

Probability and Effects of Large Stress Tensor Fluctuations

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Refs. [arXiv:1508.02359](#),
[PRD 92, 105008 \(2015\)](#);
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Physical states are generally not stress tensor eigenstates

→ stress tensor fluctuations

Effects of stress tensor fluctuations:

- 1) Force fluctuations on material bodies
- 2) Passive quantum fluctuations of spacetime geometry

Distinct from the active fluctuations from the dynamical degrees of freedom of gravity itself,
but still a quantum gravity effect

Probability distribution for quantum stress tensor fluctuations

Need to average the operator over a finite spacetime region

Expect the vacuum probability distribution to have a lower negative cutoff at the quantum inequality bound on expectation values in an arbitrary state - lowest eigenvalue of the averaged operator.

No upper cutoff, but a slowly falling tail which gives the probability of large positive fluctuations.

T = a normal ordered quadratic operator
averaged in time by $f_\tau(t)$

τ = characteristic time scale

Example: asymptotic probability distribution for
Lorentzian sampled energy density of the
EM field

C. Fewster, T. Roman & LF (2012)

$$x = T \tau^4$$

$$P(x) \sim c_0 x^{-2} e^{-a x^{1/3}} \\ x \gg 1 \quad c_0 \approx a \approx 0.96$$

Large positive fluctuations are more likely than
one might expect, and eventually vacuum
fluctuations dominate over thermal fluctuations.

More general sampling functions

$\hat{f}(\omega)$ = Fourier transform of $f_\tau(t)$

The rate of decrease of $\hat{f}(\omega)$ is crucial in determining the rate of decay of $P(x)$.

Consider $\hat{f}(\omega) = e^{-|\omega|^\alpha}$ $\tau = 1$ units

$\alpha = 1$ Lorentzian

$\alpha < 1$ a class of compactly supported sampling functions

C. Fewster & J. Louko,
S. Johnson

Switch-on behavior as $t \rightarrow 0^+$:

$$f(t) \sim D t^{-\mu} e^{-w t^{-\nu}} \quad \nu = \frac{\alpha}{1 - \alpha}$$

These compactly supported functions are infinitely differentiable, non-negative functions which are strictly zero outside of a finite interval.

If a measurement is to begin at a finite time in the past and end at a finite time in the future, these seems to be better choices than analytic functions, such as the Lorentzian or Gaussian, which have infinite tails.

Asymptotic form for the probability distribution:

$$P(x) \sim c_0 x^b e^{-ax^c} \quad \text{where} \quad c = \frac{\alpha}{3}$$

Implications:

- 1) The form of the switching function is very important.
- 2) Switching in a finite time interval can produce large stress tensor fluctuations.

Now consider averaging in space as well as time.

C. Fewster & LF (2016)

Let $g(\mathbf{x})$ be a sampling function in space and write the space and time averaged operator as, e.g.,

$$T = \int dt d^3x f(t) g(\mathbf{x}) : T_{tt}(\mathbf{x}, t) :$$

Assume that $g(\mathbf{x}) = g(|\mathbf{x}|)$ has the same functional form as $f(t)$, but with characteristic width $s < \tau$.

This leads to a transition in the rate of decrease of the tail of the probability distribution, $P(x)$, at some point $x \approx x_c$

$$1 \ll x < x_c \quad P(x) \approx c_0 x^b e^{-a x^{\alpha/3}}$$

worldline case

$$x > x_c \quad P(x) \approx c'_0 x^{b'} e^{-a' x^\alpha}$$

where $x_c \approx \left(\frac{\tau}{s}\right)^3$

Thus, if $\tau \gg s > 0$, the worldline form holds for a finite range before transitioning to a somewhat more rapid fall off, but still slower than an exponential if $\alpha < 1$

Possible physical effects of large fluctuations:

- 1) Large Ricci tensor fluctuations and hence large focussing fluctuations for timeline geodesics. The world tube of the geodesics could define the space and time sampling.
- 2) Enhanced barrier penetration rates by quantum particles. Large radiation pressure fluctuations can sometimes push particles over the barrier more quickly than they tunnel through the barrier. The shape of the barrier can define a sampling function.

H. Huang and LF (2016)

Summary

- 1) The probability distribution for stress tensor fluctuations requires averaging at least in time.
- 2) The distributions fall slower than exponentially, leading to the possible dominance of vacuum fluctuations over other effects.
- 3) Measurements in a finite time interval lead to especially slow fall off, but are sensitive to the sampling function
- 4) Spatial averaging cause more rapid fall off, but can still be slower than exponential.
- 5) Possibility of observable effects of large vacuum fluctuations, such as in quantum barrier penetration?