



MAX-PLANCK-GESELLSCHAFT



Precision Gravity Tests with Radio Pulsars *

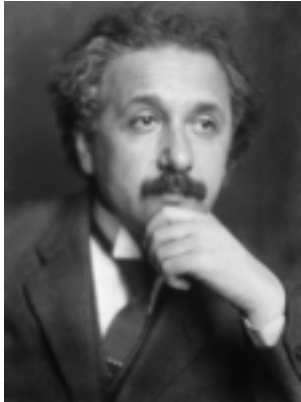
Norbert Wex

Max Planck Institute for Radio Astronomy
Bonn, Germany

GR21, New York, July 11th, 2016

* Partly modified compared to slides in the talk. Some unpublished material has been removed.

November 1915 - the completion of general relativity



Nov. 4th, 1915

Zur allgemeinen Relativitätstheorie.

VON A. EINSTEIN.

Nov. 11th, 1915

Zur allgemeinen Relativitätstheorie (Nachtrag).

VON A. EINSTEIN.

Nov. 18th, 1915

Erklärung der Perihelbewegung des Merkur aus
der allgemeinen Relativitätstheorie.

Nov. 25th, 1915

Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

$$G_{im} = -\kappa \left(T_{im} - \frac{1}{2} g_{im} T \right),$$

Damit ist endlich die allgemeine Relativitätstheorie als logisches Gebäude abgeschlossen. Das Relativitätspostulat in seiner allgemeinsten Fassung, welches die Raumzeitkoordinaten zu physikalisch bedeutungslosen Parametern macht, führt mit zwingender Notwendigkeit zu einer ganz bestimmten Theorie der Gravitation, welche die Perihelbewegung des Merkur erklärt. Dagegen vermag das allgemeine Re-

Gesamtsitzung vom 18. November 1915

Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie.

VON A. EINSTEIN.

Anomalous precession of the Mercury orbit

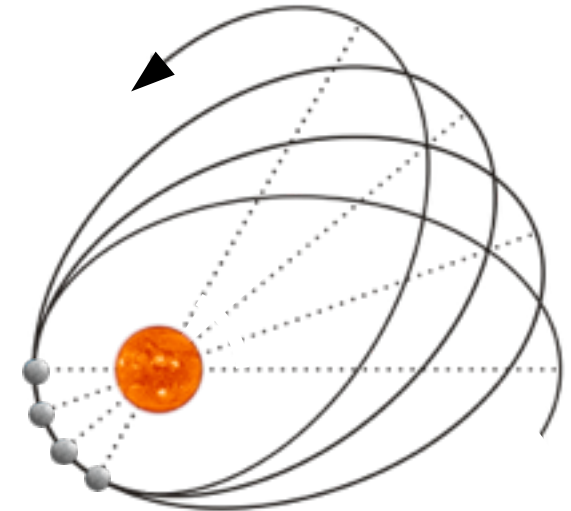
$$\varepsilon = 24 \pi^3 \frac{a^2}{T^2 c^2 (1 - e^2)} \quad (14)$$

Die Rechnung liefert für den Planeten Merkur ein Vorschreiten des Perihels um 43'' in hundert Jahren, während die Astronomen $45'' \pm 5''$ als unerklärten Rest zwischen Beobachtungen und NEWTONScher Theorie angeben. Dies bedeutet volle Übereinstimmung.

Albert Einstein to Arnold Sommerfeld (Dec 9th, 1915):

Wie kommt uns da die pedantische Genauigkeit der Astronomie zu Hilfe, über die ich mich im Stillen früher of lustig machte!"

[*"How helpful to us here is astronomy's pedantic accuracy, which I often used to ridicule secretly!"*]



Gesamtsitzung vom 18. November 1915

Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie.

VON A. EINSTEIN.

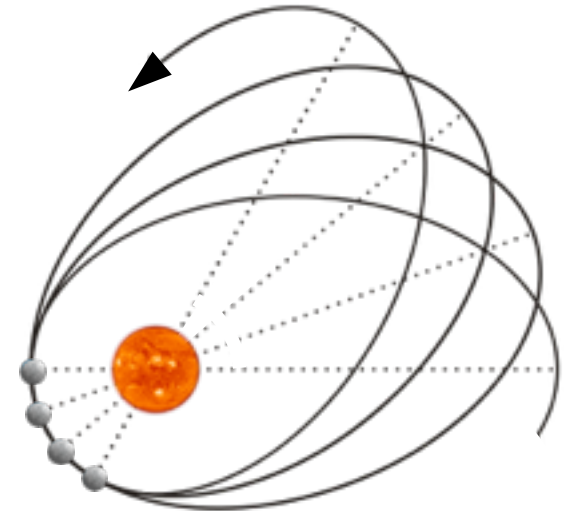
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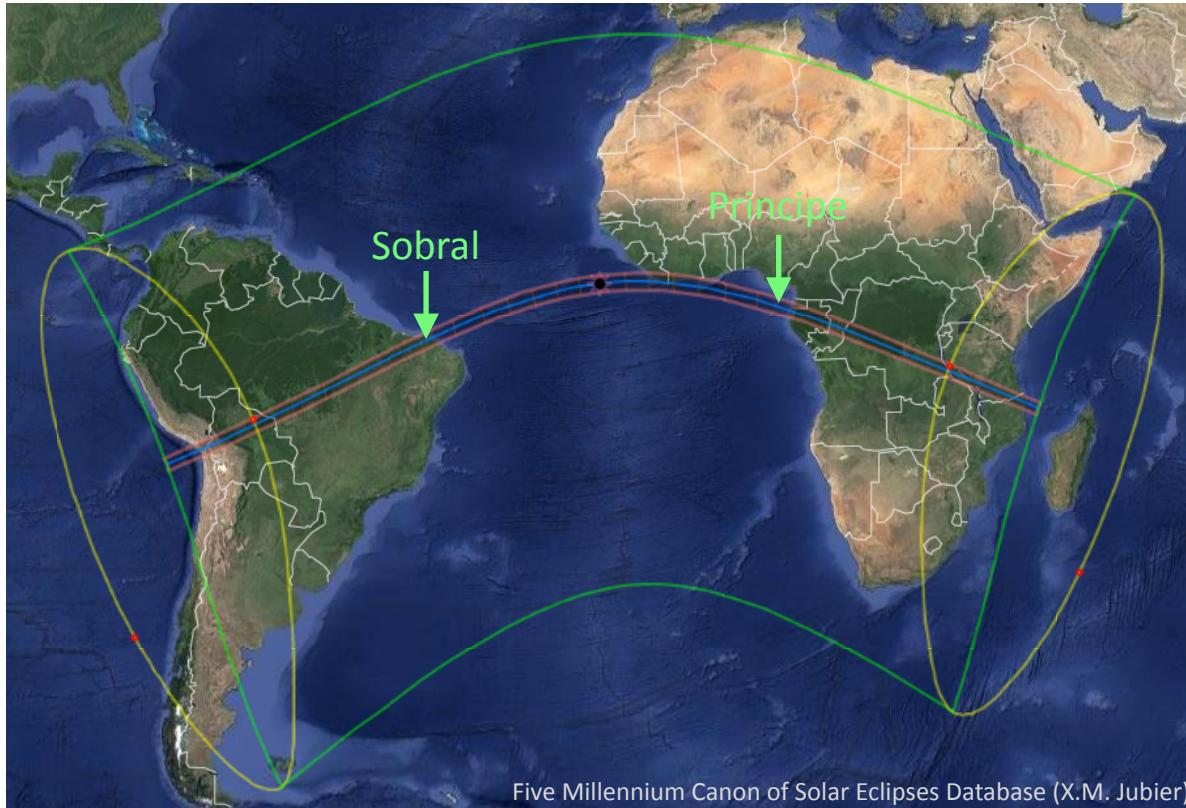
Die Rechnung liefert für den Planeten Merkur ein Vorschreiten des Perihels um 43'' in hundert Jahren, während die Astronomen $45'' \pm 5''$ als unerklärten Rest zwischen Beobachtungen und NEWTONScher Theorie angeben. Dies bedeutet volle Übereinstimmung.

Deflection of light by the Sun, gravitational redshift

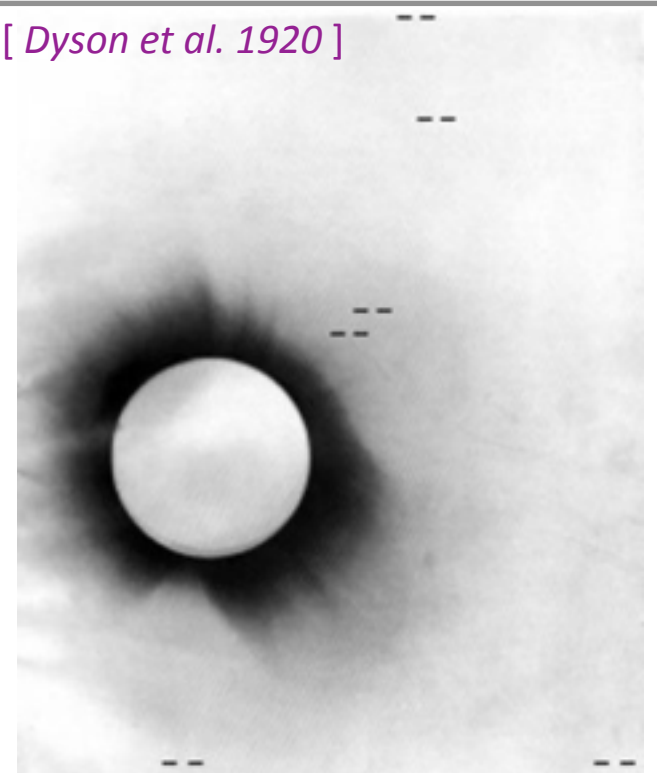
ergeben hatten. Ein an der Oberfläche der Sonne vorbeigehender Lichtstrahl soll eine Ablenkung von $1.7''$ (statt $0.85''$) erleiden. Hingegen bleibt das Resultat betreffend die Verschiebung der Spektrallinien durch das Gravitationspotential, welches durch Herrn FREUNDLICH an den Fixsternen der Größenordnung nach bestätigt wurde, ungeändert bestehen, da dieses nur von g_{44} abhängt.



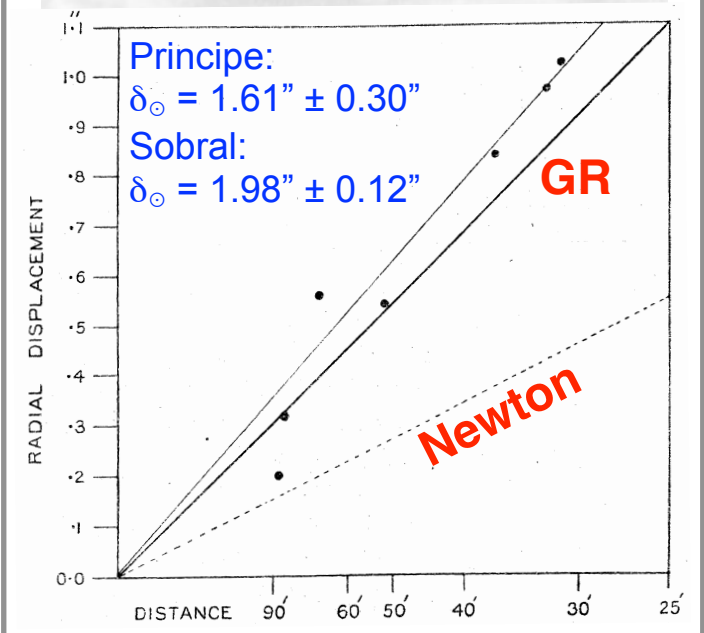
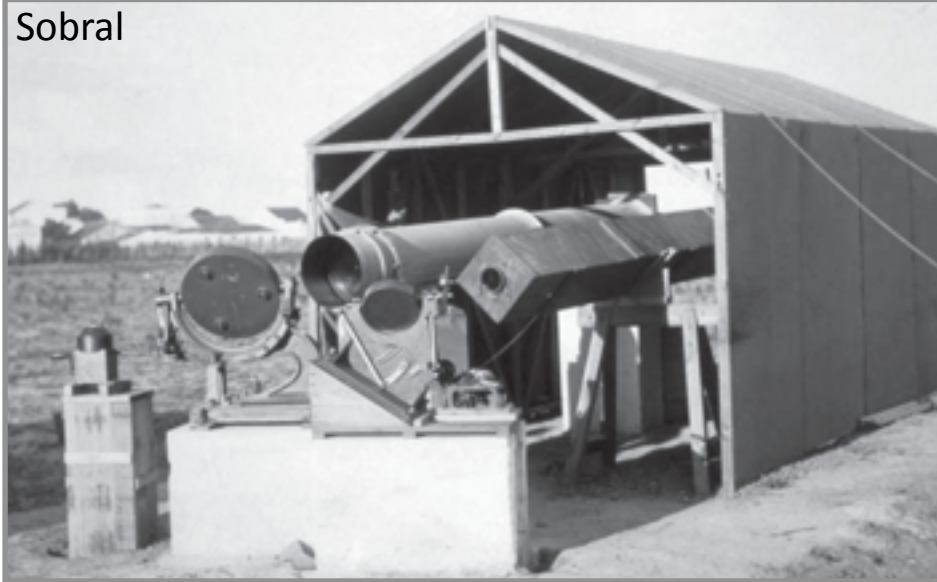
The first light deflection experiment - May 29th, 1919



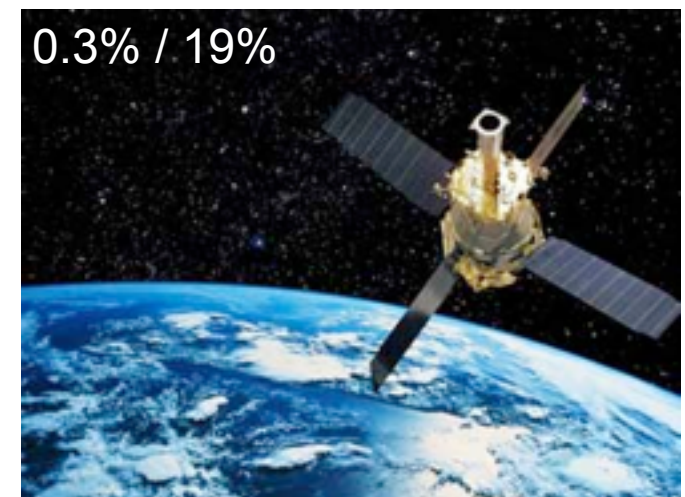
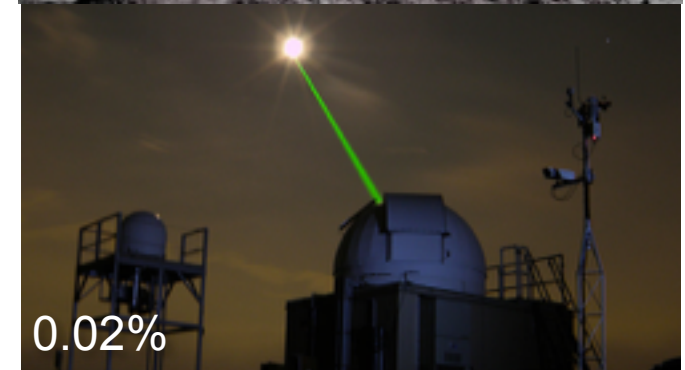
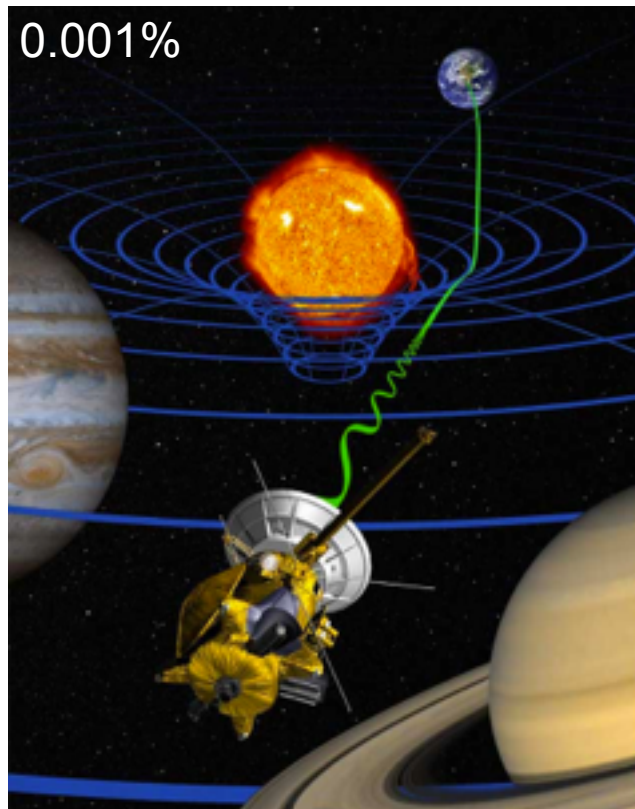
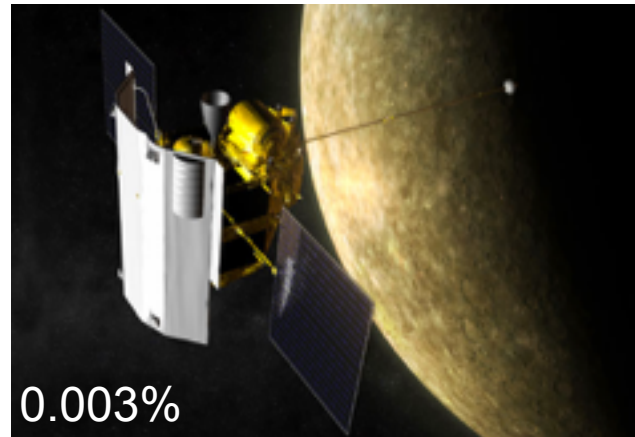
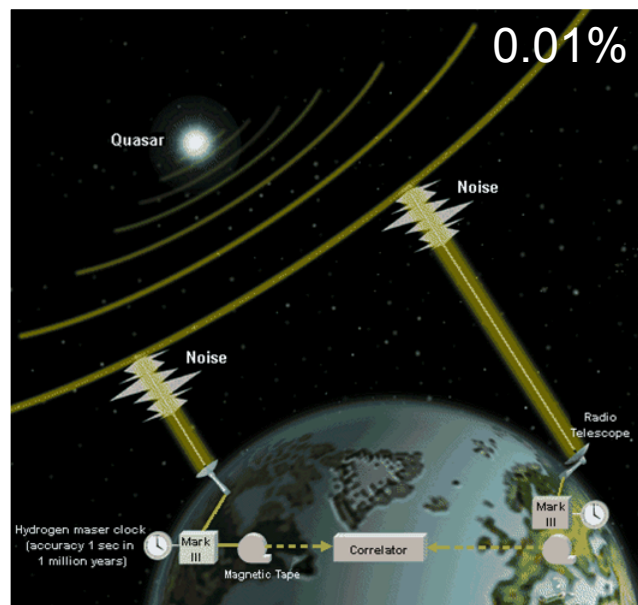
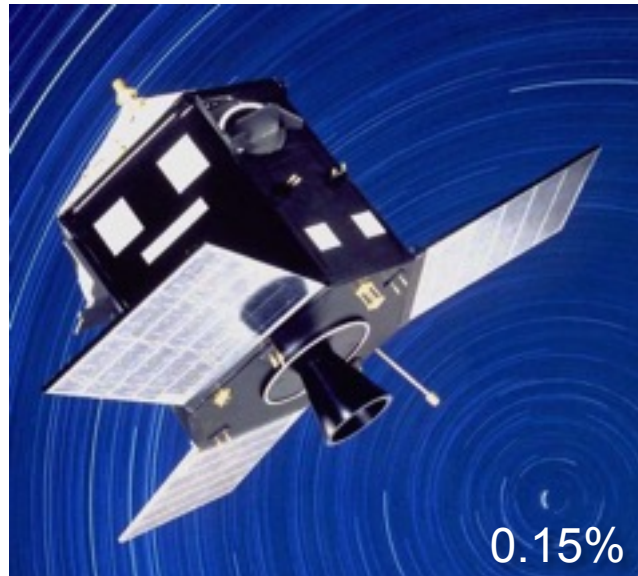
[Dyson et al. 1920]



Sobral



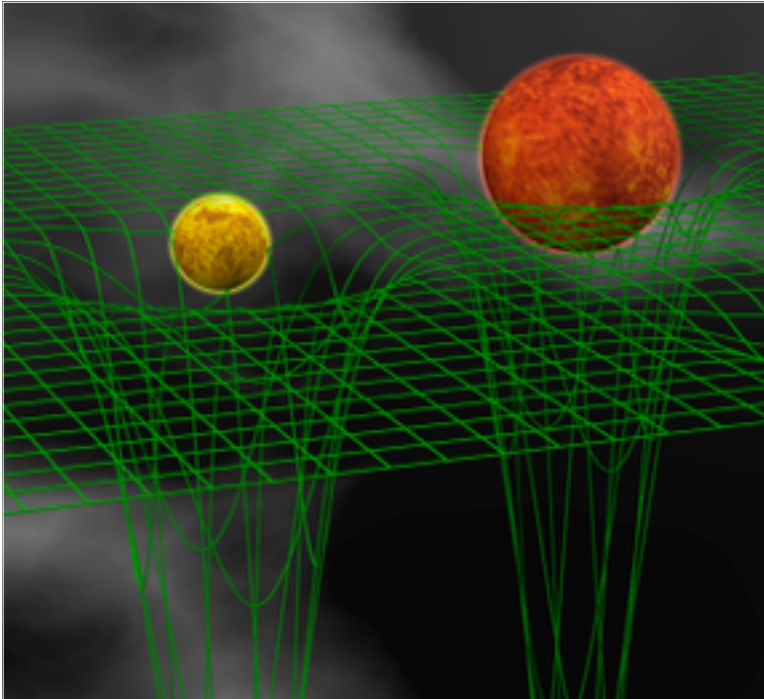
Some modern Solar system experiments



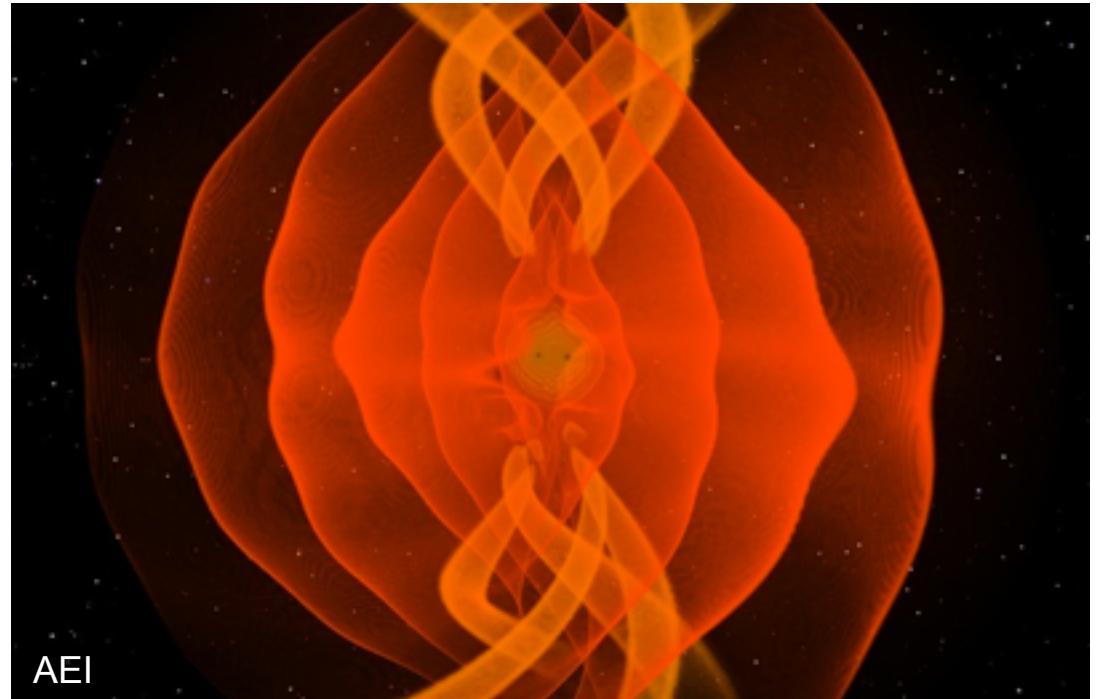
Solar system is slow-motion weak-field regime: $(v/c)^2 \sim |\Phi|/c^2 \lesssim 10^{-6}$

Two questions about gravity beyond the Solar system

Do strongly self-gravitating bodies move as predicted by GR?



Do gravitational waves exist?



Gravity regimes relevant for this talk

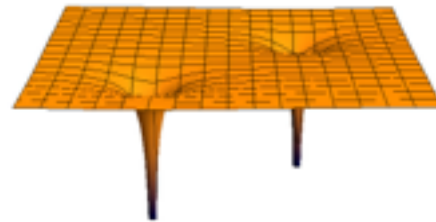
- (1) Quasi-stationary
weak-field
regime



Solar system
experiments



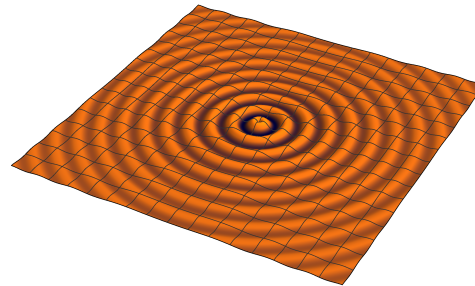
- (2) Quasi-stationary
strong-field
regime



Binary pulsar experiments



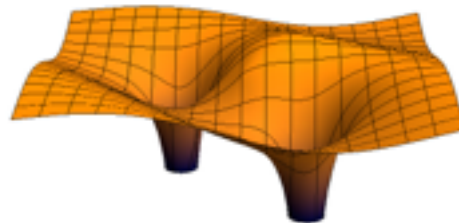
- (3) Radiative
regime



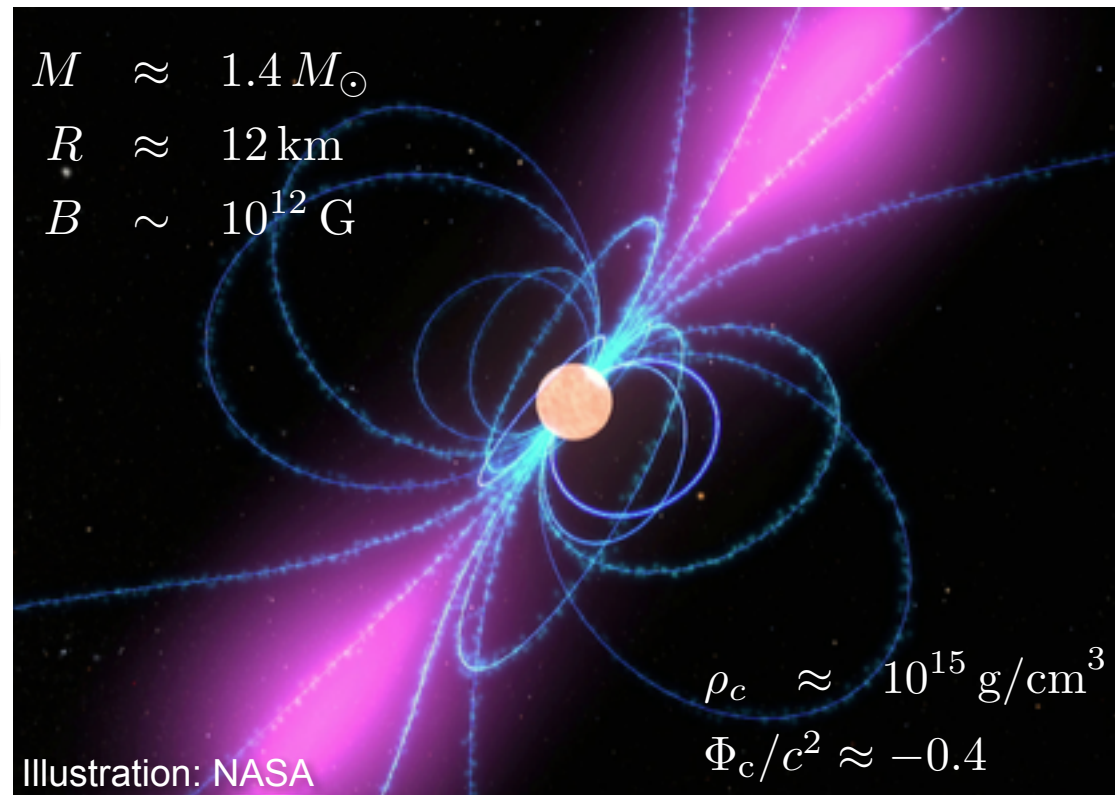
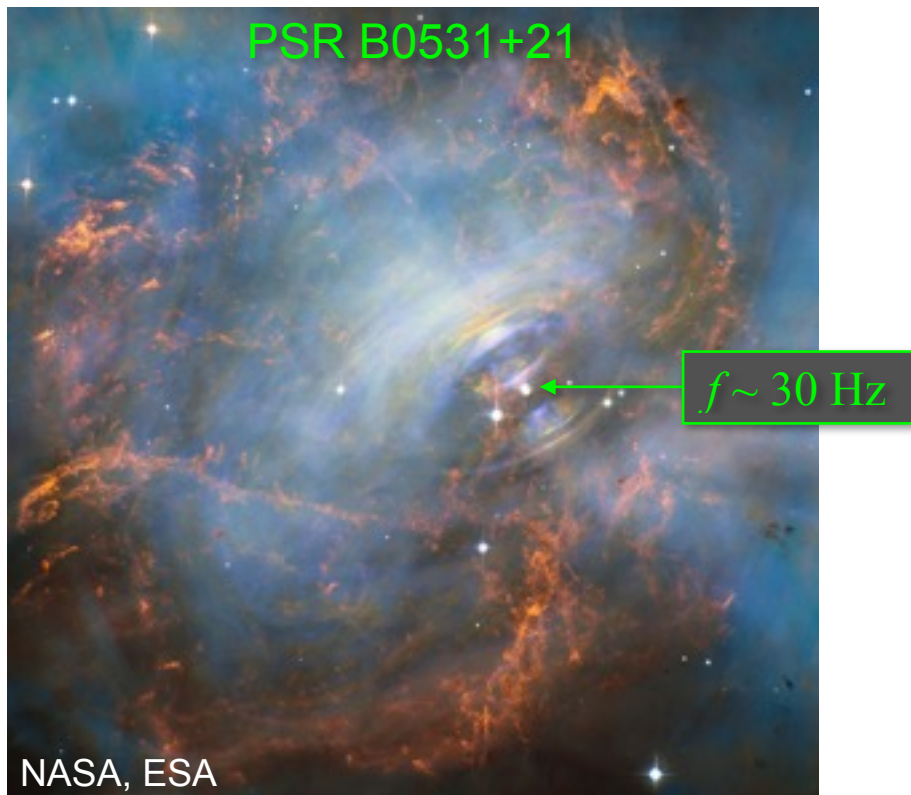
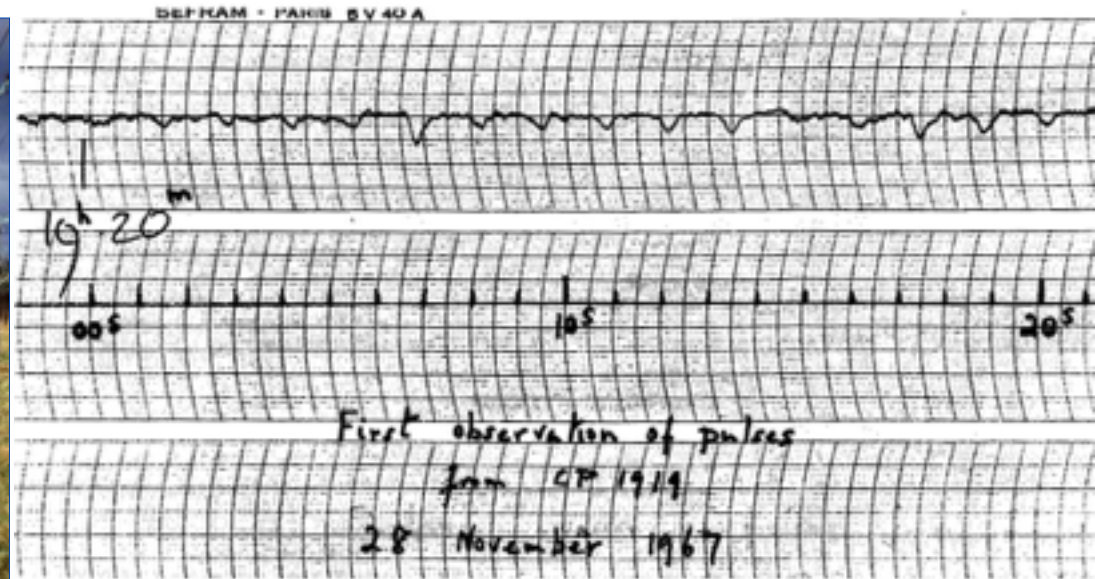
GW astronomy



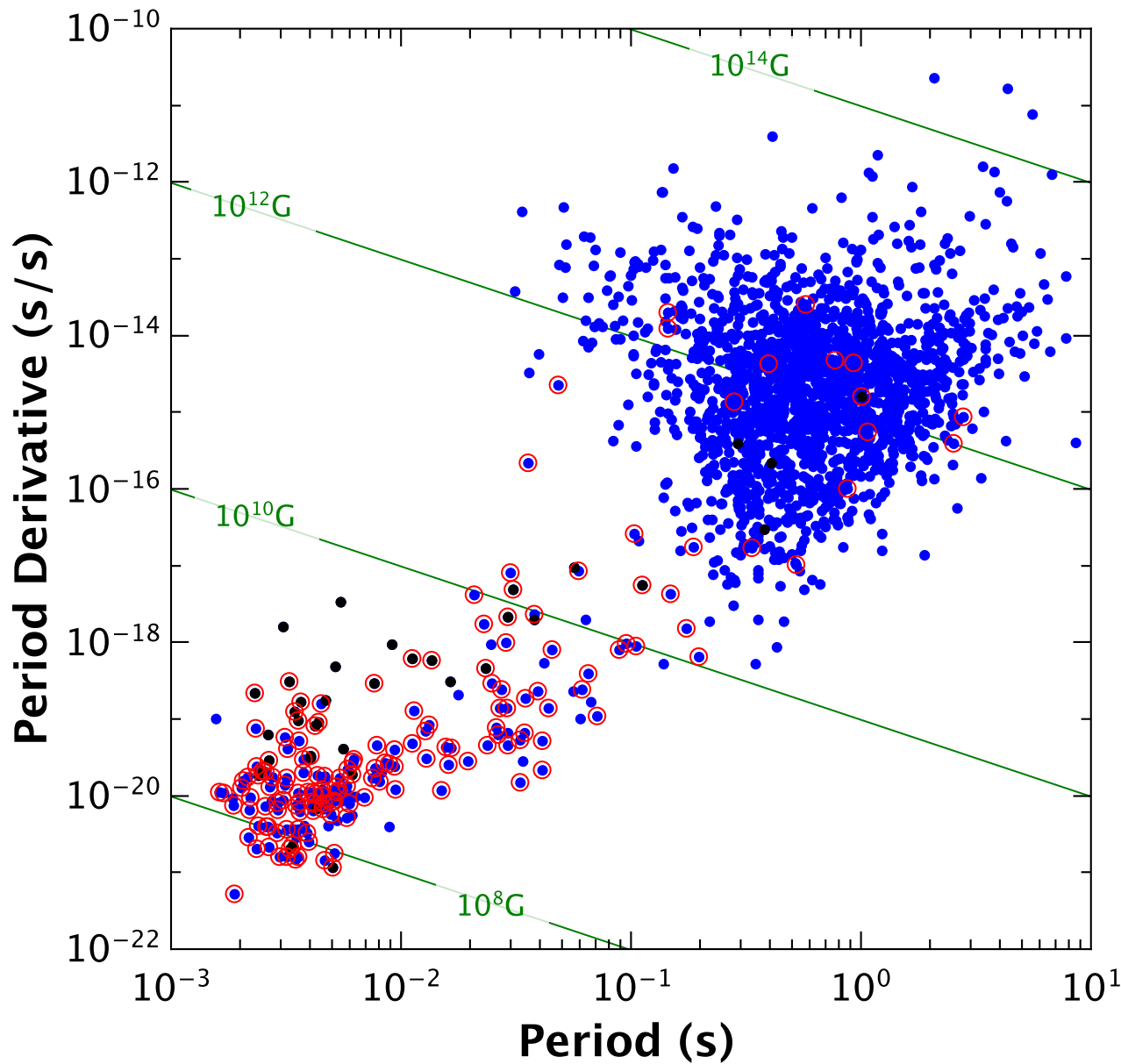
- (4) Highly dynamical
strong-field
regime



The discovery of pulsars - 1967



The radio pulsar population



~ 2500 radio pulsars

1.4 ms (PSR J1748-2446ad)

8.5 s (PSR J2144-3933)

~ 10% in binary systems

Orbital period range

95 min (PSR J0024-7204R)

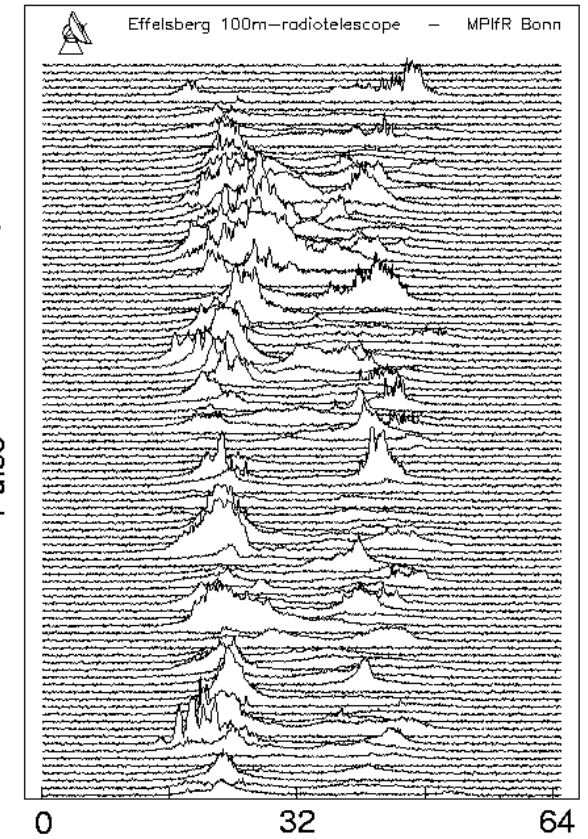
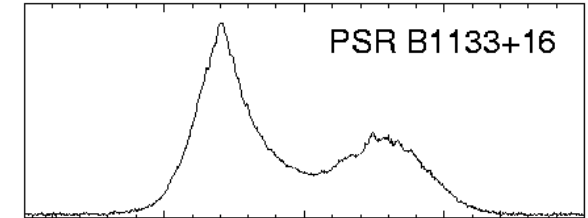
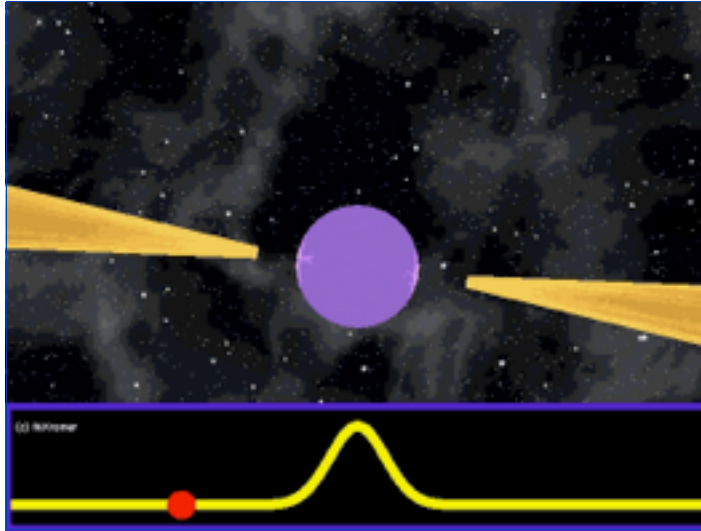
>200 yr (PSR J1024-0719)

Companions

ordinary stars,
white dwarfs,
neutron stars,
planets

Still missing: black hole

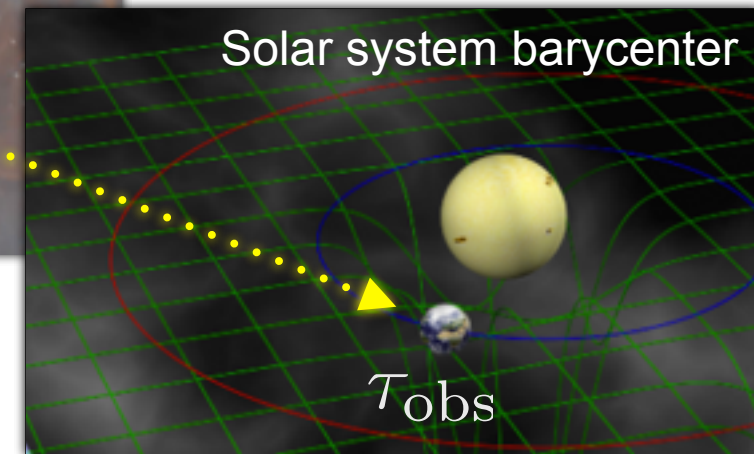
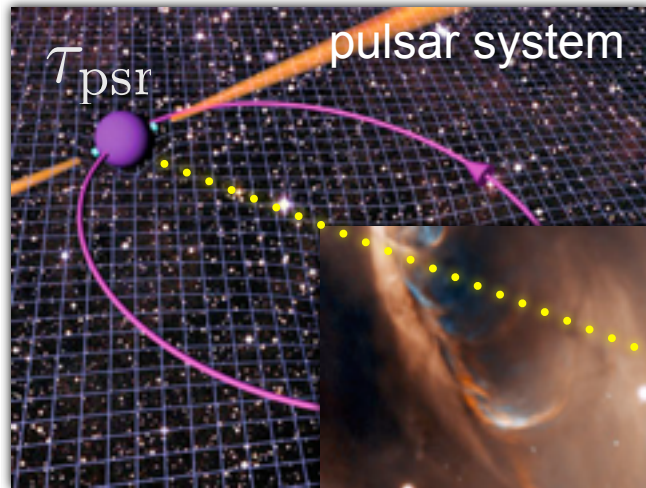
Pulsar timing – time of arrival (TOA)



Timing precision for some millisecond pulsars $< 100 \text{ ns} \rightarrow < 30 \text{ m}$

Pulsar timing - the timing model

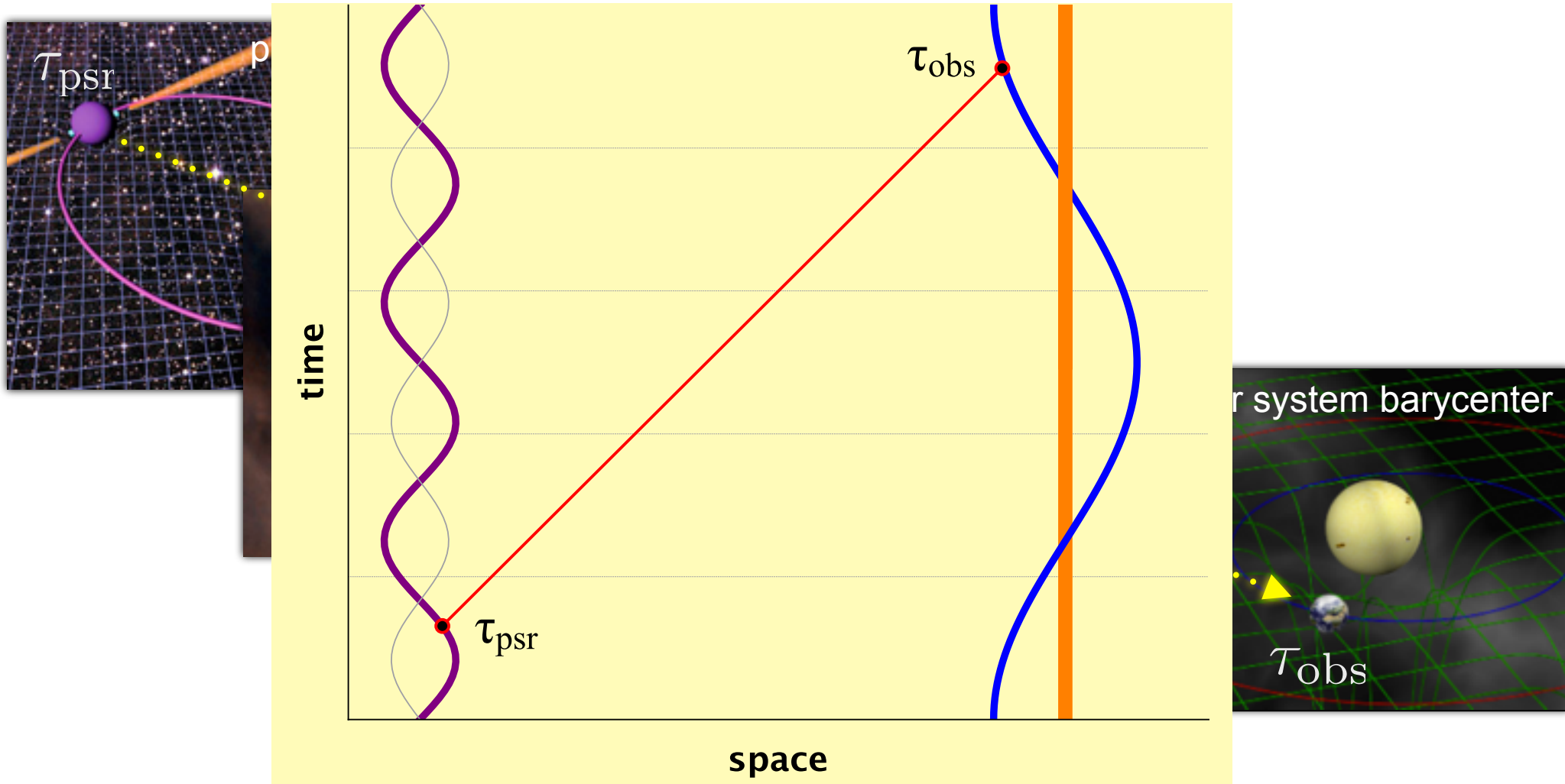
$$\tau_{\text{psr}} \propto T, \quad \phi = \phi_0 + \nu T + \frac{1}{2} \dot{\nu} T^2$$



$$\tau_{\text{psr}} = \tau_{\text{obs}} - D/f^2 + \Delta_{\text{R}\odot}(\lambda, \beta, \mu_\lambda, \mu_\beta, \pi) + \Delta_{\text{E}\odot} + \Delta_{\text{S}\odot}(\lambda, \beta) + \Delta_{\text{B}}(K, PK)$$

Pulsar timing - the timing model

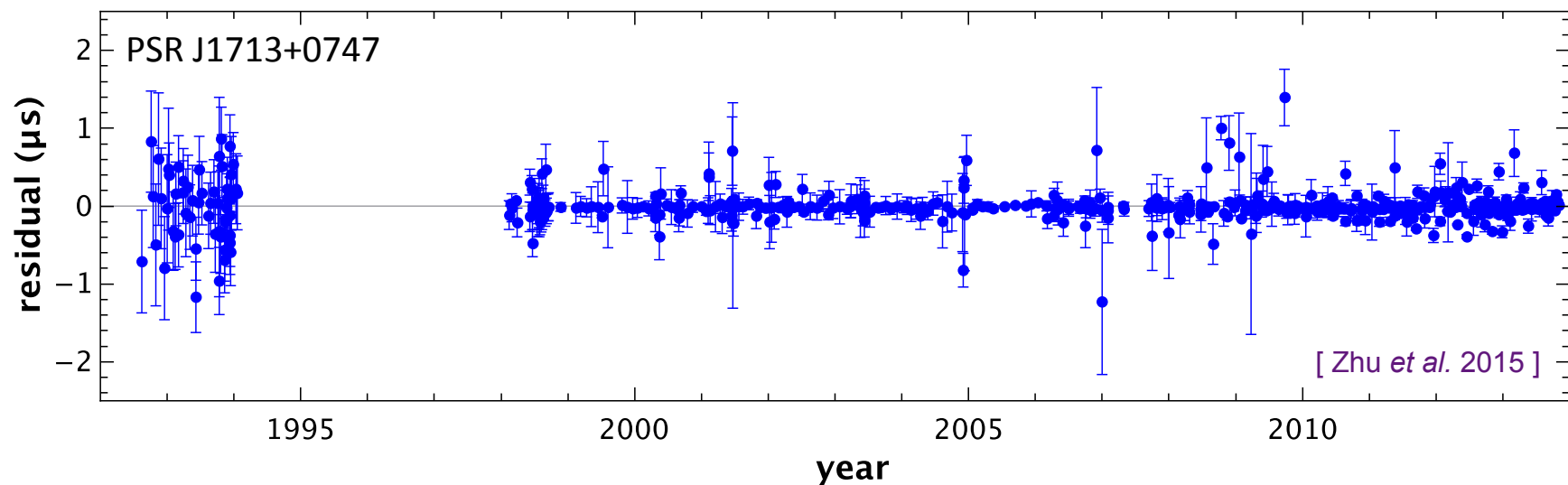
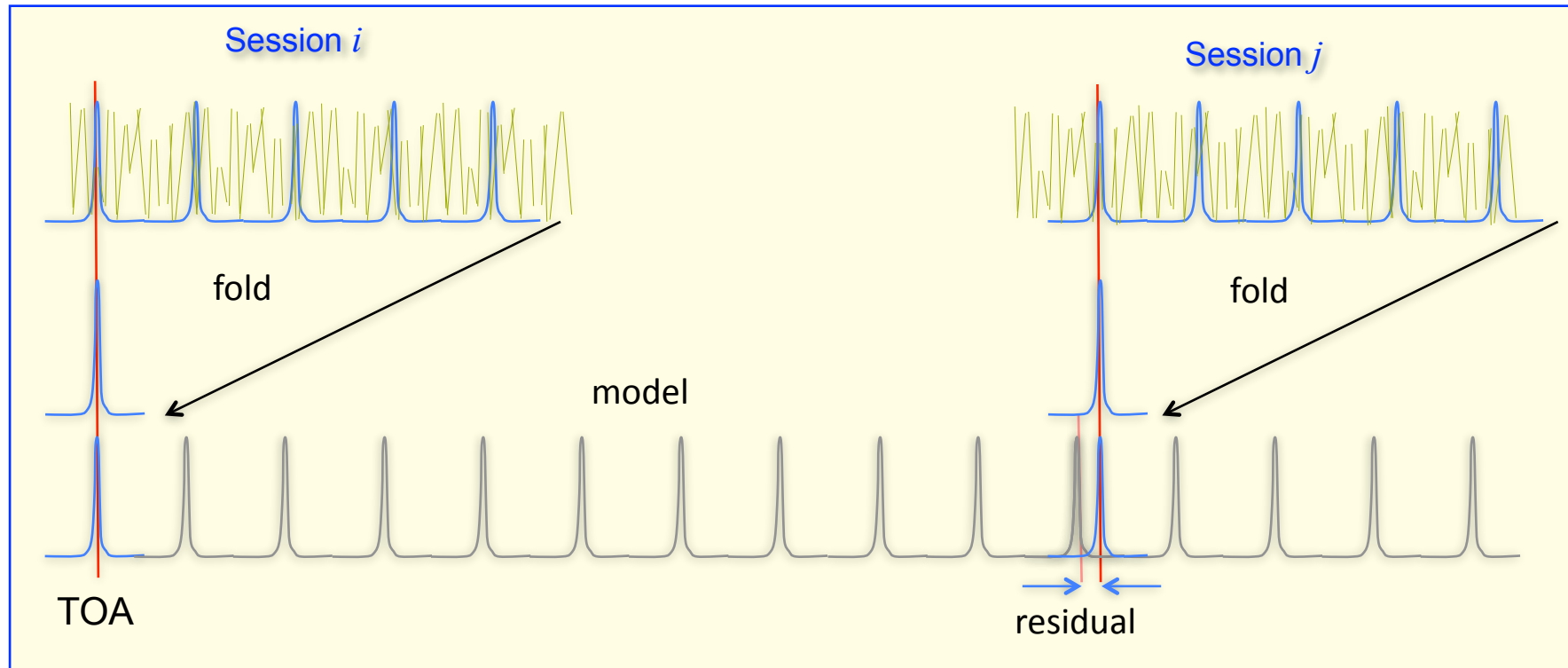
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Pulsar timing - parameter estimation

Phase-connected timing solution:



What do we mean by precision timing? Best of...

Spin parameters:

Period: 2.947108069160717(3) ms (Reardon et al. 2015)

3 atto seconds uncertainty!

Astrometry:

Position in the sky: 0.6 μ as (Reardon et al. 2015)

Proper motion: 140.911(3) mas/yr (Reardon et al. 2015)

Distance: 156.79 ± 0.25 pc (Reardon et al. 2015)

0.1 μ s uncertainty!

Orbital parameters:

Orbital period: 0.102251562472(1) days (Kramer et al. in prep.)

Projected semi-major axis 31,656,123.76(15) km (Freire et al. 2011)

Eccentricity: 0.0000749402(6) (Zhu et al. 2015)

Masses:

Masses of neutron stars: 1.33816(2) / 1.24891(2) M_{\odot} (Kramer et al. in prep.)

Mass of low-mass WD: 0.207(2) M_{\odot} (Reardon et al. 2015)

Mass of millisecond pulsar: 1.667(7) M_{\odot} (Freire et al. 2011)

Main sequence star companion: 1.029(3) M_{\odot} (Freire et al. 2011)

GR effects:

Periastron advance: 4.226598(5) deg/yr (Weisberg et al. 2010)

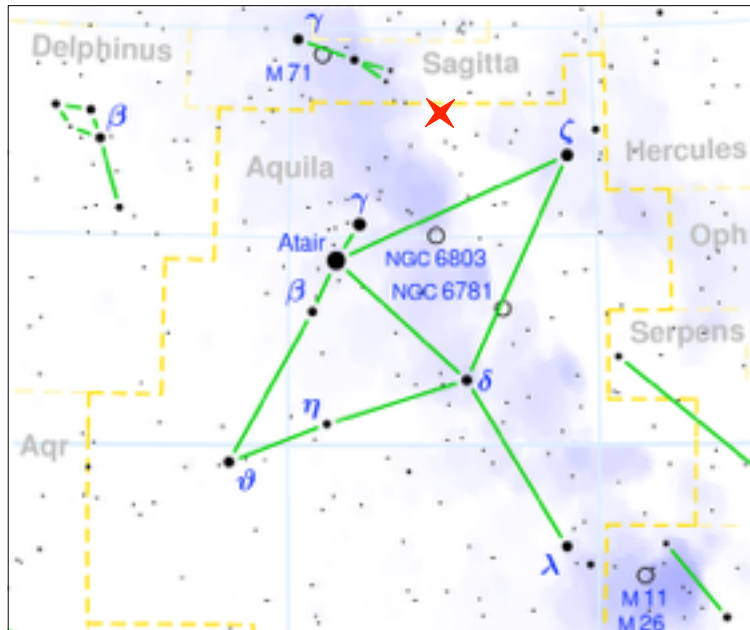
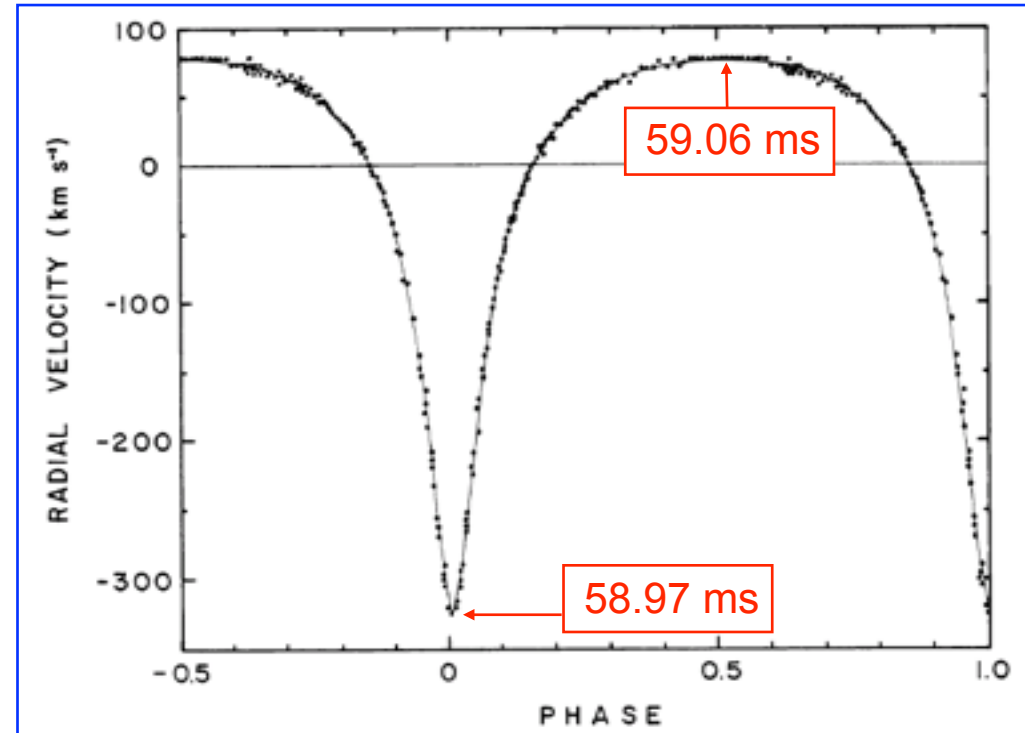
Einstein delay: 4.2992(8) ms (Weisberg et al. 2010)

Orbital GW damping: -39.384(6) μ s/yr (Kramer et al. in prep.)

The first binary pulsar - 1974



PSR B1913+16



Pulse period: 59.0 ms

Orbital period: 7.75 h

Eccentricity: 0.617

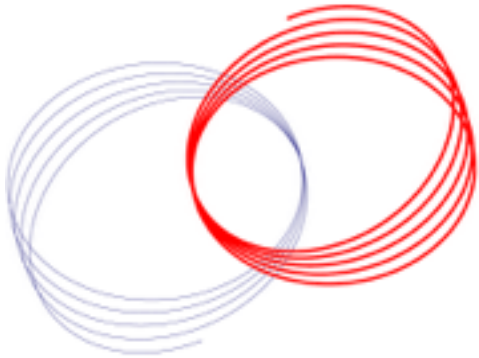
Companion: neutron star

[Hulse & Taylor 1975]

Two post-Keplerian parameters

► Advance of periastron (GR)

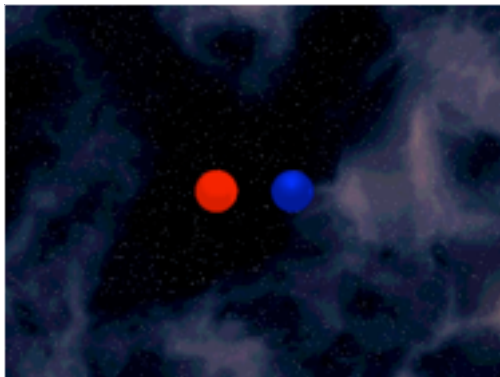
$$\dot{\omega} = 3G^{2/3}c^{-2}(P_b/2\pi)^{-5/3}(1 - e^2)^{-1}(m_1 + m_2)^{2/3}$$



Observed value:
 4.226598 ± 0.000005 deg/yr

► Time dilation (GR)

$$\gamma_E = G^{2/3}c^{-2}(P_b/2\pi)^{1/3}e m_2(m_1 + 2m_2)(m_1 + m_2)^{-4/3}$$



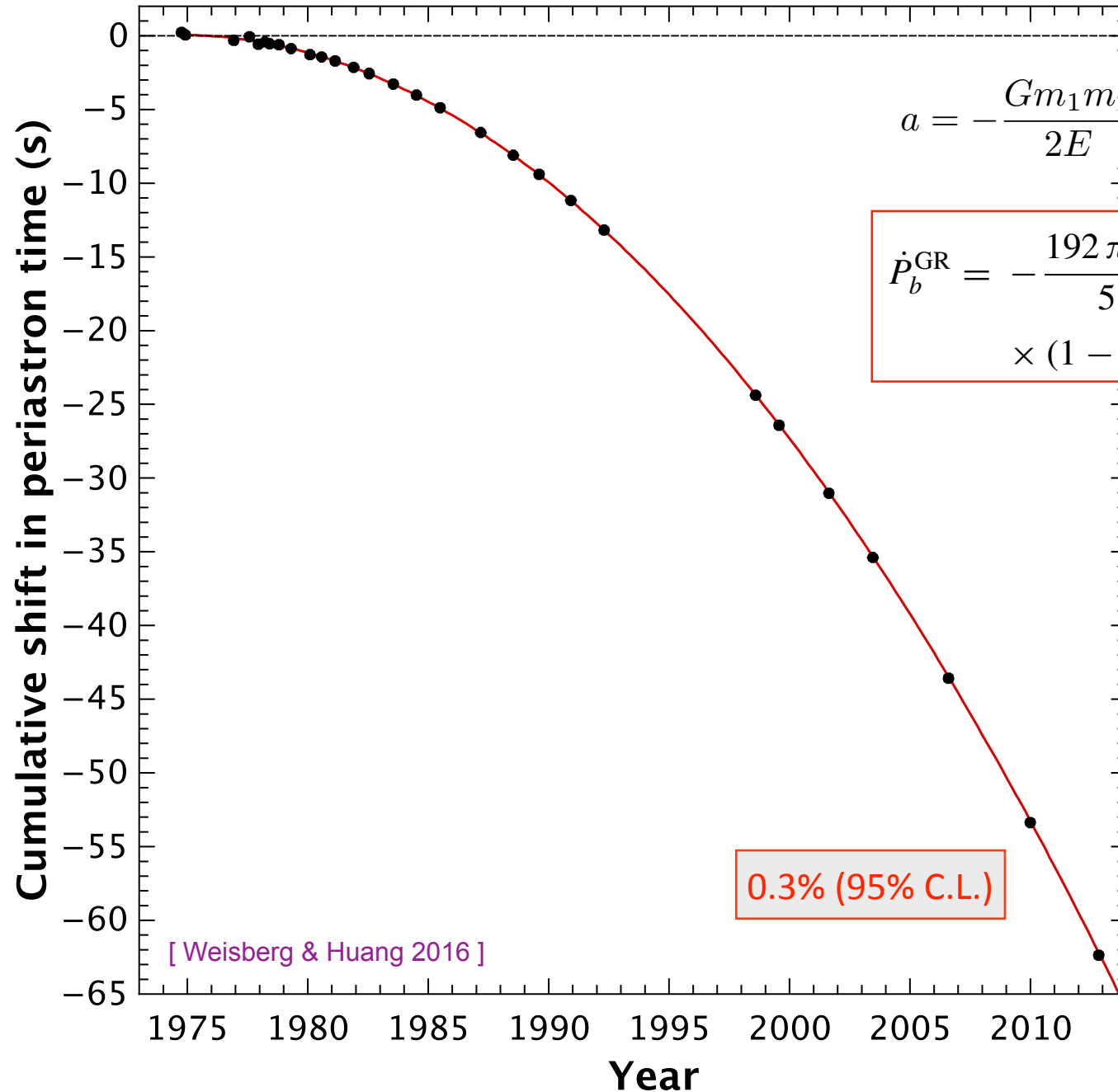
Observed value:
 4.2992 ± 0.0008 ms

► Calculated neutron star masses (GR)

$$m_1 = 1.4398 \pm 0.0002 M_{\odot} \quad m_2 = 1.3886 \pm 0.0002 M_{\odot}$$

[Weisberg *et al.* 2010]

Gravitational wave damping in the Hulse-Taylor pulsar



$$a = -\frac{Gm_1m_2}{2E}, \quad \frac{dE}{dt} = -\frac{G}{5c^5} \left\langle \sum_{ij} \ddot{Q}_{ij} \ddot{Q}_{ij} \right\rangle$$

$$\dot{P}_b^{\text{GR}} = -\frac{192\pi G^{5/3}}{5c^5} \left(\frac{P_b}{2\pi}\right)^{-5/3} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \times (1 - e^2)^{-7/2} m_1 m_2 (m_1 + m_2)^{-1/3}$$

[Peters 1964]

The mass-mass diagram for the Hulse-Taylor pulsar

Parametrized post-Keplerian formalism

[Damour 1988, Damour & Taylor 1992]

For a wide range of gravity theories:

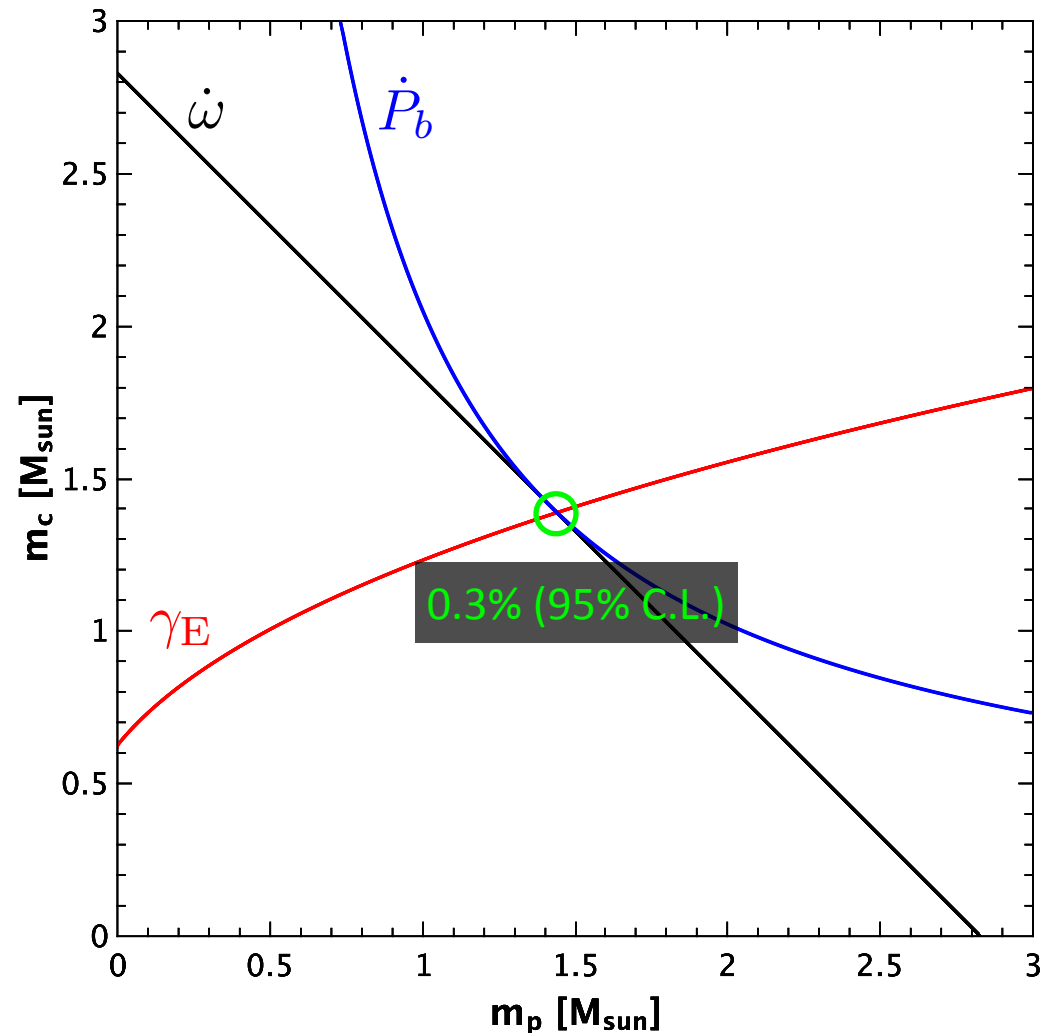
$$p_i^{\text{PK}} = f_i(\mathbf{p}^{\text{K}}; m_A, m_B)$$

K: Keplerian parameters

$$P_b, e, x, \dots$$

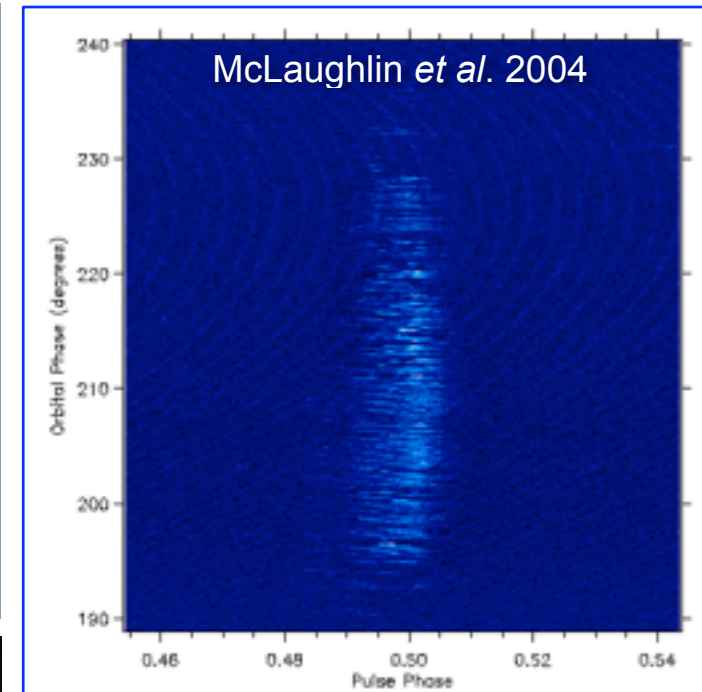
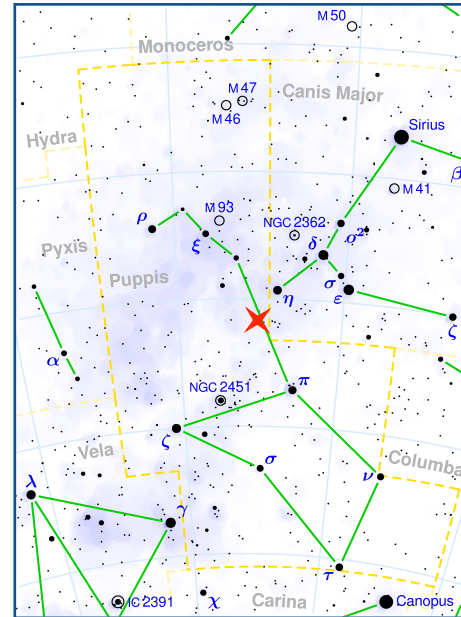
PK: post-Keplerian parameters
(order v^2/c^2 or higher)

$$\dot{\omega}, \gamma_E, \dot{P}_b, \dots$$

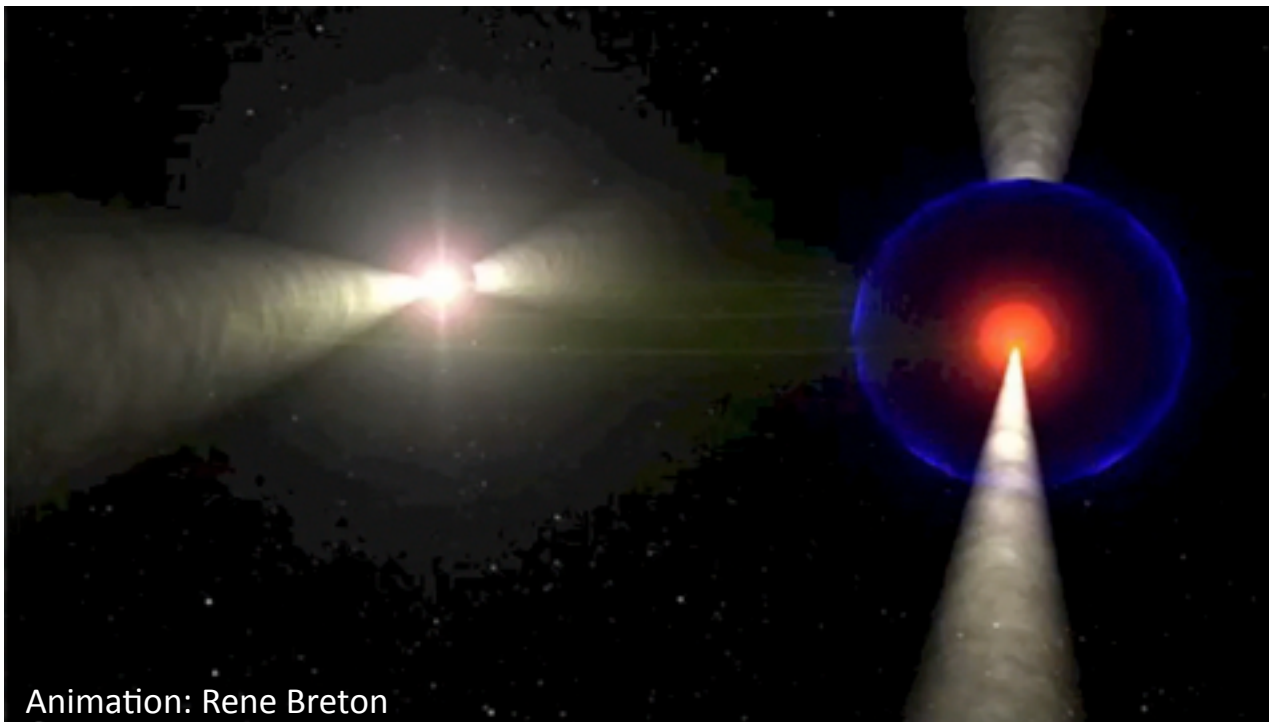


👉 talk by Joel Weisberg

The Double Pulsar PSR J0737-3039A/B



Spin periods:	23 ms / 2.8 s
Orbital period:	2.45 h
Eccentricity:	0.088



Animation: Rene Breton

[Burgay *et al.* 2003, Lyne *et al.* 2004]

Relativistic effects in the Double Pulsar

► Binary parameters from timing

Timing parameter	PSR J0737-3039A	PSR J0737-3039B
Orbital period P_b (day)	0.10225156248(5)	—
Eccentricity e	0.0877775(9)	—
Projected semimajor axis $x = (a/c)\sin i$ (s)	1.415032(1)	1.5161(16)
Longitude of periastron ω ($^\circ$)	87.0331(8)	$87.0331 + 180.0$
Epoch of periastron T_0 (MJD)	53,155.9074280(2)	—
Advance of periastron $\dot{\omega}$ ($^\circ$ /year)	16.89947(68)	[16.96(5)]
Gravitational redshift parameter γ_E (ms)	0.3856(26)	—
Shapiro delay parameter s	0.99974(−39,+16)	—
Shapiro delay parameter r (μ s)	6.21(33)	—
Orbital period derivative \dot{P}_b	$-1.252(17) \times 10^{-12}$	—

[Kramer *et al.* 2006]

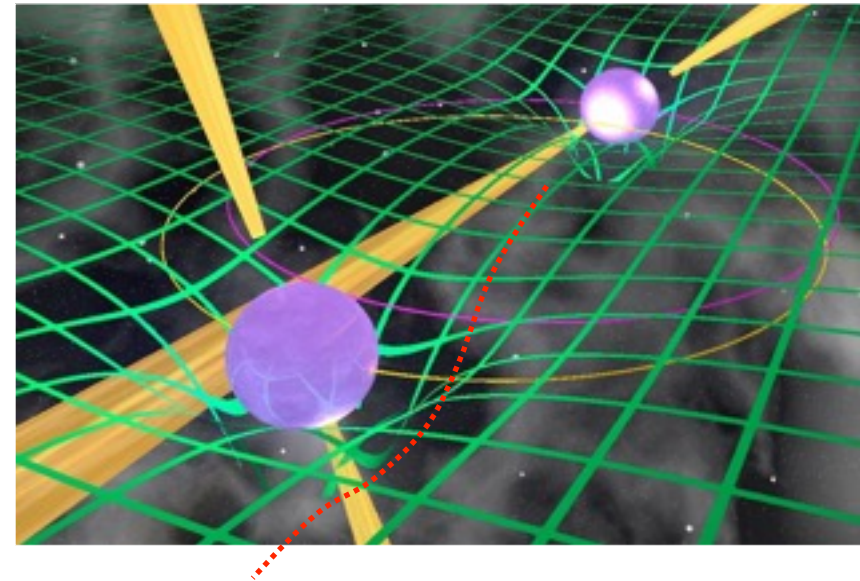
➡ **mass ratio** + 5 post-Keplerian parameters

► Spin precession of B (from eclipses)

$$\Omega_B = 4.77^{+0.66}_{-0.65} \text{ }^\circ/\text{year}$$

[Breton *et al.* 2008]

➡ 6th post-Keplerian parameter

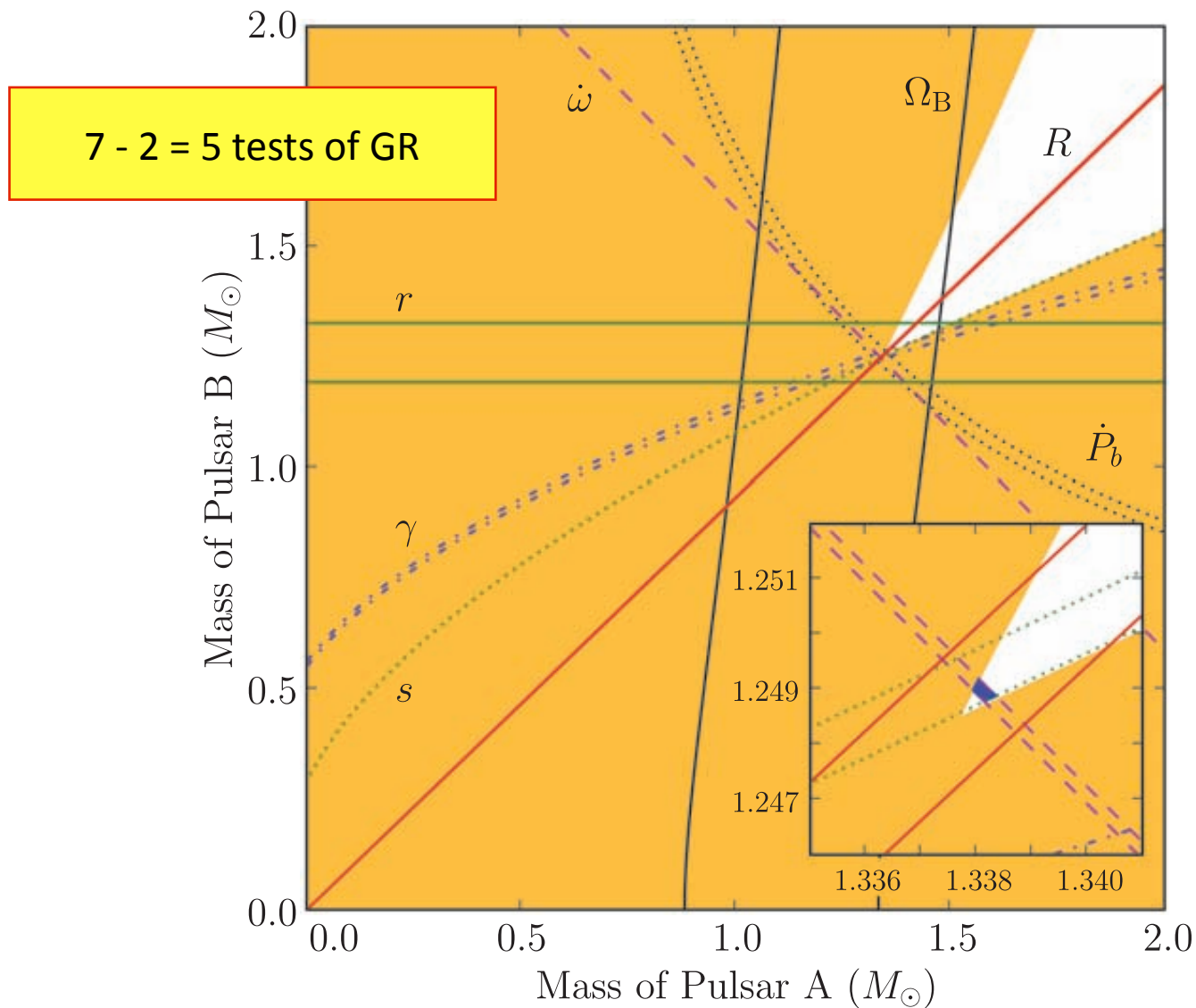


Observing the Double Pulsar

$\sim 1.3 \times 10^6$ TOAs from five different radio telescopes



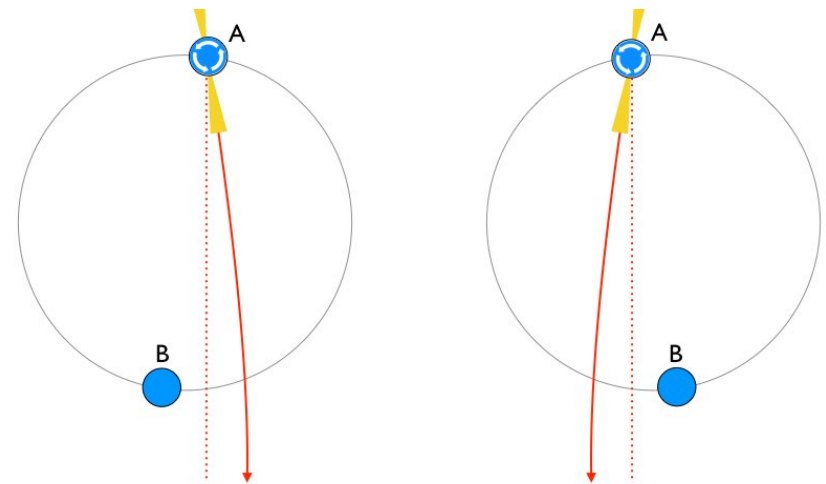
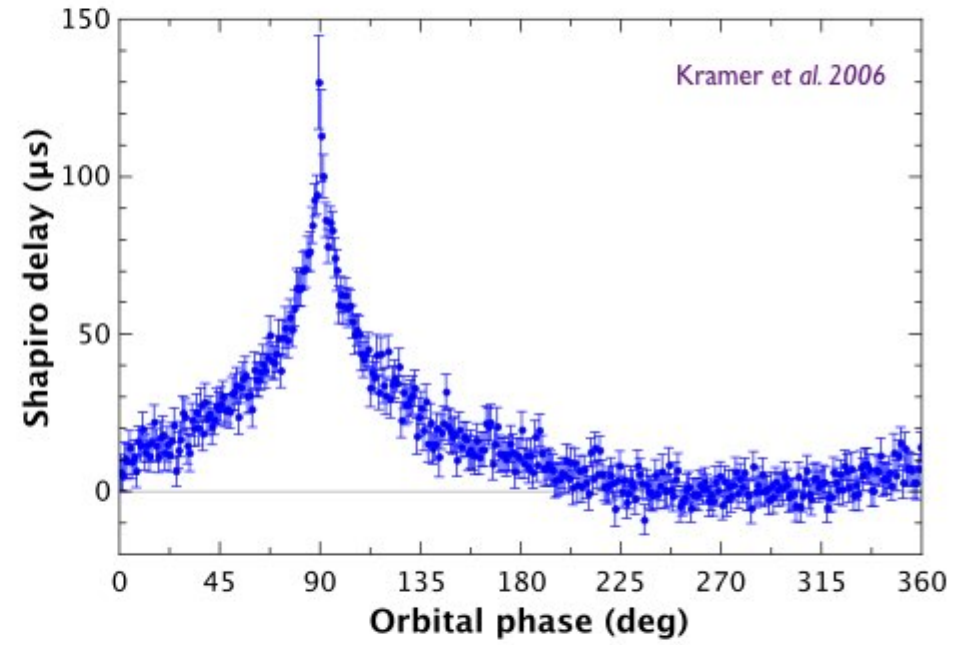
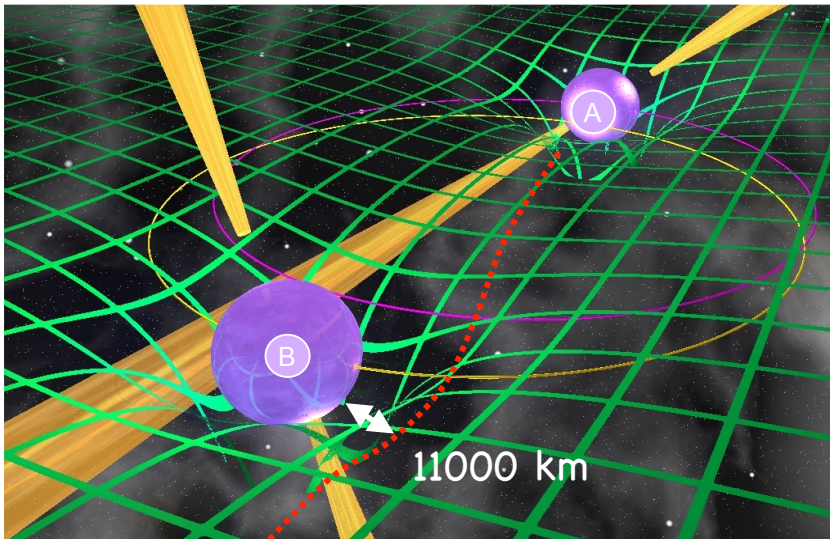
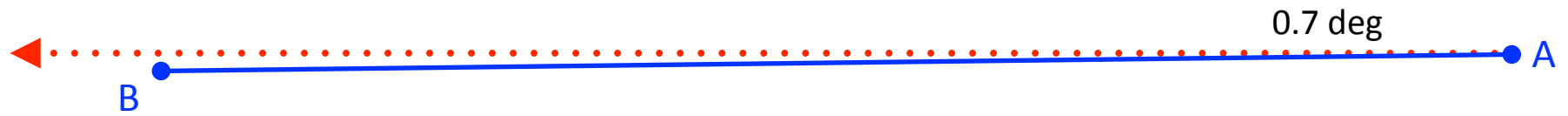
The GR mass-mass diagram of the Double Pulsar



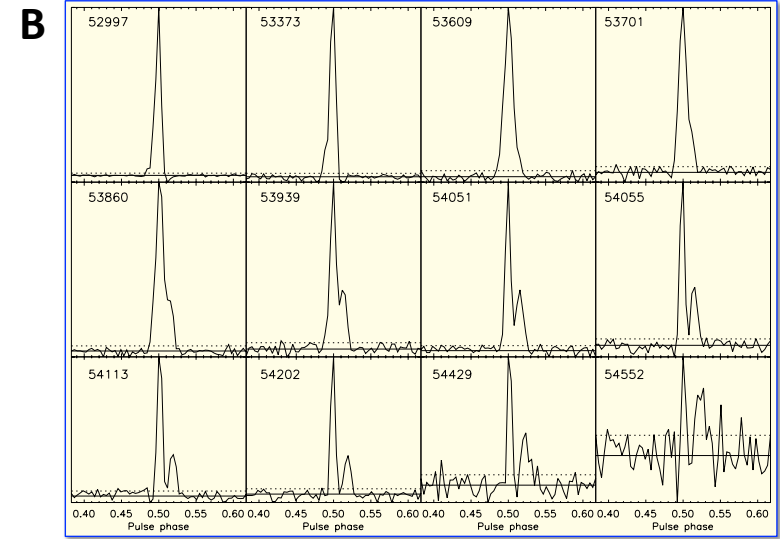
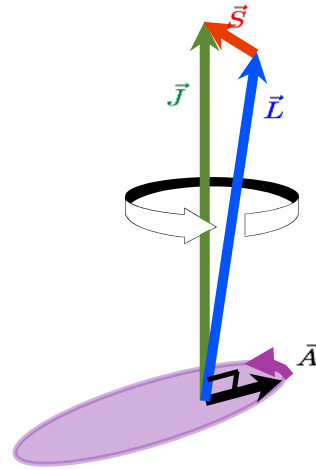
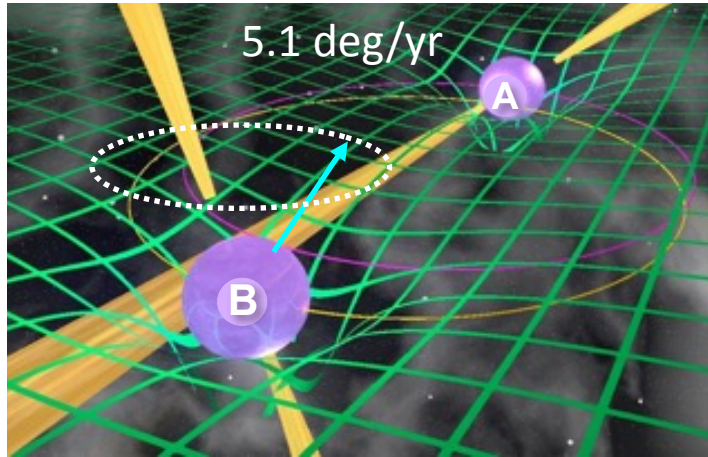
Kramer et al. 2006, Breton et al. 2008

- ➡ New version by Kramer et al. with greatly improved precision should become available soon.
- ➡ GW damping in the Double Pulsar by now tested with a precision of order 0.02% (95% C.L.).

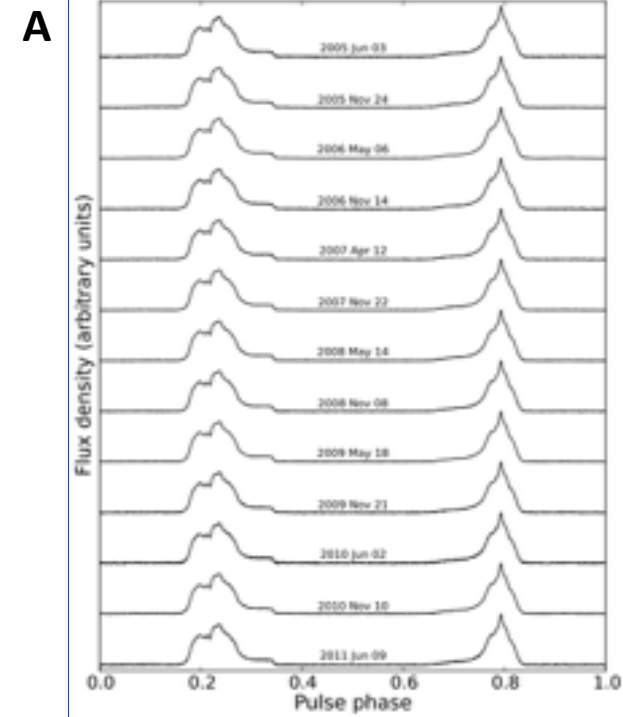
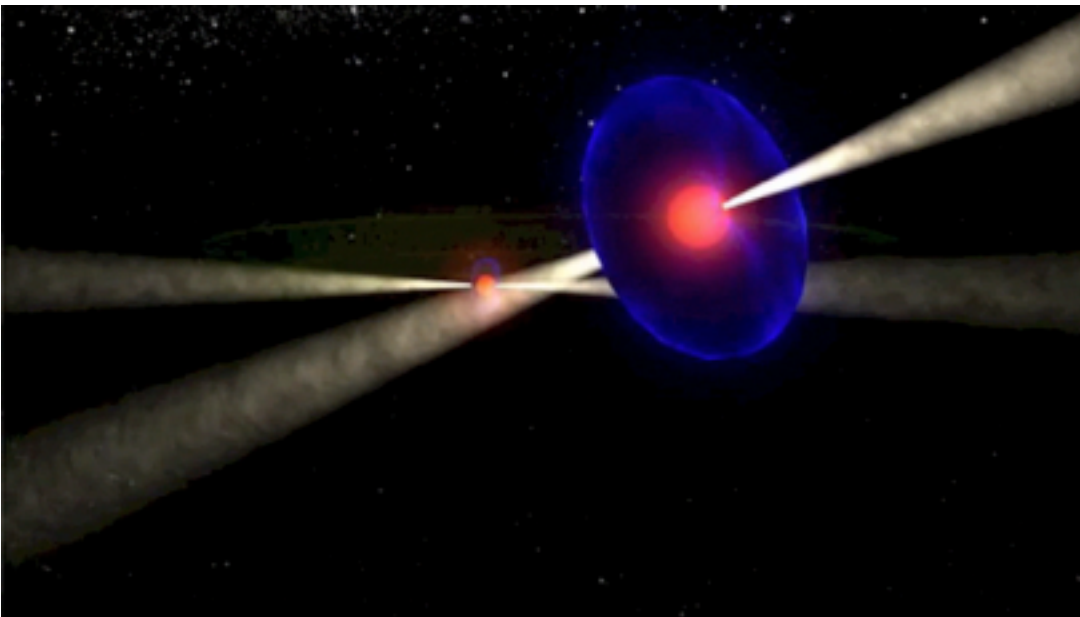
The Shapiro delay in the Double Pulsar



Relativistic spin-orbit coupling in the Double Pulsar

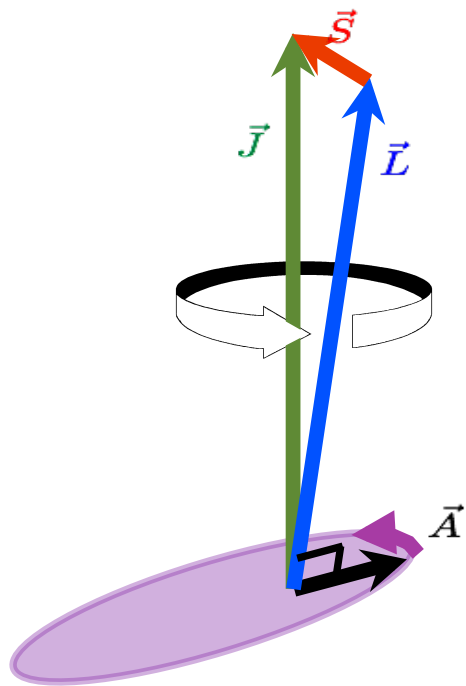


[Perera *et al.* 2010]



[Ferdman *et al.* 2013]

Lense-Thirring effect / moment of inertia of pulsar A



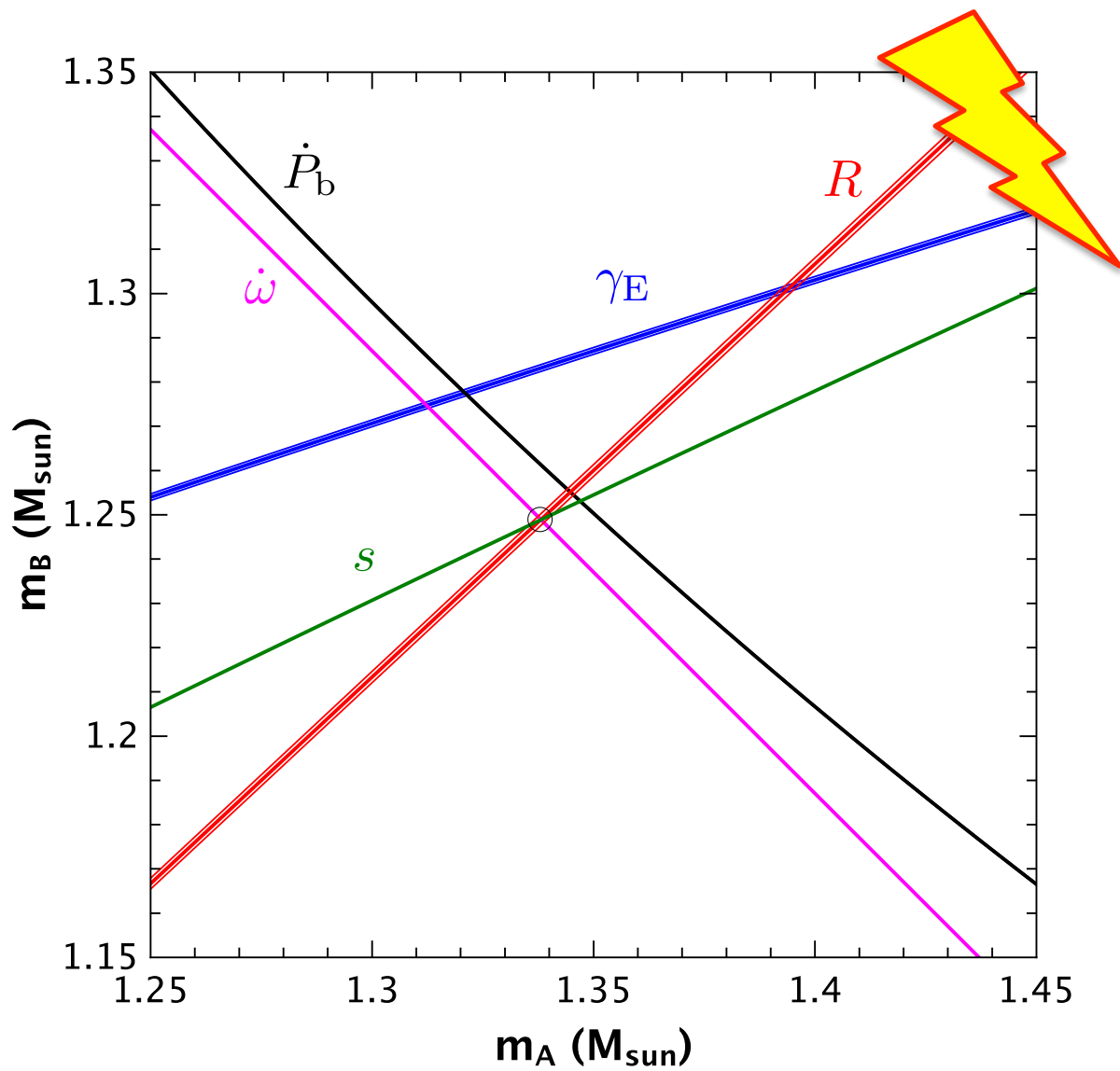
$$\dot{\omega} = \dot{\omega}_{1\text{pN}} + \dot{\omega}_{2\text{pN}} + \dot{\omega}_{\text{SO}}$$

PSR J0737-3039

$\dot{\omega}_{1\text{pN}}$	=	16.89 ...	deg/yr
$\dot{\omega}_{2\text{pN}}$	=	0.00044	deg/yr
$\dot{\omega}_{\text{SO}}$	=	$-0.00038 I_A / (10^{45} \text{ g cm}^2)$	deg/yr
$\delta\dot{\omega}_{\text{obs}}$	=	0.00002	deg/yr

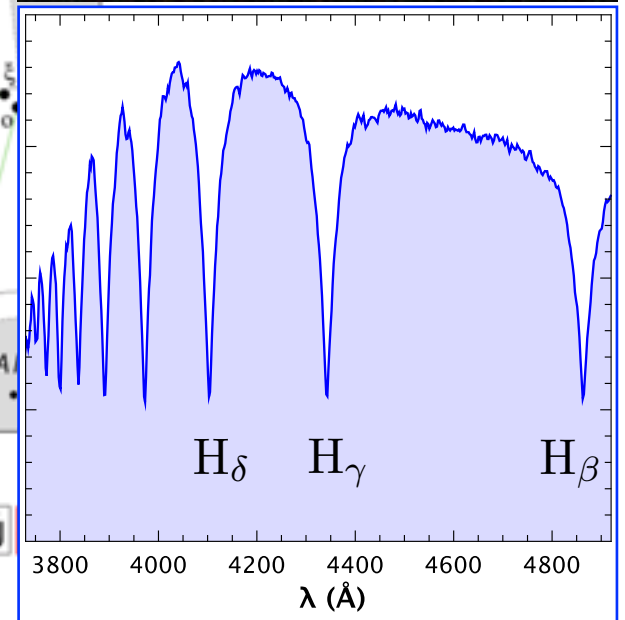
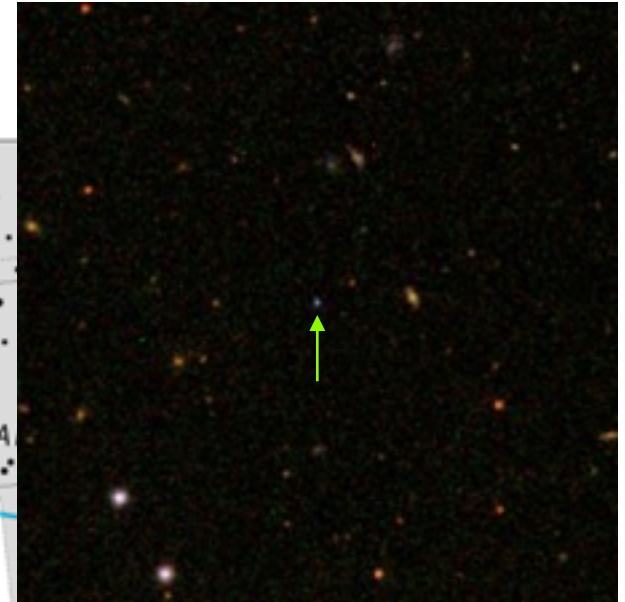
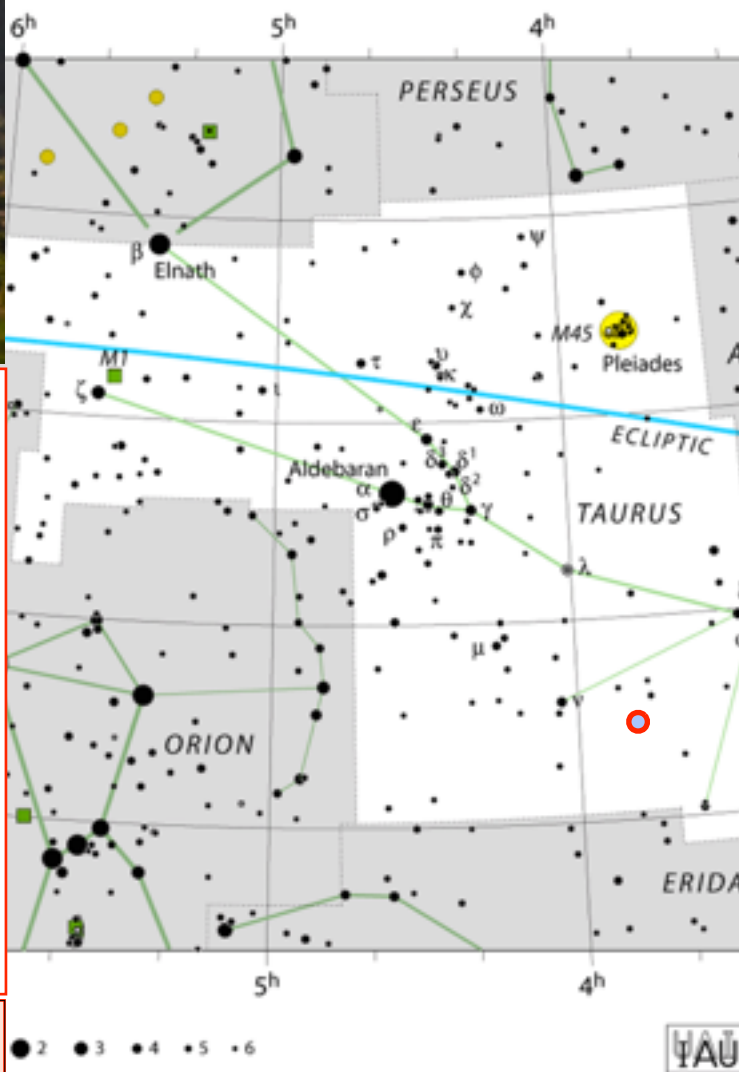
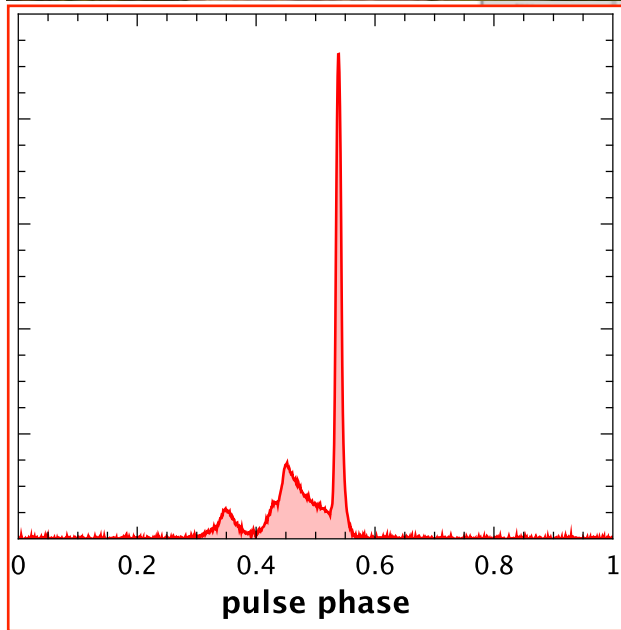
[Kramer *et al.*, in prep.; Kehl *et al.*, in prep.]

Bekenstein's TeVeS and the Double Pulsar



[Kramer *et al.*, in prep.; Wex, Esposito-Farèse *et al.*, in prep.]

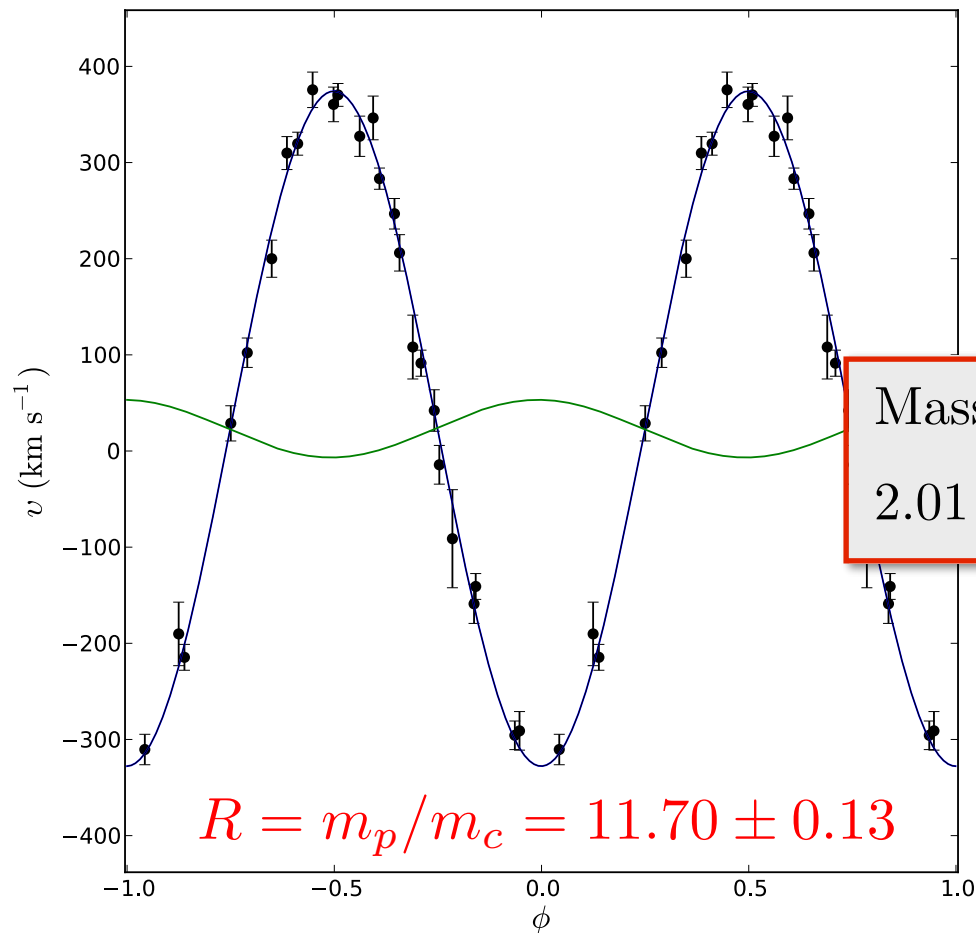
PSR J0348+0432



$$\begin{aligned}
 P &= 39.1226569017806(5) \text{ ms} \\
 P_b &= 2.45817750533(2) \text{ h} \\
 e &\lesssim 10^{-6}
 \end{aligned}$$

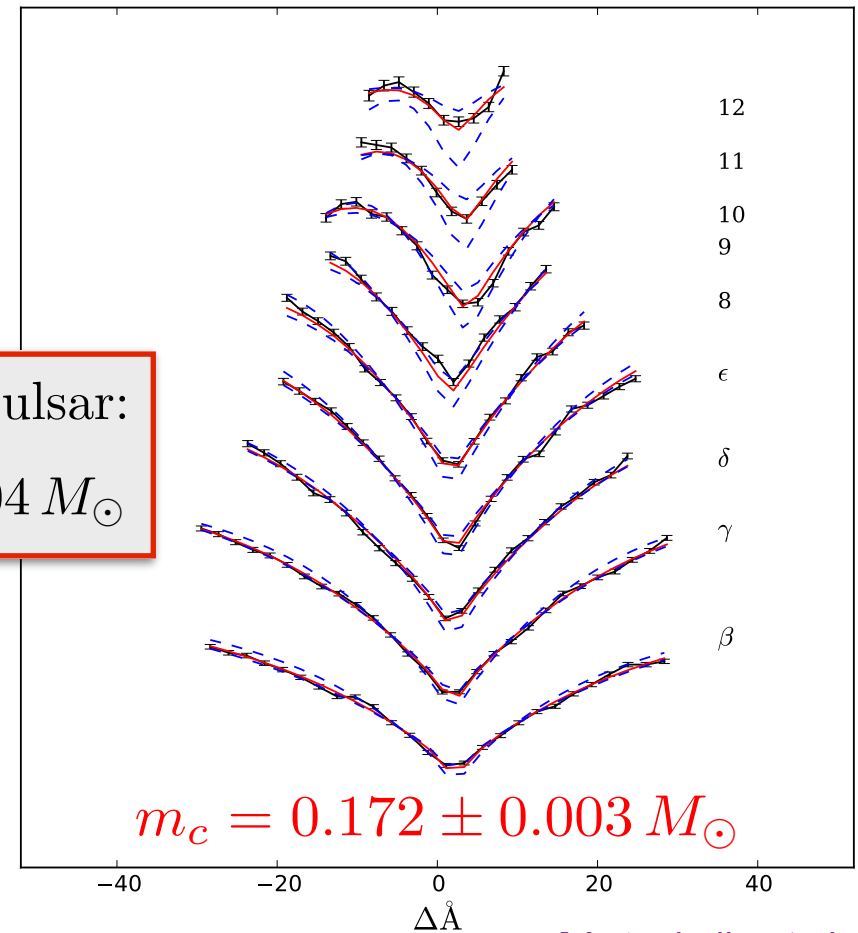
[Boyles *et al.* 2013, Lynch *et al.* 2013, Antoniadis *et al.* 2013]

High-resolution optical spectroscopy of the PSR J0348+0432 companion



Mass of pulsar:
 $2.01 \pm 0.04 M_{\odot}$

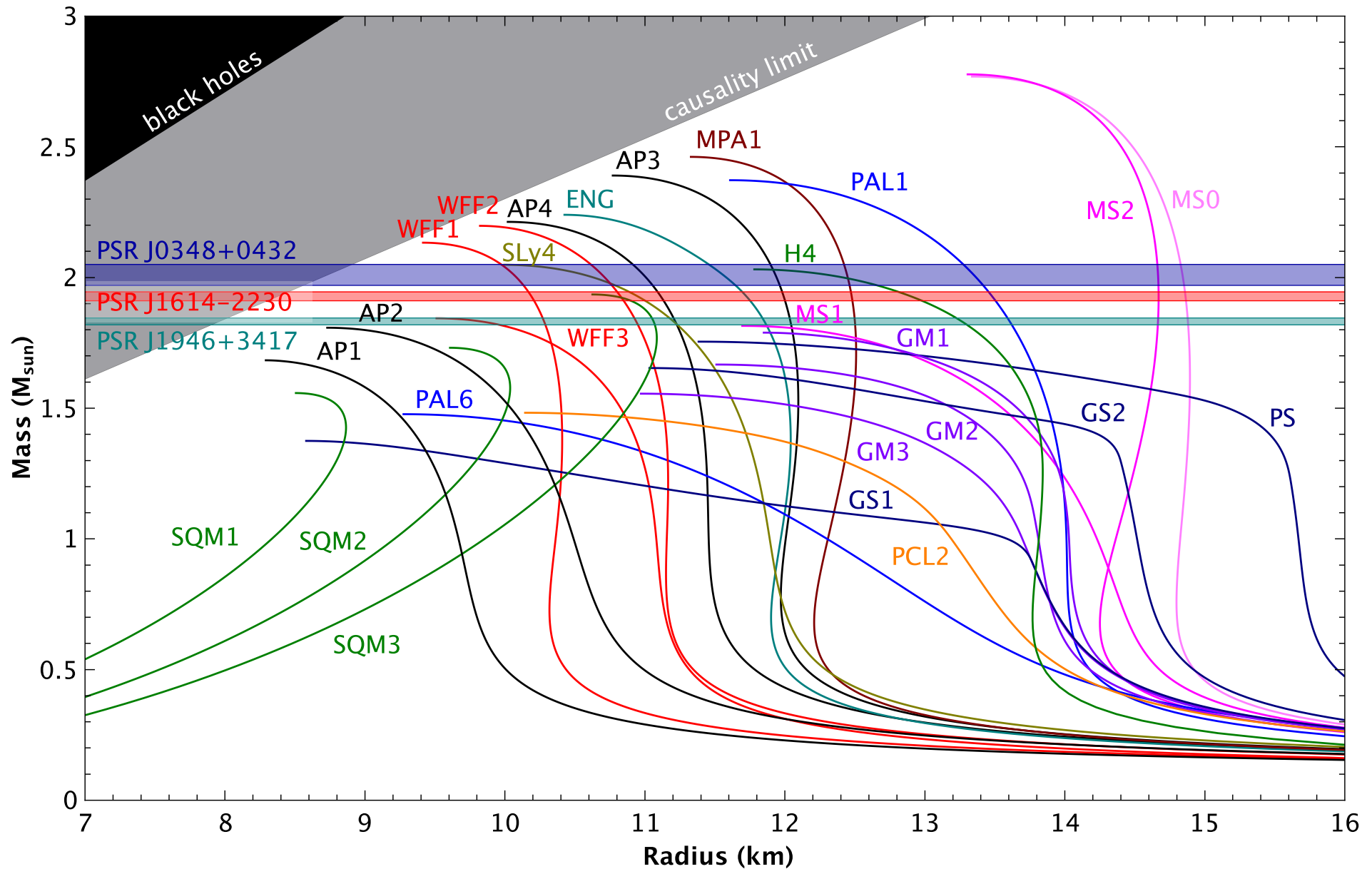
$$R = m_p/m_c = 11.70 \pm 0.13$$



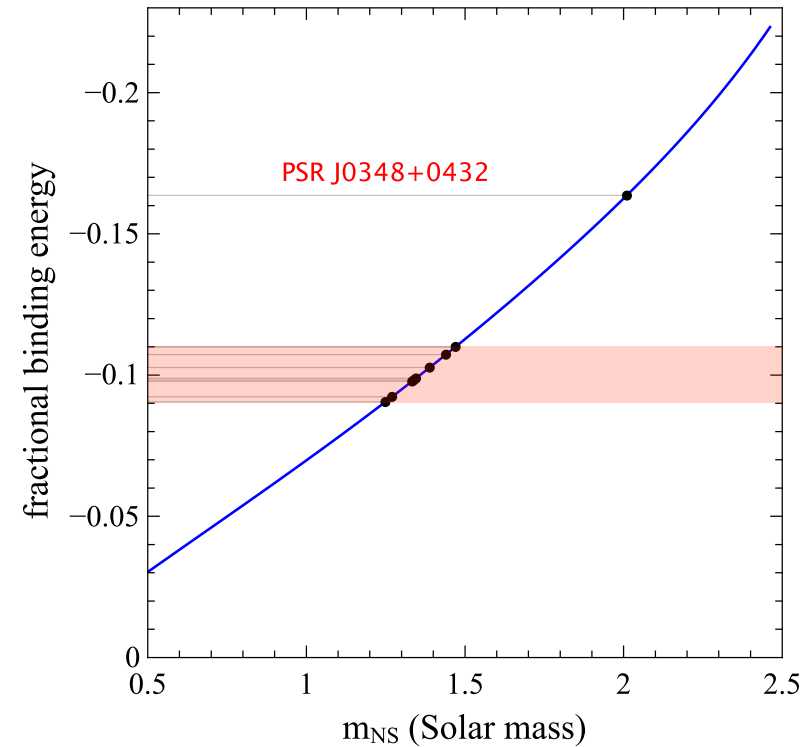
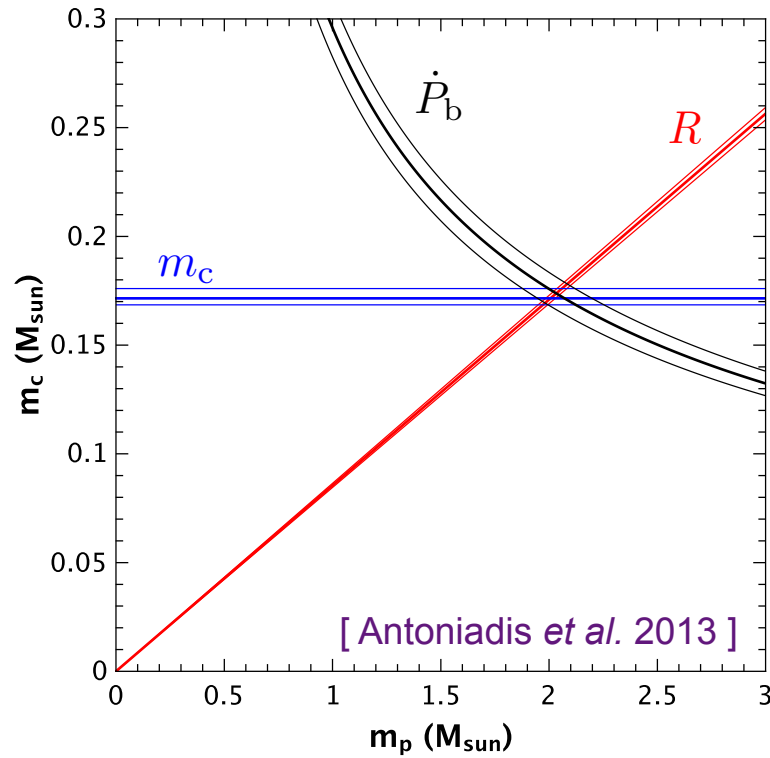
$$m_c = 0.172 \pm 0.003 M_{\odot}$$

[Antoniadis *et al.* 2013]

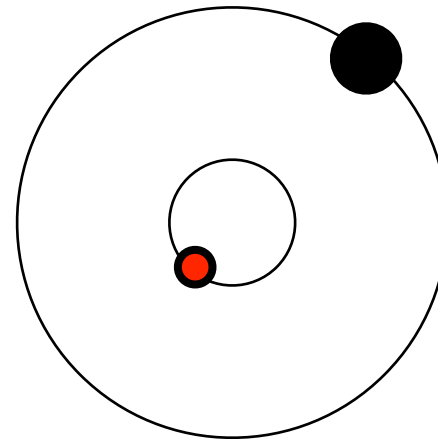
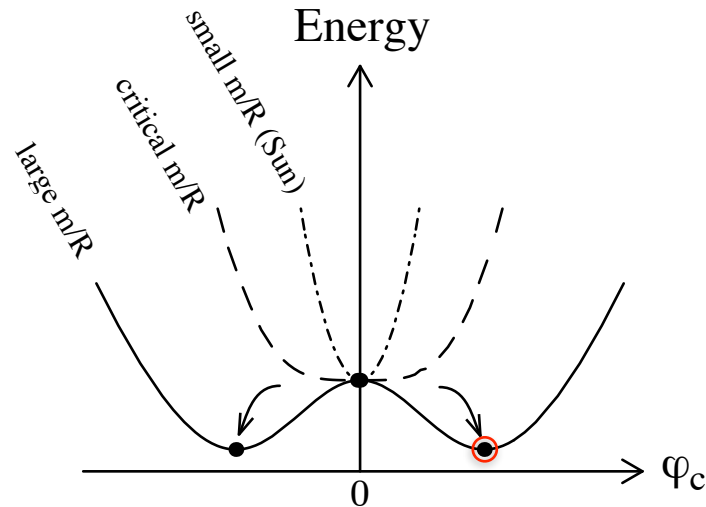
Constraining equations of state at supranuclear densities



Testing a new gravity regime



Spontaneous scalarization of neutron stars in scalar-tensor gravity



[Damour & Esposito-Farèse]

Limits on scalar-tensor gravity from pulsars

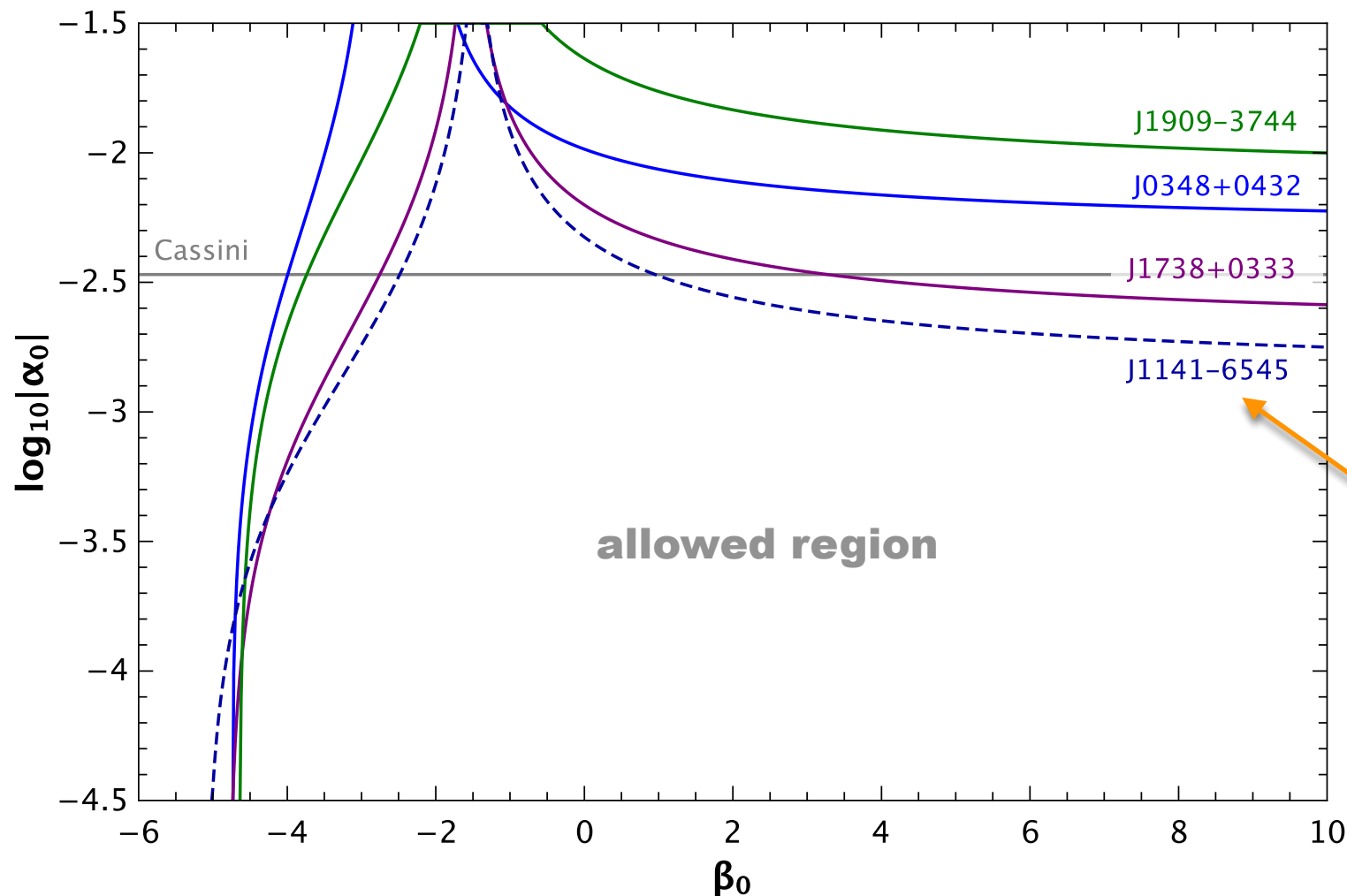
[Damour & Esposito-Farèse 1996]

$$R_{\mu\nu}^* = \frac{8\pi G_*}{c^4} \left(T_{\mu\nu}^* - \frac{1}{2} T^* g_{\mu\nu}^* \right) + 2\partial_\mu \varphi \partial_\nu \varphi - 2\cancel{V(\varphi)} g_{\mu\nu}^*$$

$$\square_{g_*} \varphi = -\frac{4\pi G_*}{c^4} (\alpha_0 + \beta_0 \varphi) T_* - \cancel{V'(\varphi)}$$

$V''(\varphi) \ll 1/(\text{size of system})^2$

Physical (Jordan) metric: $g_{\mu\nu} = g_{\mu\nu}^* \exp(2\alpha_0 \varphi + \beta_0 \varphi^2)$



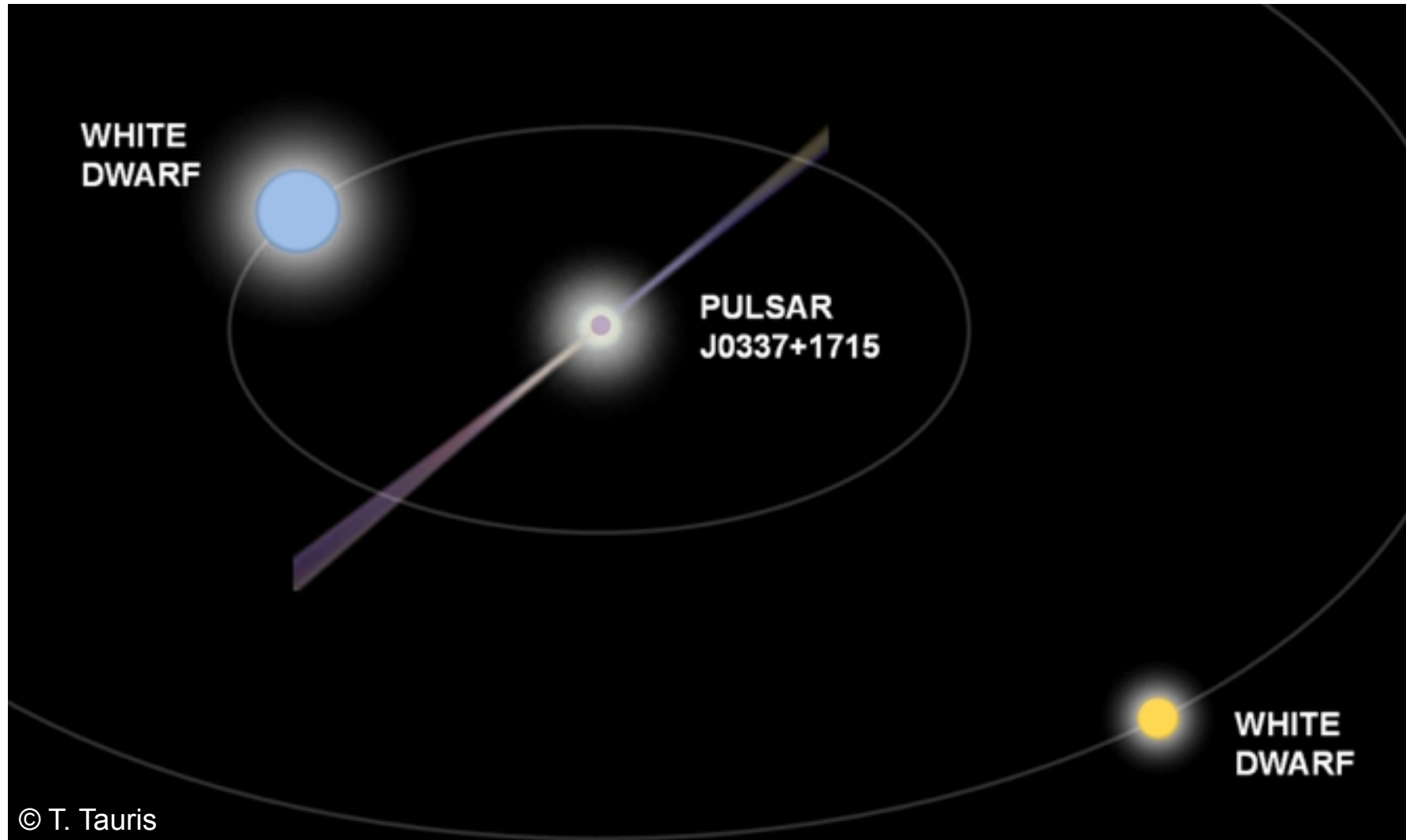
see poster by
Vivek Krishnan et al.

Triple system pulsar PSR J0337+1715 and the violation of SEP

PSR J0337+1715: $P = 2.7 \text{ ms}$, $M_{\text{PSR}} = 1.44 M_{\odot}$

Inner orbit: 1.63 d , $M_{\text{WD}} = 0.20 M_{\odot}$

Outer orbit: 327 d , $M_{\text{WD}} = 0.41 M_{\odot}$



[Ransom *et al.*, 2014]

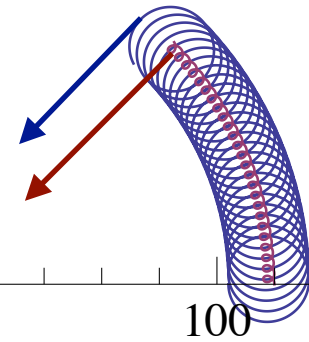
Triple system pulsar PSR J0337+1715 and the violation of SEP

PSR J0337+1715: $P = 2.7 \text{ ms}$, $M_{\text{PSR}} = 1.44 M_{\odot}$
Inner orbit: 1.63 d , $M_{\text{WD}} = 0.20 M_{\odot}$
Outer orbit: 327 d , $M_{\text{WD}} = 0.41 M_{\odot}$

X [lt-s]

-500 -400 -300 -200 -100

100



100

$$a_1 = Gm_2/r^2$$

$$a_p = \mathcal{G}_{p2}m_2/r^2$$

-100

-200

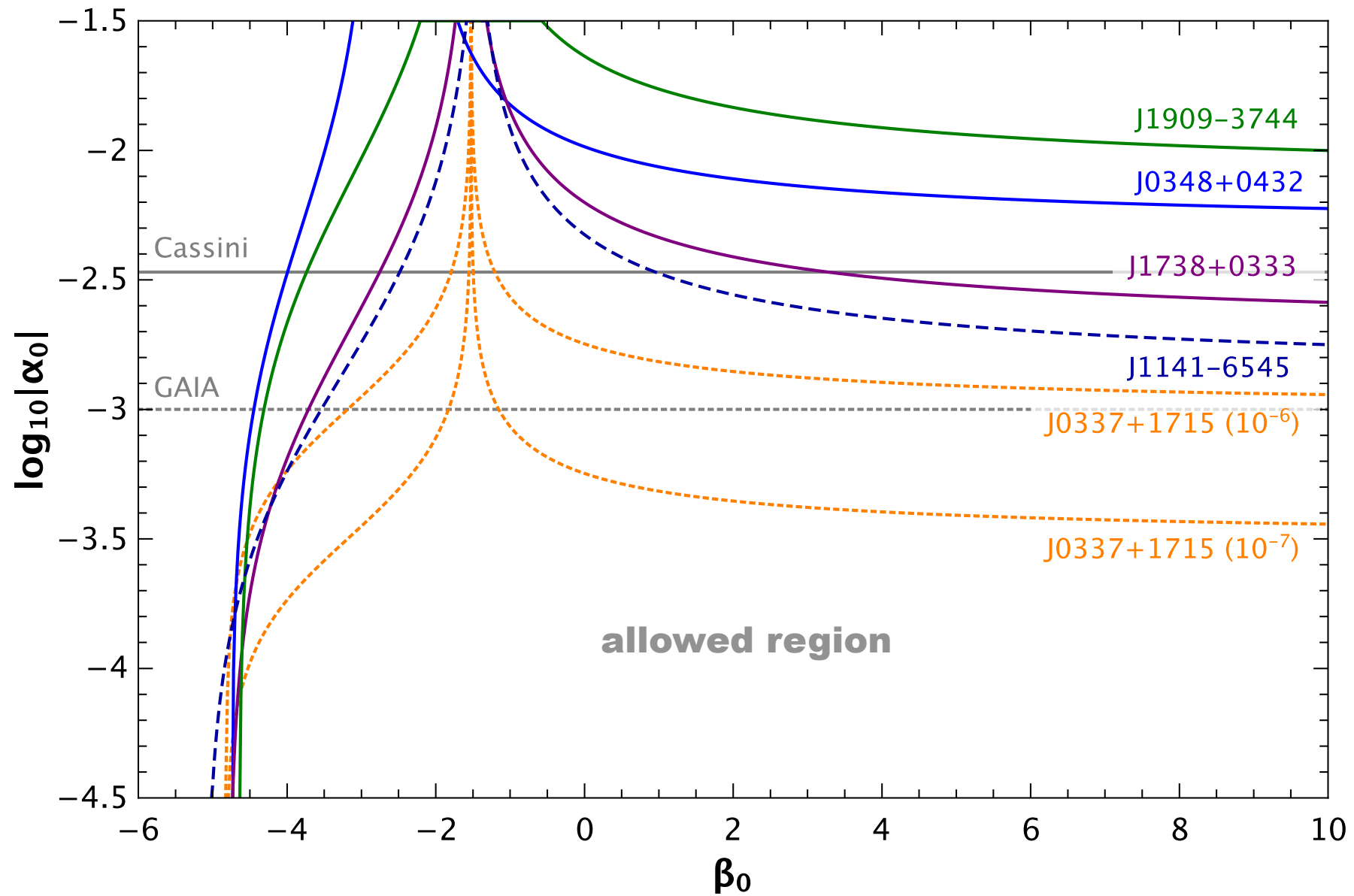
-300

Y [lt-s]



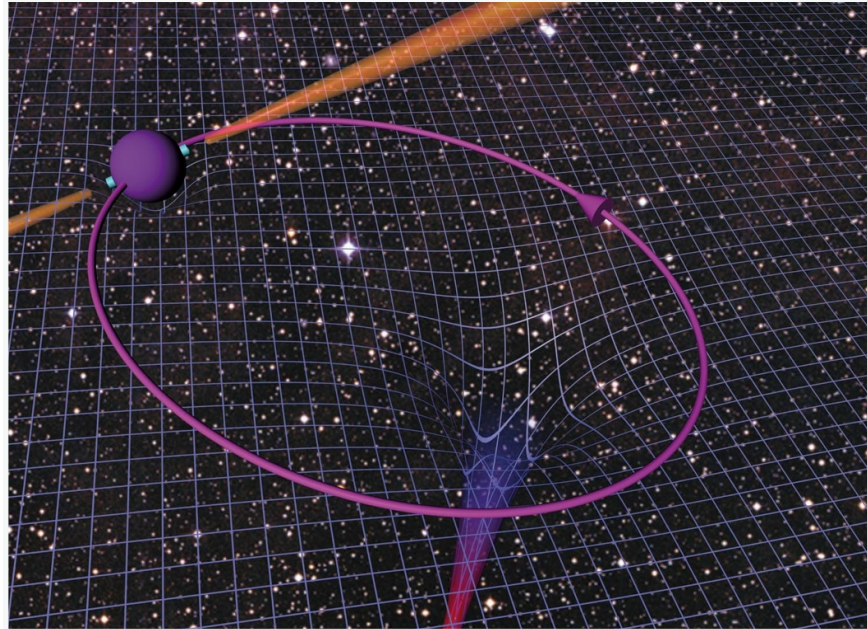
[Ransom *et al.*, 2014]

Expected limits on scalar-tensor gravity from PSR J0337+1715



[Berti et al. 2015, Shao 2016]

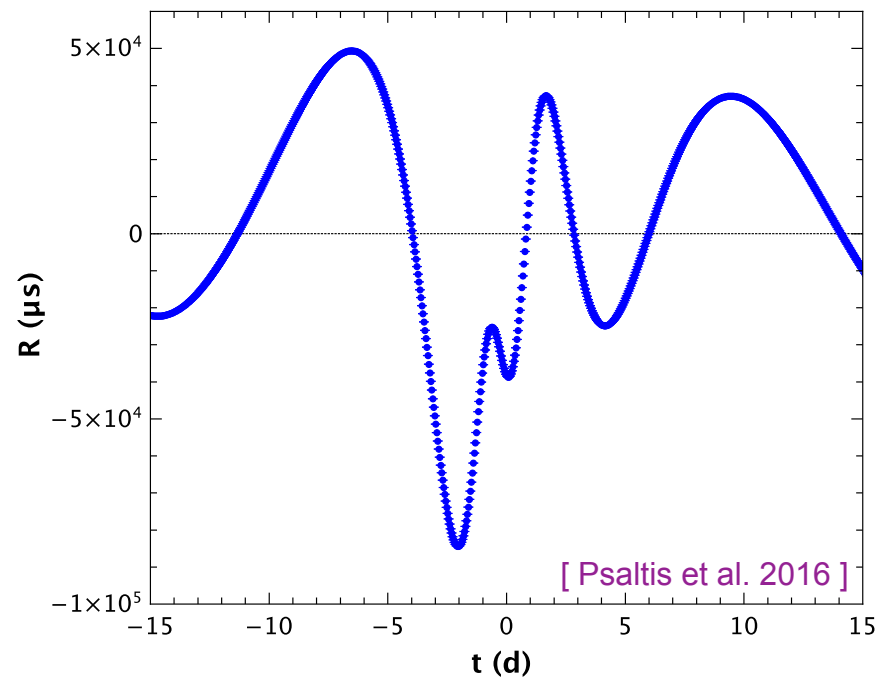
In search of a pulsar-black hole system



Probing the spacetime of Sgr A*




Residuals caused by frame dragging



Event Horizon Telescope

[e.g. Doeleman, Proceedings of Science, 2010]



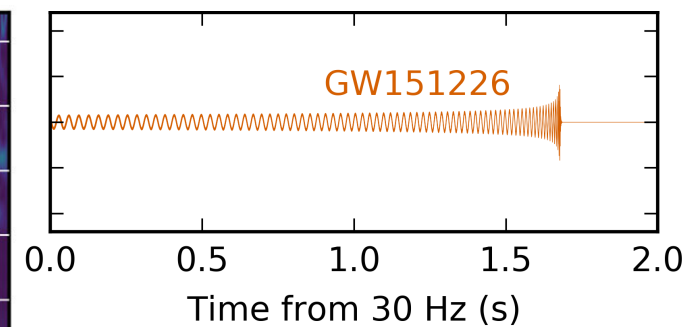
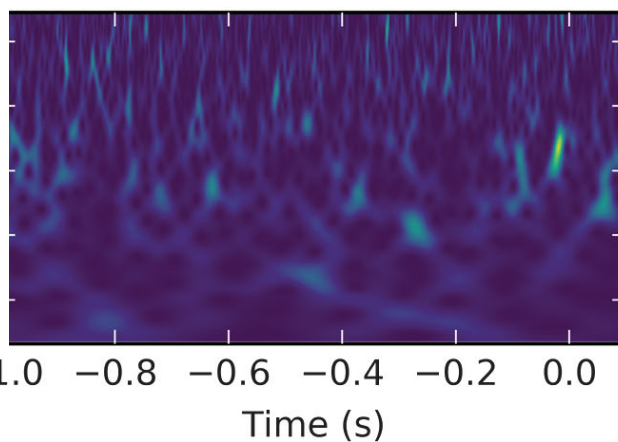
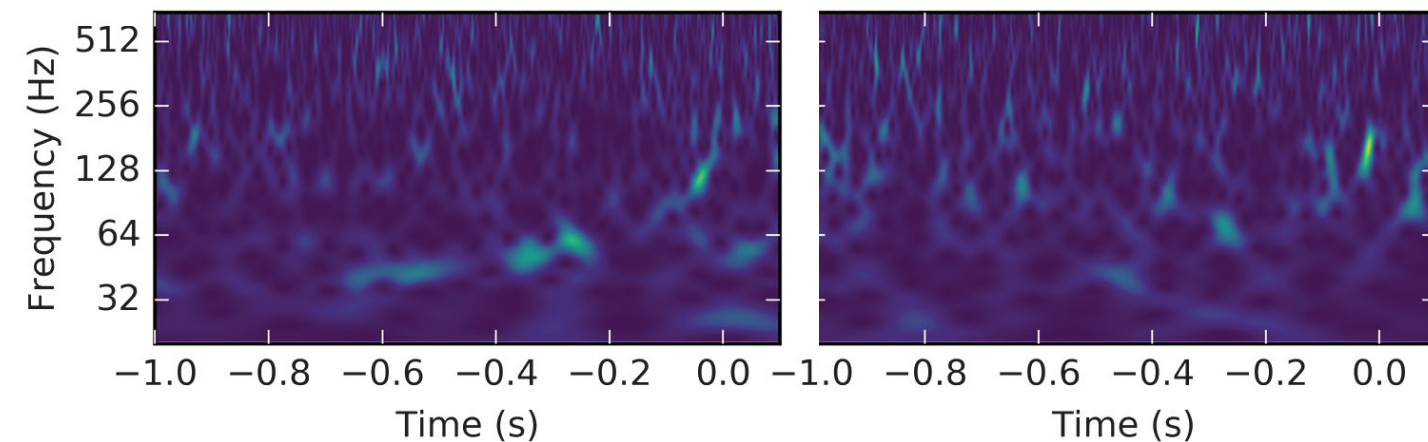
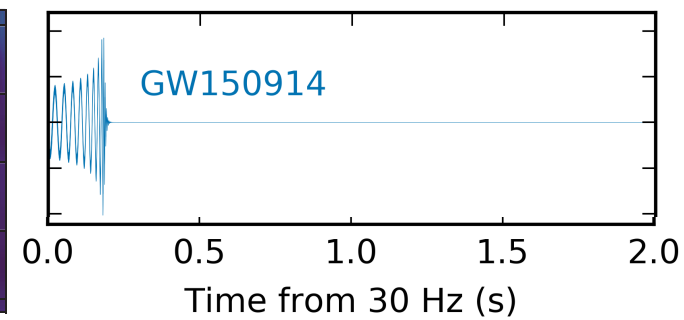
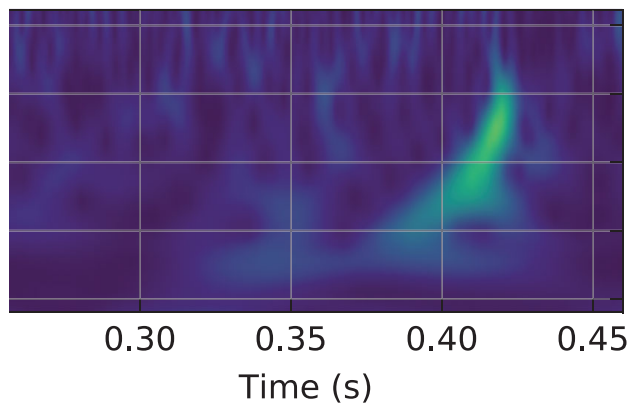
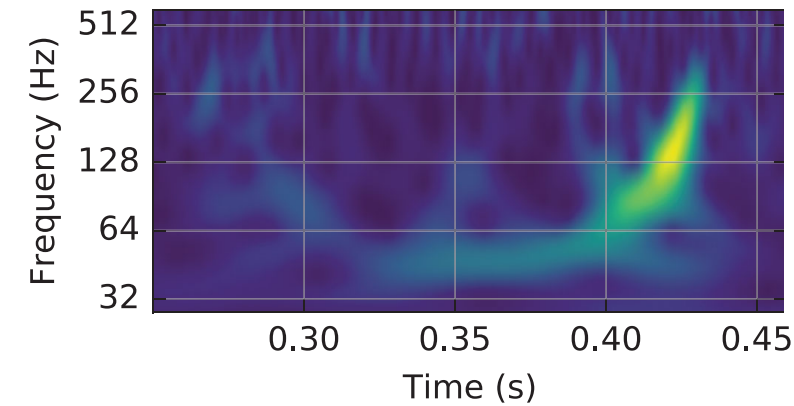
MPIfR participation, including 



The first direct observations of gravitational waves



👉 talk by Gabriela Gonzalez

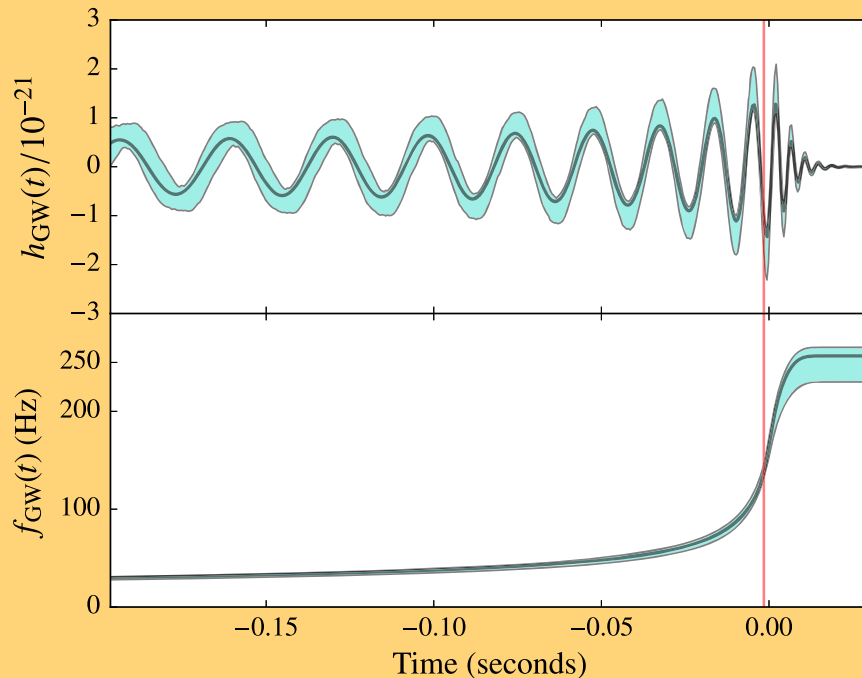


[LIGO Scientific Collaboration and Virgo Collaboration 2016]

How do pulsars compare to LIGO? - I

GW150914

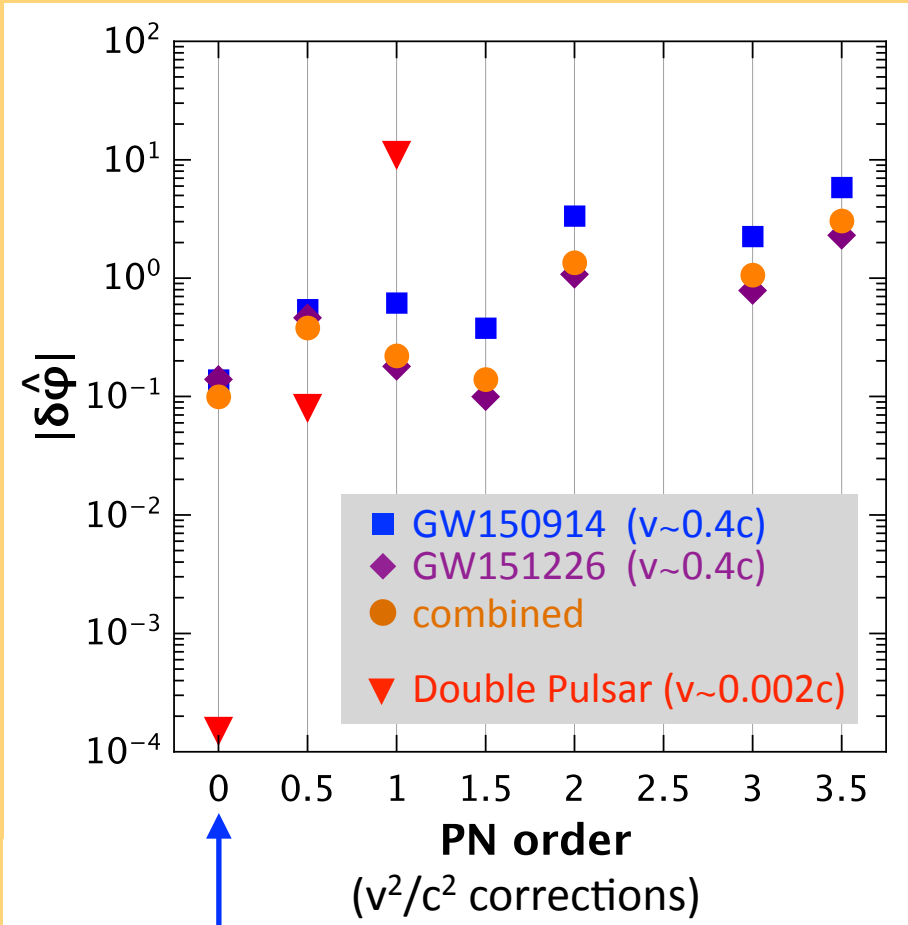
90% credible regions for the waveform and the GW frequency



GR violations are limited to less than 4%
(for effects that cannot be reabsorbed in a redefinition parameters)

[LSC/Virgo 2016]

Testing post-Newtonian corrections in the orbital phase evolution due to GW damping



Quadrupole formula

[LSC/Virgo 2016, Kramer et al. in prep.]

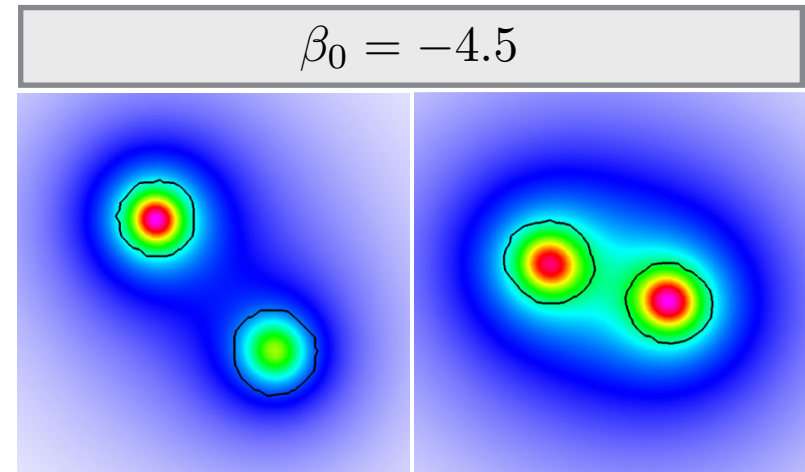
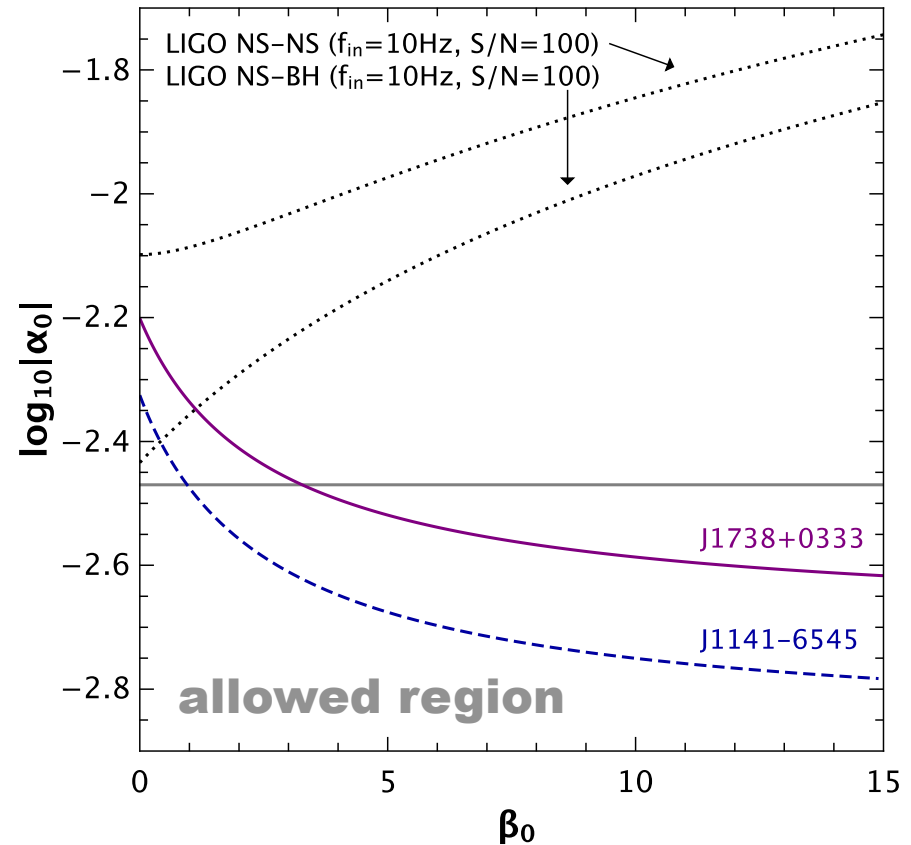
How do pulsars compare to LIGO? - II

BH-BH mergers cannot test deviations from GR that appear only in the presence of matter, e.g. JFBD-type scalar-tensor gravity

Certain alternatives to GR predict (significant) deviations only for BHs, e.g. decoupled dynamical Gauss-Bonnet (D^2GB) gravity (see Yagi et al. 2016).

For certain alternatives to GR, pulsars already provide better constraints than expected from LIGO/Virgo observations of NS-NS or NS-BH mergers.

LIGO/Virgo observations of NS-NS mergers are essential to test short range phenomena, like dynamical scalarization (Barausse et al. 2013).



[Barausse et al. 2013]

The gravitational wave spectrum

Hubble time

10 - 100 aHz

years

1 - 100 nHz

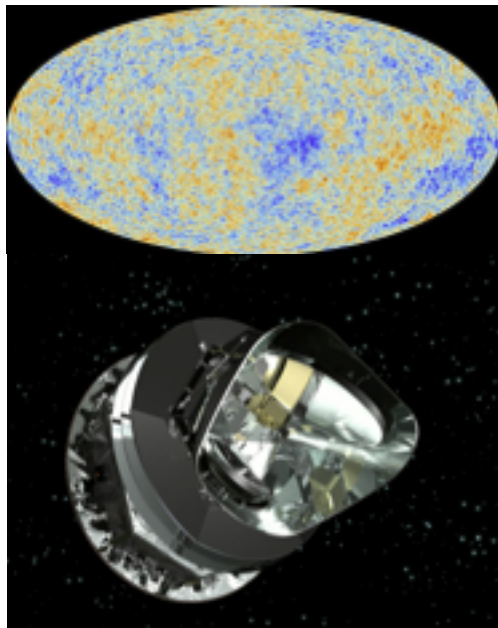
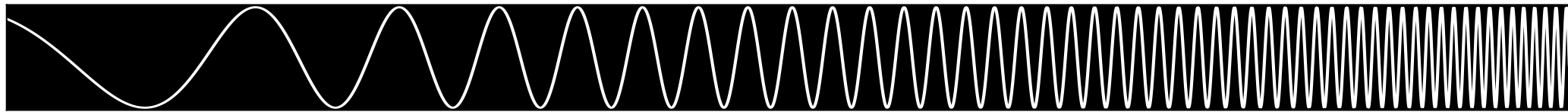
hours

0.1 - 100 mHz

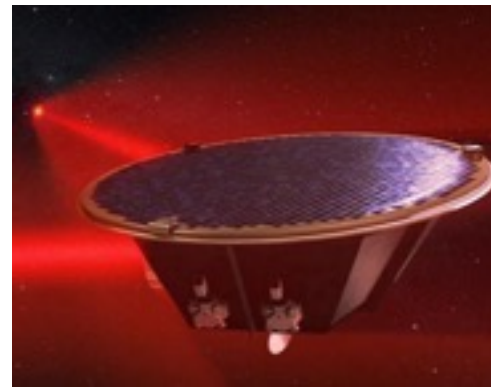
sec

20 - 2000 Hz

msec

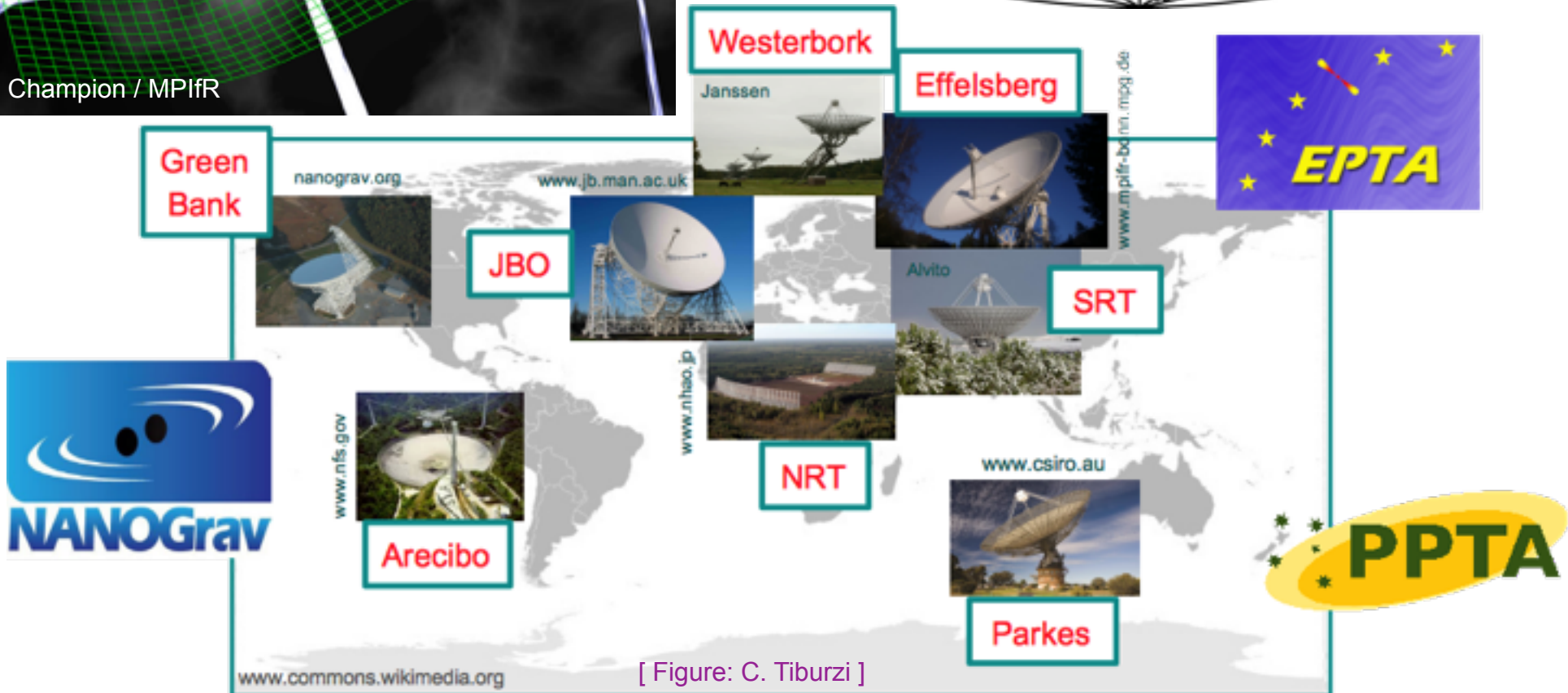
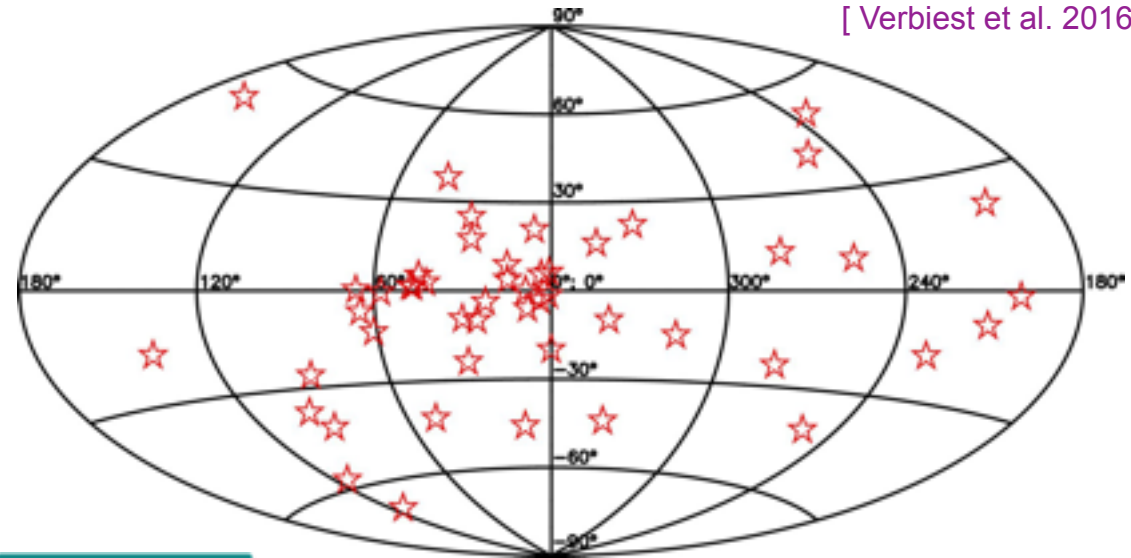
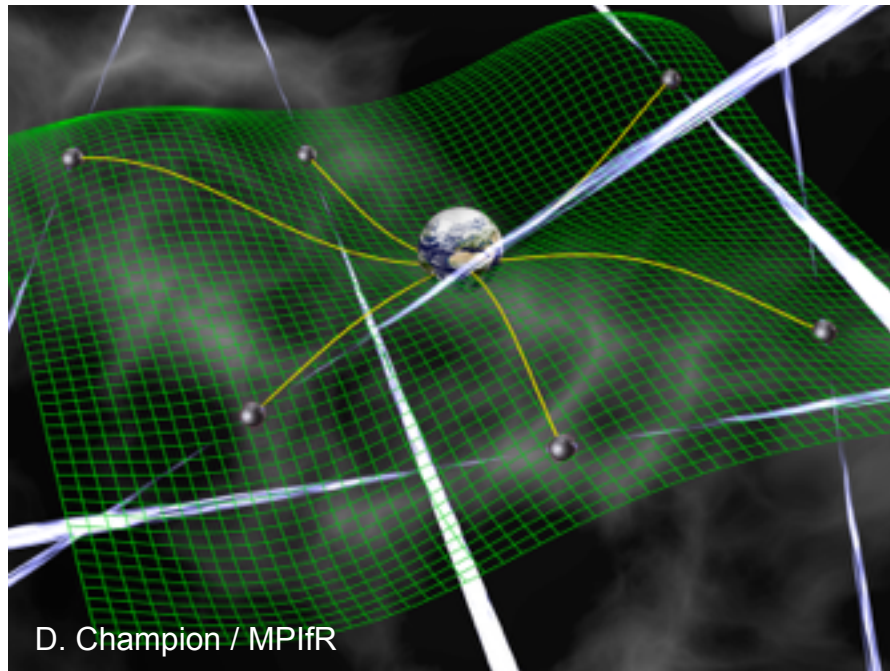


parallel session C1
Pulsar Timing Arrays



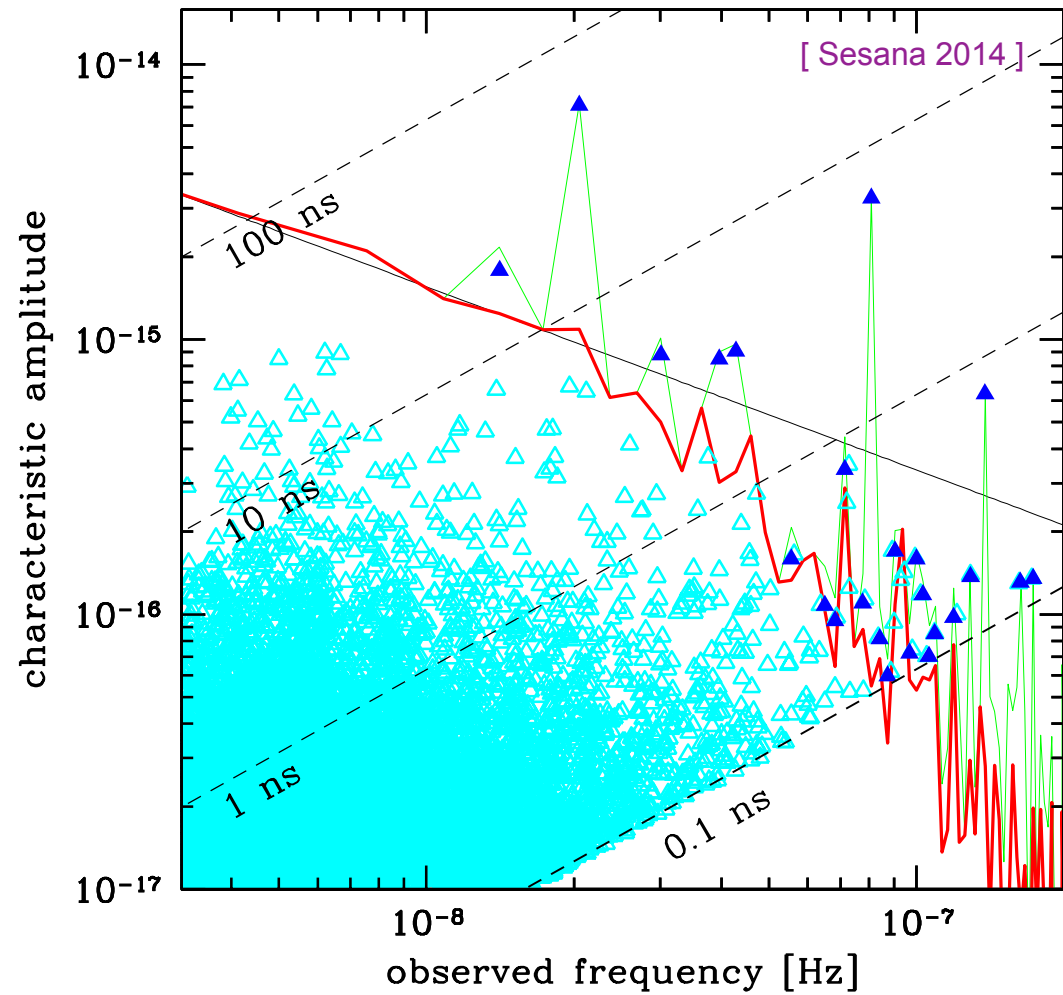
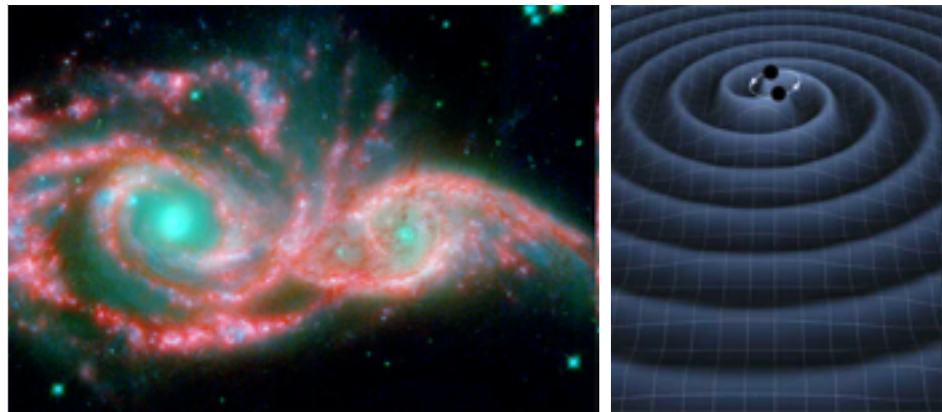
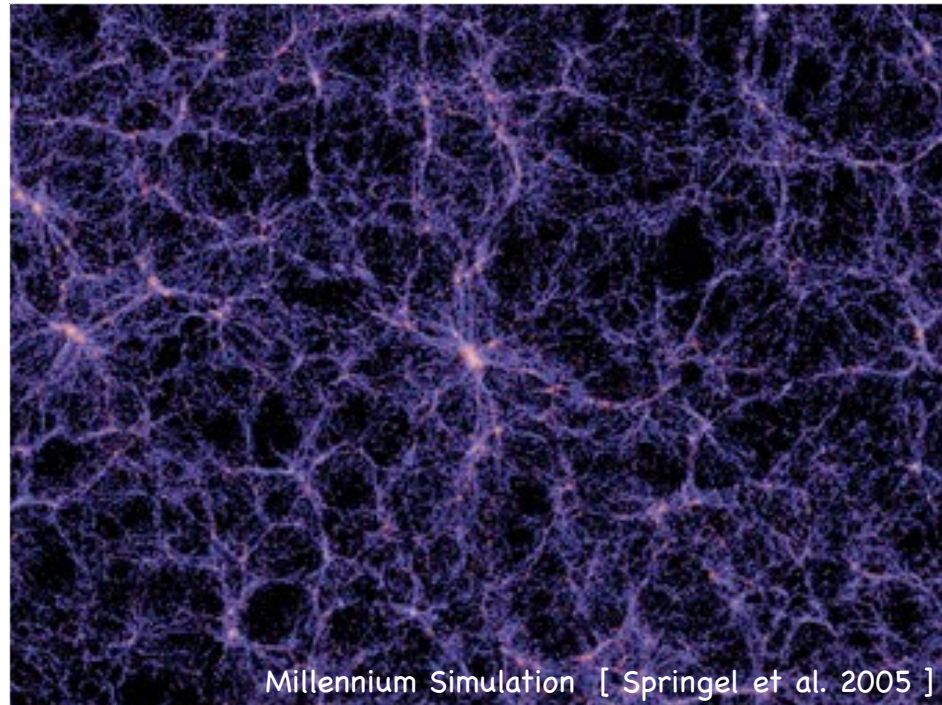
Pulsar Timing Array (PTA)

[Verbiest et al. 2016]

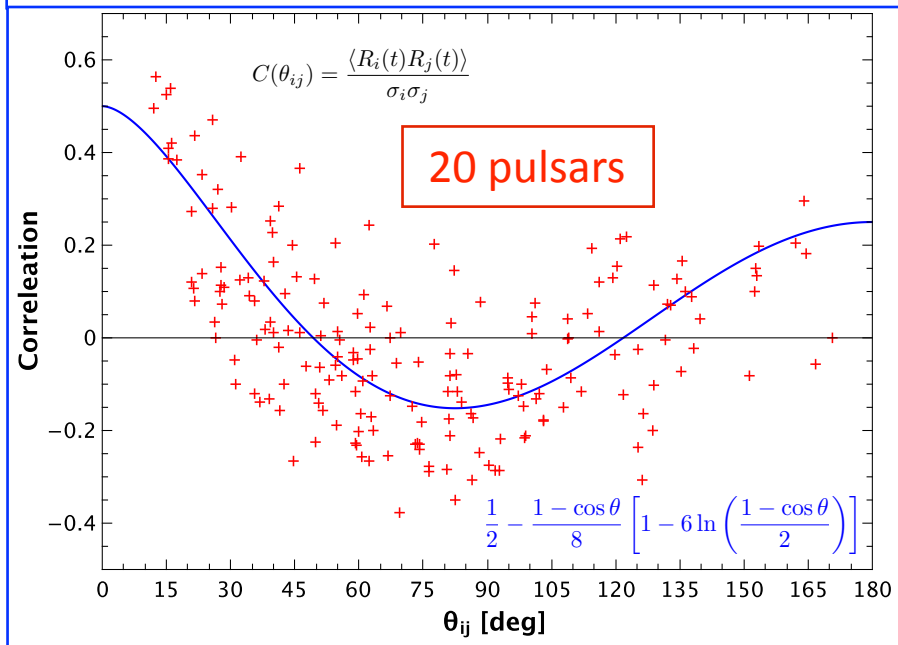
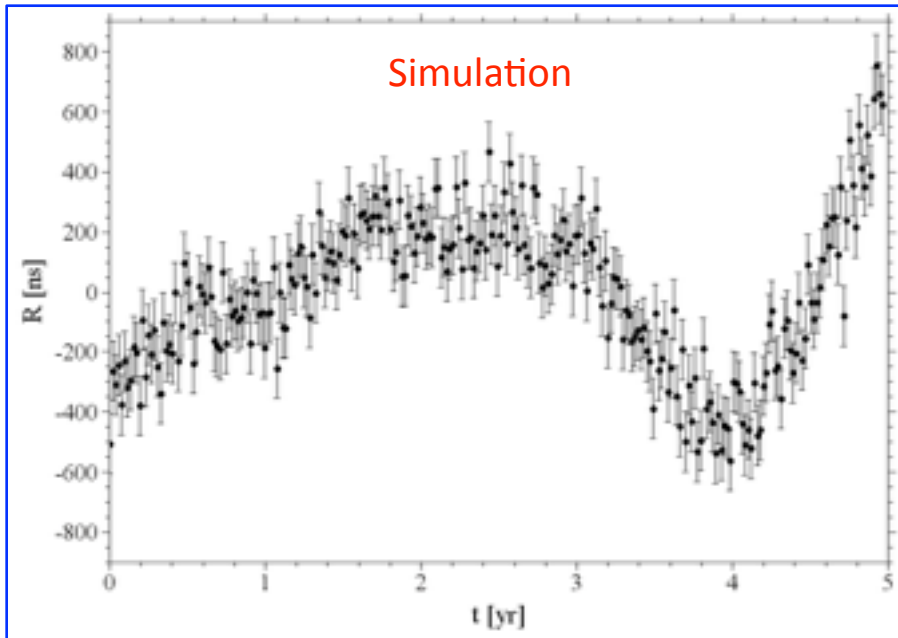


[Figure: C. Tiburzi]

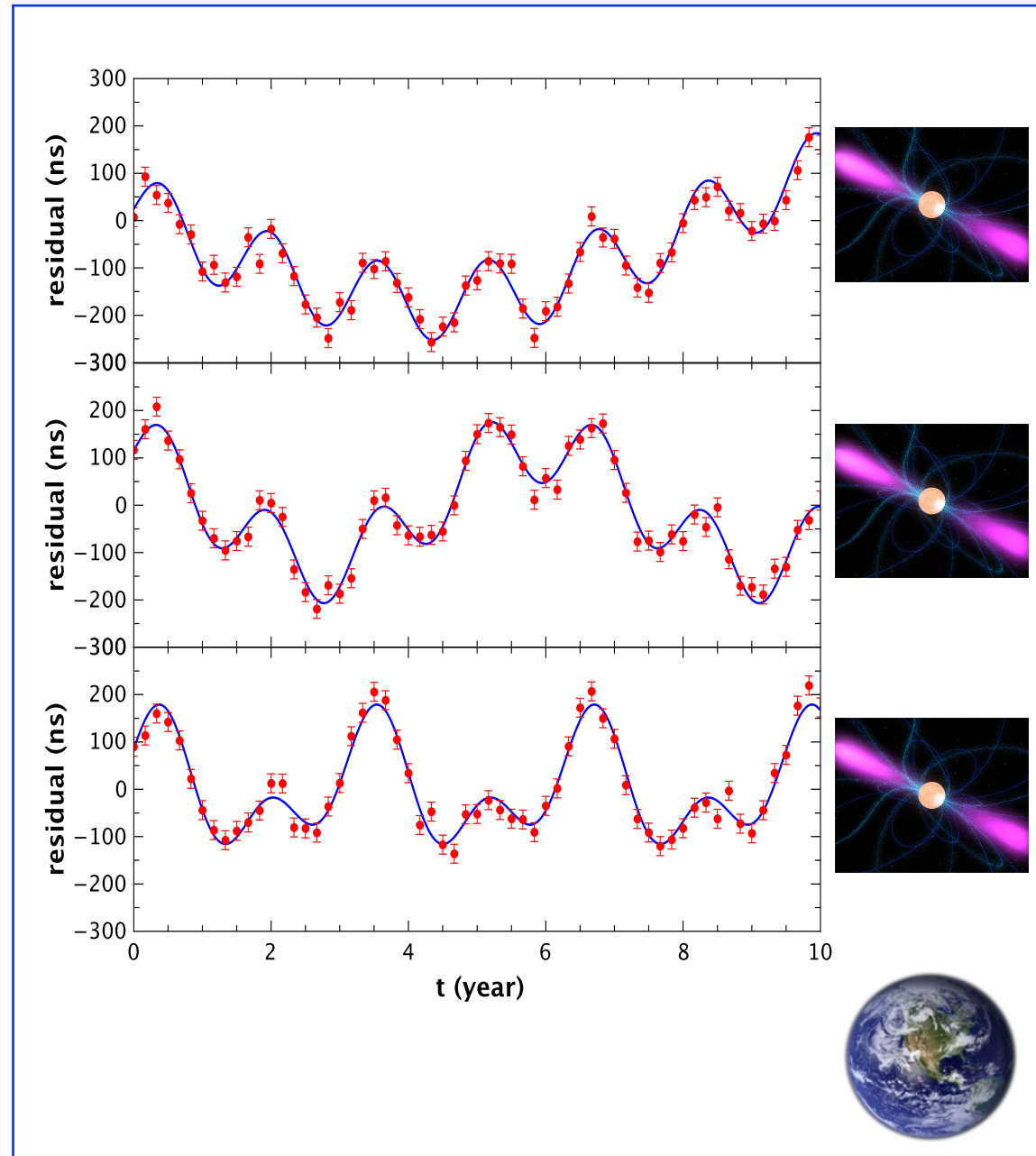
The nano-Hertz stochastic gravitational wave background



Stochastic gravitational wave background / (evolving) single source



[Hellings & Downs 1983]



[Jenet et al. 2004]

Stochastic gravitational wave background / (evolving) single source

Simulation

Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **458**, 1267–1288 (2016)
Advance Access publication 2016 February 15

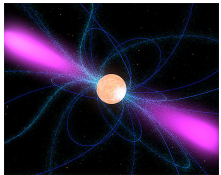
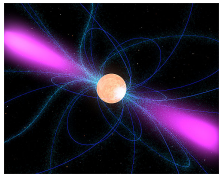
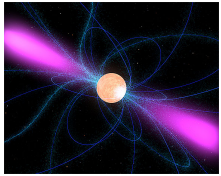
doi:10.1093/mnras/stw347

The International Pulsar Timing Array: First data release

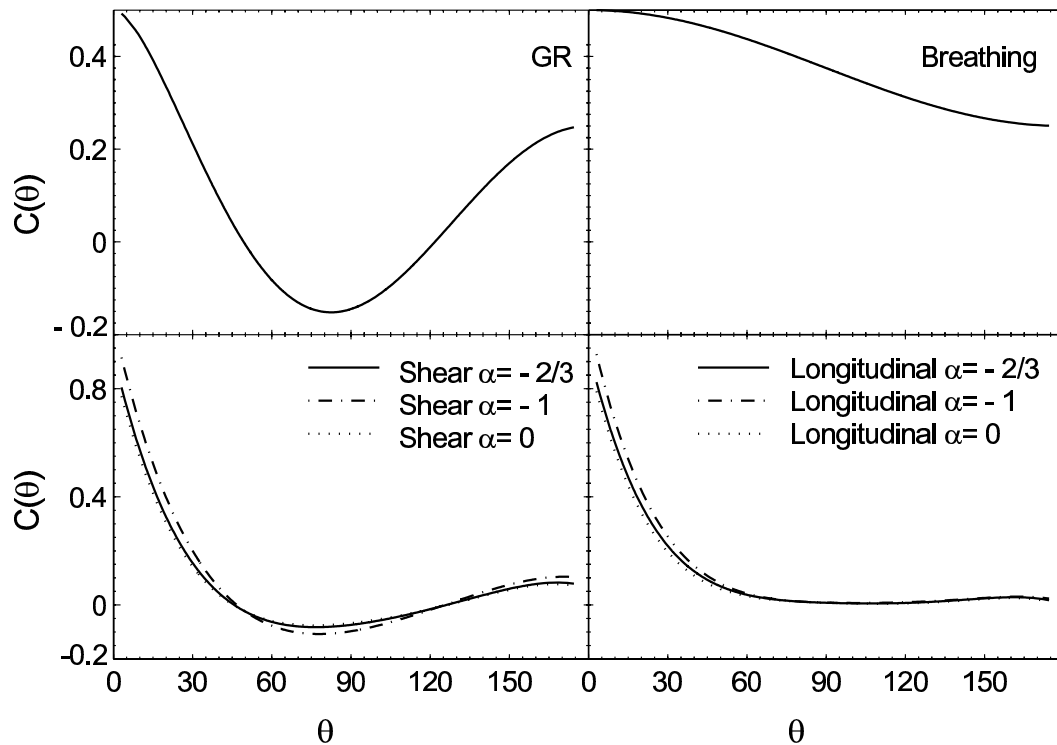
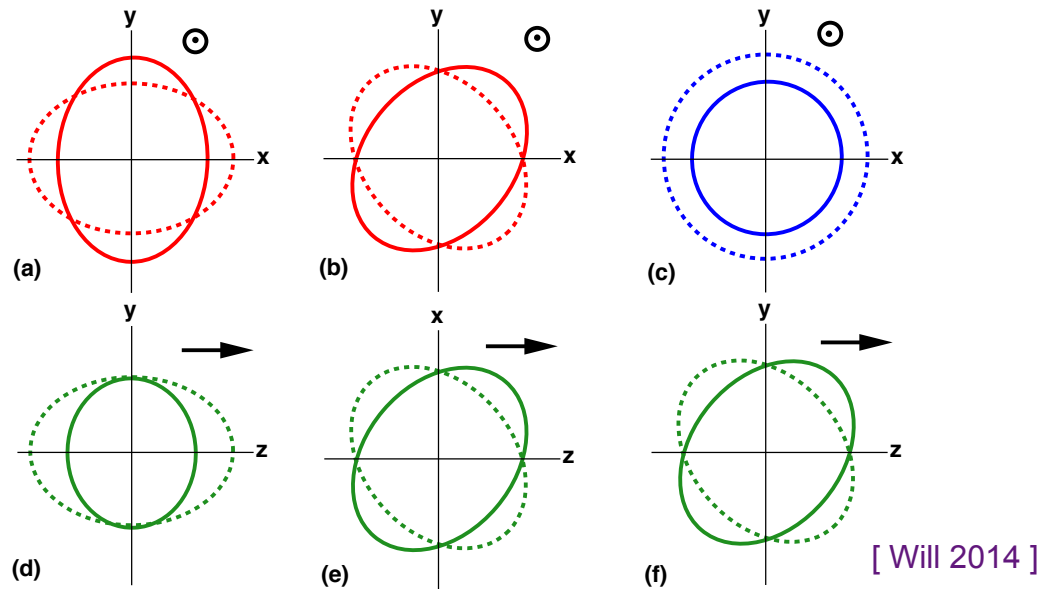
J. P. W. Verbiest,^{1,2★} L. Lentati,³ G. Hobbs,⁴ R. van Haasteren,⁵ P. B. Demorest,⁶
G. H. Janssen,⁷ J.-B. Wang,⁸ G. Desvignes,² R. N. Caballero,² M. J. Keith,⁹
D. J. Champion,² Z. Arzoumanian,¹⁰ S. Babak,¹¹ C. G. Bassa,⁷ N. D. R. Bhat,¹²
A. Brazier,^{13,14} P. Brem,¹¹ M. Burgay,¹⁵ S. Burke-Spolaor,⁶ S. J. Chamberlin,¹⁶
S. Chatterjee,^{14,17} B. Christy,¹⁸ I. Cognard,^{19,20} J. M. Cordes,¹⁷ S. Dai,^{4,21}
T. Dolch,^{22,14,17} J. A. Ellis,⁵ R. D. Ferdman,²³ E. Fonseca,²⁴ J. R. Gair,²⁵
N. E. Garver-Daniels,²⁶ P. Gentile,²⁶ M. E. Gonzalez,²⁷ E. Graikou,² L. Guillemot,^{19,20}
J. W. T. Hessels,^{7,28} G. Jones,²⁹ R. Karuppusamy,² M. Kerr,⁴ M. Kramer,^{2,9} M.
T. Lam,¹⁷ P. D. Lasky,³⁰ A. Lassus,² P. Lazarus,² T. J. W. Lazio,⁵ K. J. Lee,³¹
L. Levin,^{26,9} K. Liu,² R. S. Lynch,³² A. G. Lyne,⁹ J. McKee,⁹
M. A. McLaughlin,²⁶ S. T. McWilliams,²⁶ D. R. Madison,³³ R. N. Manchester,⁴
C. M. F. Mingarelli,^{34,2} D. J. Nice,³⁵ S. Osłowski,^{1,2} N. T. Palliyaguru,³⁶
T. T. Pennucci,³⁷ B. B. P. Perera,⁹ D. Perrodin,¹⁵ A. Possenti,¹⁵ A. Petiteau,³⁸
S. M. Ransom,³³ D. Reardon,^{30,4} P. A. Rosado,³⁹ S. A. Sanidas,²⁸ A. Sesana,⁴⁰
G. Shaifullah,^{2,1} R. M. Shannon,^{4,12} X. Siemens,⁴¹ J. Simon,⁴¹ R. Smits,⁷ R. Spiewak,⁴¹
I. H. Stairs,²⁴ B. W. Stappers,⁹ D. R. Stinebring,⁴² K. Stovall,⁴³
J. K. Swiggum,²⁶ S. R. Taylor,⁵ G. Theureau,^{19,20,44} C. Tiburzi,^{2,1} L. Toomey,⁴
M. Vallisneri,⁵ W. van Straten,³⁹ A. Vecchio,⁴⁰ Y. Wang,⁴⁵ L. Wen,⁴⁶ X. P. You,⁴⁷
W. W. Zhu² and X.-J. Zhu⁴⁶

[Hellin,

$$h_c(f) = A \left(\frac{f}{\text{yr}^{-1}} \right)^{-2/3}, \quad A < 1.7 \times 10^{-15}$$

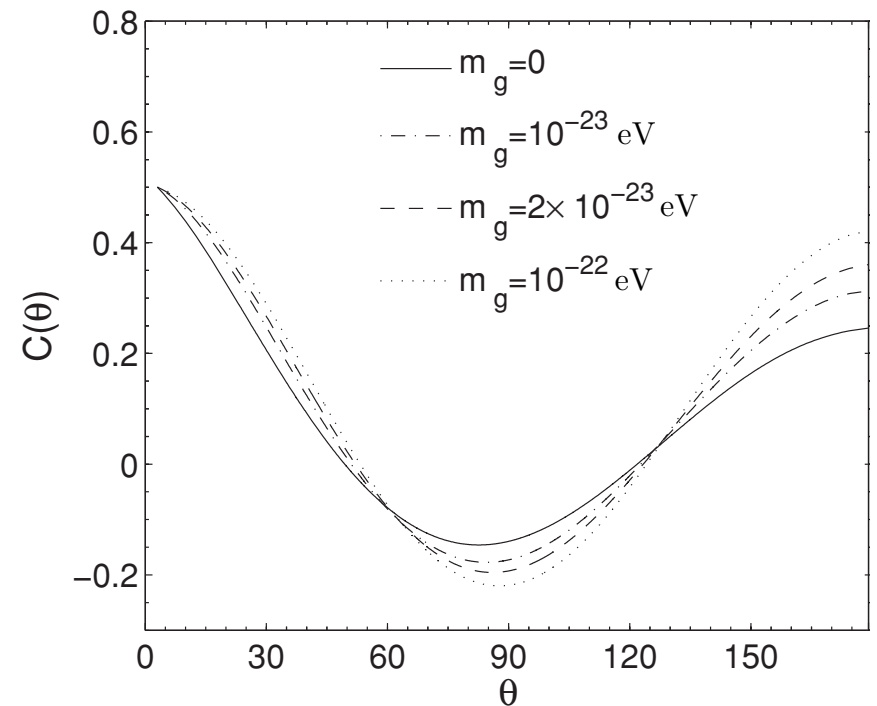


Testing the properties of nano-Hertz gravitational waves



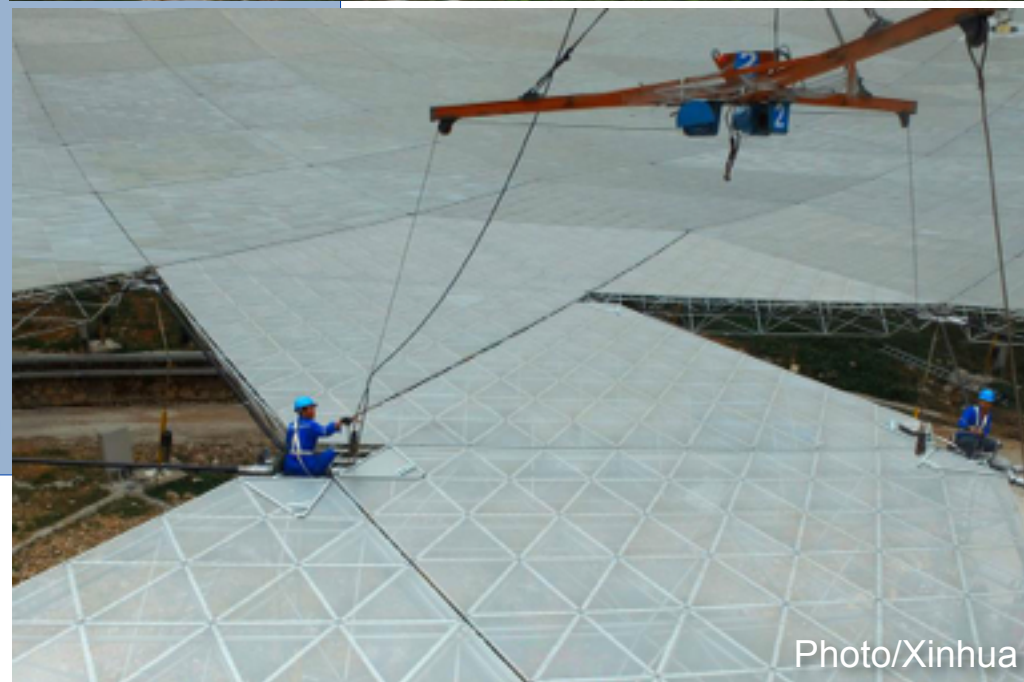
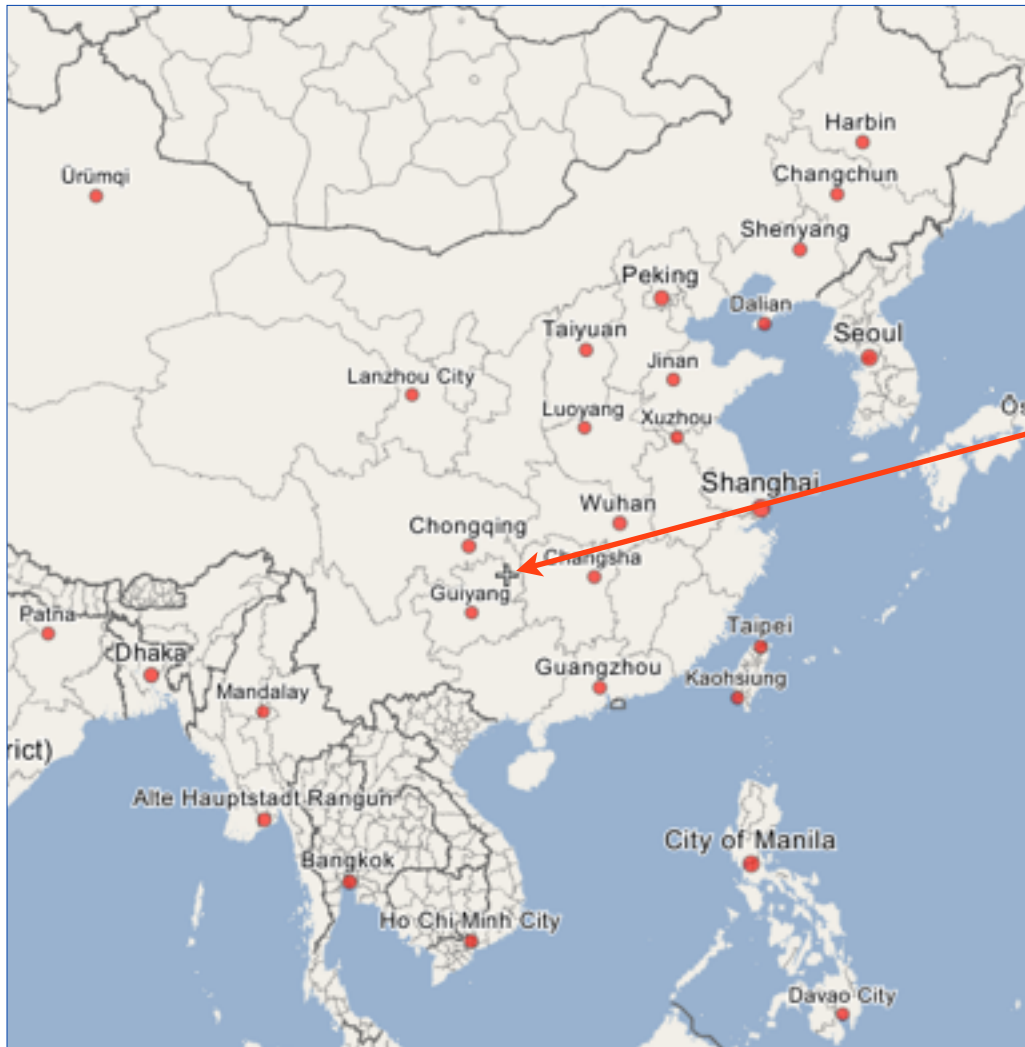
$$\mathbf{k}_g(\omega_g) = \frac{(\omega_g^2 - \omega_{\text{cut}}^2)^{\frac{1}{2}}}{c} \hat{\mathbf{e}}_z$$

$$\omega_{\text{cut}} \equiv m_g c^2 / \hbar$$



[Lee et al. 2008, Lee et al. 2010]

The Five hundred meter Aperture Spherical Telescope (FAST)



The Square Kilometre Array (SKA)

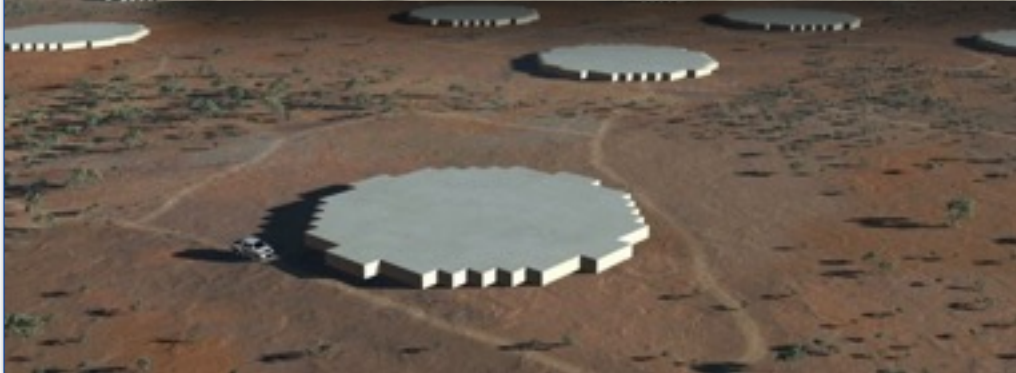
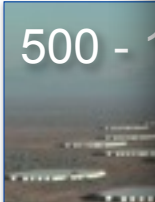
500 - 10000 (22000) MHz



MeerKAT



500 - 1



Summary

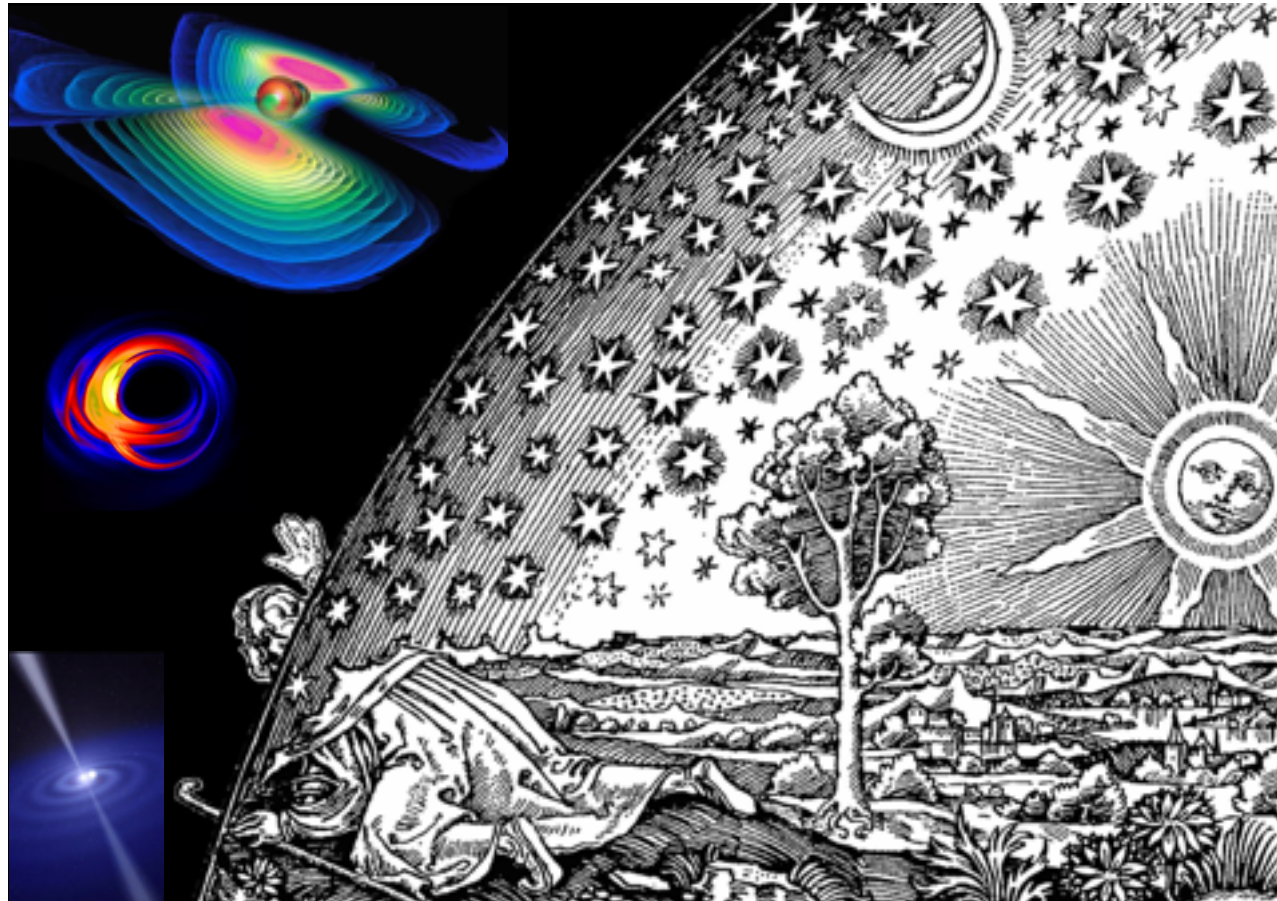


Solar system tests: 0.001%

Binary pulsar tests (quasi-stationary and radiative): 0.02%

Merging (stellar-mass) black holes: 4%

Tight constraints on many alternatives to GR



Thank you!

