Constraints as evolutionary systems

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Dedicated to the memory of Sergio Dain.

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The constraints:

- initial data: $\boxed{(h_{ij}, K_{ij})}$ on a codimension-one surface $\boxed{\Sigma}$
  - $\dim(\Sigma)$, sources, signature of the ambient space
- it is an underdetermined system: 4 equations for 12 variables

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where D_a denotes the covariant derivative operator associated with h_{ab}

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Solving the constraints:

The conformal (elliptic) method **Lichnerowicz A (1944)** and **York J W (1972)**:

- replace h_{ij} and $K_{ij} - \frac{1}{3}\, h_{ij}\, K^l{}_l$ by $\widetilde{h}_{ij}$ and $\widetilde{K}_{ij}$ as

$$h_{ij} = \phi^4\, \widetilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\, h_{ij}\, K^l{}_l = \phi^{-2}\, \widetilde{K}_{ij}$$

- using these variables the constraints are put into the **semilinear elliptic system**

$$\widetilde{D}^l \widetilde{D}_l \phi - \tfrac{1}{8}\, \widetilde{R}\, \phi + \tfrac{1}{8}\, \widetilde{K}_{ij} \widetilde{K}^{ij}\, \phi^{-7} - \tfrac{1}{12}\, (K^l{}_l)^2\, \phi^5 = 0$$

where $\widetilde{D}_l$, $\widetilde{R}$, ... $\widetilde{h}_{ij}$

- $\widetilde{K}_{ij} = \widetilde{K}^{[L]}_{ij} + \widetilde{K}^{[TT]}_{ij}$, where $\widetilde{K}^{[L]}_{ij} = \left(\widetilde{D}_i X_j + \widetilde{D}_j X_i - \frac{2}{3}\, \widetilde{h}_{ij} \widetilde{D}^l X_l \right)$

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Any method should aim to provide

- the highest possible flexibility in choosing h_{ij} and K_{ij}
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- minimize the need to impose boundary conditions
  - they influence solutions to elliptic equations everywhere in their support

The conformal method: impressive mathematical developments since 1944 but ...

- either "constancy" of ${K^l}_l$ or "smallness" of the TT part of $\widetilde{K}_{ij}$ is required
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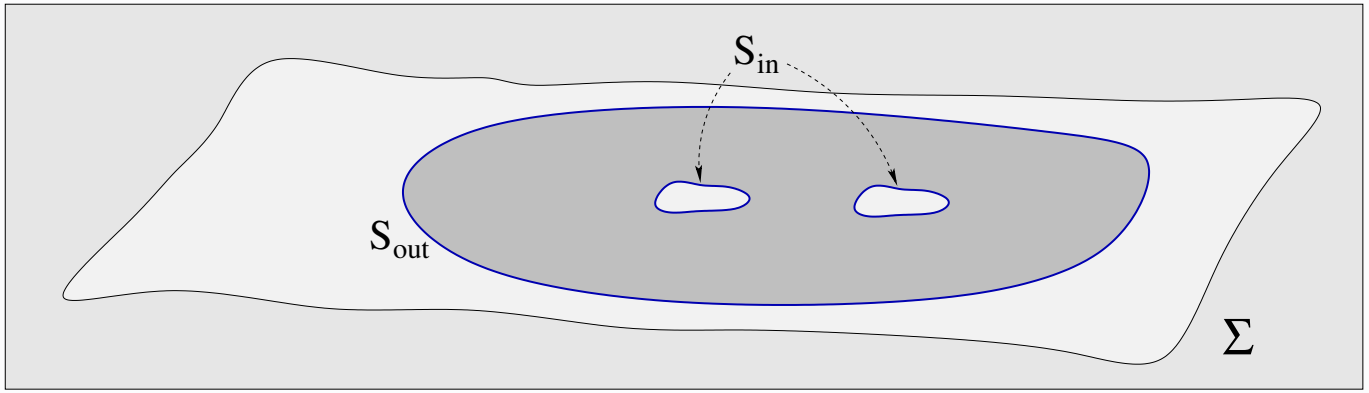

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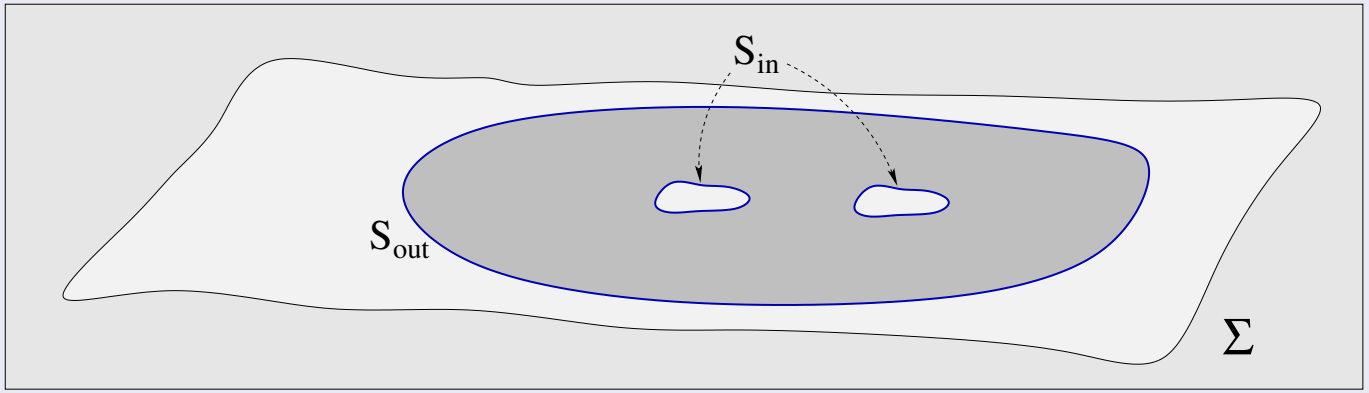

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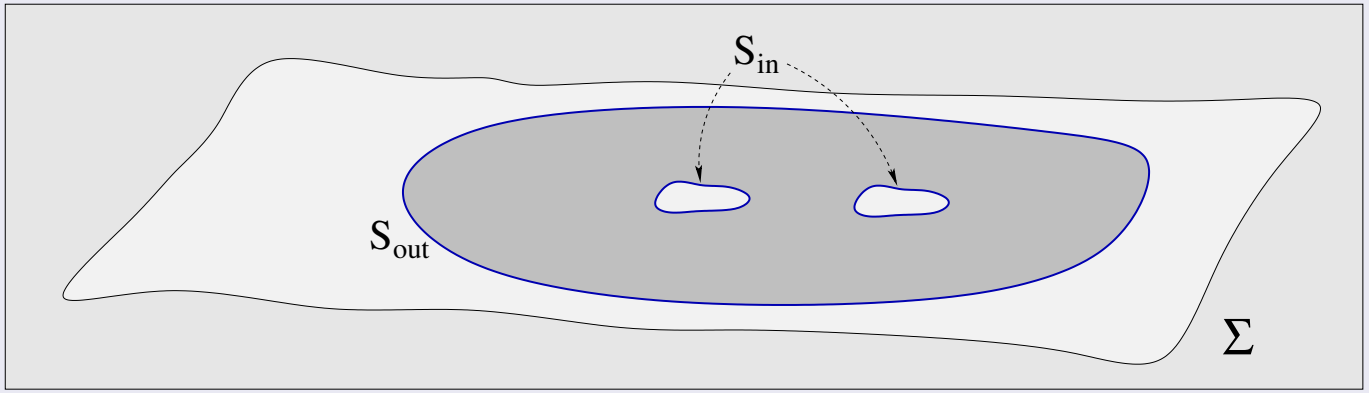

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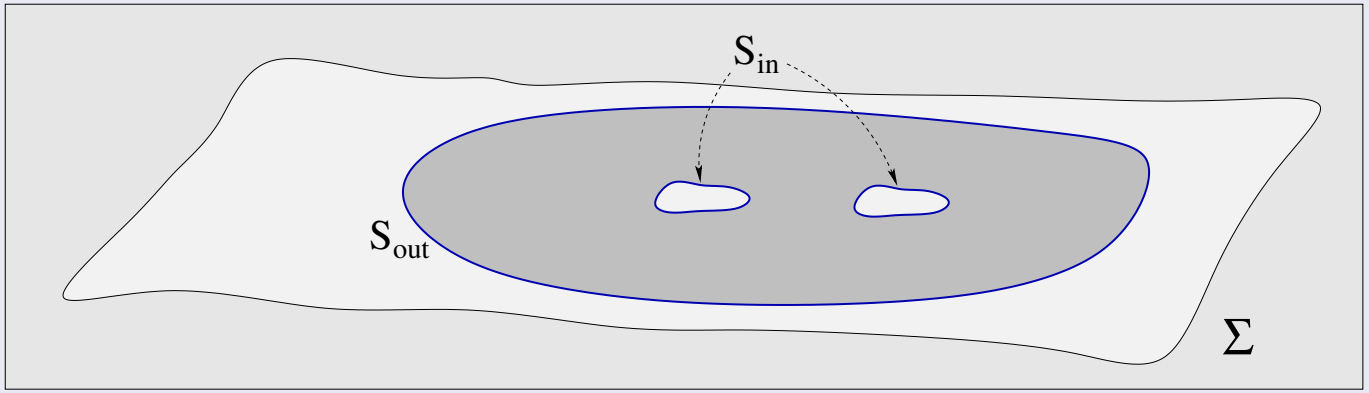

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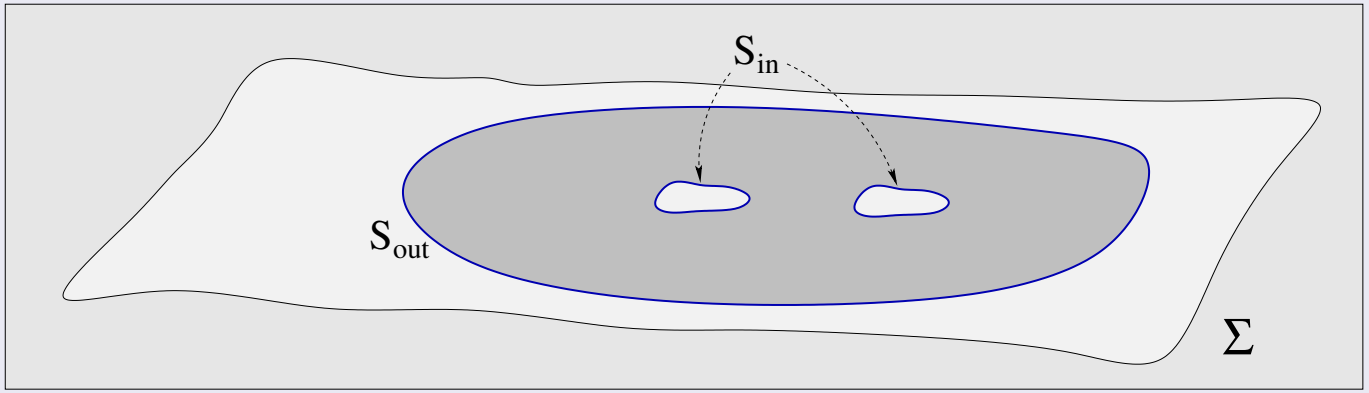

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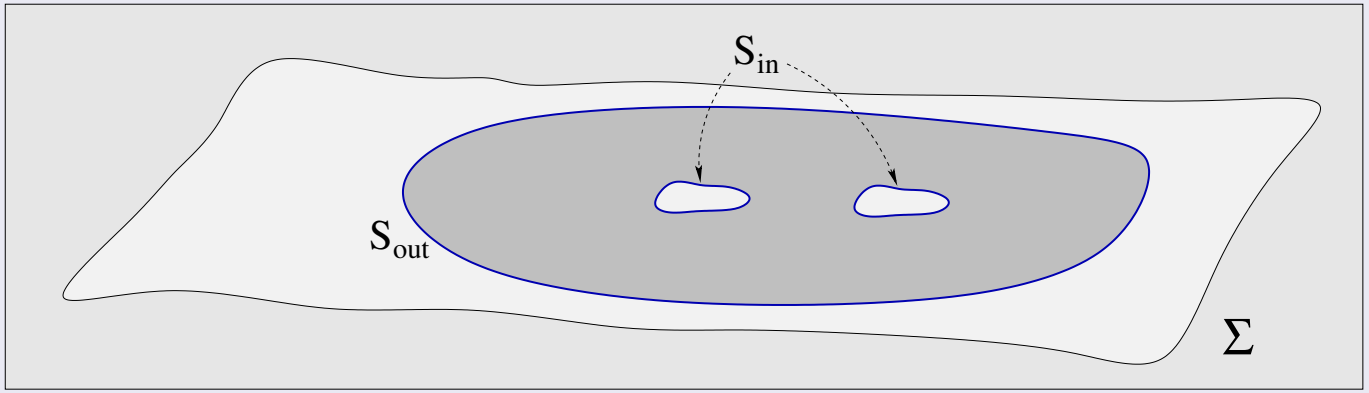

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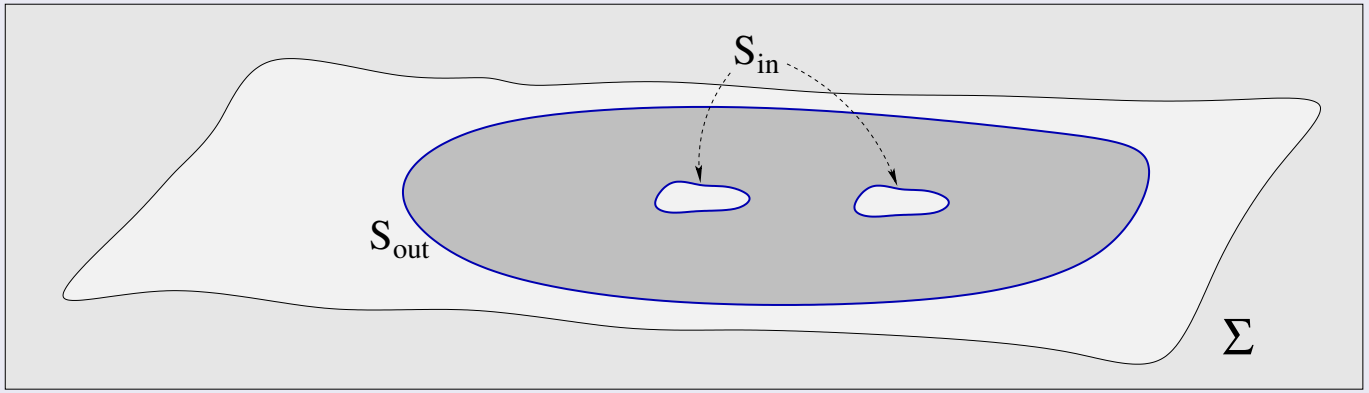

Constraints as evolutionary systems:

Outline:

1. **momentum constraint** as a **first order symmetric hyperbolic system**
2. **Hamiltonian constraint** as a **parabolic** or an **algebraic** equation
3. in either case the coupled constraints comprise an **evolutionary system**
   - **(local) existence and uniqueness** of solutions is guaranteed
   - **some global results** apply for the hyperbolic-parabolic formulation

Some of the basic properties:

- no conformal rescaling of h_{ij} and K_{ij}
  - for binary BHs an unprecedented complete control on the ADM parameters
- free of using inner boundary conditions
  - requires data for the constrained variables only on a surface surrounding the binary BH system in the asymptotic region as values in the strong field regime are then *determined* by integrating the evolutionary form of the constraints

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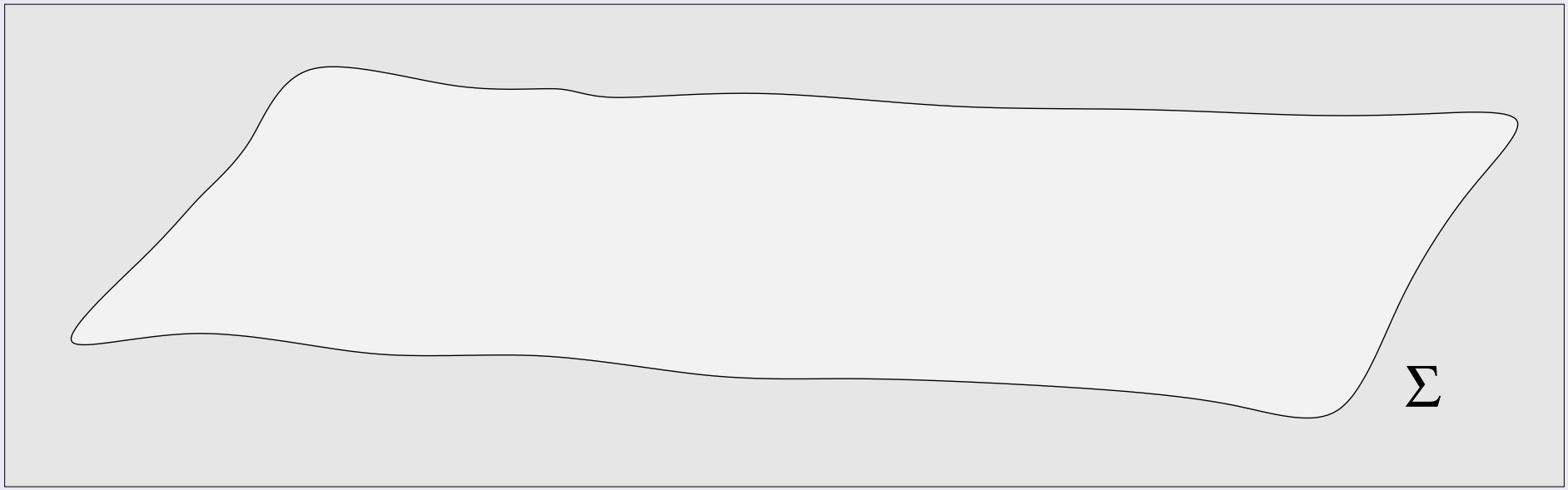
Assumptions on the topology of Σ:

- **Assume:** there exists a smooth function $\rho : \Sigma \to \mathbb{R}$ such that the $\rho = const$ surfaces (denoted also by $\mathscr{S}_\rho$) provide "a one-parameter foliation" of Σ.
- This (apart from some isolated critical points) allows to define the normal $D_i\rho$ to these level surfaces throughout Σ.



- if $\nexists$ origin in Σ it may be given as a product space: $\Sigma \approx \mathbb{R} \times \mathscr{S}$, where $\mathscr{S}$ is a 2-dimensional manifold of arbitrary topology
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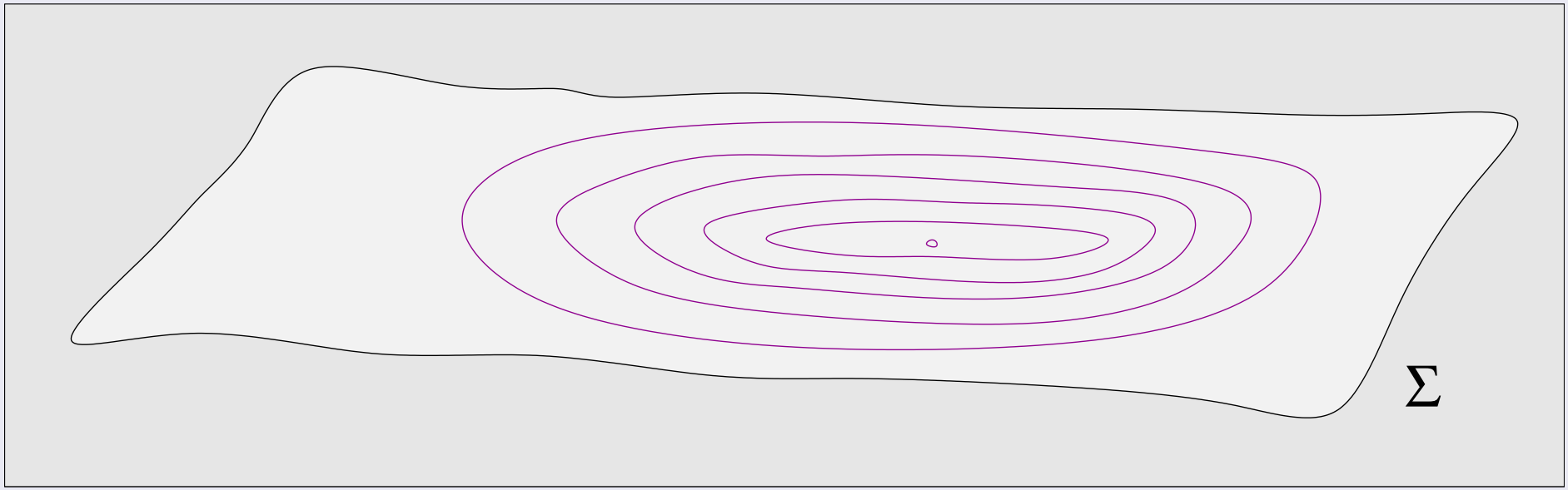

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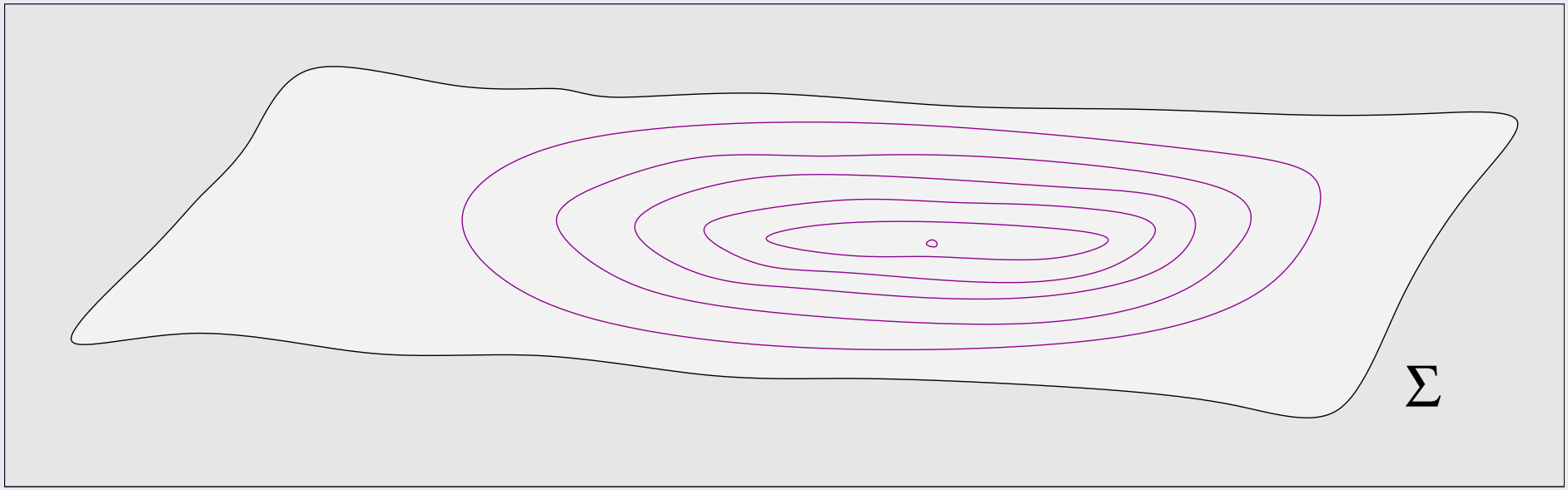

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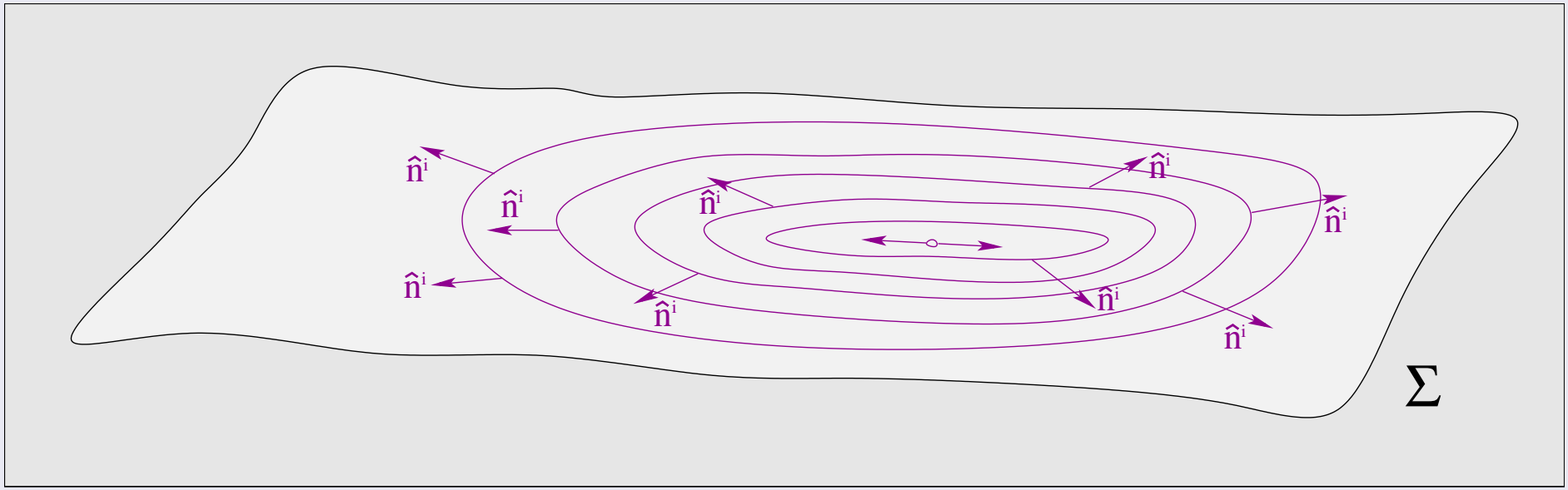

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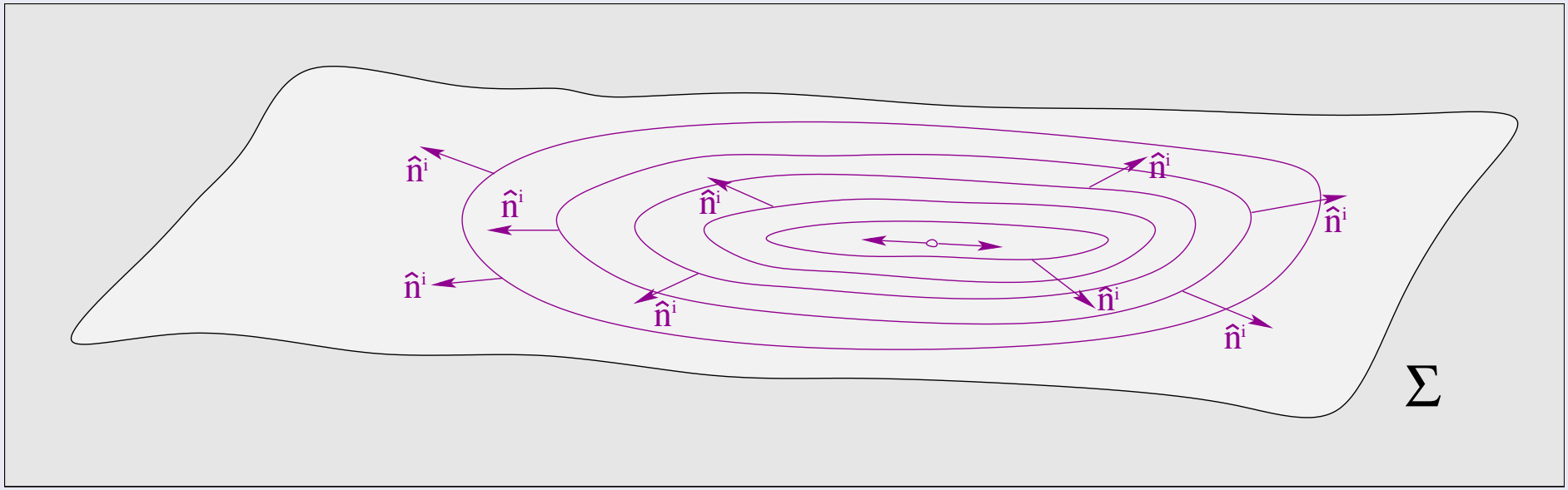

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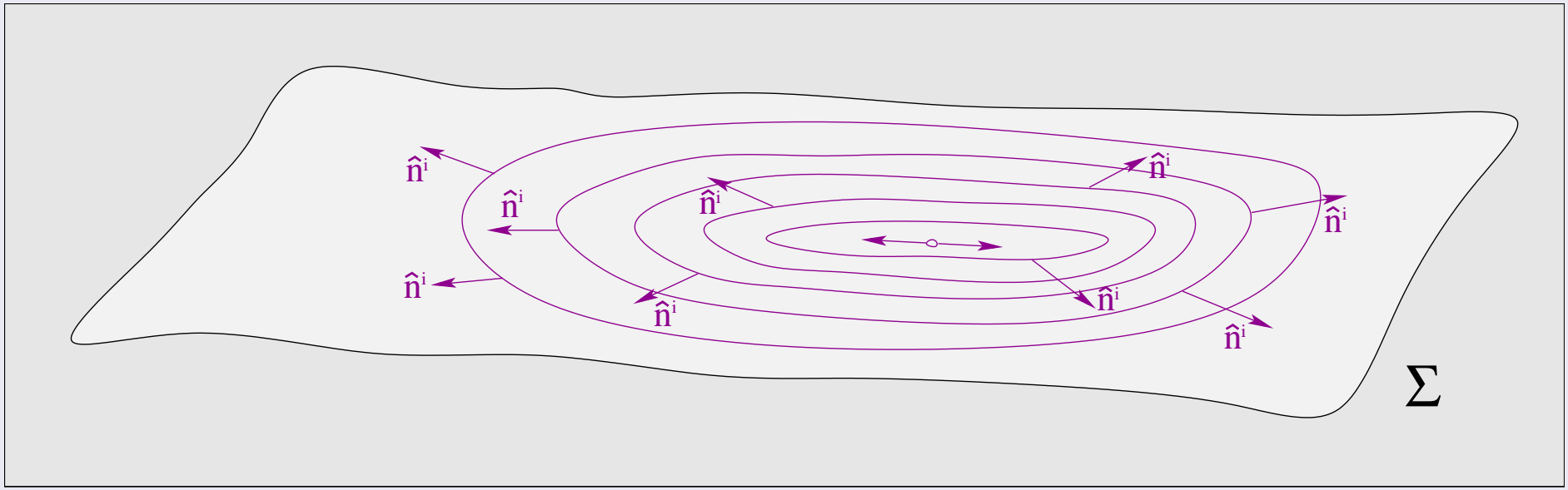

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The "evolution vector field" in a 2+1 decomposition:

- consider an 2-parameter family of smooth curves intersecting each of the $\mathscr{S}_\rho$ level surfaces precisely once
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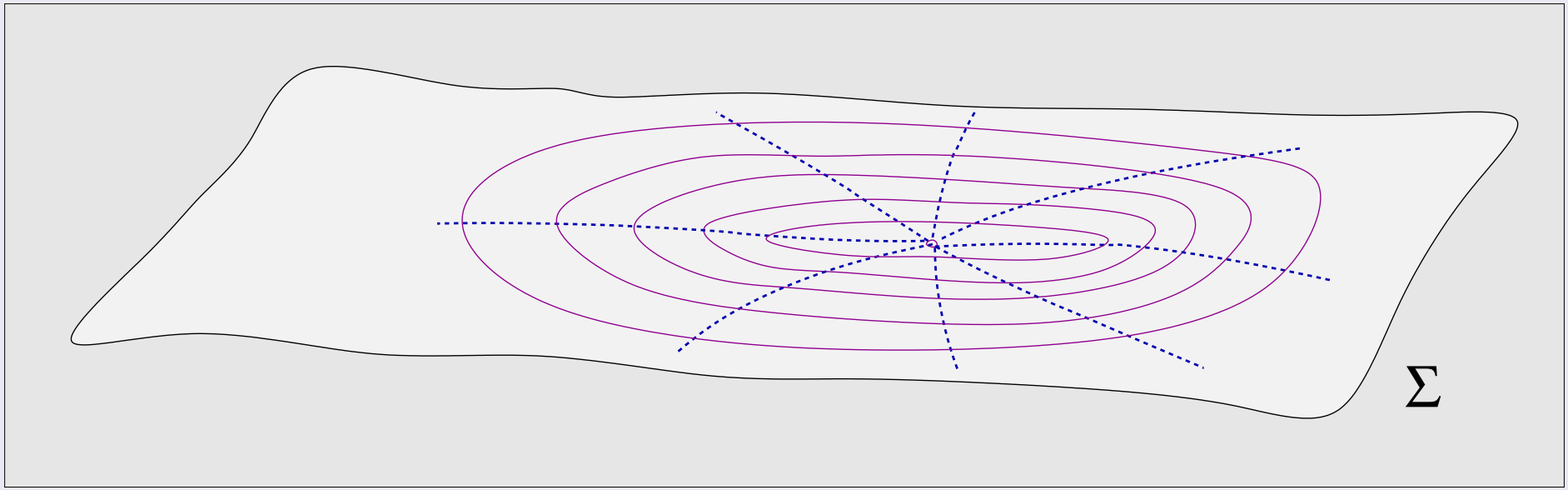

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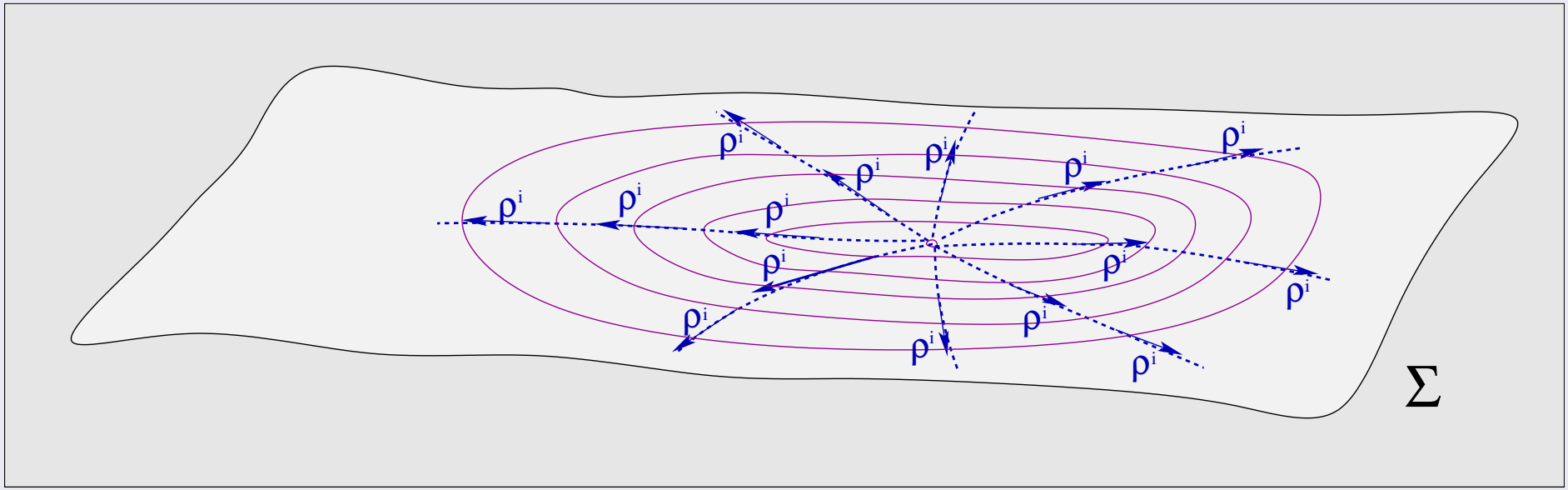

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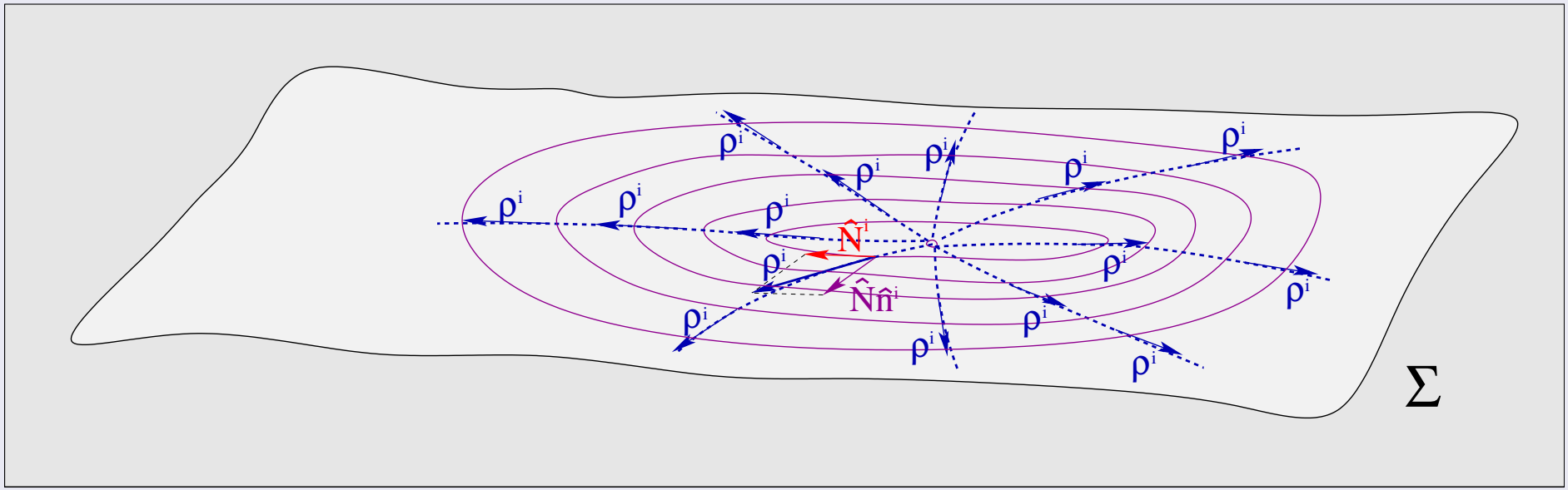

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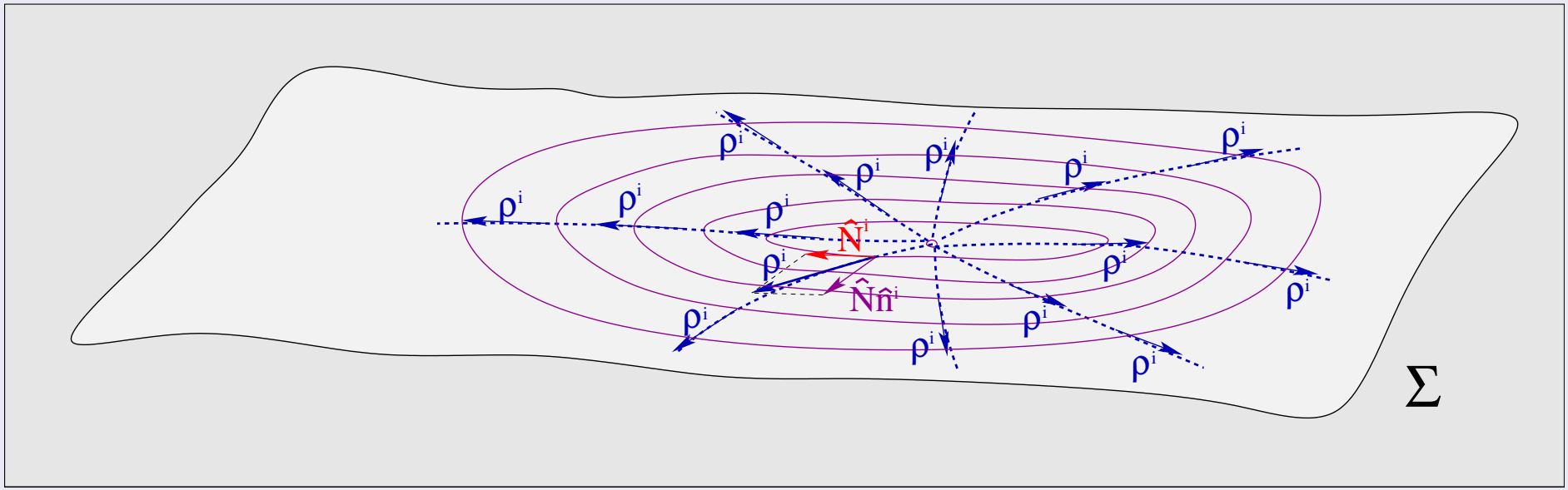

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$$\boxed{D_e K^e{}_a - D_a K^e{}_e = 0}$$

First order symmetric hyperbolic system:

$$\mathscr{L}_{\widehat{n}}\mathbf{k}_i - \tfrac{1}{2}\widehat{D}_i(\mathbf{K}^l{}_l) - \widehat{D}_i\boldsymbol{\kappa} + \widehat{D}^l\mathring{\mathbf{K}}_{li} + (\widehat{K}^l{}_l)\,\mathbf{k}_i + \boldsymbol{\kappa}\,\dot{\widehat{n}}_i - \dot{\widehat{n}}^l\,\mathbf{K}_{li} = 0 \qquad (1)$$

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$$\boxed{{}^{(3)}R + \left(K^{e}{}_{e}\right)^2 - K_{ef}K^{ef} = 0}$$

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Summary:

Tha basic results:

the constraints can be put into evolutionary systems:

1. **momentum constraint** as a **first order symmetric hyperbolic system**
2. **the Hamiltonian constraint** as a **parabolic** or an **algebraic** equation
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