Stellar objects in the quadratic regime

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Introduction

- The use of exact models with an equation of state is relevant in the study of relativistic compact stars.
- In past years there have been diverse attempts to find exact solutions of compact stars with a linear, quadratic and polytropic equation of state. However the exact models with a quadratic and a polytropic equation of state are rare because of the increased of nonlinearity in the system
- It is interesting to not just find new exact solutions with equation of state, but also to link them to observed astronomical objects.

Introduction

- Recently *Mafa Takisa et al. (2014)* have studied some physical features of linear equation of state which are consistent with the observed object such PSRJ1614-2230 $(1.97 \pm 0.08 M_{\odot})$, see (*Demorest et al. 2010)*
- By using a quadratic equation of state, we intend to study the effects quadratic term on compact objects, and particularly for the pulsar PSRJ1614-2230.

The model

The line element for a static spherically symmetric interior matter distribution has the form :

Metric

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

with $\nu = \nu(r)$ and $\lambda = \lambda(r)$ as the potentials.

The energy momentum tensor for an anisotropic charged imperfect fluid sphere is of the form

Energy momentum tensor

$$T^{ab} = \mathrm{diag}(-\rho - \tfrac{1}{2}E^2, p_r - \tfrac{1}{2}E^2, p_t + \tfrac{1}{2}E^2, p_t + \tfrac{1}{2}E^2), \quad (2)$$

The Einstein's field equations

$$G^{ab} = \frac{8\pi G}{c^4}T^{ab} \quad (3)$$

The model

By using geometrical units ($G = c = 1$) and using the expressions (1) and (2), the Einstein's field equations can written as

Einstein's field equations

$$8\pi\rho + \frac{1}{2}E^2 = \frac{1}{r^2}\left[r(1 - e^{-2\lambda})\right]', \tag{4}$$

$$8\pi p_r - \frac{1}{2}E^2 = -\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda}, \tag{5}$$

$$8\pi p_t + \frac{1}{2}E^2 = e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r}\lambda' - \frac{\lambda'}{r} - \nu\right), \tag{6}$$

$$\sigma = \frac{1}{4\pi r^2}e^{-\lambda}(r^2 E)', \tag{7}$$

where $\sigma = \sigma(r)$ is named the proper charge density and primes indicate differentiation with respect to r.

In the presence of charge the gravitational mass is defined by

Gravitational mass

$$M(r) = 4\pi \int_0^r \left(\rho(\omega) + \frac{E^2}{8\pi}\right)\omega^2 d\omega. \tag{8}$$

Physical requirements

The matter variables should satisfy the following conditions

Requirements

(a) The radial pressure $p_r(\varepsilon) = 0$ at the boundary ε.
(b) The tangential pressure $p_t > 0$ and $\rho > 0$ within the star.
(c) The speed of sound should $v^2 = \frac{dp_r}{d\rho} \leq 1$.
(d) At the centre $p_r(0) = p_t(0)$.
(e) The anisotropy $\Delta(0) = p_t(0) - p_r(0) = 0$.
(f) The energy condition $\rho - p_r - 2p_t > 0$.
(g) At the boundary $r = \varepsilon$ we require
$$e^{2\nu(\varepsilon)} = 1 - \frac{2\mathcal{M}}{\varepsilon} + \frac{Q^2}{\varepsilon^2}, \quad e^{-2\lambda(\varepsilon)} = 1 - \frac{2\mathcal{M}}{\varepsilon} + \frac{Q^2}{\varepsilon^2}, \quad M(\varepsilon) = \mathcal{M}.$$
(h) The matter distribution should satisfy an equation of state $p_r = p_r(\rho)$.

Solution

In order to solve the system (4)-(7), we make these choices

Choices

- The potential $e^{2\lambda} = \frac{1+ar^2}{1+br^2}$
- The electric field in the form $E^2 = \frac{sa^2r^4}{(1+ar^2)^2}$
- The equation of state in the quadratic form $p_r = \gamma\rho^2 + \alpha\rho - \beta$, where , a, b, s, β, α and γ are constants.

We get solution

for the potential $e^{2\nu}$

$e^{2\nu} = D^2 \left(1 + ar^2\right)^{2m} (1 + br^2)^{2n} \exp[2F(r)]$,
with *F(r)*, m and n given by

Solution

$F(r)$, m and n

- $F(r) = \gamma\left[\frac{2(2b-a)(1+ar^2)+(b-a)}{2(1+ar^2)^2}\right] - s\gamma\left[\frac{(a-b)^2(ar^2+2)}{4(a-b)(1+ar^2)^2} - \frac{a(2a+s)(1+ar^2)}{4(a-b)(1+ar^2)^2}\right] + \frac{ar^2}{16b}[s^2\gamma - 2s(1+\alpha) - 4\beta] - s\gamma\left[\frac{s(a-b)+3sb(1+ar^2)}{32(a-b)^2(1+ar^2)^2}\right]$
- $m = -\frac{s(1+\alpha)}{8(b-a)} + \frac{\alpha}{2} + \gamma[2(a-b)]^2\left[\frac{b^2}{(b-a)^3} + \frac{b}{(b-a)^2} + \frac{1}{4}\right] + \frac{s\gamma}{8(a-b)^3}\left[(a-b)[2s(a-b)+a+b] - 6ab^2 + 2b^3(2a-1)\right]$,
- $n = \frac{(1+\alpha)(a-b)}{4b} - \frac{2\alpha(a-b)}{4(b-a)} + \frac{\beta(a-b)}{4b^2} + \gamma[2(a-b)]^2\left[\frac{b^2}{(b-a)^3} + \frac{b}{(b-a)^2} + \frac{1}{4}\right] + \frac{s\gamma}{16b^2(b-a)^3}\left[a^4(s+4b) + 2b(6a^2b^2 - 2a^3b)\right] + \frac{sa^2(1+\alpha)}{8b^2(b-a)}$.

Numerical application

We make the following transformations :

$$\tilde{a} = a\mathcal{L}^2, \quad \tilde{b} = b\mathcal{L}^2, \quad \tilde{s} = s\mathcal{L}^2,$$

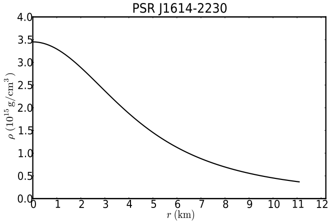
where $\mathcal{L}$ has the dimension of *length*. We choose γ such that the causality condition $v^2 = \frac{dp_r}{d\rho} \leq 1$ is satisfied and take the parameter values : $\tilde{a} = 53.34$, $\mathcal{L} = 43.245\ \mathrm{km}$, $\alpha = 0.33$, $\beta = 1.7 \times 10^{14} \mathrm{g\ cm}^{-3}$ and $\tilde{s} = 0.0$ for uncharged bodies. The parameter α has the fixed value but the parameter γ is allowed to vary. We obtain different masses, radii and central densities for different parameter values of γ. The results are given in Table 1.

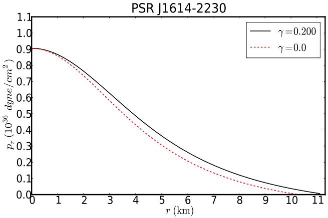
The particular interest are the underlined values $\gamma = 0.140$, $\alpha = 0.33$, $R = 10.30$, $\frac{M}{R} = 0.191$ and $\rho_c = 3.45 \times 10^{15}\mathrm{g\ cm}^{-3}$ which give the corresponding mass of the PSR J1614-2230.

TABLE: 1. Variation of mass, radius and central density in term of γ in the absence of charge. The parameter γ is variable and α is fixed.

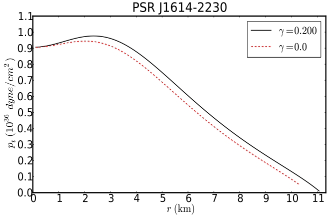
γ	$\tilde{b}$	$\tilde{s}$	α	M	M/R	R(km)	$\rho_c(\times 10^{15}$ gcm$^{-3})$	
0.100	46.44	0.0	0.33	2.55	0.230	11.07	4.0	
0.126	44.60	0.0	0.33	2.37	0.218	10.85	3.84	
0.132	42.50	0.0	0.33	2.18	0.206	10.60	3.66	
<u>0.140</u>	<u>40.01</u>	0.0	<u>0.33</u>	<u>1.97</u>	<u>0.191</u>	<u>10.30</u>	<u>3.45</u>	PSR J1614-2230
0.148	37.73	0.0	0.33	1.77	0.177	9.99	3.25	Vela X-1
0.154	36.47	0.0	0.33	1.667	0.170	9.82	3.14	PSR J1903+327
0.163	34.30	0.0	0.33	1.49	0.157	9.51	2.95	Cen X-3
0.177	31.62	0.0	0.33	1.29	0.141	9.13	2.72	SMC X-1
0.189	29.70	0.0	0.33	1.14	0.129	8.83	2.55	
0.196	28.61	0.0	0.33	1.07	0.124	8.65	2.46	
0.200	24.92	0.0	0.33	0.89	0.111	8.04	2.14	

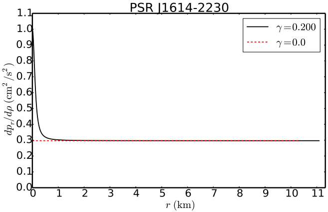
To illustrate the qualitative effect of the quadratic term of the equation of state with $\gamma \neq 0$ in the interior of PSR J1614-2230, we have plotted the energy density ρ, radial pressure p_r, tangential pressure p_t, the measure of anisotropy Δ, speed of sound $v^2 = \frac{dp_r}{d\rho}$, and the quantity $\rho - p_r - 2p_t$ in these Figures respectively for $E = 0$.

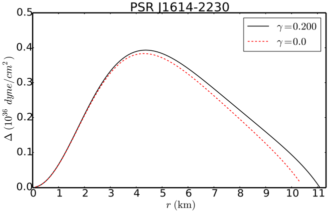




The pulsar J1614-2230







The pulsar PSR J1614-2230

We also investigate the quantitative effect of γ for the PSR J1614-2230. We compute the quantities M, R $\frac{M}{R}$ and ρ_c by allowing the parameters γ and α to be variable. The relevant values are contained in Tables 2 and 3. The underlined values in these tables represent the corresponding values that we expect for the object PSR J1614-2230 when $\gamma = 0$.

The pulsar PSR J1614-2230

For $E = 0$ and $\gamma = 0.200$, we point out a small of increase for the radius and the mass of 4% and 4.5% respectively.

TABLE: 2. Different masses and radii for PSR J1614-2230 for the uncharged case. The parameters γ and α are variable.

γ	$\tilde{b}$	$\tilde{s}$	α	M	M/R	R(km)	$\rho_c(\times 10^{15}$ gcm$^{-3})$
0.0	40.01	0.0	0.99	1.97	0.191	10.30	3.45
0.140	40.01	0.0	0.33	1.97	0.191	10.30	3.45
0.158	40.01	0.0	0.24	2.02	0.192	10.50	3.45
0.163	40.01	0.0	0.21	2.06	0.192	10.70	3.45
0.177	40.01	0.0	0.15	2.10	0.193	10.90	3.45
0.196	40.01	0.0	0.06	2.13	0.193	11.06	3.45
0.200	40.01	0.0	0.04	2.14	0.193	11.09	3.45

The pulsar PSR J1614-2230

For $E \neq 0$ and $\gamma = 0.200$ it is clear that the quadratic term γ leads to an increase of 10% and 11% in the radius and the mass of a stellar object.

TABLE: 3. Different masses and radii for PSR J1614-2230 for the charged case. The parameters γ and α are variable.

γ	$\tilde{b}$	$\tilde{s}$	α	M	M/R	R(km)	$\rho_c(\times 10^{15}\ \mathrm{gcm}^{-3})$
<u>0.0</u>	<u>40.01</u>	14.5	<u>0.99</u>	<u>2.13</u>	<u>0.231</u>	<u>9.21</u>	<u>3.45</u>
0.140	40.01	14.5	0.33	2.13	0.231	9.21	3.45
0.158	40.01	14.5	0.24	2.32	0.232	10.05	3.45
0.163	40.01	14.5	0.21	2.34	0.232	10.10	3.45
0.177	40.01	14.5	0.15	2.35	0.232	10.15	3.45
0.196	40.01	14.5	0.06	2.36	0.232	10.18	3.45
0.200	40.01	14.5	0.04	2.36	0.232	10.19	3.45

Conclusion

- We observed that for both cases $E = 0$ and $E \neq 0$, the quadratic term γ has the effect of increasing the compactification factor $\frac{M}{R}$ slightly.
- The compactification factor is in the range of $\frac{M}{R} \sim \frac{1}{10}$ to $\frac{1}{4}$; which corresponds to neutron stars and ultra-compact stars (*Tikekar and Jotania 2007)*
- We have shown the relevance of the quadratic equation of state to relativistic objects, in particular to the PSR J1614-2230.
- The solution with quadratic equation state found may be used to study physical features of a superdense object with both uncharged and charged matter.

Thank you for your attention