

Effective Phase Space of a Cosmological Scalar Field

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Outline

What I am trying to do?

The Hamiltonian Approach to Cosmology

A Simple Scenario: Canonical, Minimal Scalar Fields

Extending to Conformal Coupling and k-Inflation

Tackling Horndeski

What's the Point?

Hamiltonian Cosmology

The Basics

We want to construct a Hamiltonian - we need a time!

3 + 1 decomposition

↳ ADM formalism

In Cosmology there is a natural choice of time

↳ Frame where CMB is homogeneous and isotropic (no dipole)



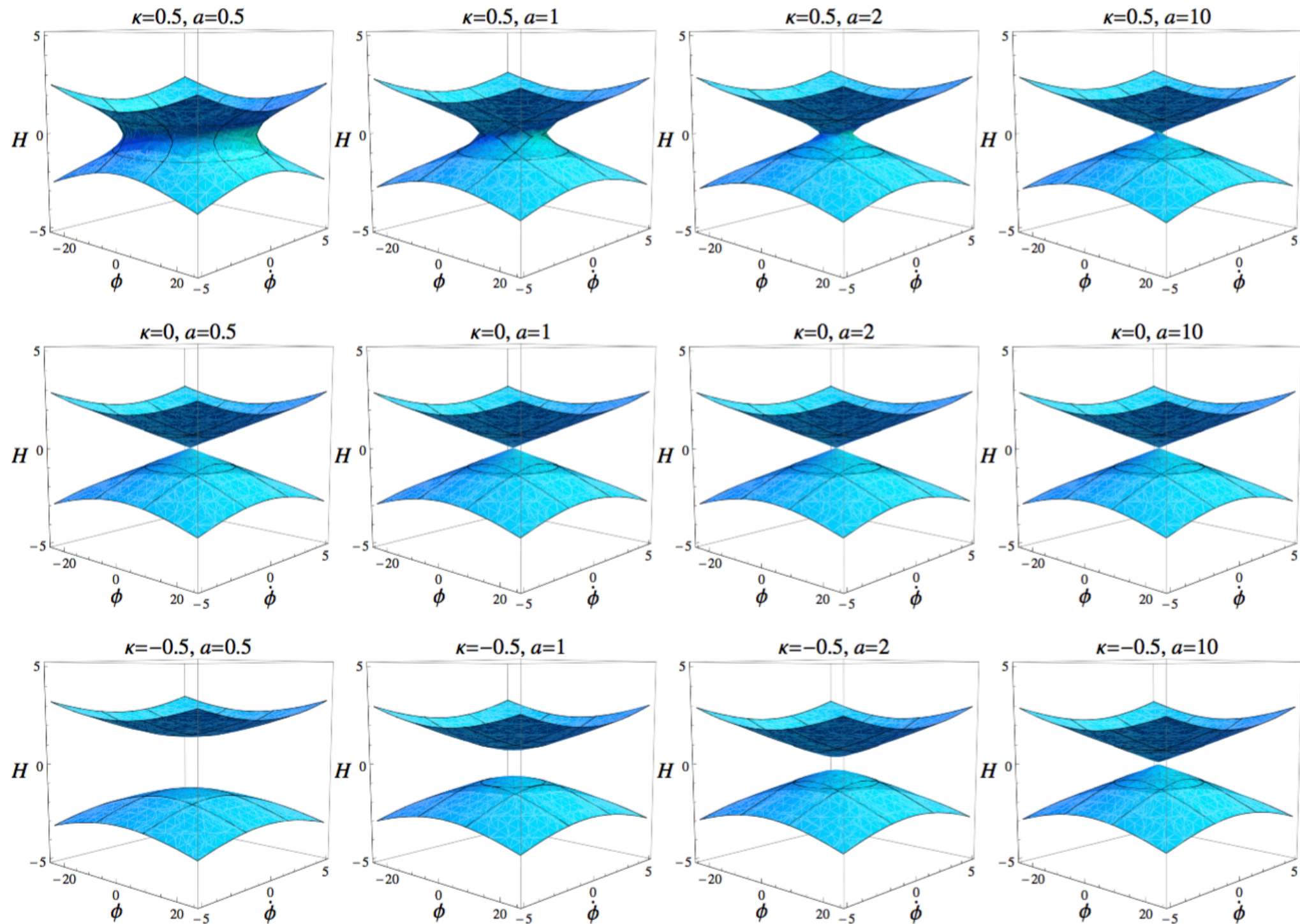
Hamiltonian Flow Vector

A geometric representation of
Hamilton's equations

$$X_{\mathcal{H}} = \frac{\partial \mathcal{H}}{\partial p_i} \frac{\partial}{\partial q^i} - \frac{\partial \mathcal{H}}{\partial q^i} \frac{\partial}{\partial p_i}$$

Tangent to the trajectory
through phase space

Hamiltonian Constraint



Vector Field Invariant Maps

$$X_p(\psi^* f) = \tilde{X}_{p'}(\psi^* f) \\ p, p' \in \psi^{-1}(q)$$

$$X_p(\psi^* f) = \tilde{X}_q(f) \\ p \in \psi^{-1}(q)$$

*Creating an effective
phase space*

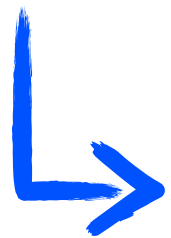
"The images of integral curves that are distinct under a vector invariant map do not intersect. Therefore, given two integral curves in M , not mapped onto each other in N , their images cannot intersect."

Remmen and Carroll, Attractor solutions in scalar-field cosmology, arXiv: 1309.2611

A Simple Scenario

What is simple?

Flat, homogeneous, isotropic



Originally inspired by background dynamics during inflation

No fun. GR and canonical scalar field

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + X + V(\phi) \right]$$

As presented in Remmen and Carroll, arXiv: 1309.2611

What do we do?

Hamiltonian

Calculate the Hamiltonian Flow Vector

$$X_{\mathcal{H}} = \frac{\partial \mathcal{H}}{\partial p_i} \frac{\partial}{\partial q^i} - \frac{\partial \mathcal{H}}{\partial q^i} \frac{\partial}{\partial p_i}$$

Reparametrize in terms of "special coordinates"

$$X_{\mathcal{H}}^{(\dot{\phi})} = X_{\mathcal{H}}^{(p_{\phi})} \frac{\partial \dot{\phi}}{\partial p_{\phi}} + X_{\mathcal{H}}^{(p_a)} \frac{\partial \dot{\phi}}{\partial p_a} \quad X_{\mathcal{H}}^{(H)} = X_{\mathcal{H}}^{(p_{\phi})} \frac{\partial H}{\partial p_{\phi}} + X_{\mathcal{H}}^{(p_a)} \frac{\partial H}{\partial p_a}$$

Apply the Hamiltonian Constraint

As presented in Remmen and Carroll, arXiv: 1309.2611

Extensions

Or What I Have Done
arXiv: 1605.05995

Conformal Couplings

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \Omega(\phi) R + X + V(\phi) \right]$$

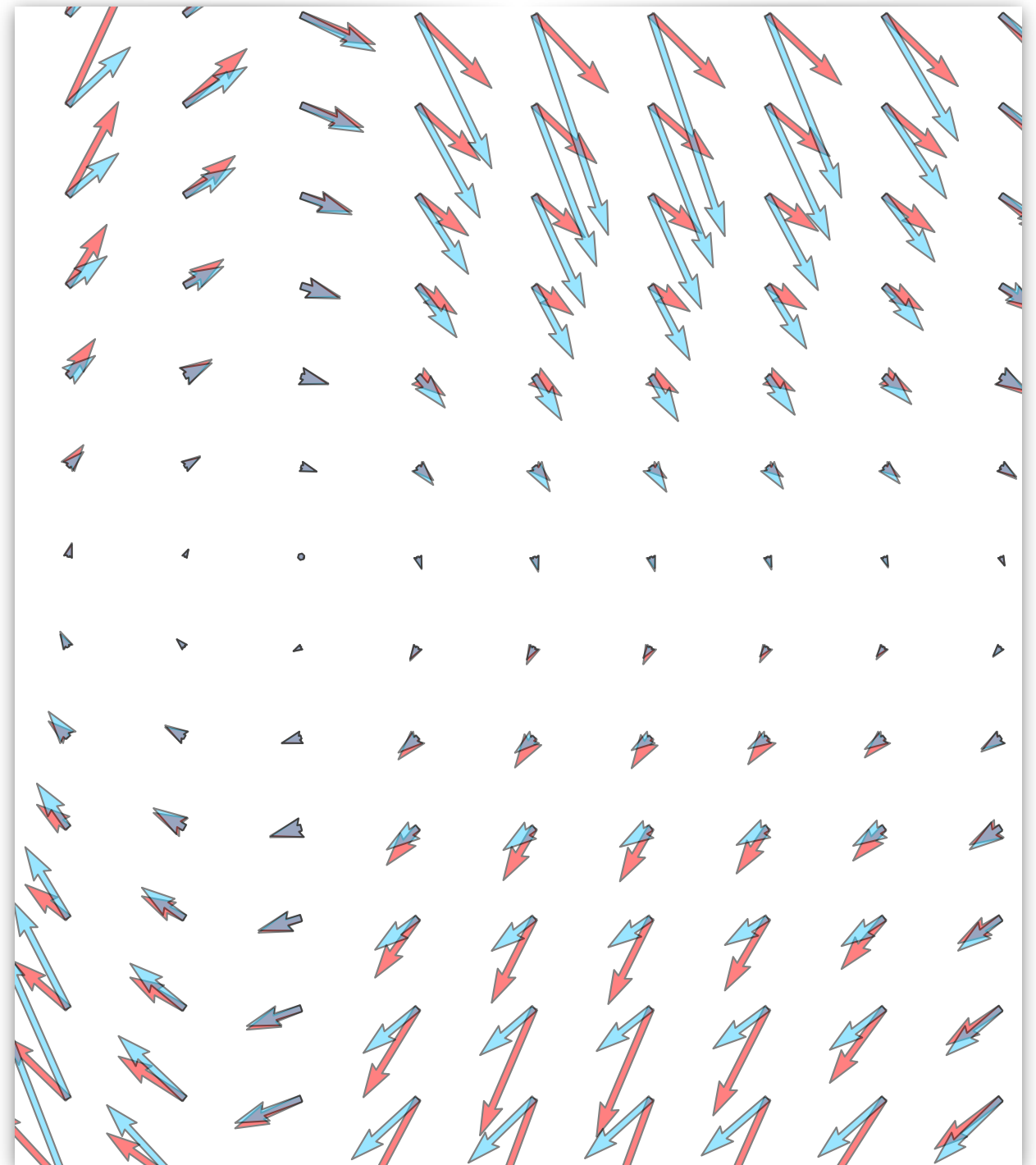
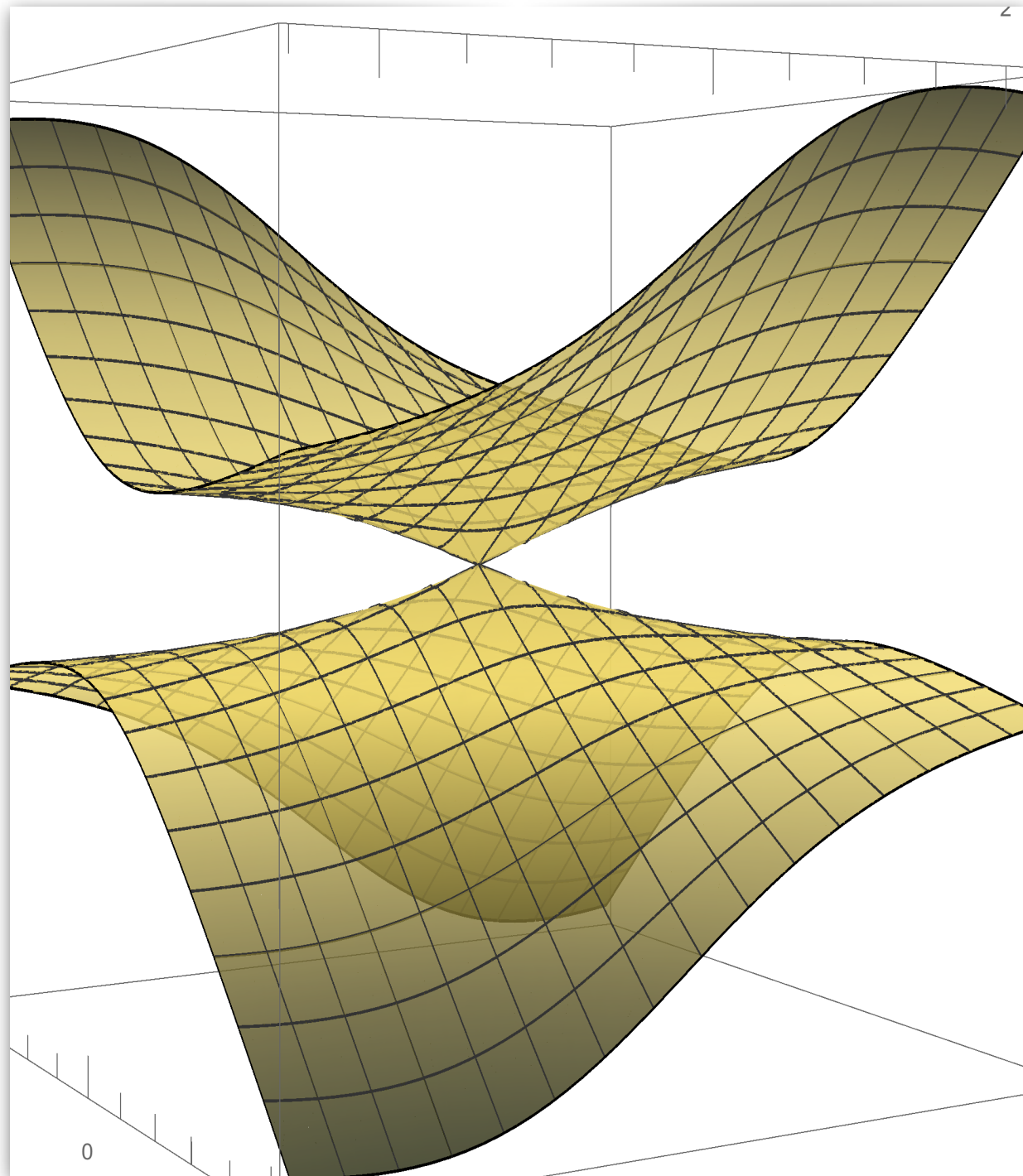
Good news: momentum and velocity still linearly related

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{a^3 \left(\dot{\phi} - 3H \partial_\phi \Omega(\phi) \right)}{N_{\text{lapse}}}$$

$$p_a = \frac{\partial L}{\partial \dot{a}} = \frac{-3a^2 \left(2H \Omega(\phi) + \dot{\phi} \partial_\phi \Omega(\phi) \right)}{N_{\text{lapse}}}$$

But there is also a complication...

Conformal Couplings



K-inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + P(\phi, X) \right]$$

Gives us a problem...

$$p_\phi = \frac{a^3 P_X \dot{\phi}}{N_{\text{lapse}}} \qquad p_a = \frac{6a^2 H}{N_{\text{lapse}}}$$

Cannot (necessarily) invert expression

$$X_{\mathcal{H}}^{(\dot{\phi})} = X_{\mathcal{H}}^{(p_\phi)} \frac{\partial \dot{\phi}}{\partial p_\phi} + X_{\mathcal{H}}^{(p_a)} \frac{\partial \dot{\phi}}{\partial p_a} \qquad X_{\mathcal{H}}^{(H)} = X_{\mathcal{H}}^{(p_\phi)} \frac{\partial H}{\partial p_\phi} + X_{\mathcal{H}}^{(p_a)} \frac{\partial H}{\partial p_a}$$

What is the answer?

Horndeski

arXiv: 1605.05995

Getting the partials

$$X_{\mathcal{H}}^{(\dot{\phi})} = X_{\mathcal{H}}^{(p_{\phi})} \frac{\partial \dot{\phi}}{\partial p_{\phi}} + X_{\mathcal{H}}^{(p_a)} \frac{\partial \dot{\phi}}{\partial p_a} \qquad X_{\mathcal{H}}^{(H)} = X_{\mathcal{H}}^{(p_{\phi})} \frac{\partial H}{\partial p_{\phi}} + X_{\mathcal{H}}^{(p_a)} \frac{\partial H}{\partial p_a}$$

We have to rewrite the partial derivatives

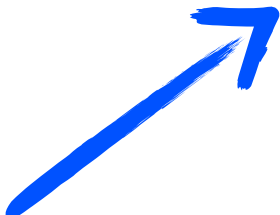
$$\frac{\partial p_i}{\partial p_j} = \frac{\partial \dot{\phi}}{\partial p_j} \frac{\partial p_i}{\partial \dot{\phi}} + \frac{\partial H}{\partial p_j} \frac{\partial p_i}{\partial H}$$

This is a system of equations we can invert

But what about the components $X_{\mathcal{H}}^{(p_{\phi})} X_{\mathcal{H}}^{(p_a)}$?

Revisiting Hamilton's Equations

$$X_{\mathcal{H}} = \frac{\partial \mathcal{H}}{\partial p_i} \frac{\partial}{\partial q^i} - \boxed{\frac{\partial \mathcal{H}}{\partial q^i} \frac{\partial}{\partial p_i}}$$

$$\frac{\partial \mathcal{H}}{\partial q_\alpha} = -\dot{p}_\alpha$$


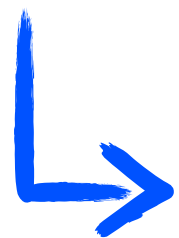
So the components $X_{\mathcal{H}}^{(p_\phi)} X_{\mathcal{H}}^{(p_a)}$ are time derivatives of momentum

What's the Point?

Why do we care?

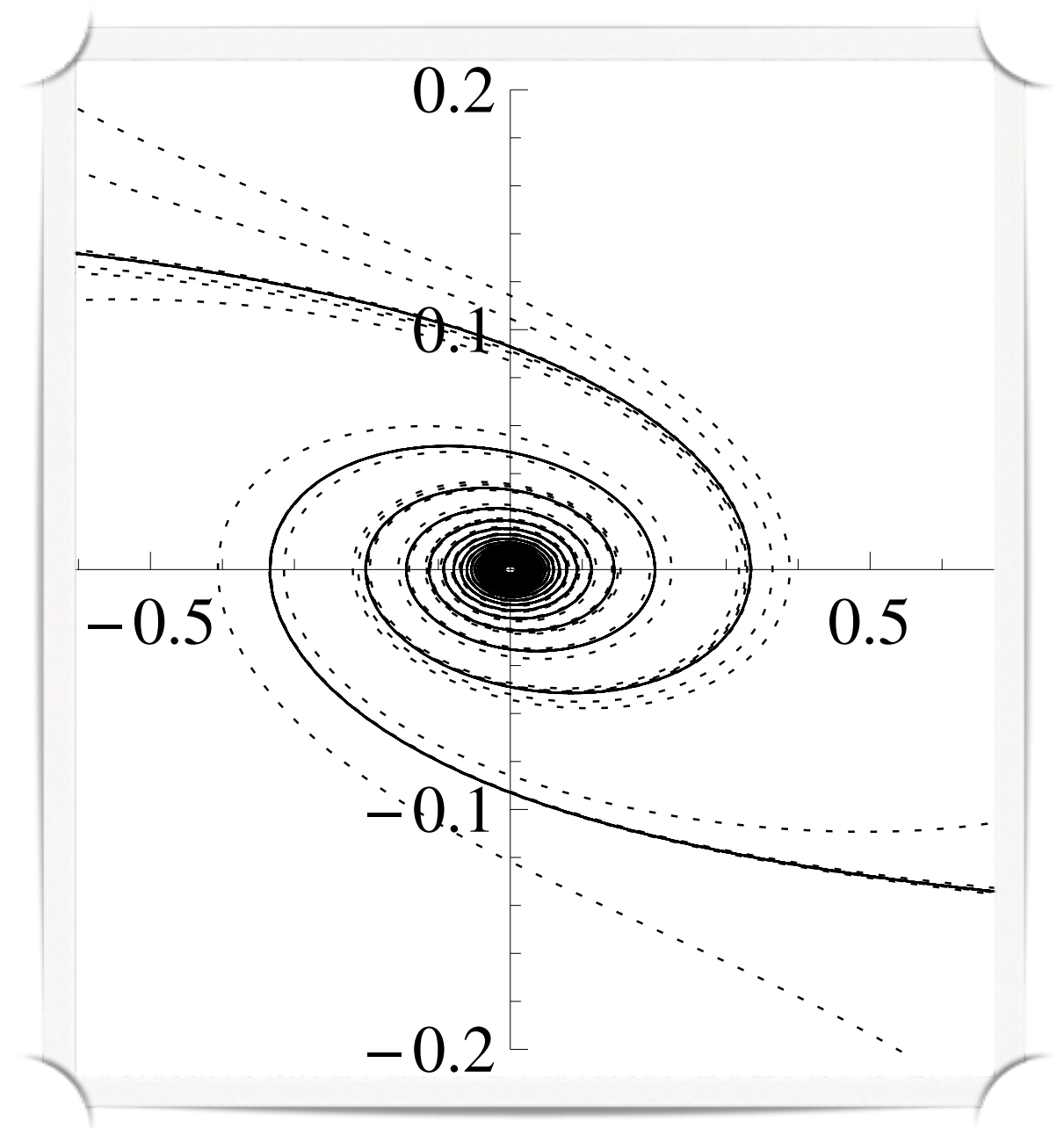
Define initial conditions

Convergence of trajectories



The slow-roll
paradigm

Measures



Remmen and Carroll, arXiv: 1309.2611