Evolution of perturbations in anisotropic loop quantum cosmology:

Are predictions of LQC robust in the presence of anisotropies?

V. Sreenath

Louisiana State University

Based on work in collaboration with Javier Olmedo and Ivan Agullo.

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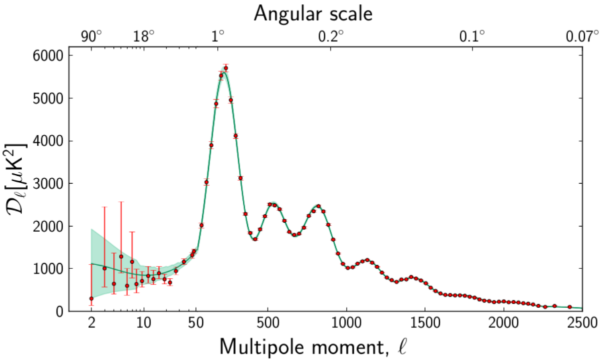
An isotropic universe



Planck mission concludes that, CMB is largely consistent with statistical isotropy, although there are a few indications of anomalies with respect to the expectations of ΛCDM [1]. As the above plot of TT correlations measured by the Planck satellite[2] indicates, we can at most accommodate departures from isotropy only at large angular scales *i.e.* at $\ell < 30$.

[1] P. A. R. Ade et al. (Planck Collaboration), arXiv:1506.07135 [astro-ph.CO].

[2] P. A. R. Ade et al., arXiv:1502.02114 [astro-ph.CO].

What happens in the presence of anisotropies?

However, it is a priori not clear why our universe should be isotropic. Hence, it is interesting to investigate the effect of anisotropies on our predictions.

- It has been found that if we consider a Bianchi I spacetime, wherein the universe gets stretched differently in different directions, inflation will dilute any shear, which quantifies the level of anisotropy, that is initially present at the onset of inflation. More specifically, in a Bianchi I spacetime, shear, σ, decays as a^{-3}.
- But this implies that, in a bouncing universe, any anisotropy that is generated initially will get amplified during the contracting phase and will become large at the bounce.
- In Loop Quantum Cosmology(LQC), a bounce is followed by inflation and hence one may argue that since inflation dilutes any amount of anisotropy that is initially present, the background will eventually become isotropic.
- However, as was discussed in the previous talks by Ivan and Brajesh, perturbations keep the memory of the bounce. Thus leading us to the question whether the presence of anisotropies alter the predictions of LQC for the primordial power spectrum.

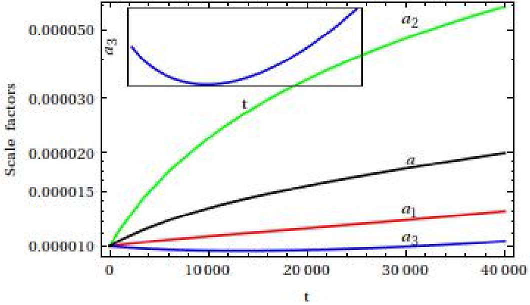
Bianchi I spacetime

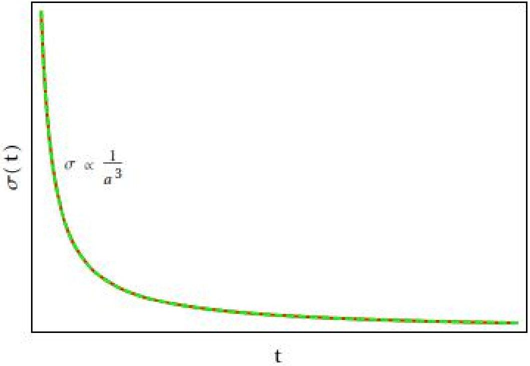
The line element in a Bianchi I spacetime is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j;\ \gamma_{ij} = \mathrm{diag}(\frac{a_1^2}{a^2}, \frac{a_2^2}{a^2}, \frac{a_3^2}{a^2}).$$

Since the universe is expanding differently in different directions, we can define shear as

$$\sigma_{ij} = \frac{\dot{\gamma}_{ij}}{2} \ \text{ and } \ \sigma^2 = \sigma_{ij}\sigma^{ij}$$





Perturbations in a Bianchi I spacetime

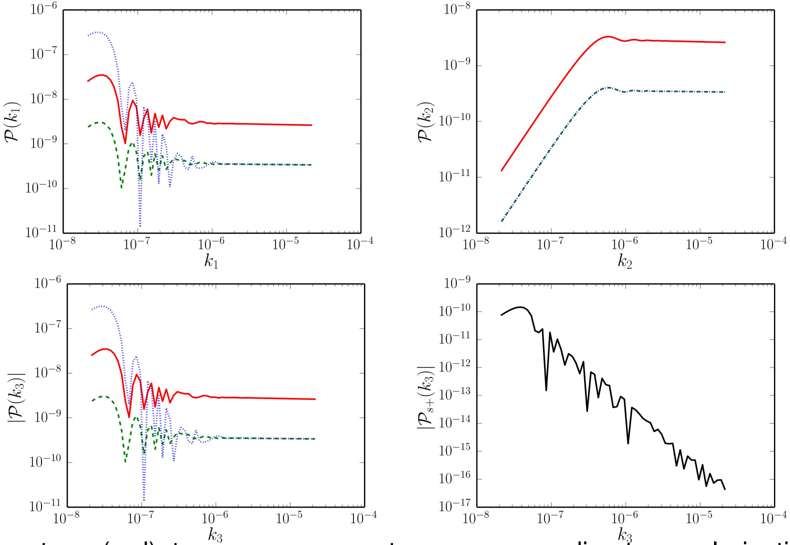
One of the striking features of a Bianchi I spacetime is that the even at the level of linear perturbation theory, the scalar and tensor modes are coupled to each other.

The consequence of this coupling is two fold:

- Cross-correlations between scalar and tensors at the level of two-point functions.
- The power spectra of two polarization are different at large scales which leave the hubble radius when the effect of shear is still significant.

The scalar power spectra, the power spectra corresponding to two polarizations and as an example cross power spectra between scalar and '+' polarization has been plotted in the next slide.

Inflationary power spectrum in a Bianchi I universe



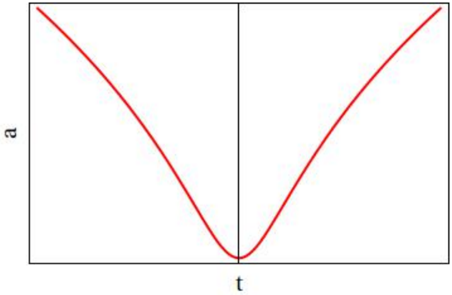
Scalar power spectrum (red), tensor power spectrum corresponding to $+$ polarization (green) and that corresponding to $\times$ polarization (blue) has been plotted for modes in different orthonormal directions. In black is the crosscorrelation between a scalar and $+$ polarization along k_3.

Inflation in loop quantum cosmology with FLRW spacetime

In loop quantum cosmology, the quantum gravity replaces the big bang singularity by a bounce. Thus a scalar field dominated universe in loop quantum cosmology, will first undergo a bounce before inflation sets in.

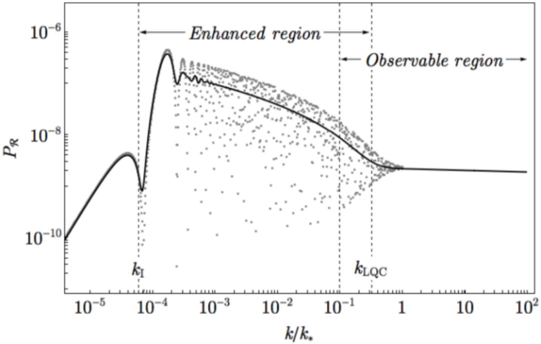
The quantum gravity modifies the Friedmann equation as follows,

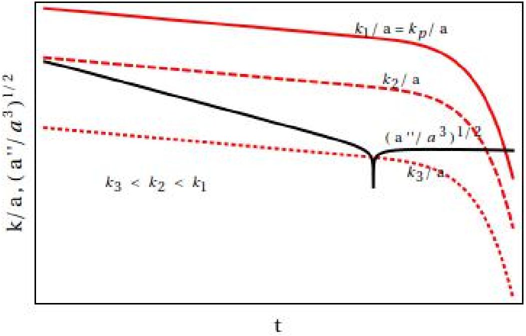
$$
\begin{aligned}
H^2 &= 8\pi G\rho\left(1-\frac{\rho}{\rho_c}\right) \\
\frac{\ddot{a}}{a} &= -4\pi G\rho\left(1-4\frac{\rho}{\rho_c}\right)-4\pi GP\left(1-2\frac{\rho}{\rho_c}\right)
\end{aligned}
$$



Power spectrum in loop quantum cosmology with FLRW spacetime

The effect of the bounce on the power spectrum is that for modes which feel the effect of curvature will have their power enhanced[3].





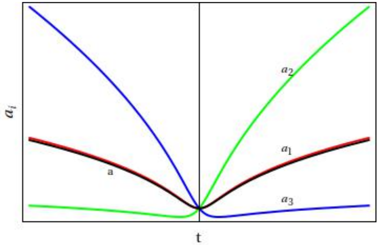
[3] Ivan Agullo and Noah A. Morris, Phys. Rev. D **92**, 124040 (2015).

Loop quantum cosmology in a Bianchi I spacetime

In a Bianchi I spacetime, the dynamics is governed by the Hamiltonian[4]

$$\mathcal{H}_{\rm eff} = -\frac{1}{8\pi G\gamma^2 V}\left(\frac{\sin(\mu_1 c_1)}{\mu_1}\frac{\sin(\mu_2 c_2)}{\mu_2}+\frac{\sin(\mu_2 c_2)}{\mu_2}\frac{\sin(\mu_3 c_3)}{\mu_3}+\frac{\sin(\mu_3 c_3)}{\mu_3}\frac{\sin(\mu_1 c_1)}{\mu_1}\right)+ \frac{P_\phi^2}{2\sqrt{p_1p_2p_3}}+\frac{1}{2}m^2\phi^2\sqrt{p_1p_2p_3}$$

where $\mu_1 = \lambda p_1/(p_2p_3)$ and so on. The inflationary dynamics in this spacetime has been studied extensively [5].



This system also produces a bounce as in FLRW but with the difference that each direction bounces at different instance.

[4] see, for instance, A. Ashtekar and E. Wilson-Ewing, Phys. Rev. D **79**, 083535 (2009).

[5] B. Gupt and P. Singh, Phys. Rev. D **86**, 024034 (2012); Class. Quant. Grav. **30**, 145013 (2013).

Perturbation theory in Bianchi I space time

- In order to study the evolution of perturbations, we will assume, as a first approximation, that most important quantum effects come through the background quantities.
- Thus, we need to derive the evolution equations for perturbations which respect the background dependence as governed by loop quantum cosmology.
- The perturbation theory in a Bianchi I universe is much more complicated because of the presence of shear as it causes the scalar, vector and tensor modes to couple to each other.
- Another feature of this perturbation theory is that, the tensor perturbations are not gauge invariant in the presence of shear. The gauge invariant perturbations in the presence of shear can be written as

$$
\begin{aligned}
Q &= \delta\varphi\left(1+\frac{3\sigma_{||}}{\kappa\pi_s}\right)+\frac{3P_\varphi}{a\kappa\pi_s}\left(\gamma_1-\frac{\gamma_2}{3}\right),\\
\gamma_+ &= \gamma_5-\frac{2a\sigma_{T+}}{P_\varphi}\delta\varphi\\
\gamma_+ &= \gamma_6-\frac{2a\sigma_{T\times}}{P_\varphi}\delta\varphi.
\end{aligned}
$$

Note that, the above expressions will lead to the gauge invariant variables in FLRW in the absence of shear. We are currently working towards deriving the second order Hamiltonian for perturbations.

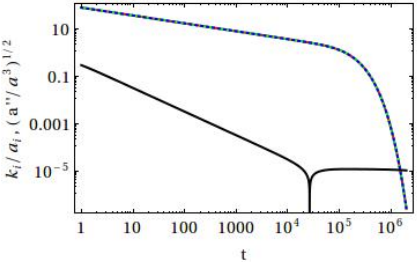
Some insights from preliminary simulations of a scalar field

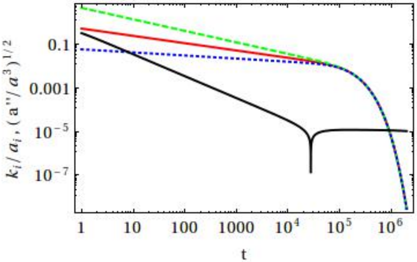
Some insights in to the behaviour of perturbations in Bianchi I universe can be achieved by simulating the behavior of a scalar field.

The key differences in the evolution of perturbations in a Bianchi I universe from that in a FLRW is as follows:

- The evolution becomes dependent on the direction of mode $\boldsymbol{k}$.
- Evolution of scalar and tensor perturbations become coupled.

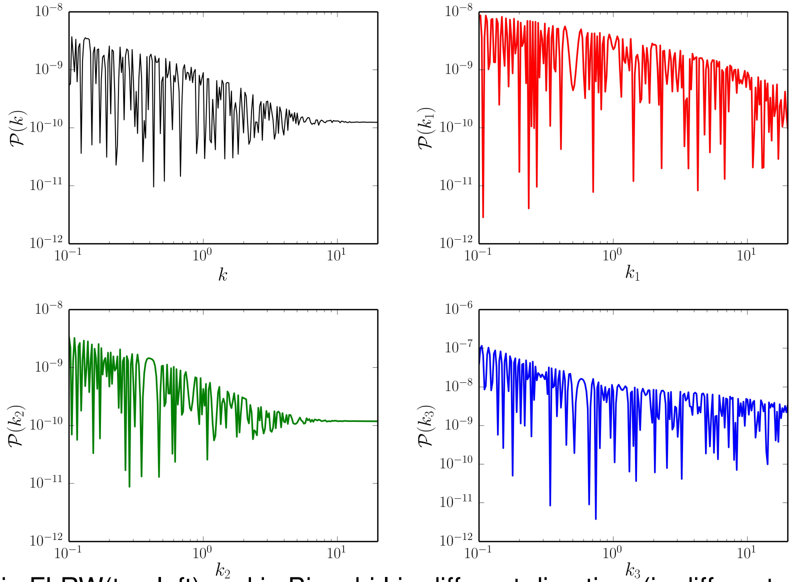
Of these, the first effect can be understood by studying the evolution of a scalar field.





Thus we see that, though all directions behave identically towards the end of evolution, near the bounce different directions feel the curvature differently at the bounce.

Direction dependence of power spectra



Power spectra in FLRW(top left) and in Bianchi I in different directions(in different colors) are given. Note the direction dependence of power spectra.

Conclusions

- Our simulations indicate that the presence of anisotropy can lead to a direction dependent power spectra.
- Since observations indicate that the power spectra should be scale invariant and isotropic at least for scales corresponding to $\ell > 30$, we should be able to put bounds on the value of shear for which the predictions of LQC are consistent with observations.
- This bound on shear may imply, for instance, one of these things:
    - The bounce is highly sensitive to shear in which case the predictions of LQC will agree with observations only when the value of shear is too low.
    - Or may be one can increase $\phi(t)|_{\mathrm{bounce}}$ with shear such that over the scales of interest the power spectra is isotropic.

Only more studies will tell us.

Thank you!