

Approximating waveforms of rapidly rotating neutron stars

Scott Lawrence, Cole Miller

Department of Astronomy, University of Maryland, College Park

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Mass-radius curves

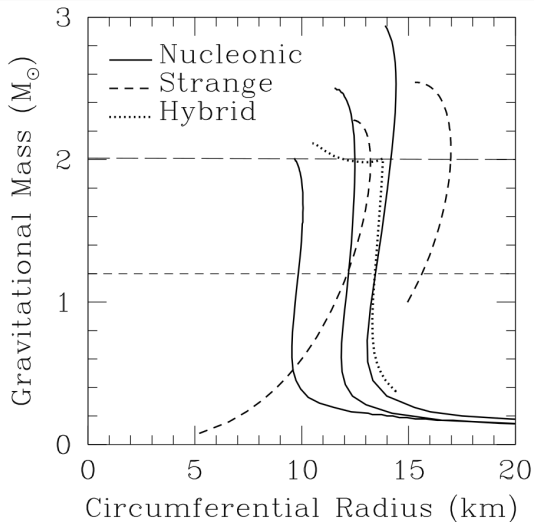
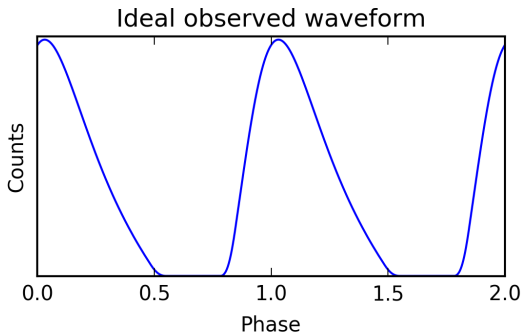


Figure from Miller 2013, [arXiv:1312.0029](https://arxiv.org/abs/1312.0029)

Fitting waveforms to get M and R



NICER/LOFT will give energy-dependent waveforms from millisecond X-ray pulsars.

Fit these observations to get M and R

Fitting waveforms to get M and R

Advantages of waveform fitting:

- ▶ Free of many systematic uncertainties
- ▶ Difficult to get a good fit with incorrect assumptions

Difficulties:

- ▶ Seven parameters: R , M , inclination, spot colatitude, spot radius, temperature, distance
- ▶ All current fitting methods require calculating many waveforms for *each* fit.

Exact waveforms: RNS + Raytracing

Compute shape of star and spacetime using the publicly-available RNS¹, for one equation of state

Trace rays from exact surface in exact spacetime

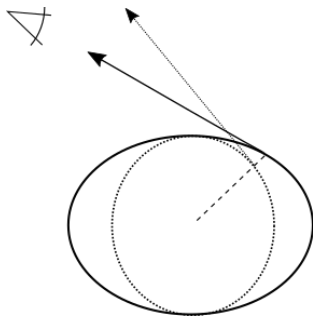
Accounts for both spacetime effects and the true shape of the star

Unacceptably slow for fitting
(~ 5 processor-hours per waveform)

¹Stergioulas and Morsink, <http://www.gravity.phys.uwm.edu/rns/>

Fast approximation: Oblate Schwarzschild

Schwarzschild spacetime with Doppler beaming, and oblate emitting surface



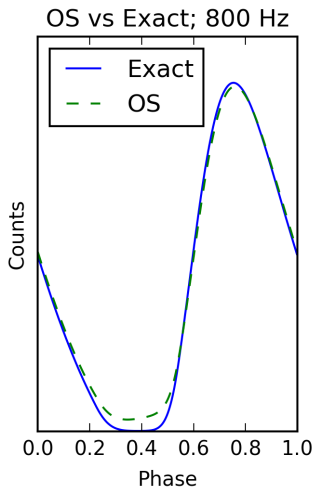
Described by Morsink et al. (2007), with updated shapes provided by AlGendy and Morsink (2014)²

²ApJ 663, 1244; ApJ 791, 78

Oblate Schwarzschild (OS) is not exact

Effects ignored by OS:

- ▶ Exact oblate shape
- ▶ Frame dragging
- ▶ Quadrupole moments (and up)



Evaluating Oblate Schwarzschild (OS)

When are the OS waveforms “close enough”?

Sufficient: for the same parameter set, if the OS waveform is statistically indistinguishable from the real thing

From NICER or LOFT, we expect $\sim 10^6$ counts.

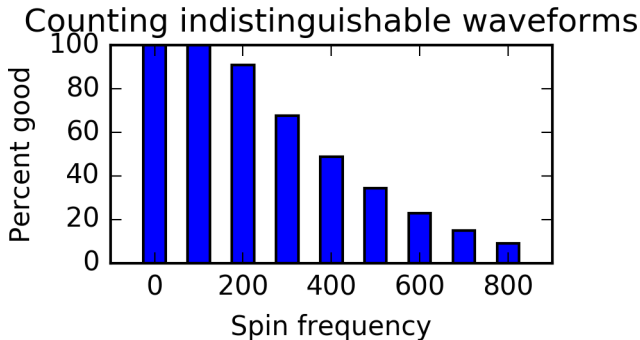
Quality of OS: Method

For a pair (exact, OS) of waveforms generated from the same parameters:

- ▶ Sample the raytraced waveform for 800k counts
- ▶ Add 200k expected counts of phase- and energy-independent noise.
- ▶ Compute χ^2 of Poisson sampled waveform compared to OS model.

Results

How many pairs of waveforms are statistically indistinguishable?



This does not guarantee that 30% of fits at 300 Hz will be biased with OS.

Summary

Oblate Schwarzschild (OS) is fast enough to perform fits.

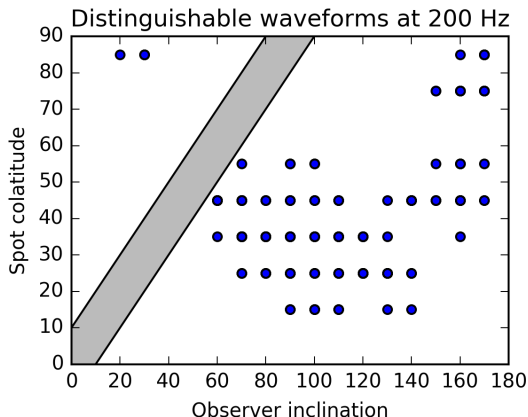
The OS approximation is sufficiently accurate at least up to 200 Hz.

Sufficient for NICER target PSR J0437 (174 Hz).

At higher spins, the OS shape may be insufficiently accurate.

When does OS fail?

At low spin frequencies, OS mostly fails when the shape of the star is important.



Fitting procedure

Seven parameters: R , M , inclination i , spot colatitude θ , spot radius $\delta\theta$, T , D

Brute force gridding

Markov-chain Monte-Carlo methods

A hybrid algorithm, specialized to millisecond pulsar waveform fits, is described in Lo, K. H. et al (2013)³

All current fitting methods require calculating many waveforms for *each* fit.

³ApJ 776 19

OS comparison procedure

For $\sim 25k$ waveform pairs:

- ▶ Add 20% noise to each raytraced waveform.
- ▶ Poisson sample the raytraced waveform for 1M counts
- ▶ Compute χ^2 of poisson sampled waveform compared to OS model.
- ▶ Minimize χ^2 over phase shift, normalization, and noise level
- ▶ Average χ^2 over 1000 instances

OS comparison waveform selection

EOSs AU, O, N, UU

R/M 4, 5, 6, 7, and 8

Spins from 0 to 800 Hz

17 inclinations (10° through 170°)

8 spot colatitudes (15° through 85°)

Spot size 10°

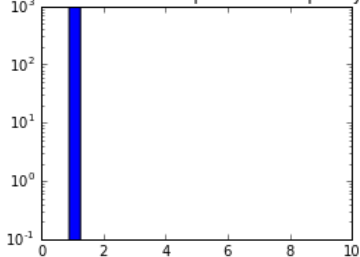
Quality of OS: Detailed results

Spin frequency	Percent indistinguishable
0 Hz	100%
100 Hz	100%
200 Hz	91%
300 Hz	68%
400 Hz	49%
500 Hz	34%
600 Hz	23%
700 Hz	15%
800 Hz	9%

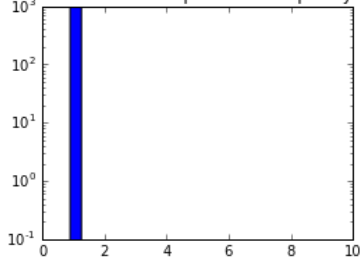
This does not guarantee that 95% of fits at 300 Hz will be biased with OS.

Quality of OS: Detailed results

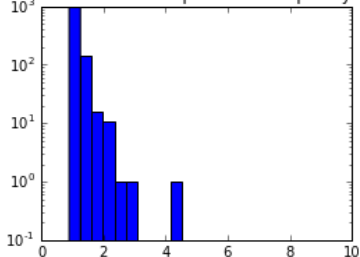
Mean Reduced Chi-Squared at Frequency 0



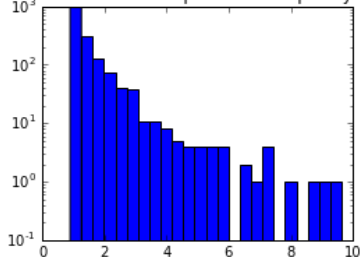
Mean Reduced Chi-Squared at Frequency 100



Mean Reduced Chi-Squared at Frequency 200

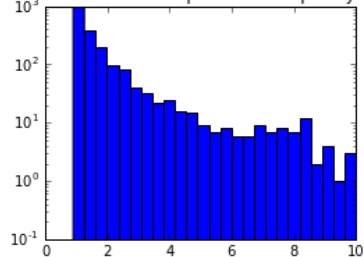


Mean Reduced Chi-Squared at Frequency 300

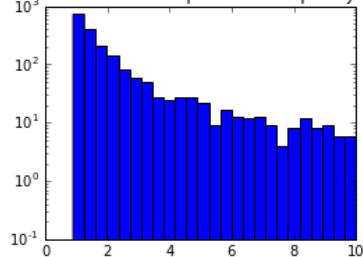


Quality of OS: Detailed results

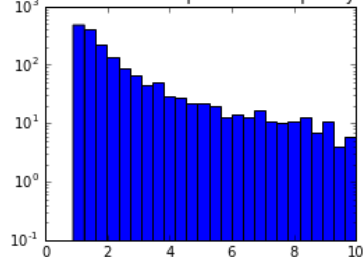
Mean Reduced Chi-Squared at Frequency 400



Mean Reduced Chi-Squared at Frequency 500



Mean Reduced Chi-Squared at Frequency 600



Mean Reduced Chi-Squared at Frequency 700

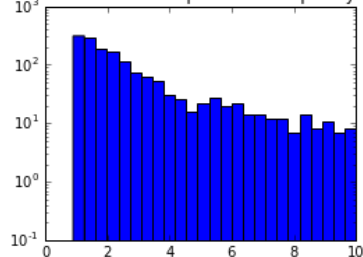


Figure parameters

Main example:

EOS UU, 600 Hz, $R/M = 5$, $\delta = 10^\circ$, $\theta = 65^\circ$, $i = 80^\circ$

OS vs exact comparison:

EOS UU, 800 Hz, $R/M = 6$, 45° spot and 100° observer

Energy-dependent waveform

