

On Symmetry Breaking and the Unruh Effect

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Why Symmetries?

D. Gaiotto, A. Kapustin, N. Seiberg, B Willet '15
arXiv:1412.5148

- Classify Operators
- If unbroken classify states
- If broken classify phases

Symmetry Realization

Fabri-Picasso Theorem: Let $|0\rangle$ be a translationally invariant vacuum and Q a generator of a symmetry $U = e^{-i\vartheta Q}$, then we have two possibilities:

- $Q|0\rangle = 0$ and the vacuum is an eigenstate of Q with null eigenvalue, so that $|0\rangle$ is invariant under U
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$$[H, Q] = 0$$

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Wigner-Weyl Realization:

$$Q|0\rangle = 0$$

$U|\phi\rangle \rightarrow |\phi'\rangle$ with degenerate energy

Nambu-Goldstone Realization:

$$|Q|0\rangle| = \infty$$

there is some X such that
 $\langle 0|[X, Q]|0\rangle \neq 0$

Fate of SSB in Rindler Space

C. T. Hill '85, Physics Letters

W. G. Unruh, N. Weiss '84, Physical Review D

In (inertial) Thermal Field Theory a broken symmetry at zero temperature usually is restored at a (finite) temperature, that is the field undergoes a phase transition. Can the Unruh temperature also induce a phase transition?

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Consider $\lambda\phi^4$ theory with spontaneous symmetry breaking. The order parameter is the VEV $\langle 0_M | \phi | 0_M \rangle$. Since the order parameter is a scalar it must be the same in all reference frames. Therefore if it is SSB in inertial quantization it must be also in Fulling's.

T. Ohsaku '04, Physics Letters B

P. Castorina, M. Finocchiaro '12, arXiv:1207.3677

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For $\lambda\phi^4$ theory $m^2 \rightarrow m^2 + \delta m^2(\lambda, \beta)$, and symmetry restoration occurs when $\frac{\partial V_{eff}}{\partial \phi} = 0$. Through the magic of K-Bessel functions, for large a , gives

$$m^2 + \frac{a^2}{4\pi^4} \frac{\lambda}{24} = 0 \Rightarrow T_c = \sqrt{\frac{-24m^2}{\lambda}}$$

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For the NO restoration camp

- No argument why the order parameter must be a scalar
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For the YES restoration camp

- The effective potential is a global property

$$S_I = i \int dx^4 \sqrt{-g} \lambda \phi^4 = i \int d\tau d\xi dx_{\perp}^2 e^{2a\xi} \lambda \phi^4.$$

What works: Algebra of Symmetries

$$[A_i^M, A_j^M] = iC_{ij}^k A_k^M,$$

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Assuming $A_M = O_R \otimes \mathbb{1}_L + \mathbb{1}_R \otimes O_L$

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From which follows

$$A_i^M = A_i^R - A_i^L$$

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- Result independent of the nature of the order parameter
- Role of Temperature Gradient vs. Intrinsic relativistic property

Unruh Effect: Probe Statement

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How about a extended probe with different phases?

Extended Probe: Definition and Mean Field

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{\Delta E}{2} \sigma^z \frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}(\tau))}{u^0(\tau)} \right) + \left(g \sigma^x \frac{\delta^{(3)}(\mathbf{x} - \mathbf{x}(\tau))}{u^0(\tau)} \phi(x) \right) - \mathcal{L}_\phi \right],$$

$$H = \frac{\Delta E}{2u^0} \sigma^z + \frac{g}{u^0} \sigma^x \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}V}} (c_{\mathbf{k}}^\dagger e^{iu^0 k_z/a} + c_{\mathbf{k}} e^{-iu^0 k_z/a}) + \sum_k \omega_k c_{\mathbf{k}}^\dagger c_{\mathbf{k}}.$$

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To study phase transition we must understand the long time behaviour of

$$H = \sum_{\langle i,j \rangle} \frac{J}{2u^0} \sigma_i^z \sigma_j^z + \sum_i \frac{g}{u^0} \sigma_i^x \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}V}} (c_{\mathbf{k}}^\dagger e^{iu^0 k_z/a} + c_{\mathbf{k}} e^{-iu^0 k_z/a}) + \sum_{\mathbf{k}} \omega_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}}.$$

$$H = \sum_i \left[\frac{\Delta E(\langle \sigma \rangle, J)}{2u^0} \sigma_i^z + \frac{g}{u^0} \sigma_i^x \sum_{\mathbf{k}} \frac{1}{\sqrt{2\omega_{\mathbf{k}}V}} (c_{\mathbf{k}}^\dagger e^{iu^0 k_z/a} + c_{\mathbf{k}} e^{-iu^0 k_z/a}) \right] + \sum_{\mathbf{k}} \omega_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}}.$$

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- Fermi's Golden Rule suggests thermalization for J sufficiently small
- We need to evaluate directly the dynamics of this Hamiltonian

- An accelerated observer will never witness the restoration of a field symmetry that is spontaneously broken in the inertial frame
- There remains the question of the physical process behind SSB in the Rindler space
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- Prof. Jose Hoyos and Prof. Frederico Brito
- Prof. Rodrigo Pereira
- CNPq & FAPESP

W. Israel '76, Physics Letters

$$G = i \sum_{\omega k_{\perp}} \theta_{\omega} (a_{\omega k_{\perp}}^{\dagger} \tilde{a}_{\omega k_{\perp}}^{\dagger} - a_{\omega k_{\perp}} \tilde{a}_{\omega k_{\perp}})$$

where $\theta_{\omega} = \arctan(e^{-\pi\omega/a})$. This kind of transformation is known as
Two-Mode Squeeze Operator

As in NG realization we have $[H, G] = 0$ but $G|0_R\rangle \neq 0$.

$U = e^{-iG}$, $U|0_R\rangle = |0_M\rangle$, but remember that $|0_R\rangle$ and $|0_M\rangle$ are not in the same Hilbert space