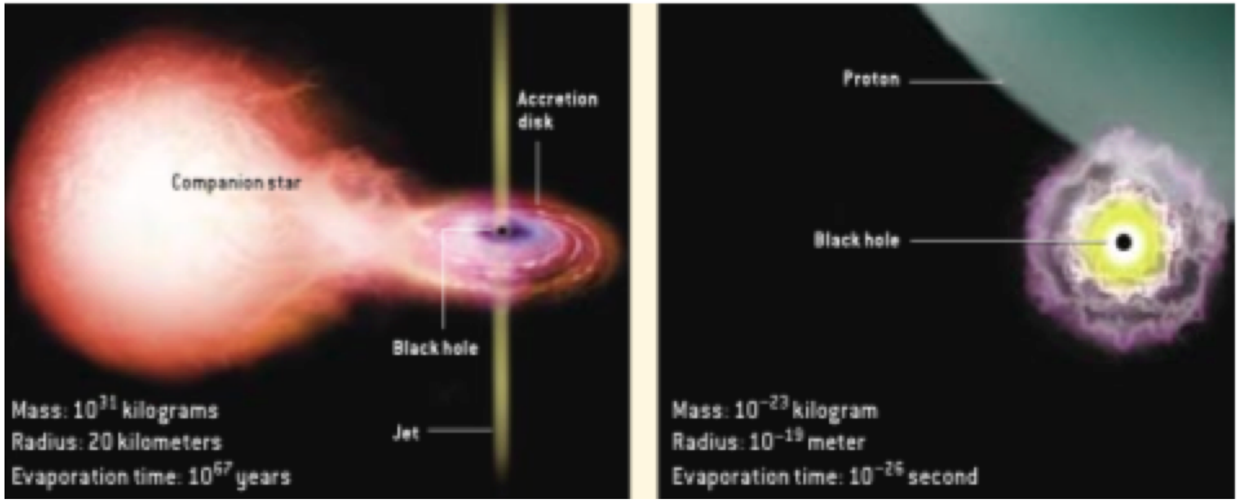
SUB-PLANCKIAN BLACK HOLES AS THE LINK BETWEEN MACROPHYSICS AND MICROPHYSICS

Bernard Carr
Queen Mary, University of London

Matthew Lake, Jonas Mureika, Piero Nicolini, Leonardo Modesto, Isabeau Premont-Schwarz, Xavier Calmet, Elizabeth Winstanley

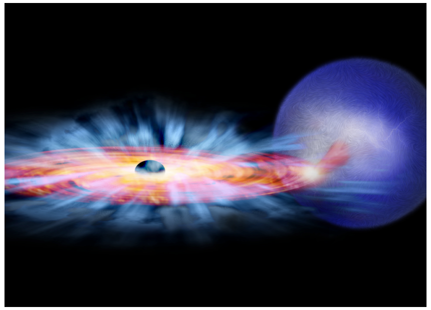


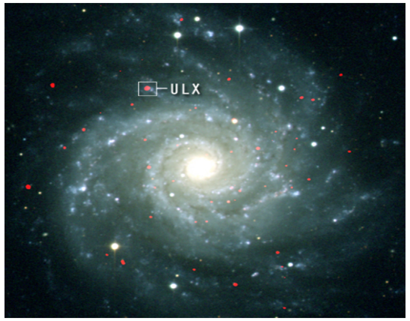
Macroscopic Microscopic

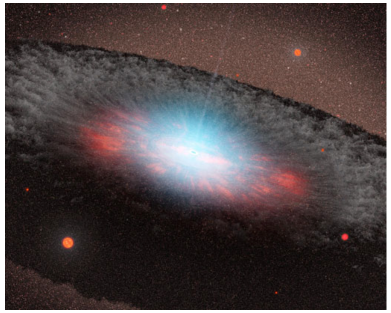
BLACK HOLE FORMATION

$R_S = 2GM/c^2 = 3(M/M_O)$ km $\Rightarrow \rho_S = 10^{18}(M/M_O)^{-2}$ g/cm^3

Good evidence that BHs form at present or recent epochs.
Stellar BH ($M \sim 10^{1\text{-}2} M_O$), IMBH ($M \sim 10^{3\text{-}5} M_O$), SMBH ($M \sim 10^{6\text{-}9} M_O$)







Small primordial black holes can only form in early Universe
cf. cosmological density $\rho \sim 1/(Gt^2) \sim 10^6 (t/s)^{-2}$ g/cm^3
=> PBHs have horizon mass at formation

$M_{PBH} \sim c^3 t/G =$
- 10^{-5}g at 10^{-43}s (minimum)
- 10^{15}g at 10^{-23}s (evaporating now)
- $10^5 M_O$ at 1s (maximum)

BLACK HOLE EVAPORATION

Black holes radiate thermally with temperature

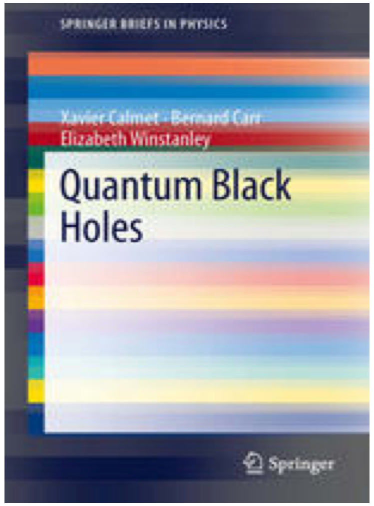
$$\mathbf{T} = \frac{hc^3}{8\pi GkM} \sim 10^{-7} \left[\frac{M}{M_0}\right]^{-1} \mathrm{K} \quad \text{(Hawking 1974)}$$

=> evaporate completely in time $\quad t_{evap} \sim 10^{64} \left[\frac{M}{M_0}\right]^{3} \mathrm{y}$

M ~ 10^{15}g => final explosion phase today (10^{30} ergs)

T > T_{CMB}=3K for M < 10^{26}g

=> "quantum" black holes



Calmet, BC, Winstanley (2014)

$R_S = 2GM/c^2$

HIGHER DIMENSIONS

Planck 10^{-5}g | $10^{22}\ M_O$ Universal

$10^{9}\ M_O$ QSO

exploding 10^{10}g

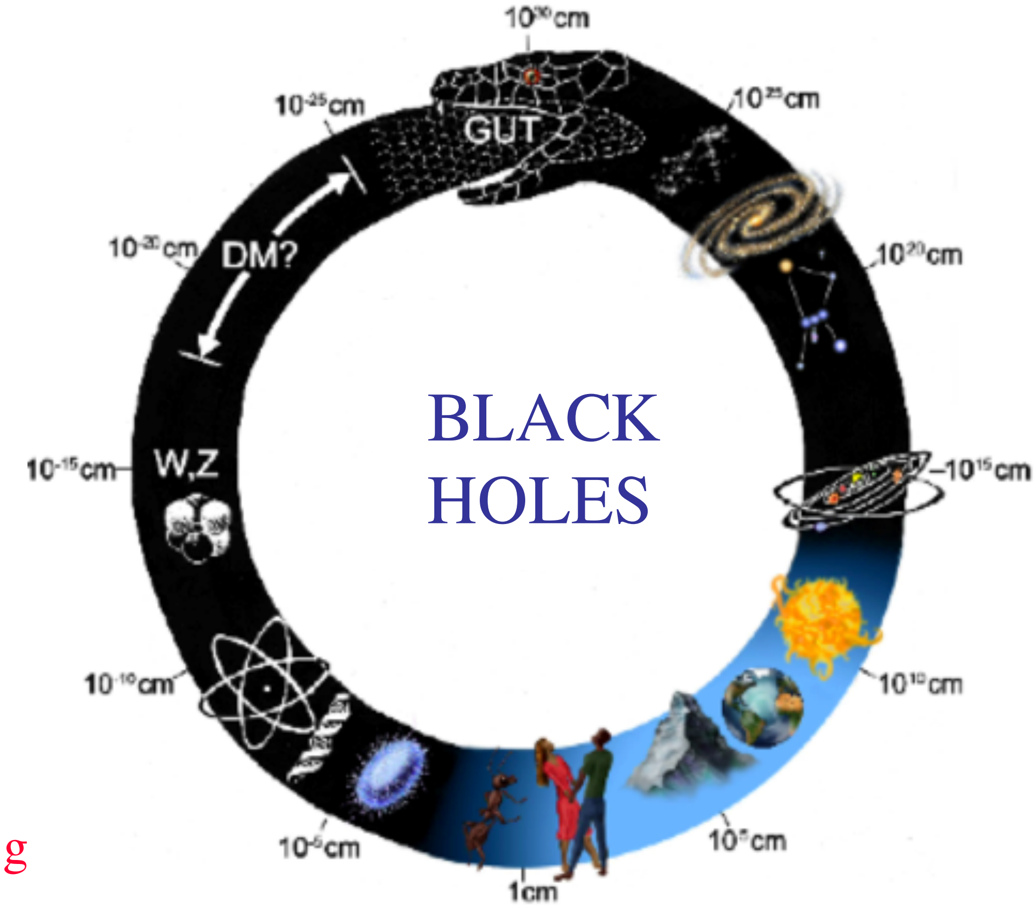
$10^{6}\ M_O$ MW

evaporating 10^{15}g

$10^{2}\ M_O$ IMBH

lunar 10^{21}g

$1\ M_O$ Stellar



terrestrial 10^{25}g

QUANTUM/CLASSICAL

UNCERTAINTY PRINCIPLE

$$\Rightarrow \Delta x > \frac{h}{(2)\Delta p} \quad \Rightarrow R_C = \frac{h}{Mc}$$ (Compton wavelength)

BLACK HOLE EVENT HORIZON

$R < R_S = 2GM/c^2$ (Schwarzschild radius)

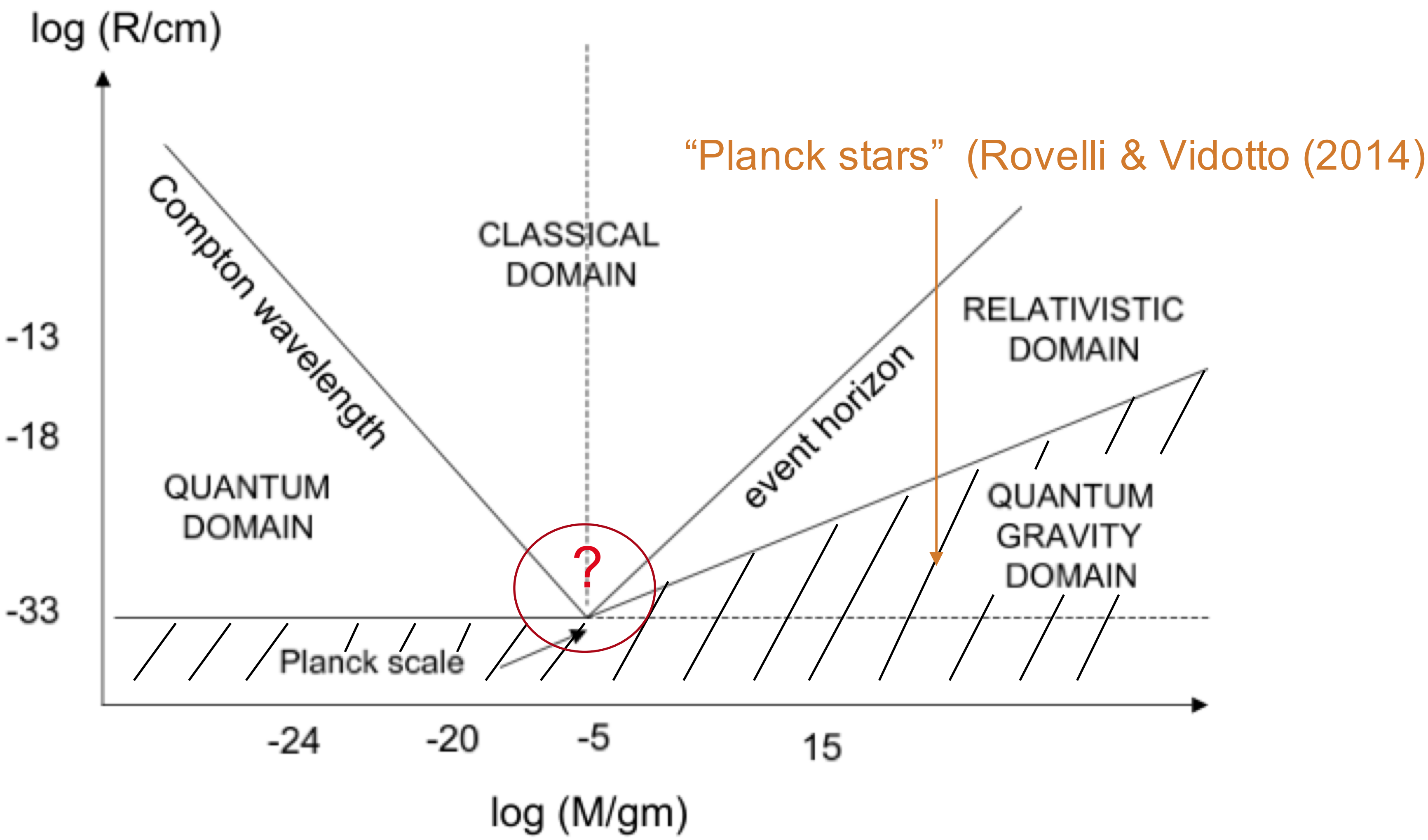
Intersect at Planck scales

$$R_P = \sqrt{Gh/c^3} \sim 10^{-33}\,cm, \quad M_P = \sqrt{hc/G} \sim 10^{-5}\,g, \quad \rho_P = \frac{c^5}{h^2 G} \sim 10^{94}\,gcm^{-3}$$

QUANTUM GRAVITY REGIME

$$\rho > \rho_P \quad \Rightarrow R < \left(\frac{3M}{4\pi\rho_P}\right)^{1/3} \sim (M/M_P)^{1/3} R_P$$

What happens to Compton and Schwarzschild lines near M_P…



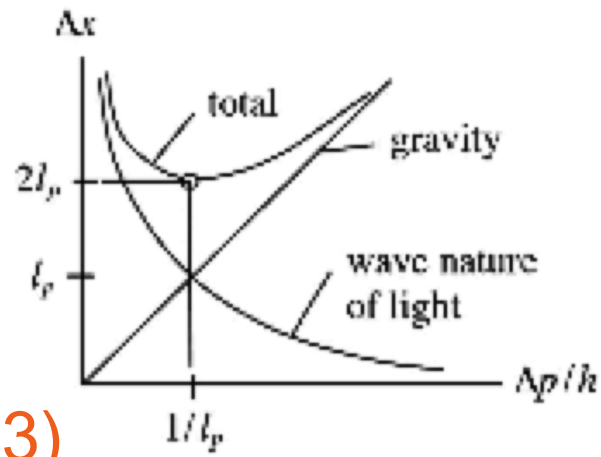
…is important feature of theory of quantum gravity.

GENERALIZED UNCERTAINTY PRINCIPLE

Heuristic argument (Adler 2010)

Gravity of photon => acceleration => uncertainty in position

=> $\Delta x > \frac{h}{\Delta p} + R_P{}^2 \frac{\Delta p}{h} > 2R_P$ (GUP)



String theory (Veneziano 1986, Witten 1996)

Minimal length considerations (Maggiore 1993)

Modifications of commutator [x,p] (Magueio & Smolin 2002)

Loop quantum gravity (Hossain et al. 2010)

Black holes (Kempf et al 1996, Scardigli 199, Calmet et al. 2004)

GENERALIZED COMPTON WAVELENGTH

Putting $\Delta x \to R$ and $\Delta p \to cM$ in $\Delta x > \dfrac{h}{\Delta p} + \alpha R_P{}^2 \dfrac{\Delta p}{h}$ gives

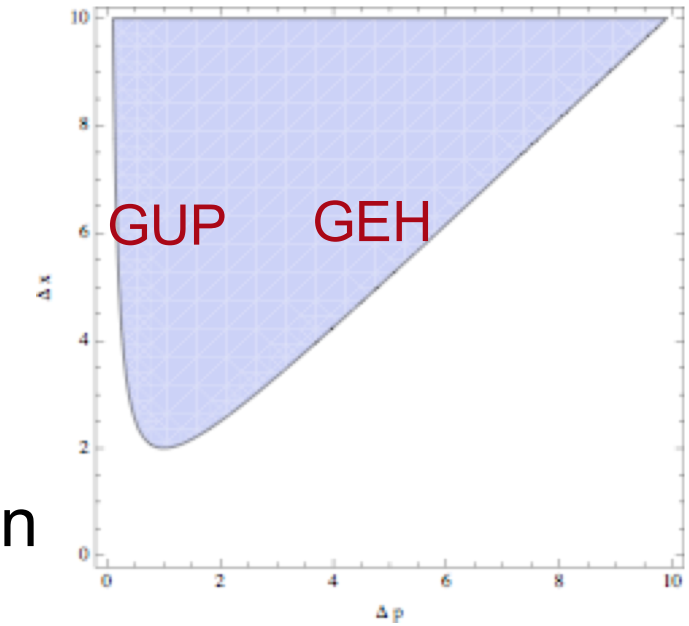
$$R > R_C{}' = \frac{h}{Mc} + \frac{\alpha G M}{c^2} \approx \begin{cases} \dfrac{h}{Mc}\left[1+\alpha\left(\dfrac{M}{M_P}\right)^2\right] & (M \ll M_P) \\ \dfrac{\alpha G M}{c^2}\left[1+\dfrac{1}{\alpha}\left(\dfrac{M_P}{M}\right)^2\right] & (M \gg M_P) \end{cases}$$

α=2?

Does Compton scale becomes Schwarzschild scale for $M \gg M_P$?

- Compton irrelevant for $M \gg M_P$ since $R_C \ll R_P$
- Cannot localize on scale below R_S for $M \gg M_P$
- BHs are intrinsically quantum: BH radiation, firewalls etc.

GENERALIZED EVENT HORIZON



Rewrite GUP in the form

$$R > R_C' = \frac{\beta h}{Mc} + \frac{2GM}{c^2}$$

For $M \gg M_P$ get generalized event horizon

$$R > R_S' = \frac{2GM}{c^2}\left[1 + \beta\left(\frac{M_P}{M}\right)^2\right]$$

(small correction to Schwarzschild)

This becomes Compton wavelength for $M \ll M_P$, suggesting

"Black Hole Uncertainty Principle Correspondence"

Generalize/unify Compton/Schwarzschild expressions such that

$$R_C' \equiv R_S' \approx \begin{cases} h/(Mc) & (M \ll M_P) \\ 2GM/c^2 & (M \gg M_P) \end{cases}$$

DO GUP UNCERTAINTIES ADD LINEARLY?

Root-mean-square error would give

$$\Delta x > \sqrt{\left(\frac{h}{\Delta p}\right)^2 + \left(\alpha R_P{}^2 \frac{\Delta p}{h}\right)^2} \Rightarrow R_C{}' = \sqrt{\left(\frac{h}{Mc}\right)^2 + \left(\frac{\alpha GM}{c^2}\right)^2} \approx \frac{h}{Mc}\left[1 + \frac{\alpha^2}{2}\left(\frac{M}{M_P}\right)^4\right]$$

$$\Rightarrow R_S{}' = \sqrt{\left(\frac{2GM}{c^2}\right)^2 + \left(\frac{\beta h}{Mc}\right)^2} \approx \frac{2GM}{c^2}\left[1 + \frac{\beta^2}{8}\left(\frac{M_P}{M}\right)^4\right]$$

For GENERAL $R_C{}'(M) \equiv R_S{}'(M)$ BHUP correspondence implies

- Form of $\Delta x(\Delta p)$ determines $R_S(M)$.
- $M \leftrightarrow 1/M$ symmetry (t-duality).
- Sub-Planckian black holes with $M<M_P$.

“Generalized Uncertainty and Self-dual Black Holes”

Carr, Modesto & Premont-Schwarz, arXiv: 1107.0708

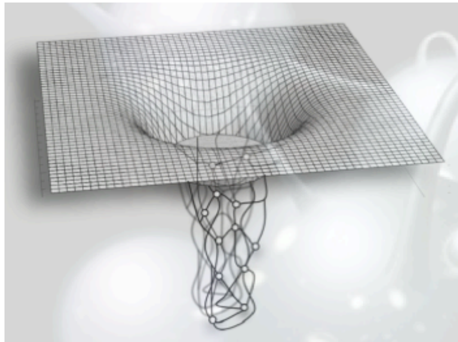
LOOP BLACK HOLES

Metric $$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^{(2)},$$

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_o^2},$$
$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_o^2)},$$
$$H(r) = r^2 + \frac{a_o^2}{r^2}.$$

where
$$r_+ = 2Gm/c^2$$
$$r_- = 2GmP^2/c^2$$
$$r_* \equiv \sqrt{r_+ r_-}$$

and $$a_o = A_{\rm min}/8\pi = \sqrt{3}\,\gamma\zeta R_P^2/2$$



Polymeric function $$P \equiv \frac{\sqrt{1+\epsilon^2}-1}{\sqrt{1+\epsilon^2}+1} \sim \epsilon^2/4 \ll 1$$

At large r
$$G(r) \to 1 - \frac{2M}{r}(1-\epsilon^2),$$
$$F(r) \to 1 - \frac{2M}{r},$$
$$H(r) \to r^2.$$

implies $M = m(1+P)^2$ (ADM mass)

Physical radial coordinate $R = \sqrt{H(r)} = \sqrt{r^2 + \frac{a_o^{\,2}}{r^2}}$

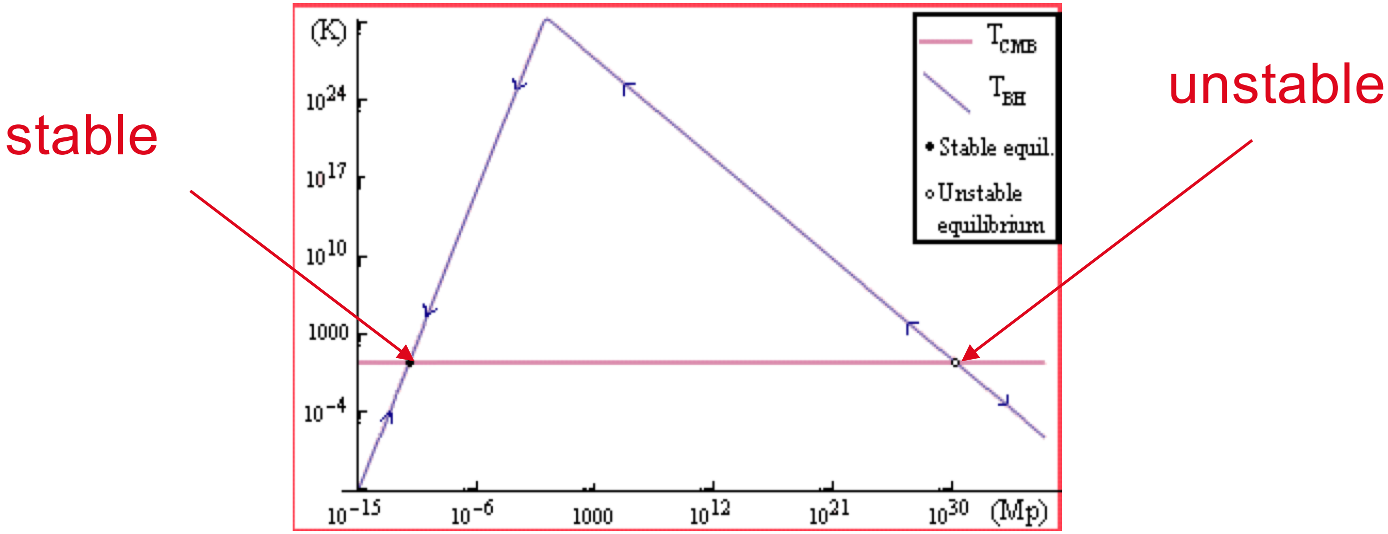
$$\Rightarrow R_S = \sqrt{\left(\frac{2Gm}{c^2}\right)^2 + \left(\frac{c^2 a_o}{2Gm}\right)^2} \approx \begin{cases} \dfrac{2Gm}{c^2} & (m > M_P) \\[2ex] \beta \, \dfrac{\sqrt{3}\gamma\varsigma}{4}\dfrac{h}{mc} & (m < M_P) \end{cases}$$

This removes the singularity, permits black holes with $m \ll M_P$, and corresponds to the quadratic GEH.

Metric has another asymp' infinity ($r=0$) with BH mass M_P^2/m

=> sub-Planckian black hole hidden within wormhole.

CAN SUB-PLANCKIAN RELICS PROVIDE DARK MATTER?



$T \propto M^3 \Rightarrow$ cooler than CMB for $M < \left(\frac{T_{CMB}}{T_P}\right)^{1/3} M_P \sim 10^{-16} g$

$\frac{dM}{dt} \propto R^{-2}T^4 \propto M^{10} \Rightarrow M(t) \propto t^{-1/9}$

=> never evaporate but current mass $M \sim \left(\frac{t_P}{t_0}\right)^{1/9} M_P \sim 10^{-12} g$

=> current temperature $T \sim \left(\frac{t_P}{t_o}\right)^{1/3} T_P \sim 10^{12} K$

=> same observational effects as PBHs with M~10^{15}g!

Sub-Planckian Black Holes and the GUP

B. Carr, J. Mureika, P. Nicolini, JHEP 07 (2015) 52, arXiv:1504.07637

Can black holes really exist below the Planck mass?

An approach to including the GUP in general relativity is to emphasize the ***duality*** in the black hole mass

$$\Delta x \sim \frac{1}{\Delta p} + \Delta p$$

$$M = M_{\mathrm{ADM}}$$

$$\Delta x_G \sim \frac{1}{M_{\mathrm{bh}}} + M_{\mathrm{bh}}$$

Can we encode this in the metric?

Gives new *M* (duality):

$$M \longrightarrow M\left(1+\frac{\beta}{2}\frac{M_{\rm Pl}^2}{M^2}\right)$$

<u>Metric is:</u>

$$\boxed{\begin{gathered} ds^2 = F(r)dt^2 - F(r)^{-1}dr^2 - r^2 d\Omega^2 \\ F(r) = 1 - \frac{2}{M_{\rm Pl}^2}\frac{M}{r}\left(1+\frac{\beta}{2}\frac{M_{\rm Pl}^2}{M^2}\right) \end{gathered}}$$

Planck mass is now critical point for which…

$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad F(r) \sim 1 - \frac{M}{r}$$

$$M \ll M_{\rm Pl} \quad \Longrightarrow \quad F(r) \sim 1 - \frac{1}{Mr}$$

Black Hole Characteristics: Horizon

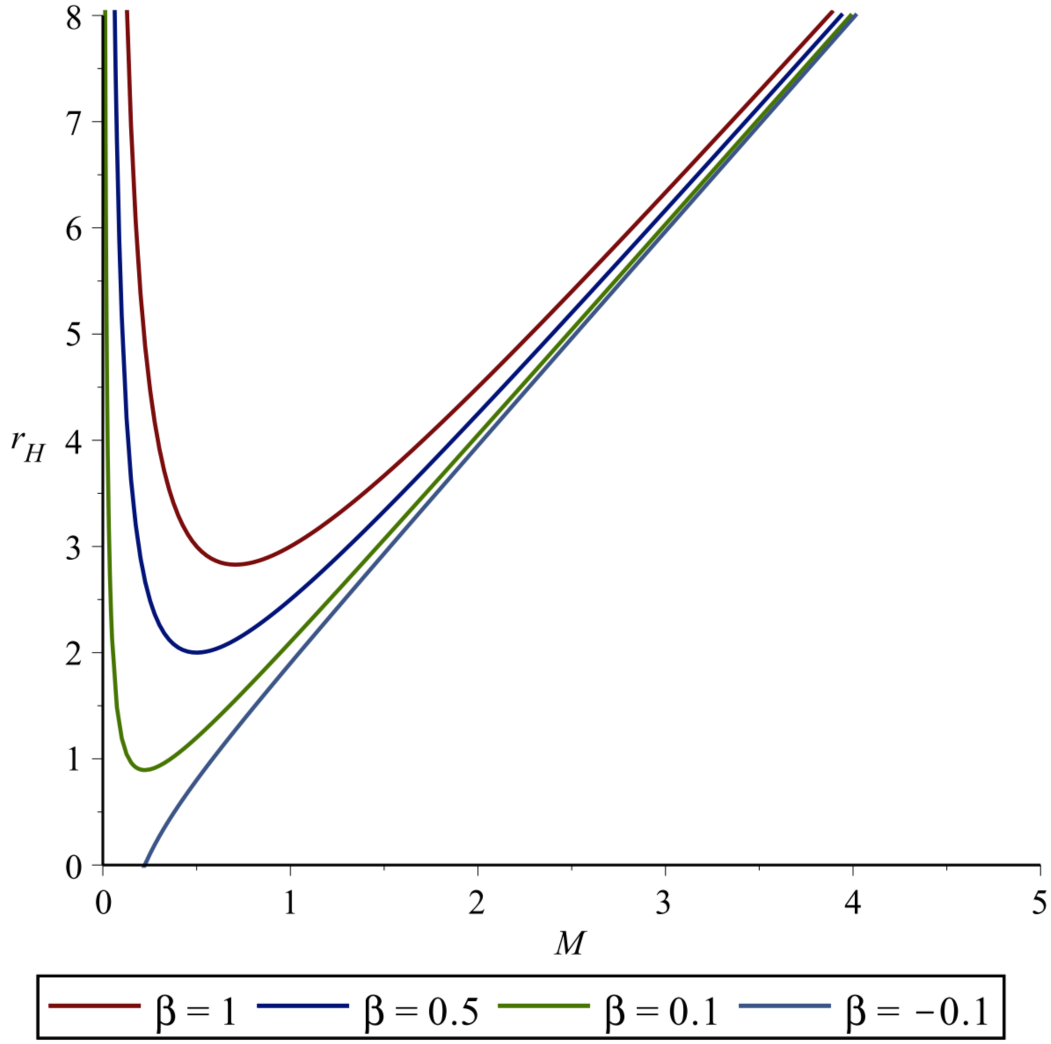
$$F(r_H) = 0$$

$$\boxed{r_H = \frac{2}{M_{\rm Pl}^2}\left(\frac{M^2 + \frac{\beta}{2}M_{\rm Pl}^2}{M}\right)}$$

$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad r_H \approx \frac{2M}{M_{\rm Pl}^2}$$

$$M \sim M_{\rm Pl} \quad \Longrightarrow \quad r_H \sim \frac{2+\beta}{M_{\rm Pl}}$$

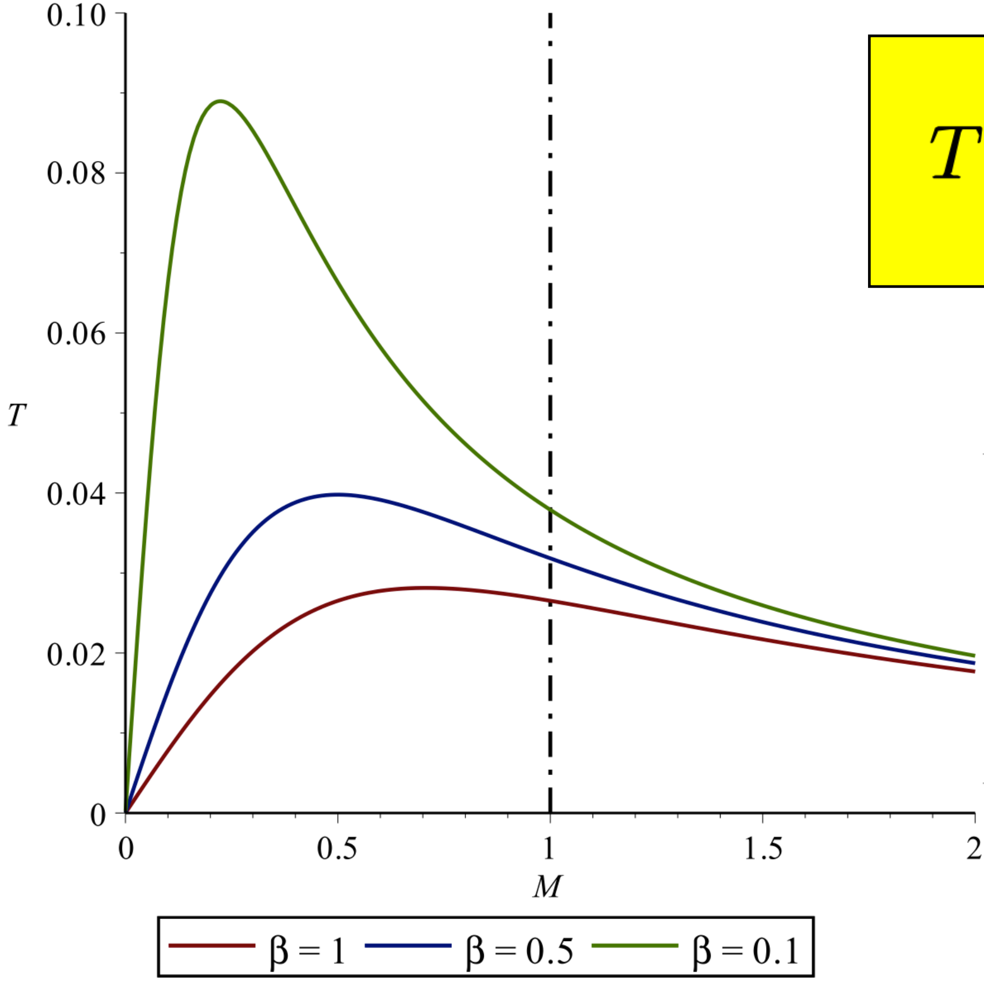
$$M \ll M_{\rm Pl} \quad \Longrightarrow \quad r_H \approx \frac{\beta}{M}$$



Black Hole Temperature

From surface gravity:

$$T = \frac{\kappa}{2\pi} \quad , \quad \kappa = \frac{1}{2}\frac{dF}{dr}(r = r_H)$$



$$\boxed{T = \frac{M_{\rm Pl}^2}{8\pi M(1 + \beta M_{\rm Pl}^2/2M^2)}}$$

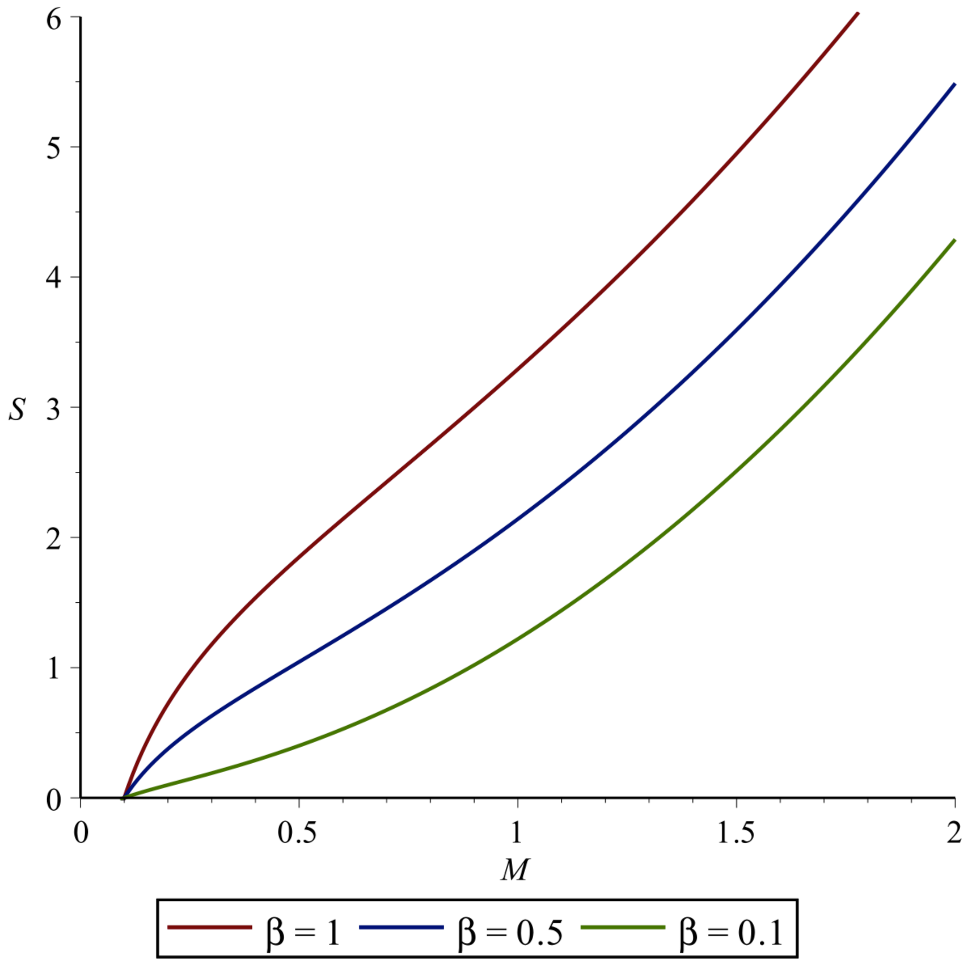
$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad T \approx \frac{M_{\rm Pl}^2}{8\pi M}$$

$$M \sim M_{\rm Pl} \quad \Longrightarrow \quad T \sim \frac{M_{\rm Pl}}{8\pi(1 + \beta/2)}$$

$$M \ll M_{\rm Pl} \quad \Longrightarrow \quad T \approx \frac{M}{4\pi\beta}$$

Black Hole Characteristics: Entropy

$$S = \int_{M_0}^{M} \frac{dM'}{T(M')} \sim \boxed{4\pi \left(\frac{M^2}{M_{\text{Pl}}^2} - \frac{M_0^2}{M_{\text{Pl}}^2} + \beta \log \frac{M}{M_0} \right)}$$



$$M \gg M_{\text{Pl}} \quad \Longrightarrow \quad S \sim M^2$$
$$M \ll M_{\text{Pl}} \quad \Longrightarrow \quad S \sim \log\left(M/M_0\right)$$

(3+1)-D vs (1+1)-D Spacetime

(3+1)-D

$$g_{tt} = 1 - \frac{2G_N M}{r}$$

$$g_{rr} = -g_{tt}^{-1}$$

$$\boxed{\begin{aligned} r_H &\sim M \\ T &\sim \frac{1}{M} \\ S &\sim M^2 \end{aligned}}$$

(1+1)-D

$$g_{tt} = 1 - G_1 M|x|$$

$$g_{xx} = -g_{tt}^{-1}$$

$$\boxed{\begin{aligned} r_H &\sim \frac{1}{M} \\ T &\sim M \\ S &\sim \log(M) \end{aligned}}$$

Sub-Planck regime

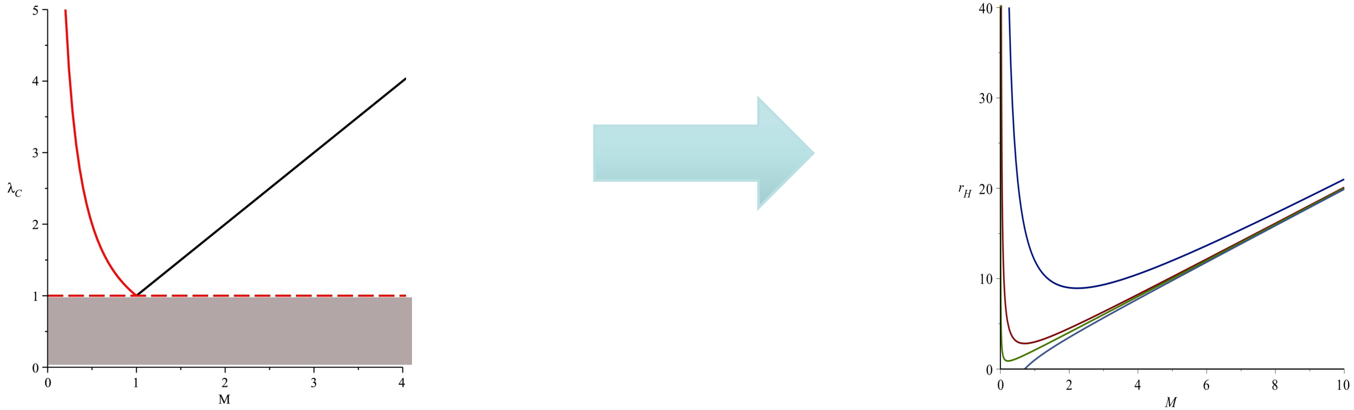
$\longrightarrow$

Dimensional reduction?

The gravitational physics of the sub-Planckian regime is governed by an *effective* (1+1)-D

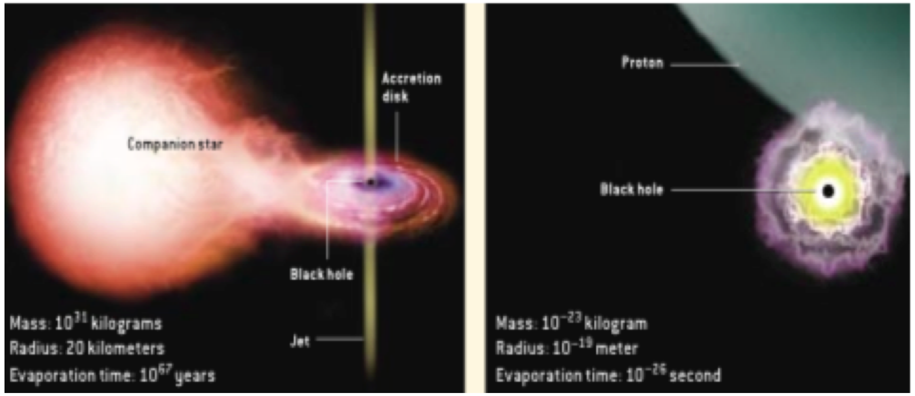
Advantages of CMN proposal

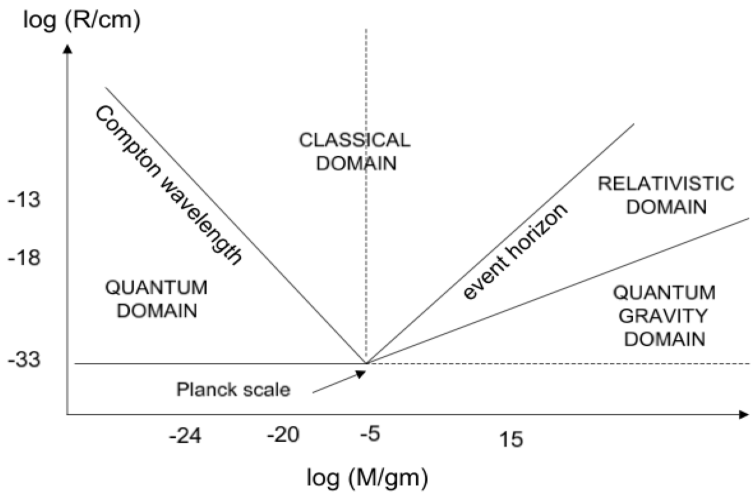
- Encoding the GUP duality in the mass gives a metric that exhibits dimensional reduction in the sub-Planckian regime
- Smooths out GUP curve, so no critical point



- Cures thermodynamic instability of evaporating Schwarzschild BHs.
- Instead of a two regimes governed by different theories (GR and QM), gives consistent theory (gravity) in different spacetime dimensions

CONCLUSIONS





- Both black holes and Generalized Uncertainty Principle provide important link between micro and macro physics.

- They are themselves linked and black holes with sub-Planckian mass may play a role in quantum gravity.

REFERENCES

B. Carr, L. Modesto & I. Premont-Schwarz, Generalized Uncertainty Principle and Self-Dual Black Holes, arXiv: 1107.0708 [gr-qc] (2011).

B. Carr, Black Holes, Generalized Uncertainty Principle and Higher Dimensions, Mod. Phys. Lett. A 28, 134001 (2013).

B. Carr, Black Hole Uncertainty Principle Correspondence, Proc KSM 2013 (2015); arXiv:1402.1427.

B. Carr, J. Mureika, P. Nicolini, Sub-Planckian black holes and Generalized Uncertainty Principle, JHEP 07 (2015) 52, arXiv:1504.07637.

M. Lake and B. Carr, Compton-Schwarzschild correspondence from extended de Broglie relations, JHEP 11 (2015) 105, arXiv:1505.06994.

M. Lake and B. Carr, Black hole uncertainty principle correspondence in higher dimensions (2015), preprint.

X. Calmet, B. Carr, E. Winstanley, *Quantum Black Holes* (Springer 2014)