

# The Gravity Dual of Boundary Causality

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# Probing Quantum Gravity with AdS/CFT

Fundamentals of QFT  $\Leftrightarrow$  Fundamentals of bulk. E.g.

- Field theory unitarity  $\Rightarrow$  unitary evolution of QG
- Field theory entanglement structure  $\Rightarrow$  assorted properties of bulk [Van Raamsdonk, Lashkari, Lin, Faulkner, Guica, Hartman, Myers, Ooguri, Rabideau, Sabelle-Garnier, Stoica, Swingle]

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Want to construct bulk *dual condition* with *minimal* assumptions about bulk theory

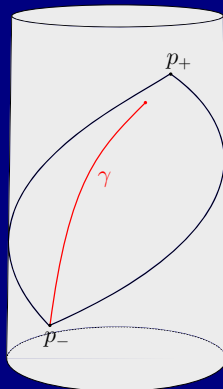
# The Boundary Causality Condition

## The Boundary Causality Condition (BCC)

Boundary points which are acausal through boundary must be acausal through bulk.

- I.e., signals cannot travel “faster” through bulk than through boundary

Forbidden:



# Partial Results

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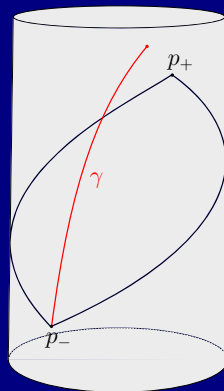
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- ANCC shouldn't be necessary, since it can be violated by quantum fields [Urban, Olum; Flanagan, Wald]
- Want a *necessary and sufficient* condition on the bulk starting from the BCC as a fundamental physical requirement



# Only Need Perturbations of Pure AdS

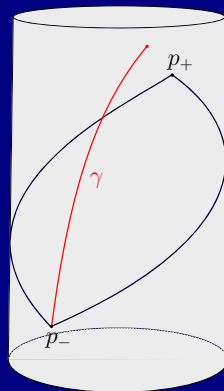
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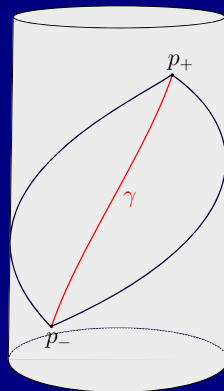
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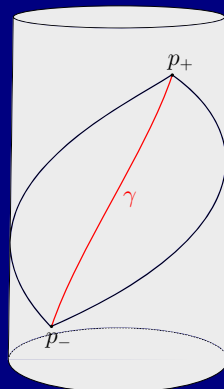


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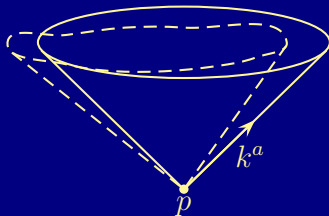
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Only known example is pure AdS: saturates the BCC “maximally”



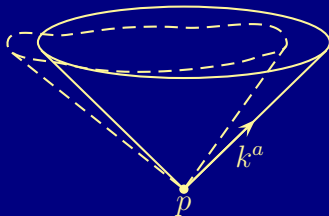
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Consider a perturbation  $\delta g_{ab}$  of pure AdS, and look at how it changes causal structure:



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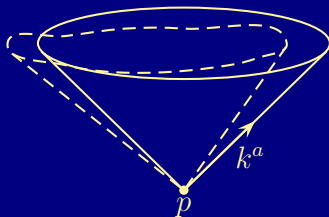
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$\left(g_{ab}^{(\text{AdS})} + \delta g_{ab}\right) k^a k^b = \delta g_{ab} k^a k^b|_p$  measures how much  $\delta g_{ab}$  “opens” or “closes” light cone at  $p$  in  $k^a$  direction

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Might expect  $\delta g_{ab} k^a k^b \geq 0$  for all null  $k^a$  to be sufficient to ensure BCC

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If  $\delta g_{ab}$  is a regular,  $C^2$  perturbation of pure AdS which leaves the causal structure of the boundary unchanged, then

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# Future Directions

- Background-dependence is annoying (though note that lots of work focuses on perturbations from vacuum)
- Can we find a background-independent condition whose linearization about AdS is equivalent to the BCC?
- Can we express result in terms of well-understood physical quantities, like [Kleban, McGreevy, Thomas; Page, Surya, Woolgar]?
- What if bulk causal structure isn't sharp? (E.g. superposition of geometries) - can BCC be made “fuzzy” to take this into account?