Squeezed vacua in loop quantum gravity

Jonathan Guglielmon

In collaboration with E. Bianchi, L. Hackl, N. Yokomizo

Institute for Gravitation and the Cosmos
The Pennsylvania State University

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Overview

Goal of this talk: Introduce squeezed vacua in LQG as a new class of states that contain controllable correlations

Motivation: Want states that have a simple parameterization and contain the type of long-range correlations expected of semiclassical states

Outline

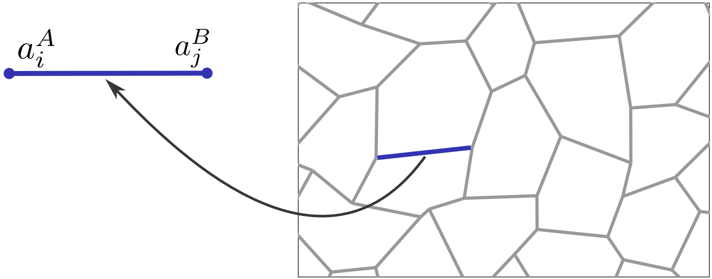
1. Review bosonic representation
2. Introduce squeezed vacua on the bosonic lattice
3. Implement projection via loop expansion
4. Discuss properties of squeezed vacua

Review of bosonic representation

Consider a graph Γ with N nodes and L links

Associate a pair of bosonic oscillators a_i^A to each end-point of each link

- Index $A = 0, 1$ runs over *pairs*
- Index $i = 1, \ldots, 2L$ runs over *end-points* of links



- Yields a bosonic Hilbert space H_B with $4L$ degrees of freedom

Girelli, Livine, (2005); Borja, Freidel, Garay, Livine, (2011); Livine, Tambornino (2012)

Review of bosonic representation

- For each end-point i:

$$\vec{J}_i = \frac{1}{2}\vec{\sigma}_{AB} a_i^{A\dagger} a_i^B \qquad I_i = \frac{1}{2}\delta_{AB} a_i^{A\dagger} a_i^B$$

- For each link ℓ:

$$h_\ell{}^A{}_B = (2I_t + 1)^{-1/2}(\epsilon^{AC} a^\dagger_{t\,C} a^\dagger_{s\,B} - \epsilon_{BC} a_t^A a_s^C)(2I_s + 1)^{-1/2}$$

- $H_\Gamma \subset H_B$ satisfying two constraints:
  (1) Area matching (links): $C_\ell = I_{s(\ell)} - I_{t(\ell)} \approx 0$
  (2) Gauge invariance (nodes): $\vec{G}_n = \sum_{i\in n} \vec{J}_i \approx 0$

Girelli, Livine, (2005); Borja, Freidel, Garay, Livine, (2011); Livine, Tambornino (2012)

Squeezed vacua on the bosonic lattice

Squeezed vacua in H_B:

- Let $|0\rangle$ be the vacuum associated with the a_i^A
- Consider the following set of complex matrices:

$$\mathcal{D} = \{\gamma \in \mathrm{Mat}(4L, \mathbb{C}) | \gamma = \gamma^T \text{ and } \mathbb{1} - \gamma\gamma^\dagger > 0\}$$

Squeezed vacuum state

Given $\gamma \in \mathcal{D}$, we have the squeezed vacuum state

$$|\gamma\rangle = \exp\left(\tfrac{1}{2}\gamma_{AB}^{ij} a_i^{A\dagger} a_j^{B\dagger}\right)|0\rangle$$

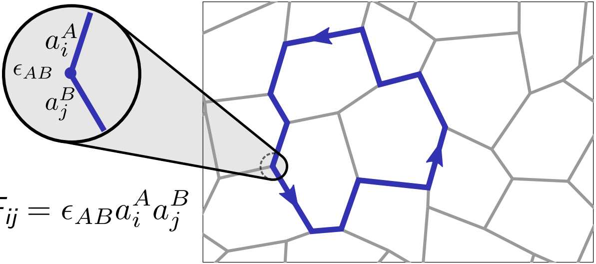
- Squeezing matrix γ encodes correlations
- Need to project: $P_\Gamma|\gamma\rangle \in H_\Gamma$

Bianchi, Hackl, Yokomizo (2015)

Loop expansion of P_Γ

Implement the projection using a loop expansion:

- Consider a non-repeating loop α in the graph Γ



$$F_{ij} = \epsilon_{AB} a_i^A a_j^B$$

- Define a loop operator F_α as

$$F_\alpha \equiv \prod_{\langle i,j \rangle \in \alpha} F_{ij}$$

Bianchi, Guglielmon, Hackl, Yokomizo (2016)

Loop expansion of P_Γ

- Define a *multiloop* Φ as a collection of loops with multiplicities:
$$\Phi = \{\alpha_1^{m_1}, \ldots, \alpha_k^{m_k}\}$$
- Corresponding multiloop operator:
$$F_\Phi \equiv \prod_{\alpha \in \Phi} (F_\alpha)^{m_\alpha}$$
- $F_\Phi^\dagger$ acts on the vacuum $|0\rangle$ to excite the multiloop

Loop expansion of P_Γ

The states $F_\Phi^\dagger|0\rangle$ satisfy both constraints and are complete in H_Γ

Multiloop resolution of the identity

The projector $P_\Gamma : H_B \to H_\Gamma$ can be written

$$P_\Gamma = \sum_\Phi \frac{1}{\prod_n (j_n+1)! \prod_\ell (2j_\ell)!} F_\Phi^\dagger |0\rangle\langle 0| F_\Phi$$

where $j_n = \sum_{\ell \in n} j_\ell$

Apply to squeezed vacua $|\gamma\rangle \in H_B$ to obtain

$$|\Gamma, \gamma\rangle = P_\Gamma |\gamma\rangle \in H_\Gamma$$

Loop expansion of squeezed vacua

Overlap of $F^\dagger_\Phi|0\rangle$ with a squeezed state $|\gamma\rangle$ is

$$\mu_\Phi(\gamma) \equiv \langle 0|F_\Phi|\gamma\rangle = \int \bar{Z}_\Phi \exp\left(-z_i^A \bar{z}^i_A + \frac{1}{2}\gamma^{ij}_{AB} z_i^A z_j^B\right) \prod_{i,A} \frac{dz_i^A d\bar{z}_i^A}{\pi}$$

where Z_Φ is given by

$$Z_\Phi \equiv \prod_{\alpha\in\Phi} \prod_{\langle i,j\rangle\in\alpha} (\epsilon_{AB} z_i^A z_j^B)^{m_\alpha}$$

Projected squeezed vacuum

The projected squeezed vacuum state $|\Gamma,\gamma\rangle = P_\Gamma|\gamma\rangle$ can be written

$$|\Gamma,\gamma\rangle = \sum_\Phi \frac{\mu_\Phi(\gamma)}{\prod_n (j_n+1)! \prod_\ell (2j_\ell)!} F^\dagger_\Phi|0\rangle$$

Properties of squeezed vacua

Properties of the states $|\Gamma, \gamma\rangle$:

- Overcomplete in H_Γ

$$P_\Gamma = \int |\Gamma, \gamma\rangle\langle\Gamma, \gamma| \, d\mu(\gamma)$$

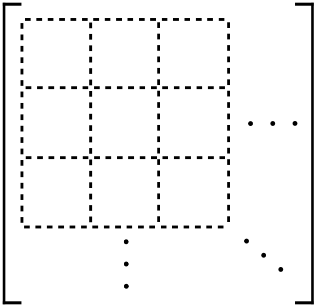
  where $d\mu(\gamma)$ is the Haar measure on $Sp(4L, \mathbb{R})$
- Enough freedom to control 1-point and 2-point functions
- Have a compact generating function

Properties of squeezed vacua

- Enough freedom to control 1-point and 2-point functions:

  (a) Divide squeezing matrix into blocks (i.e. by links or nodes)

$$\gamma =$$



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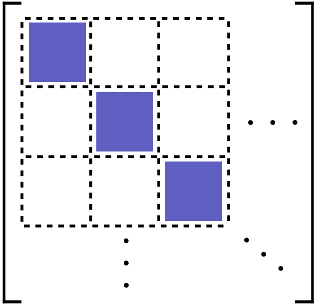
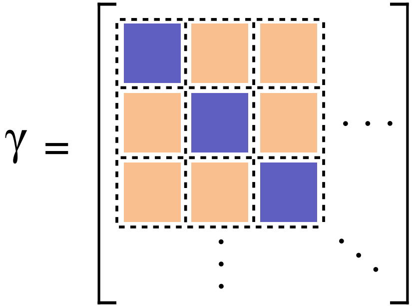


  (b) Use diagonal blocks to control **1-point function**

Properties of squeezed vacua

- Enough freedom to control 1-point and 2-point functions:

  (a) Divide squeezing matrix into blocks (i.e. by links or nodes)



  (b) Use diagonal blocks to control **1-point function**
  (c) Use off-diagonal blocks to control **2-point function**
  (also affect 1-point function)

Properties of squeezed vacua

- Generating function:
  (a) Decompose $|\Gamma, \gamma\rangle$ by spin:

$$|\Gamma, \gamma\rangle = \sum_{j_\ell} \frac{1}{\prod_n (j_n + 1)! (\prod_\ell (2j_\ell)!)^2} |\Gamma, \gamma, j_\ell\rangle$$

  (b) Assign a dummy variable x_i to each end-point of each link and form the following $4L \times 4L$ node-wise block-diagonal matrix:

$$(\mathcal{M}^\dagger)^{AB}_{ij} = \begin{cases} \epsilon^{AB} F^\dagger_{ij} x_i x_j & i, j \in \mathit{node} \\ 0 & \mathit{otherwise} \end{cases}$$

Generating function for squeezed vacua

Squeezed vacua projected at fixed spins j_ℓ can be written

$$|\Gamma, \gamma, j_\ell\rangle = \prod_\ell \left(\frac{\partial^2}{\partial x_{s(\ell)} \partial x_{t(\ell)}} \right)^{2j_\ell} \det(\mathbb{1} - \gamma \mathcal{M}^\dagger)^{-1/2} |0\rangle \Big|_{x_i = 0}$$

Summary of talk

Summary:

- Introduced squeezed vacua in LQG using bosonic representation
- Implemented projection with a loop expansion
- Squeezed vacua are overcomplete
- Squeezing matrix γ parameterizes the states and encodes correlations
- States can be written in compact form using a generating function