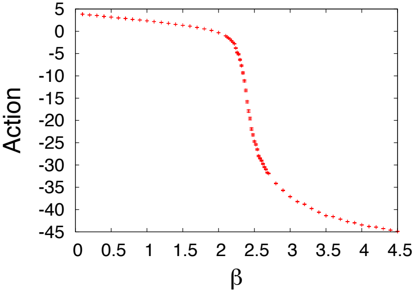
  - N_n: # of n-element order intervals

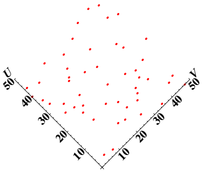


  - Mesoscale: $\epsilon = \left(\frac{l_p}{l_k} \right)^2 \in (0,1]$
  - $f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n (1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2}$
  - $\epsilon = 1$: $\frac{1}{\hbar} S^{(2)}(C) = N - 2N_0 + 4N_1 - 2N_2$

$\langle S \rangle$ vs β for $N = 50, \epsilon = 0.12$



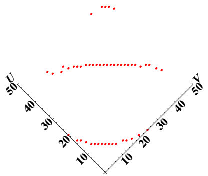
Continuum Phase



- The typical causal set is a 2d "random order": U and V chosen randomly and independently.
- 2d random orders dominate when $\beta = 0$. –Peter Winkler, Order 1, 317, (1985), El-Zahar and N.W. Sauer, Order 5, 239, (1988)
- 2d random order $\sim$ Minkowski spacetime ${}^2M$. –G. Brightwell, J. Henson, S.Surya, Class.Quant.Grav. 25, 2008

$$\boxed{\text{Continuum Phase} \sim \text{2d causal diamond}}$$

Crystalline Phase

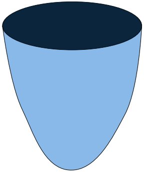


- The typical causal set is layered, with a large number of links, but much fewer small intervals.
- It is distinctly non-manifold like.

$$\boxed{\text{Crystalline Phase} \not\sim \text{continuum}}$$

The Hartle-Hawking Prescription in CST

- Continuum Proposal: $\Psi_0(h_{ab}, \Sigma) = A \sum_M \int dg \; \exp^{-\frac{1}{\hbar} I_E(g)}, \qquad \partial M = \Sigma, g|_\Sigma = h$
  - Path integral over Riemannian geometries on M.
  - M is compact with a "final" boundary geometry (Σ, h_{ab}).
  - Initial spatial "zero" geometry, *"a single point, which captures the idea of a universe emerging from nothing."'*



The Hartle-Hawking Prescription in CST

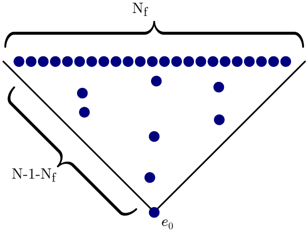
- Continuum Proposal: $\Psi_0(h_{ab},\Sigma) = A\sum_M \int dg\ \exp^{-\frac{1}{\hbar}I_E(g)}, \qquad \partial M = \Sigma, g|_\Sigma = h$

- CST Proposal: $\Psi_0^{(N)}(\mathcal{N}_f,\beta) \equiv A \sum_{c\in\Omega_N} \exp^{-\frac{1}{\hbar}\beta S(c)}, \qquad |Max(C)| = \mathcal{N}_f$

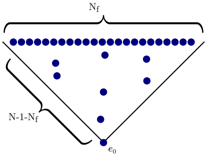
  - The sum is over "discrete Lorentzian" geometries or causal sets, $c \in \Omega_N$ of finite cardinality.

  - Initial spatial geometry is a single element to the past of all other elements in c. This *is* "a single point" from which the universe emerges.

  - The final geometry is a *maximal antichain* $\mathcal{A}_f$ whose only characteristic feature is cardinality which is therefore fixed $|\mathcal{A}_f| = \mathcal{N}_f$.



$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}, \qquad \mathcal{N}_f = N - p$$



- **p = 1** : $\Psi_0(N-1) = A \exp^{-\beta R}$, $R = 2\epsilon N(1-2\epsilon) + 4\epsilon^2$

The 2d Hartle-Hawking Wave Function: Analytic Results

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}, \qquad \mathcal{N}_f = N - p$$

- **p** $=$ **2** : $\Psi_0(N-2) = \frac{A \exp^{-\beta(R-Q)}}{(1-\exp^{\beta Q})^2}\left(N-2-(N-1)\exp^{\beta Q} + \exp^{\beta Q(N-1)}\right)$, $Q = 4\epsilon^2(1-3\epsilon)$.

$$\Psi_0^{(N)}(\mathcal{N}_f,\beta) \equiv A \sum_{c\in\Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}, \qquad \mathcal{N}_f = N - p$$

- **p = 3** :
  - antichain

$$\Psi_0^{(a,i)}(N-3) = A\exp^{-\beta R} \sum_{\ell_1=1}^{N-3}\sum_{\ell_2=1}^{N-3}\sum_{m=m_0}^{m_f} (N-2-\ell_1-\ell_2+m)\exp^{\beta Pm}\exp^{\beta Q(\ell_1+\ell_2)}$$

  - chain

$$\Psi_0^{(a,ii)}(N-3) = A\exp^{-\beta R} \sum_{\ell_1=1}^{(N-3-1)}\sum_{\ell_2=1}^{(N-3-\ell_1)}\sum_{\widetilde{m}=0}^{(N-3-\ell_1-\ell_2)} (N-2-\ell_1-\ell_2-\widetilde{m})\exp^{\beta Q(\ell_1+\ell_2)}$$

$$P = 24\epsilon^4, \quad m_0 = max(1,\ell_1+\ell_2-N+3), \quad m_f = min(\ell_1,\ell_2)$$

The 2d Hartle-Hawking Wave Function: Analytic Results

$$\Psi_0^{(N)}(\mathcal{N}_f, \beta) \equiv A \sum_{c \in \Omega_{2d}} \exp^{-\frac{1}{\hbar}\beta S_{2d}(c)}, \qquad \mathcal{N}_f = N - p$$

- **p** $>$ **3** : Analytically challenging/impossible!

$$\Psi_0(\mathcal{N}_f) = A\mathcal{Z}_\beta(\mathcal{N}_f) = A\mathcal{Z}_0(\mathcal{N}_f)\exp\left(-\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle\right)$$

- Calculation of $\langle S_\beta(\mathcal{N}_f)\rangle$ using MCMC methods.
- Numerical Integration: $\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle$
- Estimation of $\mathcal{Z}_0(\mathcal{N}_f)$.
- Normalise to get A.
- Calculations performed for $N = 50, \epsilon = 0.12, 0.5, 1$.

Numerical Simulations

$$\Psi_0(\mathcal{N}_f) = A\mathcal{Z}_\beta(\mathcal{N}_f) = A\mathcal{Z}_0(\mathcal{N}_f)\exp\left(-\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle\right)$$

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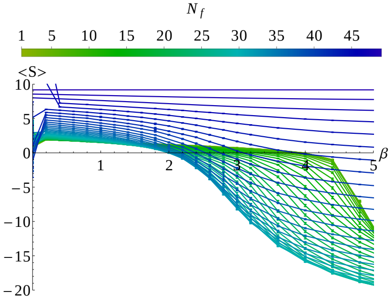
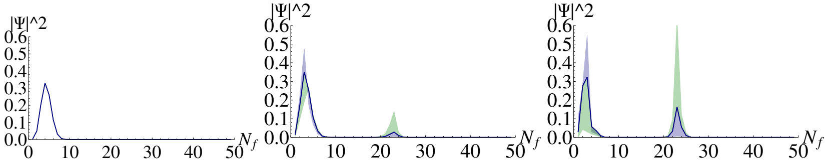


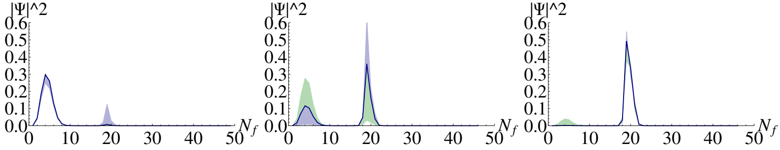
  - Phase transition for smaller $\mathcal{N}_f$.
  - As N_f increases to $N-1$, the phase transition is wiped out
  - Minimum value of β_c at $\mathcal{N}_f \sim 30$: $\beta_c(\mathcal{N}_f)$ is not a monotonic function.

- Numerical Integration: $\int_0^\beta d\beta' \langle S_{\beta'}(\mathcal{N}_f)\rangle$

The Hartle-Hawking Wave Function.



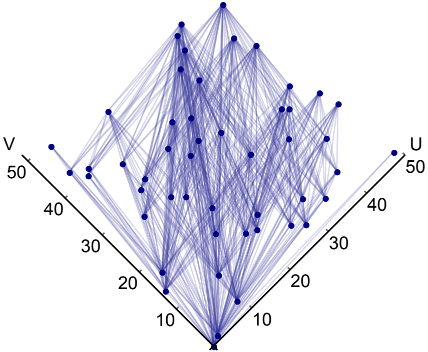
(a) $\beta = 0$ (b) $\beta = 7.5, \epsilon = 0.12$ (c) $\beta = 8.5, \epsilon = 0.12$



(d) $\beta = 0.10, \epsilon = 0.5$ (e) $\beta = 0.11, \epsilon = 0.5$ (f) $\beta = 0.12, \epsilon = 0.5$

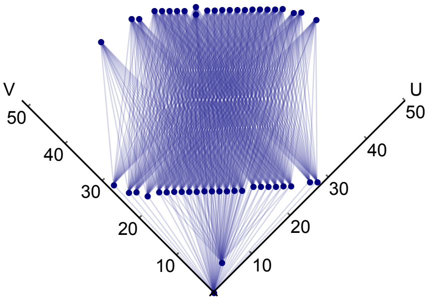
The Two Peaks

- **The Peak at** $\mathcal{N}_f \sim 4$



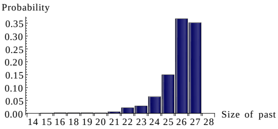
The Two Peaks

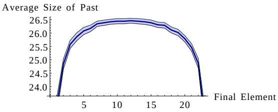
- **The Peak at** $\mathcal{N}_f \sim 23$



Features of the second peak geometry

- Rapid expansion from a single element to a large spatial slice: $\mathcal{N}_f/height \sim 6$.
- Homogeneity determined from causal past.





- Causal pasts of final elements maximally overlap
- Non-manifold like.

Initial Conditions for the Universe from Quantum Gravity?

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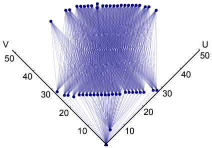

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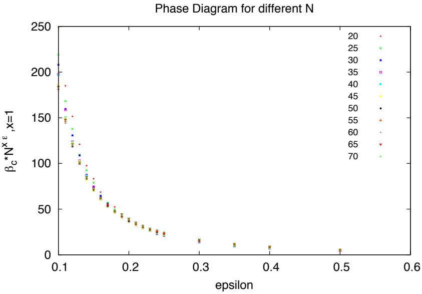
Initial Conditions for the Universe from Quantum Gravity?

- Nature of phase transition: First order for $\epsilon = 0.02 - -0.5$.



- Action is non-extensive, but there is a scaling: $\beta\langle S\rangle/N^2$.

$$\boxed{\text{Thermodynamic Limit Exists}}$$

Large N behaviour

L.Glaser, D.O'Connor, S. Surya, in preparation

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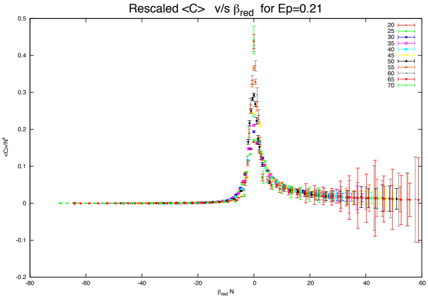

Thermodynamic Limit Exists

Summary and Open Questions and Speculations

- A concrete *illustration* of how physically interesting initial conditions can arise from a theory of quantum gravity.
- Importance of non-continuum structures.
- 2d is NOT 4d, but there are universal dimension-independent features:
  - β parameter: In $d > 2$ rescales the Planck volume.
  - Large β limit is dominated by the Action $\Rightarrow$ Crystalline Phase will dominate.
  - Speculations:

Simulations on RRI HPC cluster. Supported in part by FQXi via Theiss Research

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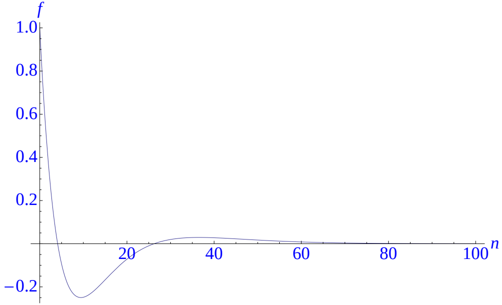

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  - Speculations: $N \sim$ Age of Universe. ϵ could flow to smaller values for which spacetime emerges.

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