

# Explicit algebraic classification of Robinson–Trautman and Kundt geometries

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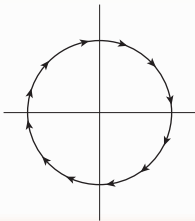
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## How to define the RT & K class?

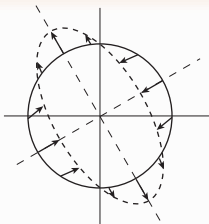
Transverse deformations of a null geodesic congruence generated by a vector field  $k$ :

**twist**



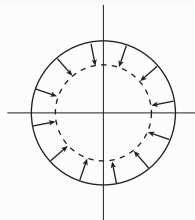
$$A^2 = -k_{[a;b]}k^{a;b}$$

**shear**



$$\sigma^2 = k_{(a;b)}k^{a;b} - \frac{1}{D-2} (k^a_{;a})^2$$

**expansion**



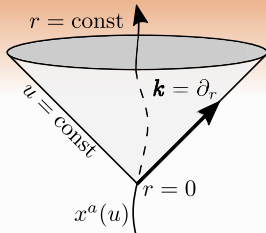
$$\Theta = \frac{1}{D-2} k^a_{;a}$$

**RT & K** class  $\Leftrightarrow$  geometries admitting **non-twisting** and **shear-free** null geodesic congruence



## How to describe the RT & K geometry?

Twist-free condition:  $A^2 = 0 \Leftrightarrow \exists$  null foliation with  $\mathbf{k}$  normal (tangent)



- $u = \text{const}$  – uniquely labels null hypersurfaces
- $\mathbf{k} = \partial_r$  – generator of *non-twisting* null congruence
- $r$  – affine parameter along *non-twisting* null congruence
- $u = \text{const}$  &  $r = \text{const}$ :  $(D - 2)$ -dim transverse space with metric  $g_{pq}$

$(r, u, x^p) \Leftrightarrow$  adapted coordinates and metric:

$$ds^2 = g_{pq}(r, u, x) dx^p dx^q + 2g_{up}(r, u, x) dx^p du - 2dudr + g_{uu}(r, u, x) du^2$$

Shear-free condition:  $\sigma^2 = 0 \Leftrightarrow g_{pq} = \rho^2(r, u, x) h_{pq}(u, x)$  with  $\frac{\rho_{,r}}{\rho} = \Theta$

**RT** subclass  $\Leftrightarrow \Theta \neq 0$

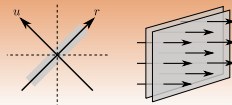
**Kundt** subclass  $\Leftrightarrow \Theta = 0$



## Why are the Kundt spacetimes interesting?

- pp-waves: admit a covariantly constant null vector field  $k \Rightarrow r$ -independent (Brinkmann, 1925)

$$ds^2 = h_{pq} dx^p dx^q + 2e_p dx^p du - 2dudr + c du^2$$

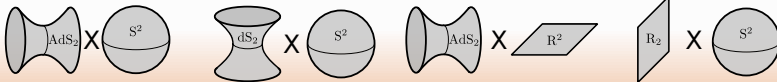


- VSI spacetimes: scalar curvature invariants of all orders vanish  $\Rightarrow g_{pq}(u, x) = \delta_{pq}$

$$ds^2 = \delta_{pq} dx^p dx^q + 2(e_p + rf_p) dx^p du - 2dudr + (ar^2 + br + c) du^2 \quad (\text{Coley et al., 2006})$$

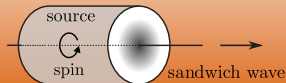
- direct product spacetimes: Bertotti–Robinson, Nariai, Plebanski–Hacyan backgrounds

$$ds^2 = h_{pq} dx^p dx^q - 2dudr + ar^2 du^2 \quad (\text{see Chap. 7 in Griffiths and Podolský, 2012})$$



- gyratons: toy models for null particles with spin (Bonnor, 1970, Frolov et al., 2005)

$$ds^2 = h_{pq} dx^p dx^q + 2e_p dx^p du - 2dudr + c du^2$$





## Why are the RT spacetimes interesting?

Four dimensions:  $D = 4$

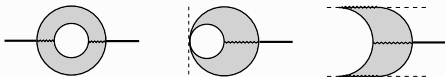
(see Stephani et al., 2003)

- Schwarzschild–(anti-)de Sitter black holes

$$ds^2 = r^2(d\vartheta^2 + \sin^2 \vartheta d\phi^2) - 2du dr - \left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right) du^2$$



- C-metric: accelerated black holes
- expanding gravitational waves: counterpart to *non*-expanding Kundt waves



- alternative *spherically symmetric* BH in the Einstein–Weyl theory

(Lü et al., 2015)

Higher dimensions:  $D > 4$

(see Ortaggio et al., 2013)

- generalisation of Schwarzschild-like black holes
- analogy to C-metric does NOT exist in  $D > 4$  (within Einstein's theory)
- expanding vacuum gravitational waves do NOT exist in  $D > 4$  (within Einstein's theory)



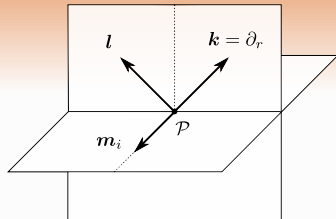
## What is the natural reference frame?

Real null frame:  $\{\mathbf{k}, \mathbf{l}, \mathbf{m}_i\}$

- $\mathbf{k}, \mathbf{l}$ : future null vectors:  $\mathbf{k} \cdot \mathbf{l} = -1$
- $\mathbf{m}_i$ :  $D - 2$  spatial vectors:  $\mathbf{m}_i \cdot \mathbf{m}_j = \delta_{ij}$

Lorentz transformations  $\Leftrightarrow$  freedom in  $\{\mathbf{k}, \mathbf{l}, \mathbf{m}_i\}$

- null rotation with  $\mathbf{k}$  fixed
- null rotation with  $\mathbf{l}$  fixed
- boost in the  $\mathbf{k} - \mathbf{l}$  plane
- spatial rotation in the space of  $\mathbf{m}_i$



Natural frame in the RT & K class:

$$\mathbf{k} = \partial_r \quad \mathbf{l} = \frac{1}{2} g_{uu} \partial_r + \partial_u \quad \mathbf{m}_i = m_i^p (g_{up} \partial_r + \partial_p)$$

- the vector  $\mathbf{k} \Leftrightarrow$  generator of the *optically privileged null congruence*  $\partial_r$
- the coefficients  $m_i^p : g_{pq} m_i^p m_j^q = \delta_{ij} \Leftrightarrow \mathbf{m}_i \cdot \mathbf{m}_j = \delta_{ij}$



## How to efficiently express the Weyl tensor?

Frame components – notation analogous to standard NP scalars:

$$\text{b.w. } +2: \quad \Psi_{0ij} = C_{abcd} k^a m_i^b k^c m_j^d \sim \Omega_{ij}$$

$$\text{b.w. } +1: \quad \Psi_{1Ti} = C_{abcd} k^a l^b k^c m_i^d \sim \Psi_j$$

$$\text{b.w. } 0: \quad \Psi_{2S} = C_{abcd} k^a l^b l^c k^d \sim \Phi$$

$$\Psi_{2ij} = C_{abcd} k^a l^b m_i^c m_j^d \sim \Phi_{ij}^A$$

$$\text{b.w. } -1: \quad \Psi_{3Ti} = C_{abcd} l^a k^b l^c m_i^d \sim \Psi'_j$$

$$\text{b.w. } -2: \quad \Psi_{4ij} = C_{abcd} l^a m_i^b l^c m_j^d \sim \Omega'_{ij}$$

$$\Psi_{1ijk} = C_{abcd} k^a m_i^b m_j^c m_k^d$$

$$\Psi_{2ijkl} = C_{abcd} m_i^a m_j^b m_k^c m_l^d$$

$$\Psi_{2Tij} = C_{abcd} k^a m_i^b l^c m_j^d$$

$$\Psi_{3ijk} = C_{abcd} l^a m_i^b m_j^c m_k^d$$

Irreducible components (in  $D > 4$ ):

$$\tilde{\Psi}_{1ijk} \equiv \Psi_{1ijk} - \frac{1}{D-3} (\delta_{ij} \Psi_{1Tk} - \delta_{ik} \Psi_{1Tj}) \sim \tilde{\Psi}_{ijk}$$

$$\tilde{\Psi}_{2T(ij)} \equiv \Psi_{2T(ij)} - \frac{1}{D-2} \delta_{ij} \Psi_{2S} \sim \tilde{\Phi}_{ij}^S$$

$$\tilde{\Psi}_{2ijkl} \equiv \Psi_{2ijkl} - \frac{2}{D-4} (\delta_{ik} \tilde{\Psi}_{2T(jl)} + \delta_{jl} \tilde{\Psi}_{2T(ik)} - \delta_{il} \tilde{\Psi}_{2T(jk)} - \delta_{jk} \tilde{\Psi}_{2T(il)}) - \frac{2(\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk})}{(D-2)(D-3)} \Psi_{2S} \sim \tilde{\Phi}_{ijkl}$$

$$\tilde{\Psi}_{3ijk} \equiv \Psi_{3ijk} - \frac{1}{D-3} (\delta_{ij} \Psi_{3Tk} - \delta_{ik} \Psi_{3Tj}) \sim \tilde{\Psi}'_{ijk}$$

See Ortoggio et al. (2013) for more details.





## What is the classification scheme?

Principal alignment (sub)types  $\Leftrightarrow \exists$  (multiple) WAND  $k$

type	vanishing components								
G	no null frame exists in which all components $\Psi_{0ij}$ vanish (only in $D > 4$ )								
I	$\Psi_{0ij}$								
I(a)	$\Psi_{0ij}$	$\Psi_{1T^i}$							
I(b)	$\Psi_{0ij}$	$\tilde{\Psi}_{1ijk}$							
II	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$						
II(a)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$					
II(b)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\tilde{\Psi}_{2T^{(ij)}}$					
II(c)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\tilde{\Psi}_{2ijkl}$					
II(d)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2ij}$					
III	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$	$\tilde{\Psi}_{2T^{(ij)}}$	$\tilde{\Psi}_{2ijkl}$	$\Psi_{2ij}$		
III(a)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$	$\tilde{\Psi}_{2T^{(ij)}}$	$\tilde{\Psi}_{2ijkl}$	$\Psi_{2ij}$	$\Psi_{3T^i}$	
III(b)	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$	$\tilde{\Psi}_{2T^{(ij)}}$	$\tilde{\Psi}_{2ijkl}$	$\Psi_{2ij}$	$\tilde{\Psi}_{3ijk}$	
N	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$	$\tilde{\Psi}_{2T^{(ij)}}$	$\tilde{\Psi}_{2ijkl}$	$\Psi_{2ij}$	$\Psi_{3T^i}$	$\tilde{\Psi}_{3ijk}$
O	$\Psi_{0ij}$	$\Psi_{1T^i}$	$\tilde{\Psi}_{1ijk}$	$\Psi_{2S}$	$\tilde{\Psi}_{2T^{(ij)}}$	$\tilde{\Psi}_{2ijkl}$	$\Psi_{2ij}$	$\Psi_{3T^i}$	$\tilde{\Psi}_{3ijk}$

See Ortogio et al. (2013) for more details.



## What is the explicit form of the Weyl scalars?

$$\Psi_{0ij} = 0$$

$$\Psi_{1Ti} = m_i^p \frac{D-3}{D-2} N_p$$

$$\Psi_{2S} = \frac{D-3}{D-1} P$$

$$\Psi_{2ij} = m_i^p m_j^q F_{pq}$$

$$\Psi_{3Ti} = m_i^p \frac{D-3}{D-2} V_p$$

$$\Psi_{4ij} = m_i^p m_j^q \left( W_{pq} - \frac{1}{D-2} g_{pq} W \right)$$

$$\tilde{\Psi}_{1ijk} = 0$$

$$\tilde{\Psi}_{2T(ij)} = m_i^p m_j^q \frac{1}{D-2} \left( Q_{pq} - \frac{1}{D-2} g_{pq} Q \right)$$

$$\tilde{\Psi}_{2ijkl} = m_i^m m_j^p m_k^n m_l^q {}^S C_{mpnq}$$

$$\tilde{\Psi}_{3ijk} = m_i^p m_j^m m_k^q \left( X_{pmq} - \frac{2}{D-3} g_{p[m} X_{q]} \right)$$

where  $Q \equiv g^{pq} Q_{pq}$

$X_q \equiv g^{pm} X_{pmq}$

$W \equiv g^{pq} W_{pq}$

and

- $e_{pq} = g_{u(p||q)} - \frac{1}{2} g_{pq,u}$      $E_{pq} = g_{u[p,q]} + \frac{1}{2} g_{pq,u}$      $f_{pq} = g_{u(p,r||q)} + \frac{1}{2} g_{up,r} g_{uq,r}$
- $||$  stands for the *covariant derivative* w.r.t. transverse metric  $g_{pq}$
- ${}^S C_{mpnq}$ ,  ${}^S R_{pq}$  and  ${}^S R$ : the *Weyl tensor*, *Ricci tensor* and *Ricci scalar* w.r.t.  $g_{pq}$

Coefficients  $N_p$ ,  $P$ ,  $Q_{pq}$ ,  $F_{pq}$ ,  $V_p$ ,  $X_{pmq}$  and  $W_{pq}$  are ...



## What is the explicit form of characteristic functions?

$$N_p = \left( -\frac{1}{2} g_{up,r} + \Theta g_{up} \right)_{,r} + \Theta_{,p}$$

$$P = \left( \frac{1}{2} g_{uu,r} - \Theta g_{uu} \right)_{,r} + \frac{g_{,r}}{(D-2)(D-3)} - \frac{1}{4} \frac{D-4}{D-2} g^{mn} g_{un,r} g_{un,r} + \frac{1}{D-2} \left( g^m g_{un,rr} + g^{mn} g_{un,r} |_{|n} \right) \\ - \frac{2}{D-2} g^m g_{un} \Theta_{,r} - 2 \Theta_{,u} - \frac{4}{D-2} g^m \Theta_{,n} - \Theta^2 \frac{D-4}{D-2} g^m g_{un} + \Theta \left( \frac{D-6}{D-2} g^m g_{un,r} - \frac{2}{D-2} g^{mn} g_{un} |_{|n} \right)$$

$$Q_{pq} = {}^S R_{pq} + (D-4) \left[ \frac{1}{2} (f_{pq} + g_u (p g_q)_{u,r}) - (\Theta_{,r} - \Theta^2) g_{up} g_{uq} - 2 g_u (p \Theta_{,q}) - \Theta (g_u (p |_{|q}) + 2 g_u (p g_q)_{u,r}) \right]$$

$$F_{pq} = g_u [p, q]_{,r} - g_u [p g_q]_{u,rr} + 2 \Theta (g_u [p g_q]_{u,r} - g_u [p, q]_{,r})$$

$$V_p = \frac{1}{2} \left[ \frac{1}{2} g_{uu} g_{up,rr} - g_{uu,rp} + g_{up,ru} - \frac{1}{2} g^m g_{un,r} g_{up,r} + g^{mn} g_{un,r} E_{np} - g_{up} (g_{uu,rr} - \frac{1}{2} g^{mn} g_{un,r} g_{un,r}) \right] + \frac{1}{2} g_{up} g_{uu} \Theta_{,r} + g_{up} \Theta_{,u} + \frac{1}{2} g_{uu} \Theta_{,p} \\ + \frac{1}{D-3} \left[ \frac{1}{2} g^m g_{un} g_{up,rr} + g^{mn} e_m [n g_p]_{u,r} - g^m g_u [n, p]_{,r} + \frac{1}{2} g^m (g_u [p, r] |_{|n} + f_{pn}) - g^{mn} (g_m [p, u] |_{|n} + g_u [m, p] |_{|n}) - \frac{1}{2} g_{up} (g^m g_{un,rr} + g^{mn} f_{mn}) \right] \\ - \Theta \left[ \frac{1}{2} g_{uu} g_{up,r} - g_{uu,p} + g_{up,u} - g^m g_u [n g_p]_{u,r} + g^m E_{np} - g_{up} g_{uu,r} + \frac{1}{D-3} (3 g^m g_u [n g_p]_{u,r} - 3 g^m g_u [n, p]_{,r} - \frac{1}{2} g_{up} g^{mn} g_{mn,u} + \frac{1}{2} g^m g_{np,u}) \right]$$

$$X_{pmq} = g_p [m, u]_{|q} + g_u [q, m] |_{|p} + g_{up} g_u [m g_q]_{u,rr} + e_p [m g_q]_{u,r} - g_u [q g_m]_{u,r} |_{|p} - g_{up} g_u [m, r]_{|q} - \frac{1}{2} g_u [q g_m]_{u,r} g_{up,r} \\ + \Theta \left( 3 g_u [q g_m]_{u,r} g_{up} + g_u [q g_m]_{p,u} + g_u [q g_m]_{u} |_{|p} - g_{up} |_{|[m g_q]_u} - 2 g_u [q, m] g_{up} \right)$$

$$W_{pq} = -\frac{1}{2} g_{uu} |_{|p} |_{|q} - \frac{1}{2} g_{pq,uu} + g_u (p, u)_{|q} - \frac{1}{2} g_{uu,r} e_{pq} + \frac{1}{2} g_{uu, (p g_q)_{u,r}} - g_{uu,r} (p g_q)_u + \frac{1}{2} g_{uu} g_u (p, r)_{|q} + \frac{1}{2} g_{uu} g_u (q g_p)_{u,rr} - \frac{1}{2} g_{uu,rr} g_{up} g_{uq} \\ + g_u (q g_p)_{u,ru} + \frac{1}{4} g^{mn} (g_{un} g_{up} g_{up,r} g_{uq,r} + g_{un,r} g_{up,r} g_{up} g_{uq}) - \frac{1}{2} g^{mn} g_{un} g_{up,r} g_u (q g_p)_{u,r} + g^{mn} (E_{mp} E_{nq} + g_{un,r} E_n (p g_q)_u - g_{un} E_n (p g_q)_{u,r}) \\ + \Theta (g_{up} g_{uq} g_{uu,r} + g_{uu, (p g_q)_u} - g_{uu} g_u (p g_q)_{u,r} - 2 g_u (p g_q)_{u,u} - \frac{1}{2} g_{uu} g_{pq,u})$$

... which completely determine Weyl tensor of general RT & K geometries



## What is the most general type of RT & K geometries?

The RT & K line element:

$$ds^2 = \exp \left( 2 \int \Theta(r, u, x) dr \right) h_{pq}(u, x) dx^p dx^q + 2g_{up} dx^p du - 2du dr + g_{uu} du^2$$

- $\Psi_{0ij} = 0 \Rightarrow$  **algebraic type I** or more special,  $k = \partial_r$  is a WAND
- $\tilde{\Psi}_{1ijk} = 0 \Rightarrow$  **all RT & K geometries are of type I(b)  $\equiv$  I**, or more special

How to continue?

**Algebraically special RT & K spacetimes** (type II=I(ab) or more special)

- perform the null rotation with  $l$  fixed to find **more suitable frame**  
(see JP, RŠ (2016) for the  $D = 4$  case)
- discuss the algebraic structure w.r.t. **optically privileged WAND**  $k = \partial_r$   
(see JP, RŠ (2013, 2015) in any  $D$ )

$$\text{1st step: } \text{type II} \Leftrightarrow \Psi_{1Ti} = m_i^p \frac{D-3}{D-2} \left[ \left( -\frac{1}{2} g_{up,r} + \Theta g_{up} \right)_{,r} + \Theta_{,p} \right] = 0 \quad \text{etc.}$$

To discuss specific conditions in the most general case (see JP, RŠ 2015) is not very illustrative.

$\Rightarrow$  **Explicit examples.**



## RT spacetimes of a general Ricci type

Free scalar field in Einstein's theory (see Tahamtan and Svátek, 2015)

Field equations:  $R_{ab} = \phi_{,a}\phi_{,b}$  and  $\square\phi = 0$

$$ds^2 = \frac{r^2 U^2 - C^2}{U p^2} (dx^2 + dy^2) - 2du dr - \frac{k + rU_{,u}}{U} du^2 \quad \text{with}$$

$$p = p(x, y) \quad k = k(x, y) \quad C, \gamma, \omega, \eta \dots \text{positive constants}$$

$$U(u) = \gamma \exp(\omega^2 u^2 + \eta u) \quad k = \Delta \log p \quad \Delta k = 4C^2 \omega^2 \quad \phi(r, u) = \frac{1}{\sqrt{2}} \log \frac{rU - C}{rU + C}$$

- *expansion:*  $\Theta = r U^2 (r^2 U^2 - C^2)^{-1}$
- *general Ricci type:*  $R_{ab} k^a k^b \neq 0$
- (at least) *Weyl type II:*  $\Psi_{1T^i} = 0 \Leftrightarrow g_{ui} = 0$  and  $\Theta_{,i} = 0$  ( $k$  is a double WAND)

$$\Psi_{2S} = \frac{2}{3} C^2 U \frac{r U_{,u} - k}{(r^2 U^2 - C^2)^2} \quad \dots \text{vanishing for } k = 0 = U_{,u} \quad \text{BUT } \nexists \text{ nontrivial } U(u)$$

$\dots$  **no** type III, N type D  $\Leftrightarrow k = \text{const}$

$$\Psi_{3T^i} = -\frac{r U^{3/2} p}{2(r^2 U^2 - C^2)^{3/2}} k_{,i}$$

$$\Psi_{422} = \frac{p^2}{4(r^2 U^2 - C^2)} ((k_{,22} - k_{,33}) + 2k_{,2} (\log p)_{,2} - 2k_{,3} (\log p)_{,3})$$

$$\Psi_{423} = \frac{p^2}{2(r^2 U^2 - C^2)} (k_{,23} + k_{,2} (\log p)_{,3} + k_{,3} (\log p)_{,2})$$



## Vacuum RT spacetimes in Einstein's theory

The expansion has to be  $\Theta = r^{-1}$  and the line element is:

$$ds^2 = r^2 h_{pq} (dx^p + e^p du)(dx^q + e^q du) - 2 du dr - \left[ \underbrace{\frac{\mathcal{R}}{(D-2)(D-3)}}_a + \frac{b(u)}{r^{D-3}} - \underbrace{\frac{2}{D-2} (e^p ||_p - \frac{1}{2} h^{pq} h_{pq,u})}_c r - \frac{2\Lambda}{(D-1)(D-2)} r^2 \right] du^2$$

Constraints implied by the field equations:

$$\mathcal{R}_{pq} = \frac{h_{pq}}{D-2} \mathcal{R} \quad (D-4) \mathcal{R}_{,p} = 0$$

$$h_{pq,u} = 2 e_{(p||q)} + c h_{pq} \quad h^{mn} a_{||m||n} + \frac{1}{2} (D-1)(D-2) b c + (D-2) b_{,u} = 0$$

type	D = 4	D > 4
II(a)	$b = 0$	$b = 0 \Leftrightarrow \text{D(a)}$
II(b)	always	always $\Leftrightarrow \text{D(b)}$
II(c)	always	$\mathcal{C}_{mpnq} = 0 \Leftrightarrow \text{D(c)}$
II(d)	always	always $\Leftrightarrow \text{D(d)}$
III	II(abcd)	
III(a)	$\mathcal{R}_{,p} = 0$	equivalent to O
III(b)	always	equivalent to O
N	III(ab)	
O	$c_{  p  q} = \frac{1}{D-2} h_{pq} h^{mn} c_{  m  n}$	equivalent to D(ac)
D	$c_{  p  q} = \frac{1}{D-2} h_{pq} h^{mn} c_{  m  n}$ and $\mathcal{R}_{,p} = 0$	always D(bd)



## Type II and N Kundt waves on D and O backgrounds

The Kundt line element:

$$ds^2 = g_{pq} dx^p dx^q - 2 du dr + (a r^2 + c) du^2$$

where  $g_{pq} = g_{pq}(x)$   $a = \text{const}$   $c = c(u, x)$

type	necessary and sufficient conditions
II(a)	${}^S R = -(D-2)(D-3)a$
II(b)	${}^S R_{pq} = \frac{1}{D-2} g_{pq} {}^S R$
II(c)	${}^S C_{mpnq} = 0$
II(d)	always
N	II(abcd)
O	N with $c_{  p  q} = \frac{1}{D-2} g_{pq} \Delta c$
D	$c_{  p  q} = \frac{1}{D-2} g_{pq} \Delta c$
D(a)	D with II(a)
D(b)	D with II(b)
D(c)	D with II(c)
D(d)	D with II(d)

- multiple WAND  $k = \partial_r$
- (possibly double) WAND  $l = \frac{1}{2}(a r^2 + c)\partial_r + \partial_u$
- for  $a = 0$ :  $pp$ -waves without off-diagonal terms  $g_{up}$



## What have we done?

The algebraic structure of a fully general Robinson–Trautman and Kundt family (geometries without twist and shear) in any dimension  $D$  was investigated.

Without employing any specific field equations:

- coordinate components of the Riemann, Ricci and Weyl tensors were explicitly calculated
- the Weyl tensor components were expressed in an adapted null frame
- Robinson–Trautman and Kundt geometries are of type I(b) (or more special)
- possible algebraically special types w.r.t. multiple WAND  $k = \partial_r$  were identified  
JP and RŠ (2015)
- existence of other WANDs (principal null directions) was discussed in  $D = 4$   
JP and RŠ (2016)

More details can be found in:

- Explicit algebraic classification of Kundt geometries in any dimension  
JP and RŠ, *Class. Quantum Grav.* **30**, 125007 (2013)
- Algebraic structure of Robinson–Trautman and Kundt geometries in arbitrary dimension  
JP and RŠ, *Class. Quantum Grav.* **32**, 015001 (2015)
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Thank you for your attention!