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# From Quantum to Classical Instability in Relativistic Stars

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André G. S. Landulfo (Center for  
Natural and Human Sciences/  
UFABC)

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*In collaboration with W. Lima (York), G. Mastsas (ift), and D. Vanzella (usp)*

# UNSTABLE RELATIVISTIC SYSTEMS

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Fernando Botero: Equilibrist with Umbrella



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# UNSTABLE RELATIVISTIC SYSTEMS

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**Amplitude**  $\sim \sqrt{\hbar}$



Fernando Botero: Equilibrist with Umbrella



# UNSTABLE RELATIVISTIC SYSTEMS

**Relativistic stars may become unstable due to quantum effects  
[The vacuum awaking effect]:**

- **W. Lima, D. Vanzella, PRL (2010),**
- **W. Lima, G. Matsas, D. Vanzella, PRL (2010)**

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- **W. Lima, R. Mendes, G. Matsas, D. Vanzella, PRD (2013)**
- **R. Mendes, G. Matsas, and D. Vanzella, PRD (2014)**

**Amplitude**

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**- J. Santiago, AL, W. Lima, R. Mendes, G. Matsas, and D. Vanzella, PRD (2015)**

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# AWAKING THE VACUUM

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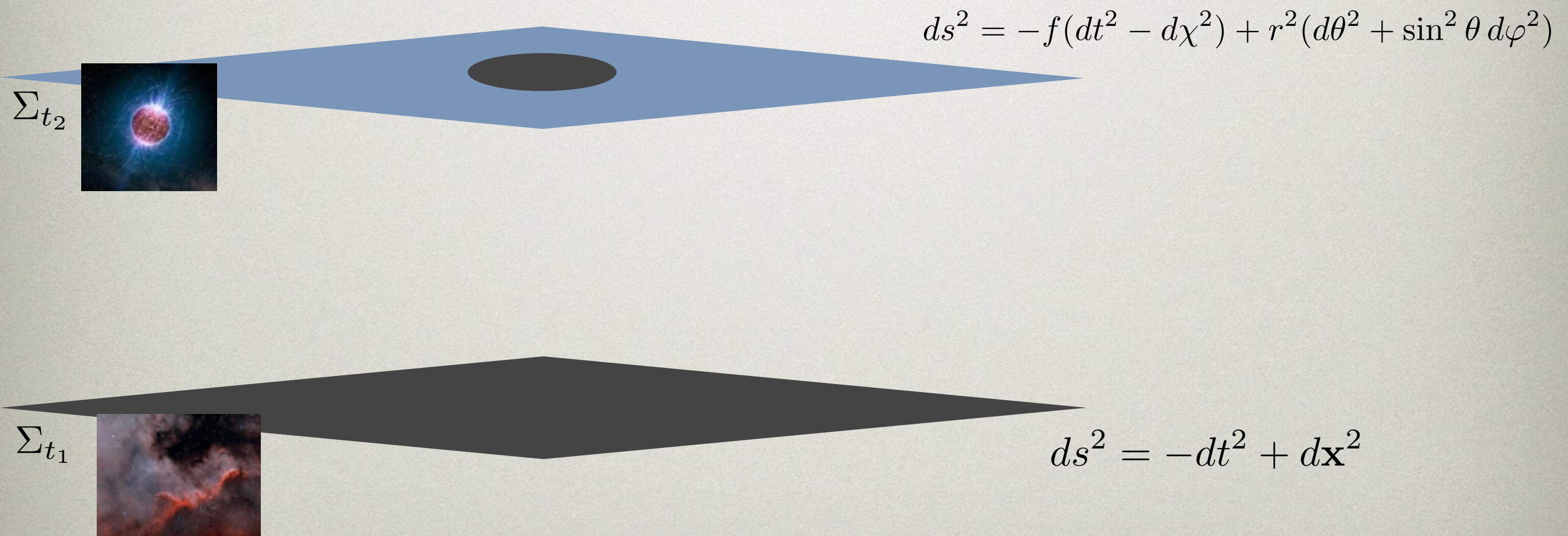
$\Sigma_{t_1}$

$$ds^2 = -dt^2 + d\mathbf{x}^2$$



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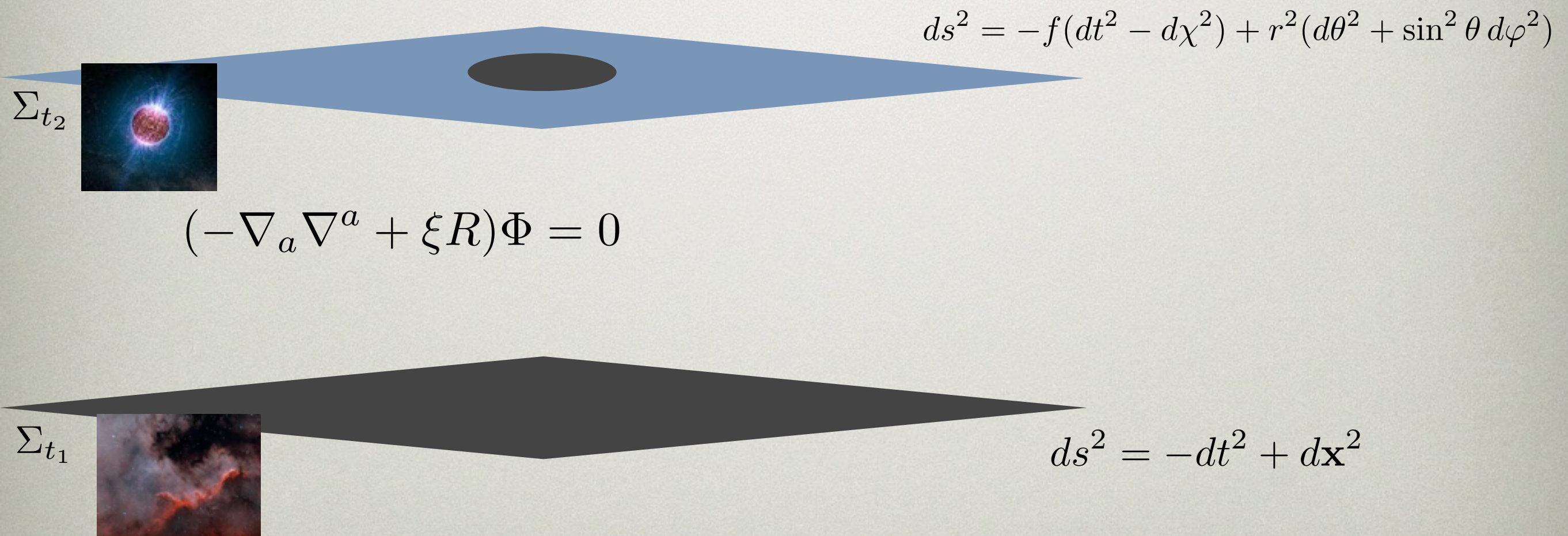
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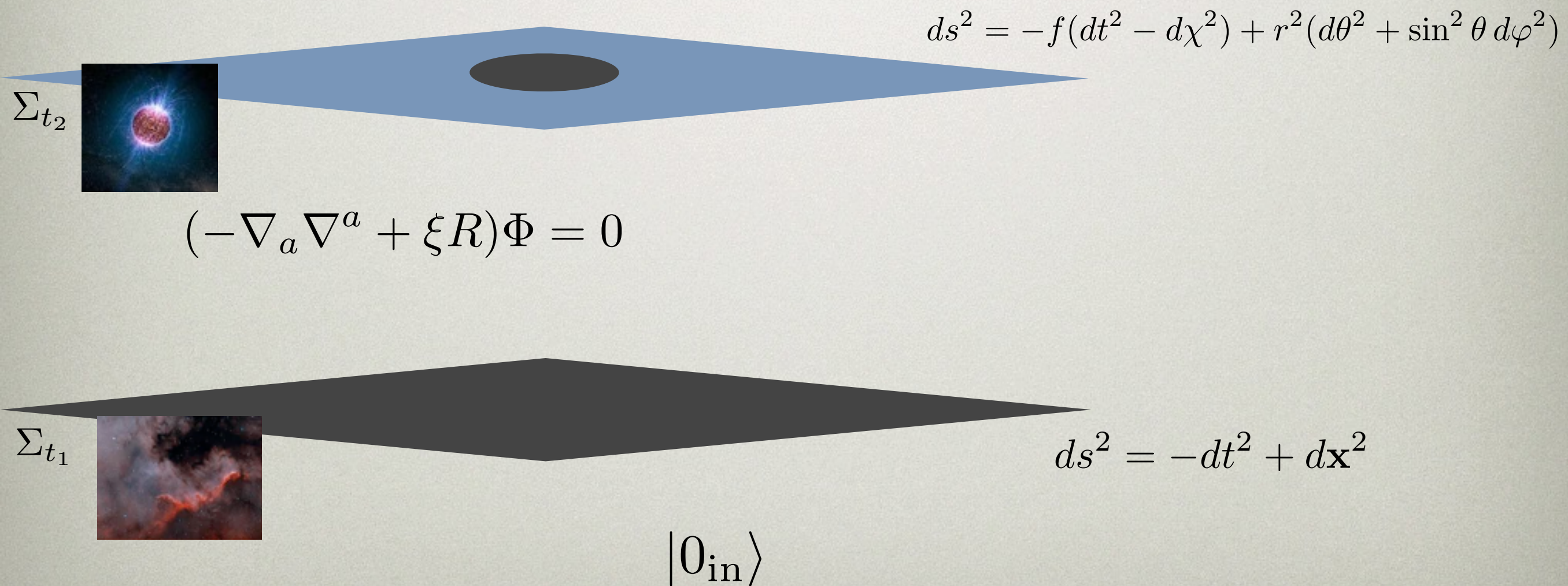
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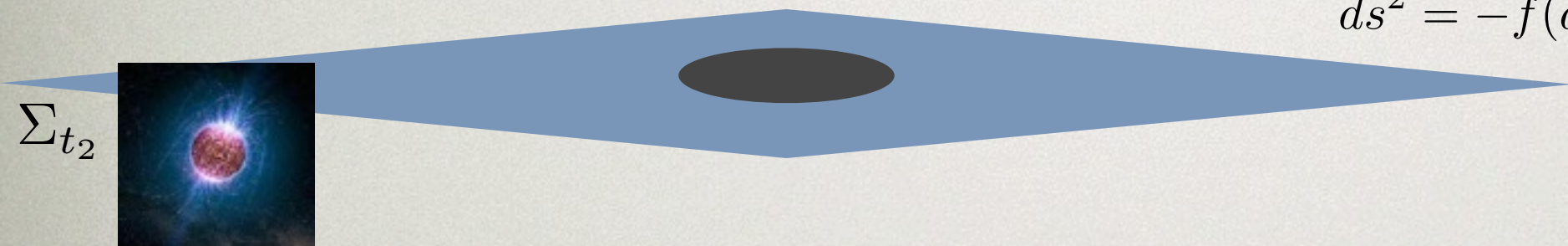


# AWAKING THE VACUUM

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$$\langle \hat{\Phi}^2 \rangle_{\text{in}} \sim \frac{\kappa e^{2\bar{\Omega}t}}{2\bar{\Omega}} \left( \frac{\bar{F}(\mathbf{x})}{f_{\text{int}}(\mathbf{x})} \right)^2 [1 + \mathcal{O}(e^{-\epsilon t})]$$

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$



$$(-\nabla_a \nabla^a + \xi R)\Phi = 0$$



$$ds^2 = -dt^2 + d\mathbf{x}^2$$

$$|0_{\text{in}}\rangle$$

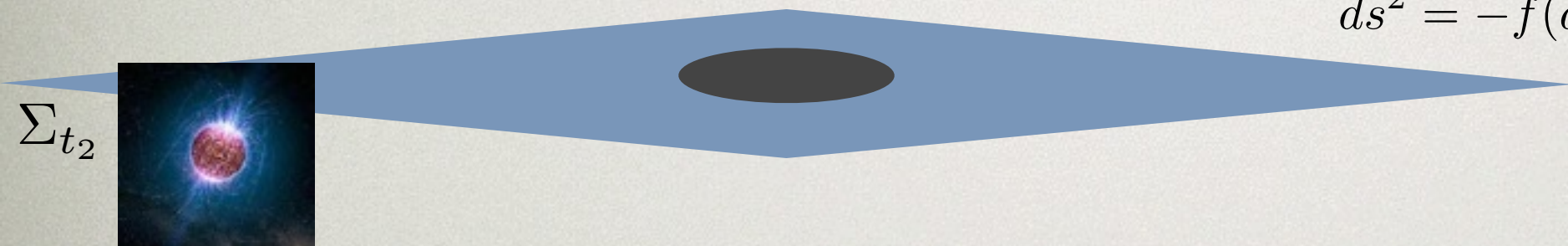


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$$\begin{aligned} \langle \hat{\rho} \rangle_{\text{in}} \sim & \kappa \frac{\bar{\Omega} e^{2\bar{\Omega}t}}{16\pi\sqrt{f}} \left\{ \frac{1-4\xi}{2r^2} \frac{d}{d\chi} \left( r^2 \frac{d}{d\chi} \left( \frac{F_{\bar{\Omega}0}}{\bar{\Omega}r} \right)^2 \right) \right. \\ & \left. + \frac{\xi}{\bar{\Omega}^2 r^2} \frac{d}{d\chi} \left( \frac{F_{\bar{\Omega}0}^2}{f} \frac{df}{d\chi} \right) \right\} [1 + \mathcal{O}(e^{-\epsilon t})]. \end{aligned}$$

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$



$\Sigma_{t_2}$

$$(-\nabla_a \nabla^a + \xi R)\Phi = 0$$



$\Sigma_{t_1}$

$$ds^2 = -dt^2 + d\mathbf{x}^2$$

$|0_{\text{in}}\rangle$



# VACUUM DOMINANCE

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- ✧ Vacuum energy density

$$\frac{\rho_v}{\rho} \sim \left( \frac{\ell_P}{R} \right)^2 \times \exp(2t/R) \longrightarrow t_{\text{br}} \sim R \ln(R/\ell_P) \quad \rho_v \sim \rho$$



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- \* In a few milliseconds the vacuum take control over the energy density of the compact object

$$t_{\text{br}} \sim 10^{-3} s$$

$$\rho_v \sim 10^{14-15} \text{ g/cm}^3$$



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- ❖ The unstable phase must be detained by backreaction effects which should be responsible to bring the vacuum back to some stationary regime.



# AWAKING THE VACUUM

- ✧ P. Pani, V Cardoso, E. Berti, J. Read, and M. Salgado, PRD (2011)

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

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- ✧ T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)
- ✧ T. Harada, Prog. Theor. Phys, (1993), PRD (1997)
- ✧ M. Ruiz et. al., PRD (2012)
- ✧ R. Mendes and N. Ortiz, PRD (2016)

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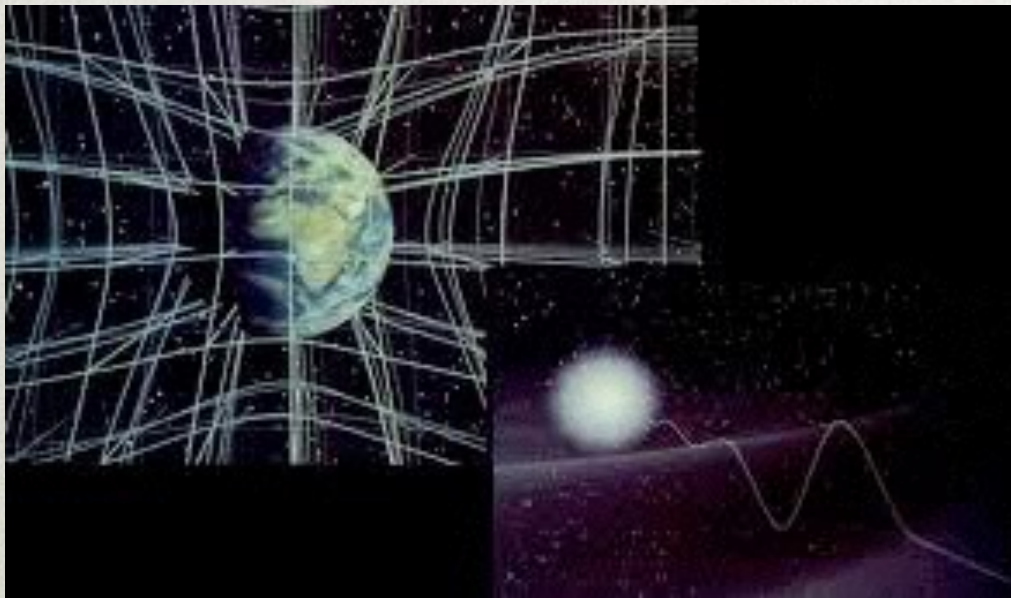


# AWAKING THE VACUUM

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✧ Einstein semiclassical equation:

$$R_{ab} - \frac{1}{2}g_{ab}R = 8\pi\langle T_{ab}\rangle_\omega$$





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**This approach, however, have many difficulties, e.g.,**

**(1) One does not expect it to be valid when the fluctuations of the stress energy tensor are “large” when compared to its mean value**

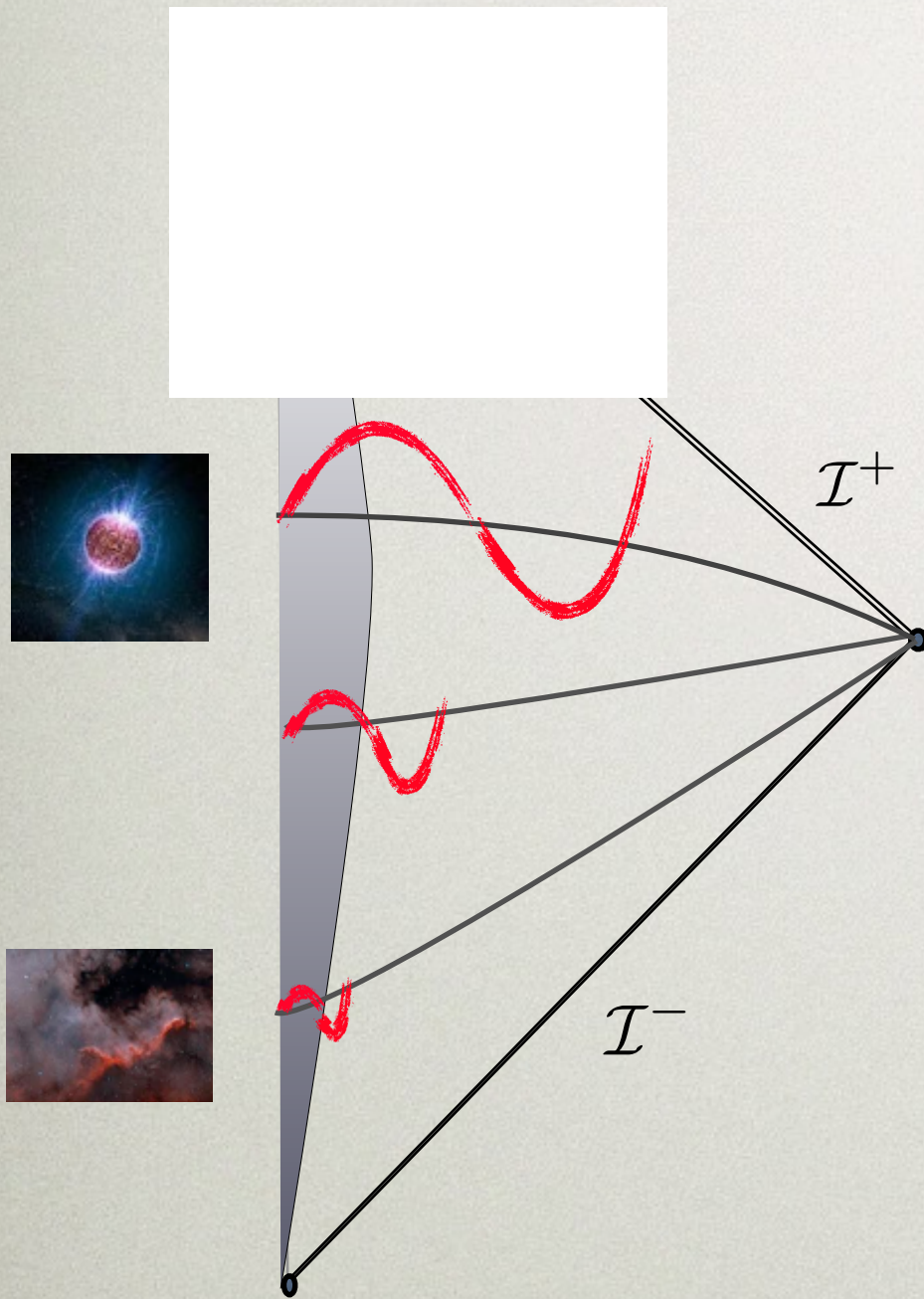
**(2) Even if/when it is valid, it is, in general, prohibitively difficult to solve it.**

**⋮**



# AWAKING THE VACUUM

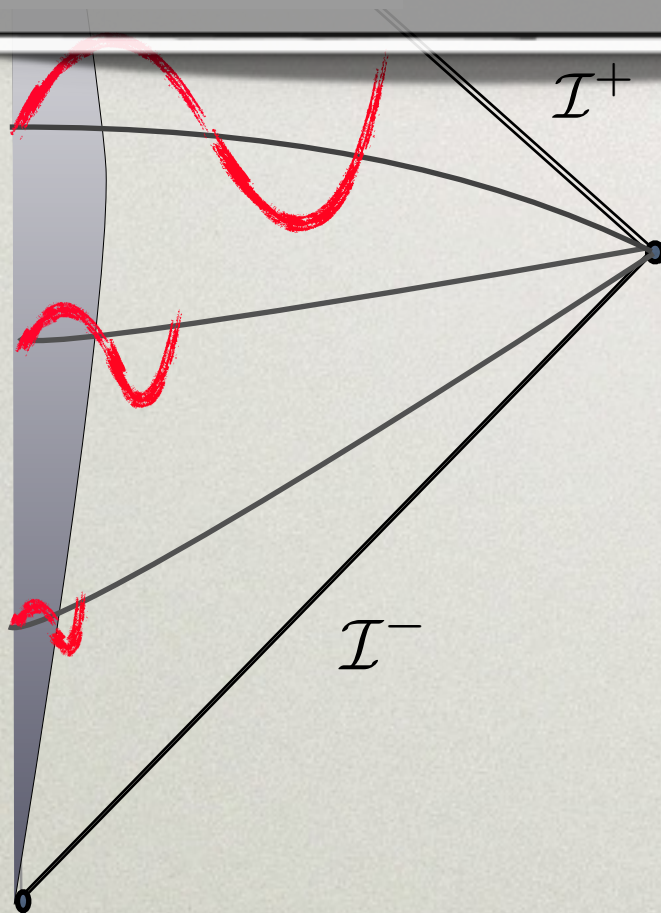
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# AWAKING THE VACUUM

it seems reasonable to believe that quantum fluctuations amplified enough to menace the stability of relativistic stars cannot remain "quantum" for too long.

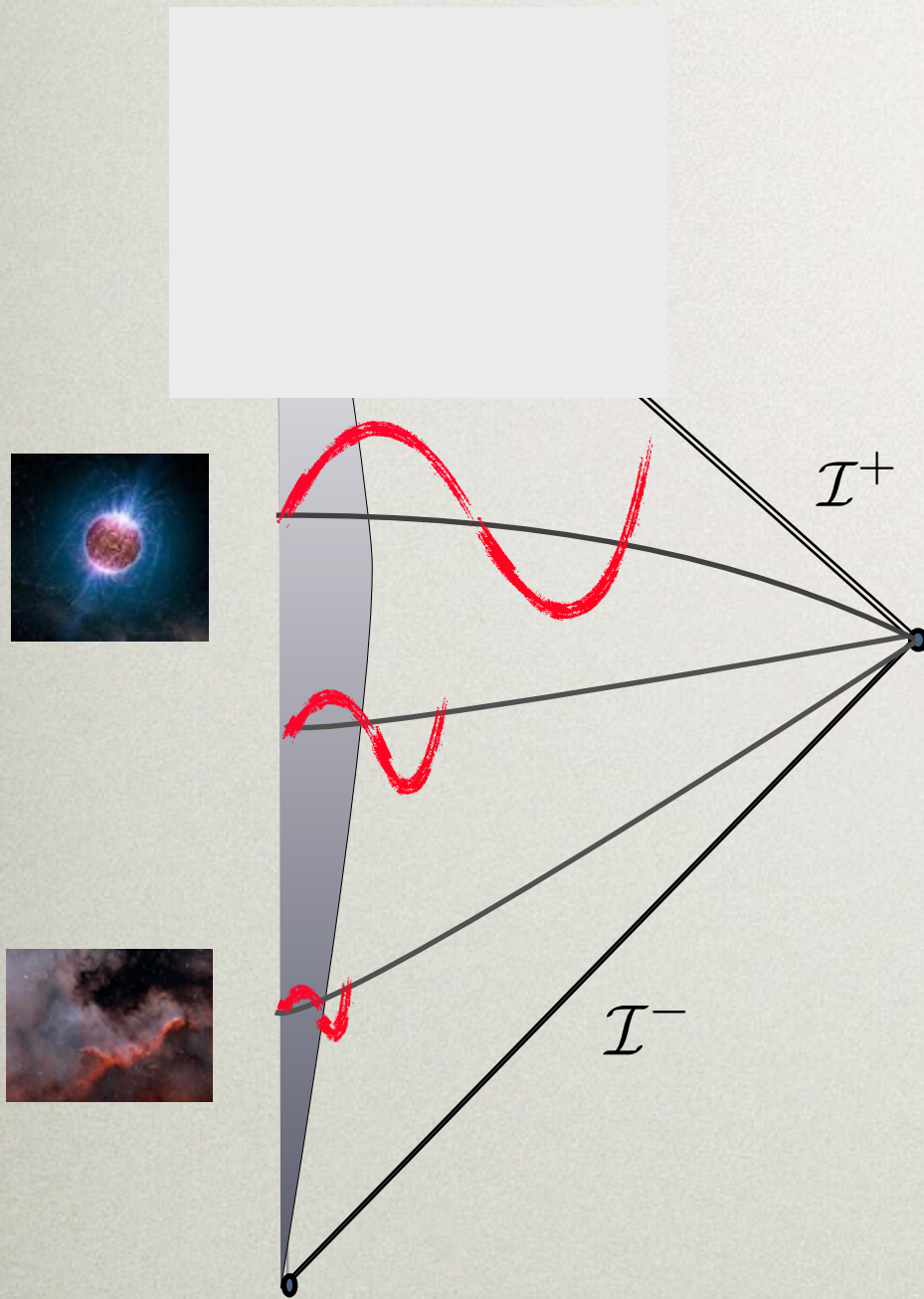


If the quantum phase ends before vacuum fluctuations dominate the system, we expect backreaction to be well described by the CLASSICAL general-relativistic equations.



# AWAKING THE VACUUM

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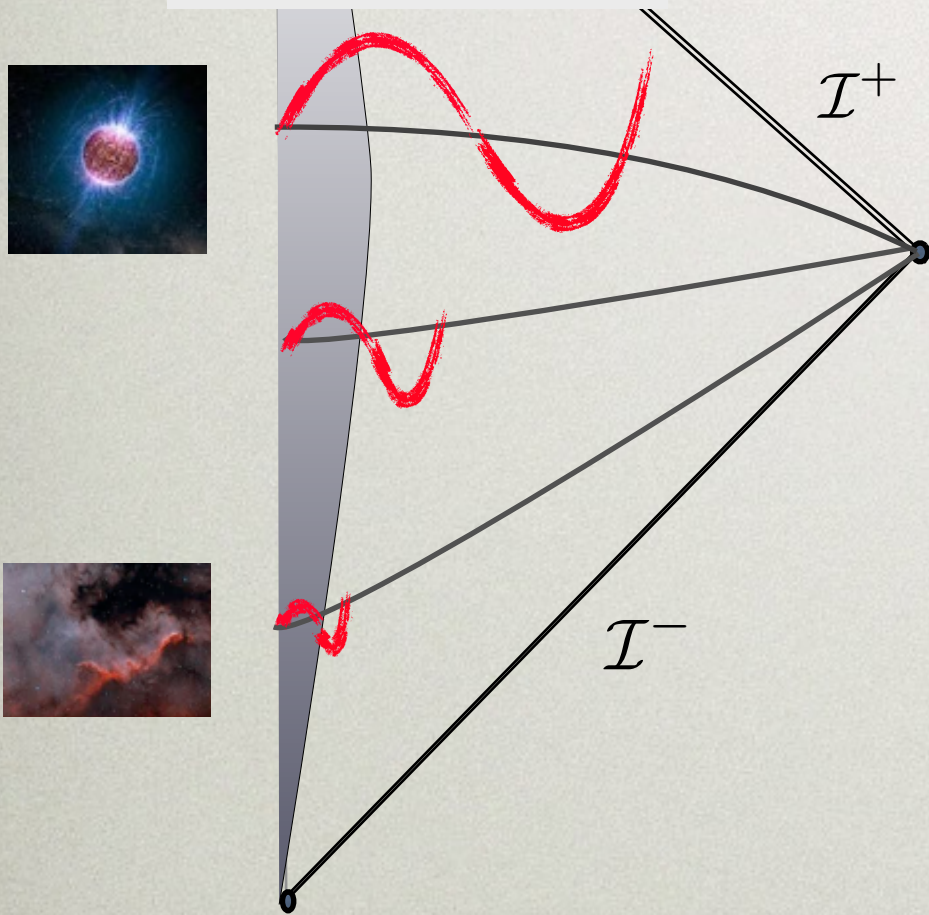




# AWAKING THE VACUUM

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- ✧ For this quantum to classical transition to occur one must have:



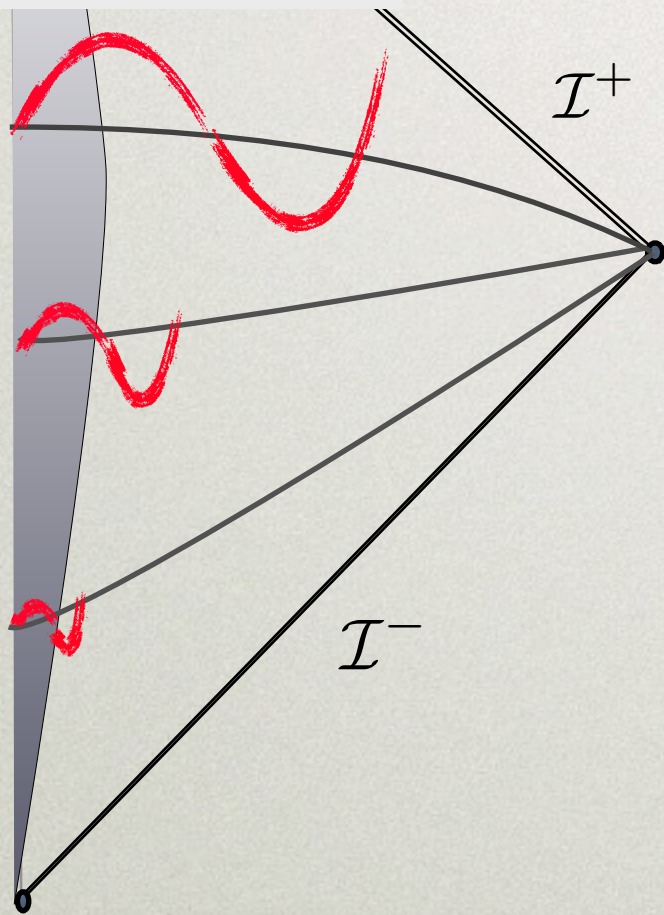


# AWAKING THE VACUUM

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✧ For this quantum to classical transition to occur one must have:

(1) The appearance of certain “classical” correlations



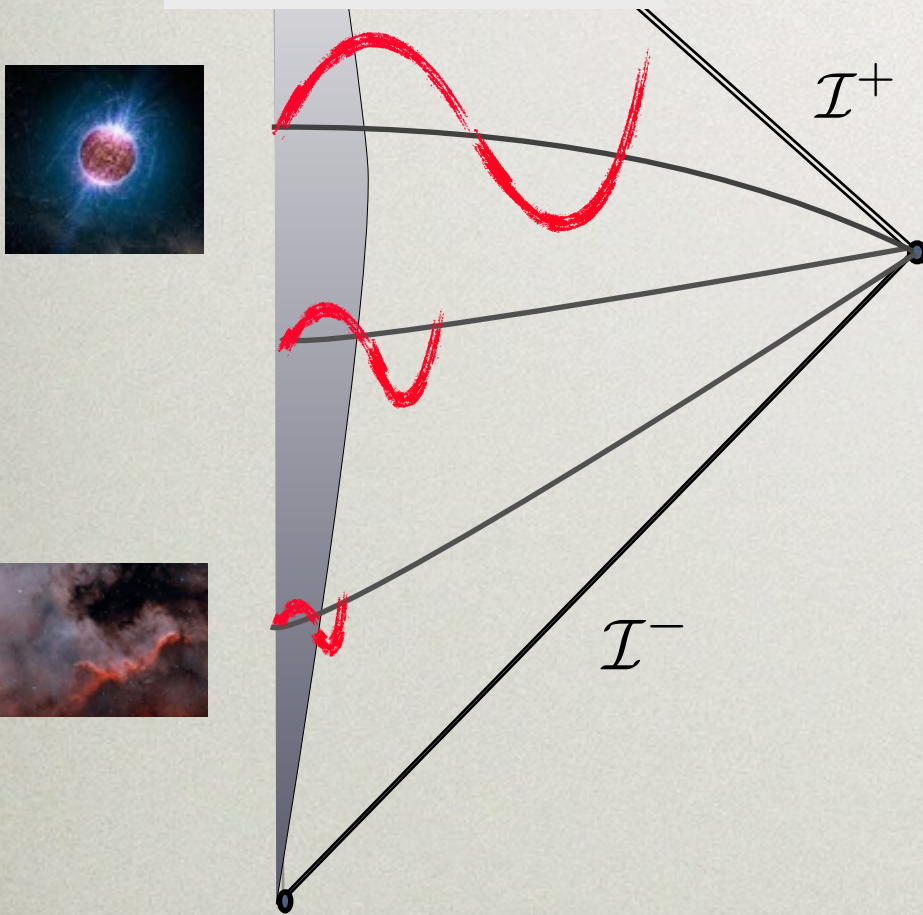


# AWAKING THE VACUUM

✧ For this quantum to classical transition to occur one must have:

(1) The appearance of of certain “classical” correlations

(2) Loss of quantum coherence





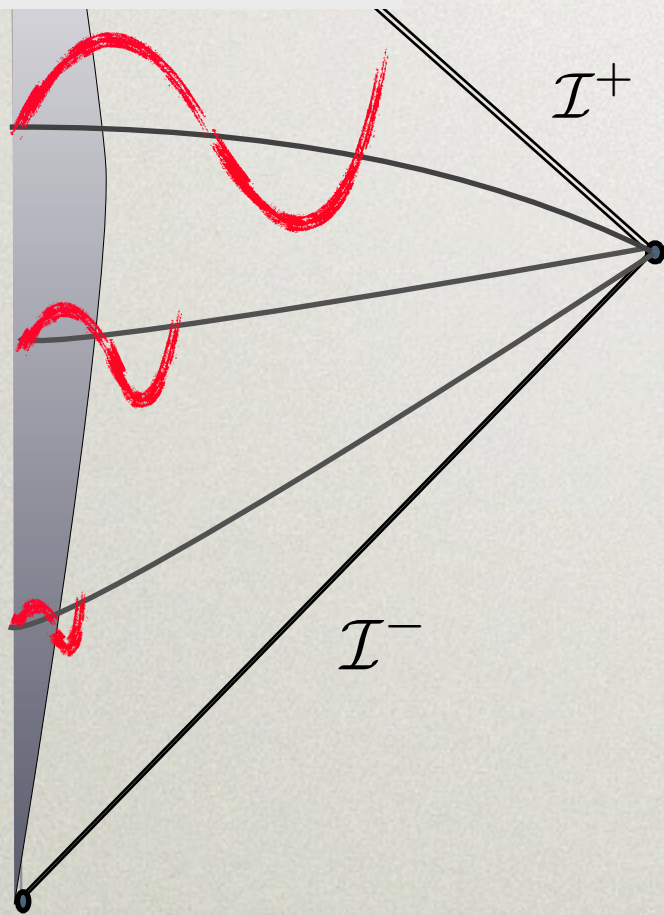
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System  
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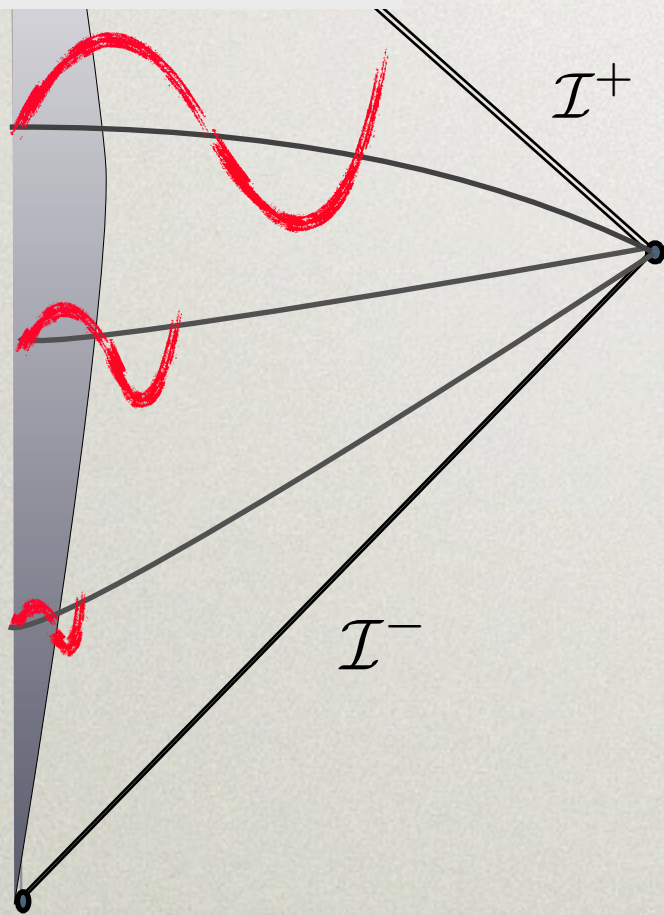
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System  $\mathcal{S}$  +





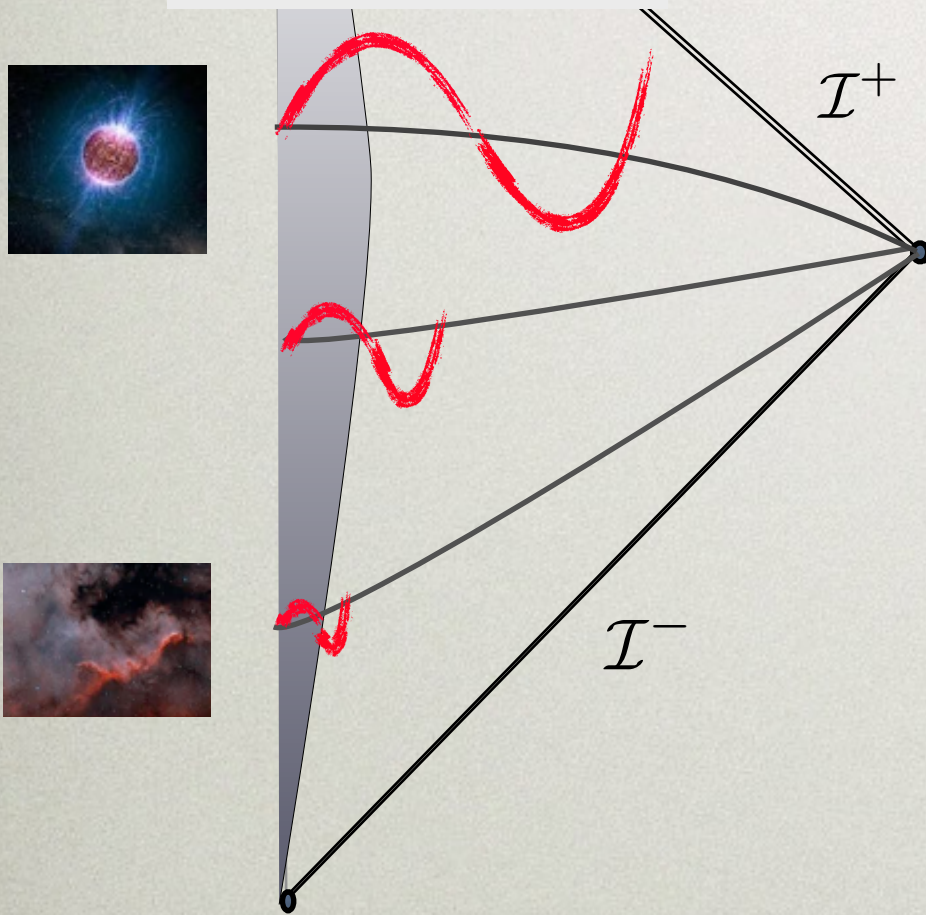
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System      +      Environment  
 $\mathcal{S}$                        $\mathcal{E}$





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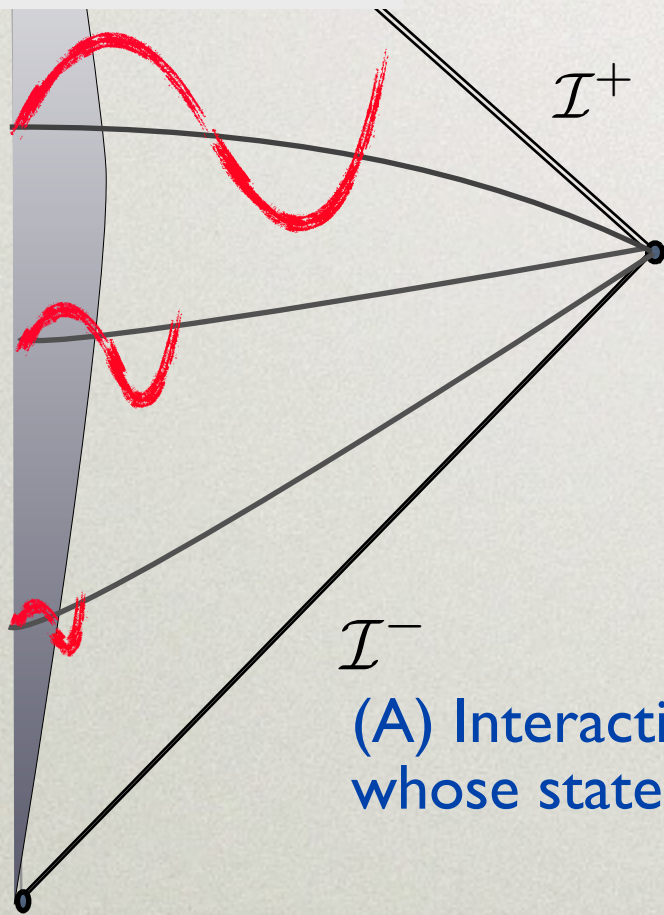
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System      +      Environment  
 $\mathcal{S}$                        $\mathcal{E}$

(A) Interaction between  $\mathcal{S}$  and  $\mathcal{E}$  selects a preferred basis of  $\mathcal{S}$  whose states are “stable” (in spite of the interaction with  $\mathcal{E}$ )





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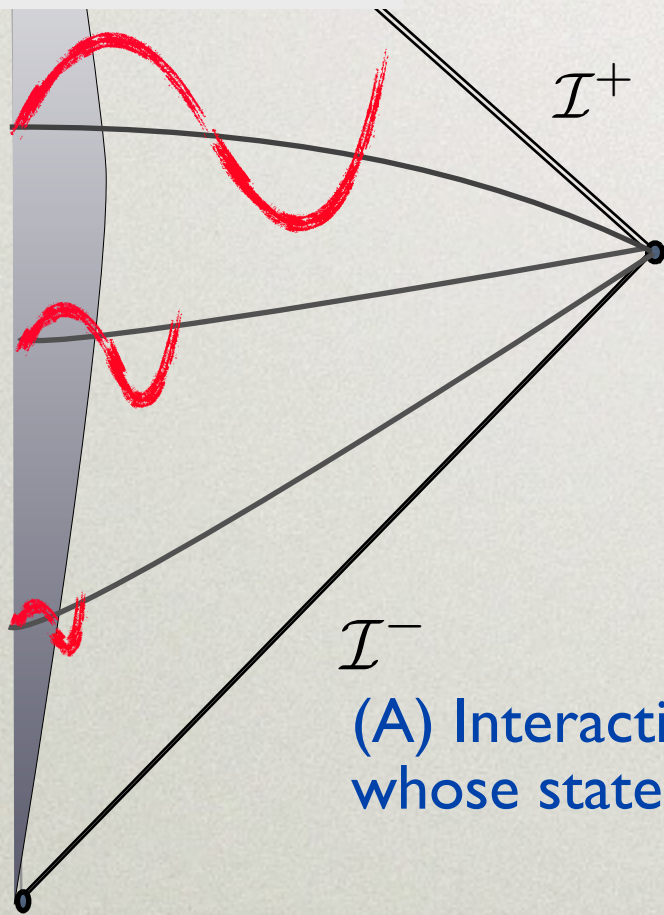
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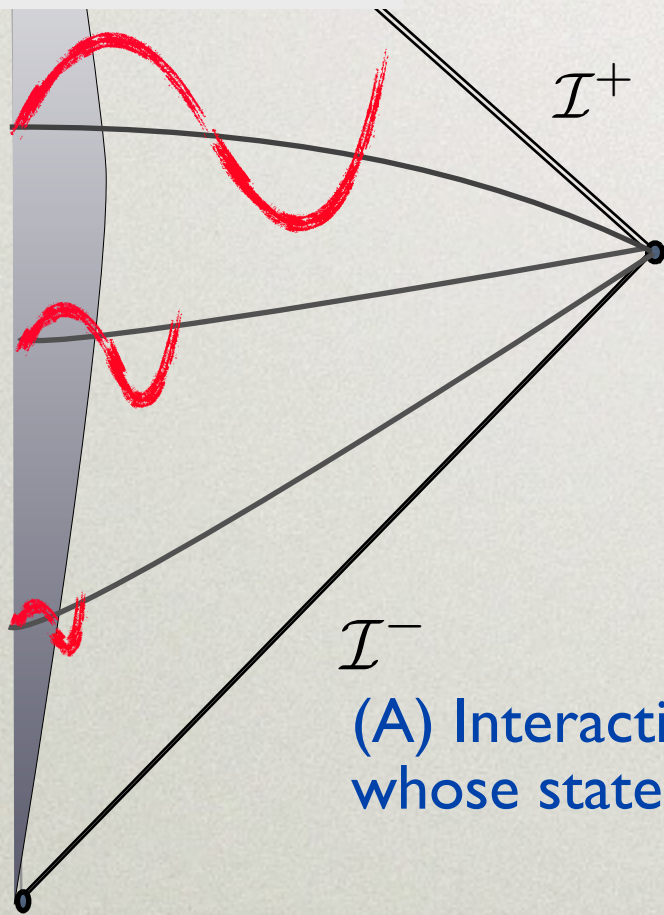
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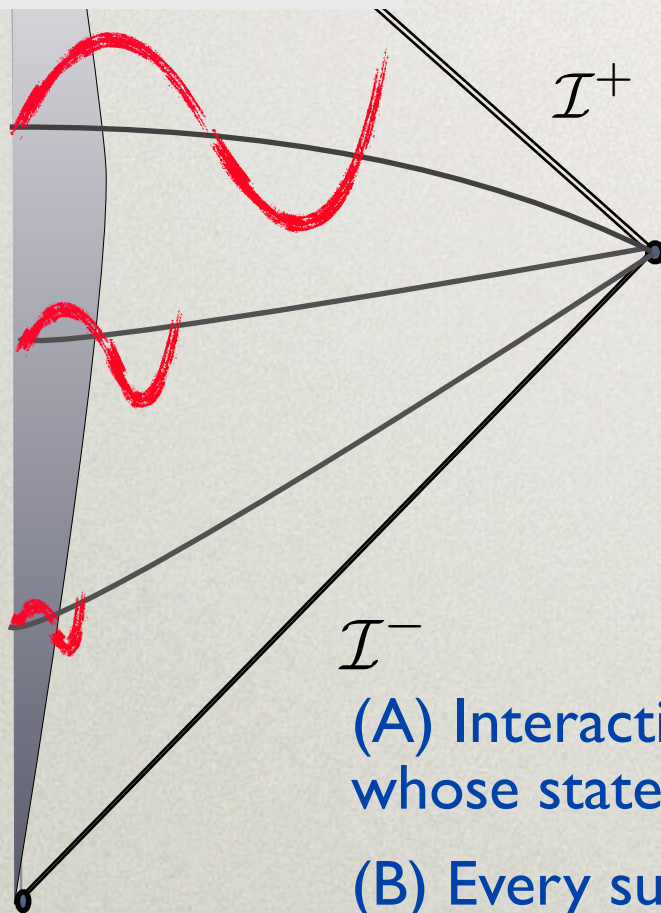
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System  $\mathcal{S}$  + Environment  $\mathcal{E}$

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(B) Every superposition of pointer states evolves to a statistical mixture of the pointer states (i.e. every density matrix become diagonal in the pointer state basis)





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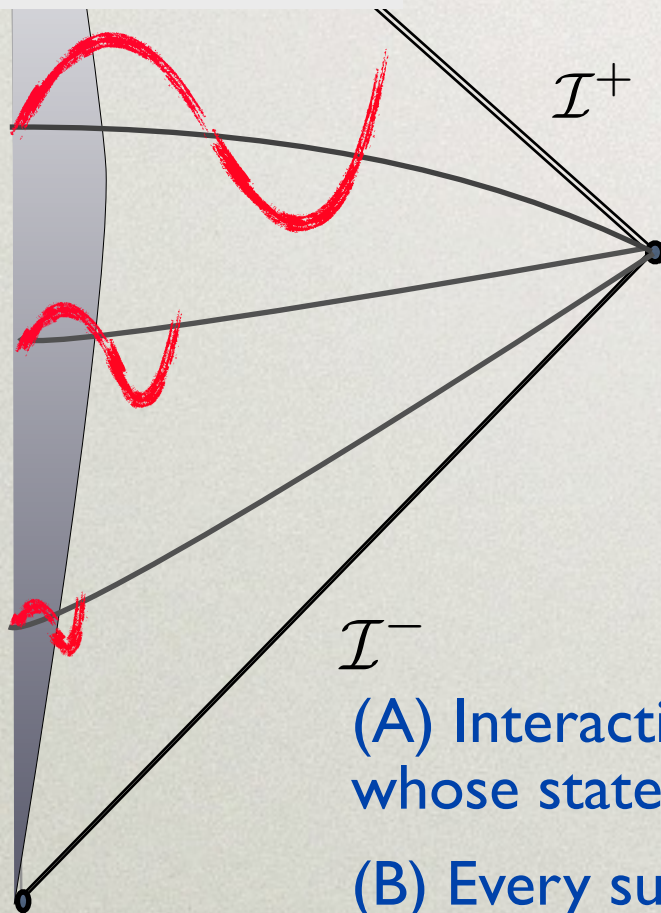
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W. Zurek, RMP (2003), W. Zurek, Nature Phys. (2009)

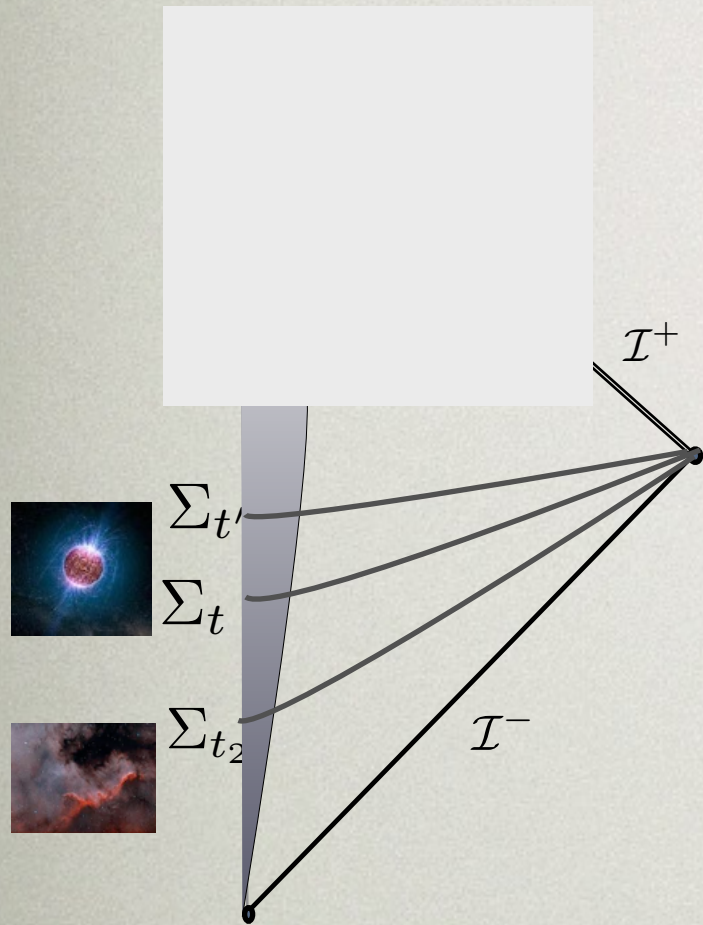




# FROM QUANTUM TO CLASSICAL

*A. Landulfo, W. Lima, G. Mastsas, and D. Vanzella, PRD (2015)*

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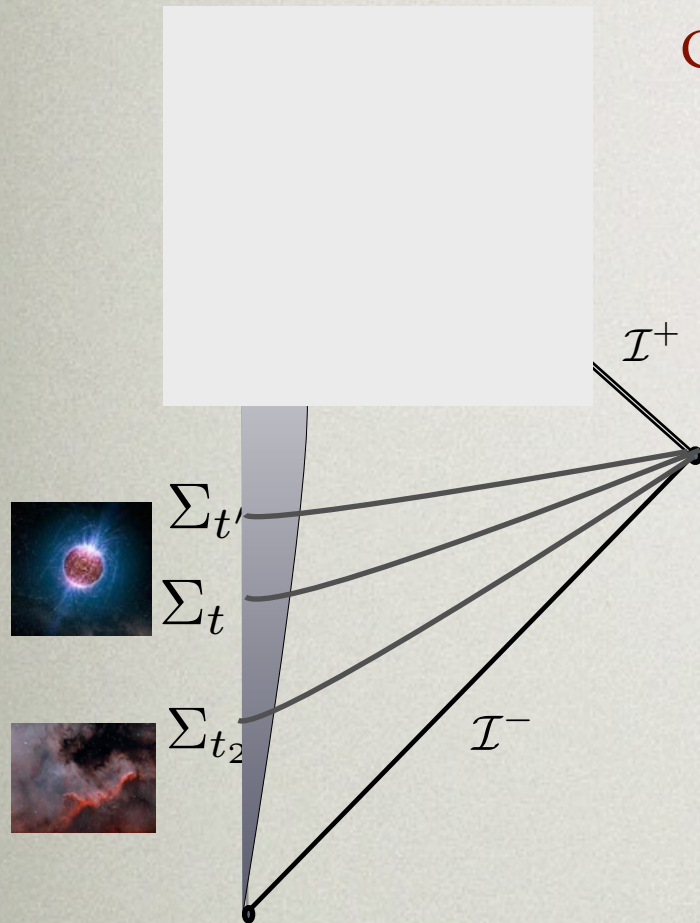
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Complete orthonormal set of  
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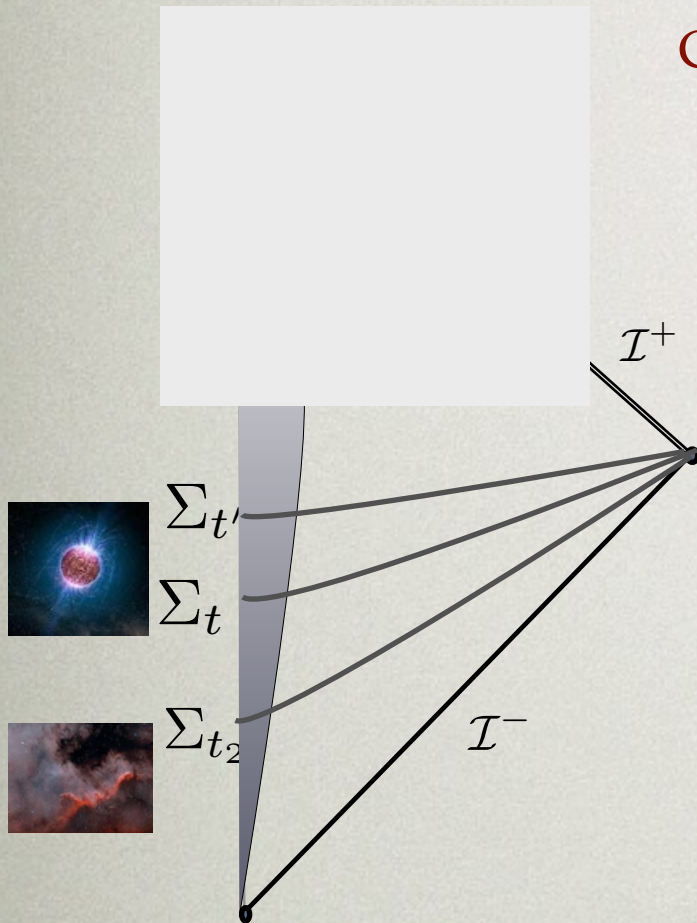
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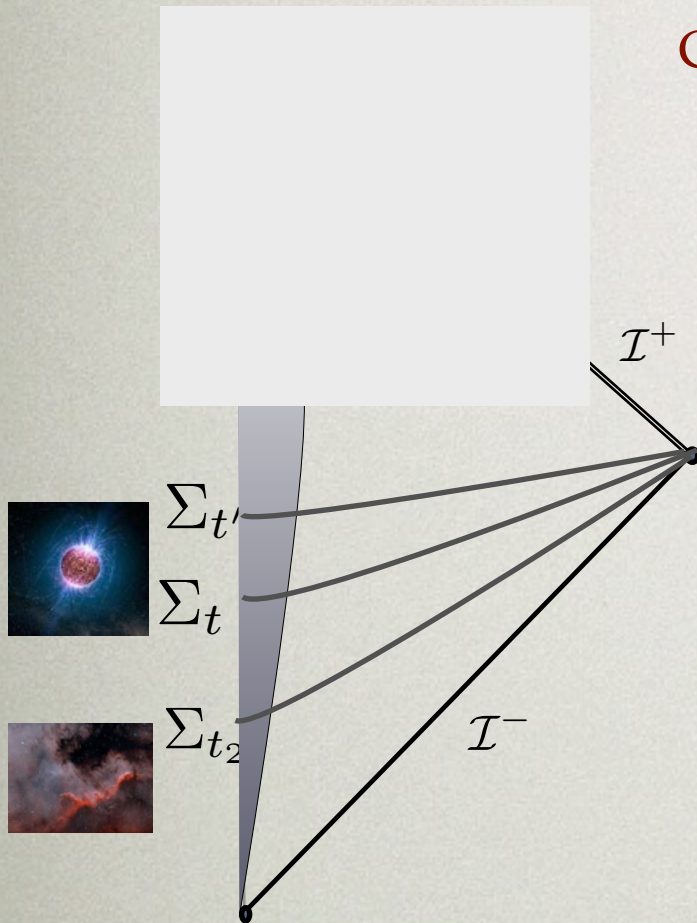
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Hamiltonian:

$$\hat{H} = \hat{H}_s + \hat{H}_u$$





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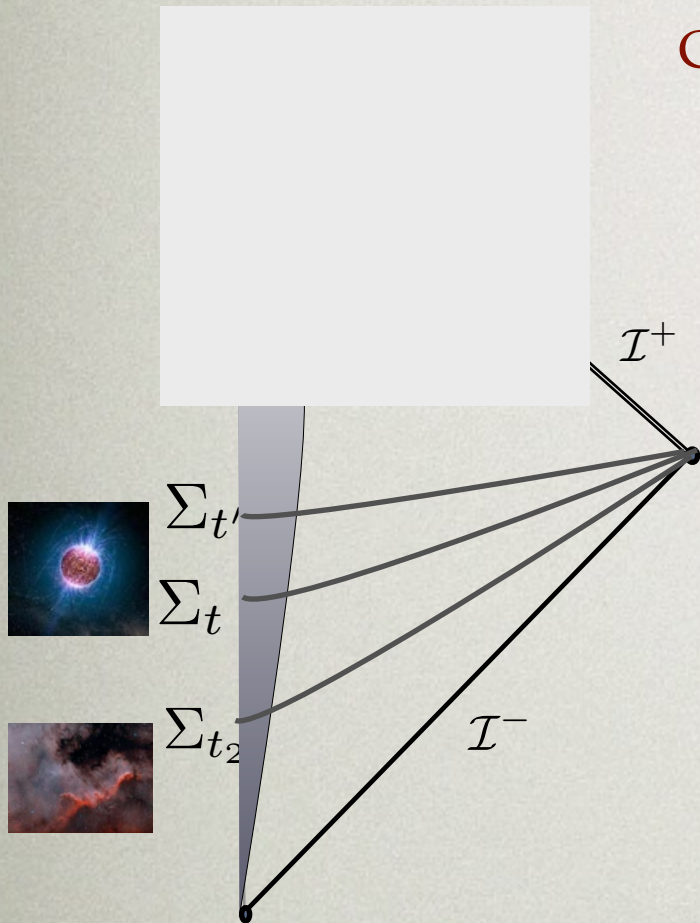
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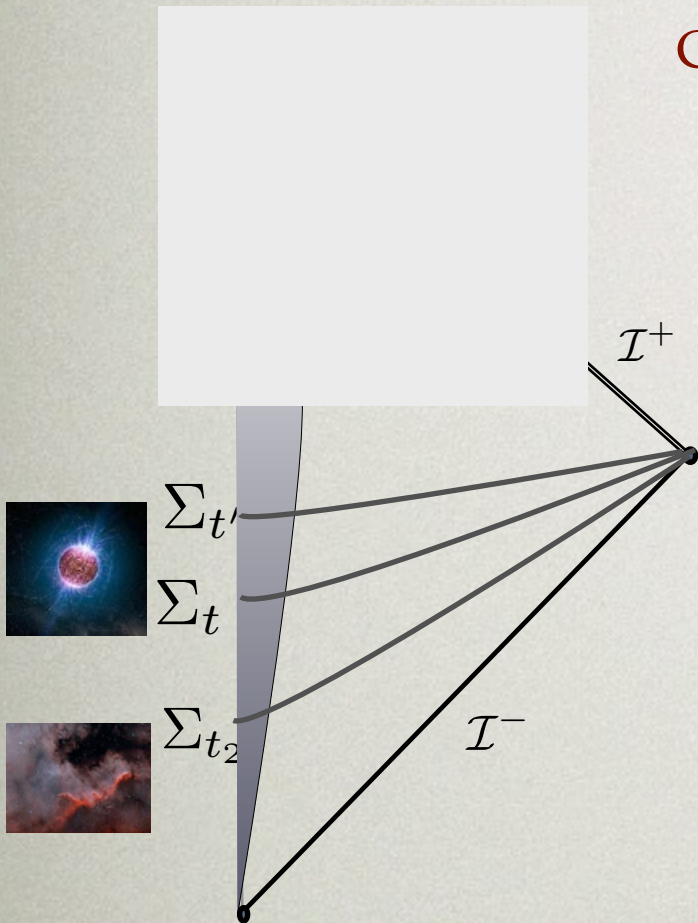
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$$\hat{H}_u = -\frac{\Omega}{2} (\hat{a}_{\Omega} \hat{a}_{\Omega} + \hat{a}_{\Omega}^{\dagger} \hat{a}_{\Omega}^{\dagger}) \rightarrow \hat{H}_u = \frac{1}{2} \hat{p}_{\Omega}^2 - \frac{\Omega^2}{2} \hat{q}_{\Omega}^2$$

Upside down harmonic oscillator

$$\hat{q}_{\Omega} \equiv \frac{1}{\sqrt{2\Omega}} (\hat{a}_{\Omega} + \hat{a}_{\Omega}^{\dagger}) \quad \hat{p}_{\Omega} \equiv -i\sqrt{\frac{\Omega}{2}} (\hat{a}_{\Omega} - \hat{a}_{\Omega}^{\dagger})$$





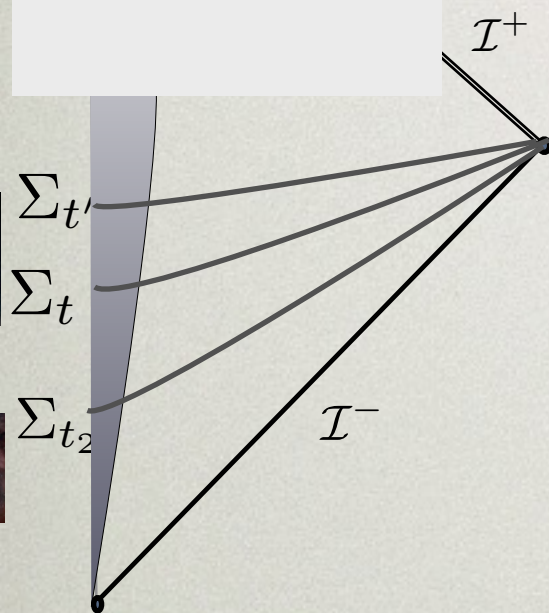
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$$\hat{\Phi} = \sum_{l\mu} \int d\varpi [\hat{b}_{\varpi l\mu} v_{l\mu}^{(+)} + \hat{b}_{\varpi l\mu}^{\dagger} v_{l\mu}^{(-)}] + \left[ \cosh(\Omega t) \hat{q}_{\Omega} + \frac{\sinh(\Omega t)}{\Omega} \hat{p}_{\Omega} \right] \frac{F_{\Omega}(\chi)}{r(\chi)} Y_{00}(\theta, \varphi)$$

Hamiltonian:

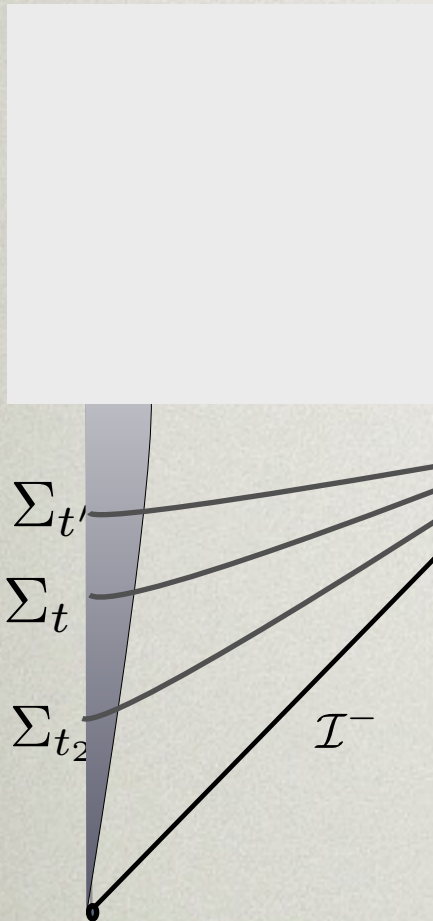
$$\hat{H} = \hat{H}_s + \hat{H}_u$$

$$\hat{H}_s = \frac{1}{2} \sum_{l\mu} \int d\varpi [\hat{b}_{\varpi l\mu} \hat{b}_{\varpi l\mu}^{\dagger} + \hat{b}_{\varpi l\mu}^{\dagger} \hat{b}_{\varpi l\mu}] \varpi$$

$$\hat{H}_u = -\frac{\Omega}{2} (\hat{a}_{\Omega} \hat{a}_{\Omega} + \hat{a}_{\Omega}^{\dagger} \hat{a}_{\Omega}^{\dagger}) \rightarrow \hat{H}_u = \frac{1}{2} \hat{p}_{\Omega}^2 - \frac{\Omega^2}{2} \hat{q}_{\Omega}^2$$

Upside down harmonic oscillator

$$\hat{q}_{\Omega} \equiv \frac{1}{\sqrt{2\Omega}} (\hat{a}_{\Omega} + \hat{a}_{\Omega}^{\dagger}) \quad \hat{p}_{\Omega} \equiv -i\sqrt{\frac{\Omega}{2}} (\hat{a}_{\Omega} - \hat{a}_{\Omega}^{\dagger})$$





# FROM QUANTUM TO CLASSICAL

---

## ❖ Unstable Phase

$\Sigma_t$



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Wigner Function

$$W(t, q, p) \equiv \frac{1}{2\pi} \int_{-\infty}^{+\infty} dy \varrho(t, q - y/2, q + y/2) e^{ipy},$$



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- ❖ For pure states  $|\psi\rangle$

$$\int_{-\infty}^{+\infty} dq W(q, p) = |\tilde{\psi}(p)|^2, \quad \int_{-\infty}^{+\infty} dp W(q, p) = |\psi(q)|^2$$



# FROM QUANTUM TO CLASSICAL

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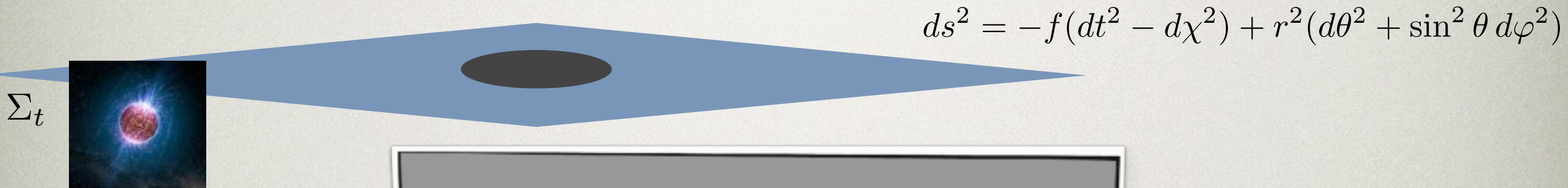
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- ❖ For quadratic potentials (like our upside down harmonic oscillator)



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$$\partial_t W(t, q, p) = \{H(q, p), W(t, q, p)\}$$



# FROM QUANTUM TO CLASSICAL

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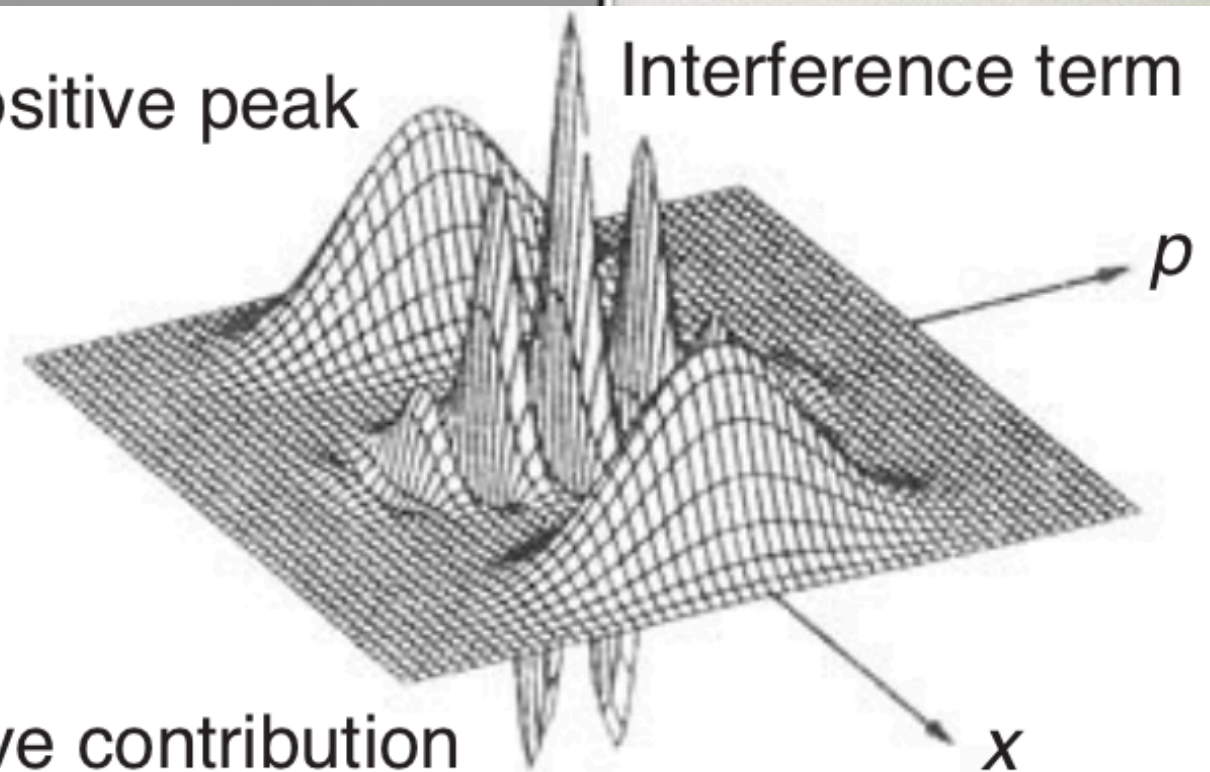
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**It is a very  
classicaliz**

Positive peak

Interference term



Negative contribution

W. Zurek, Decoherence and the Transition from Quantum to Classical—Revisited

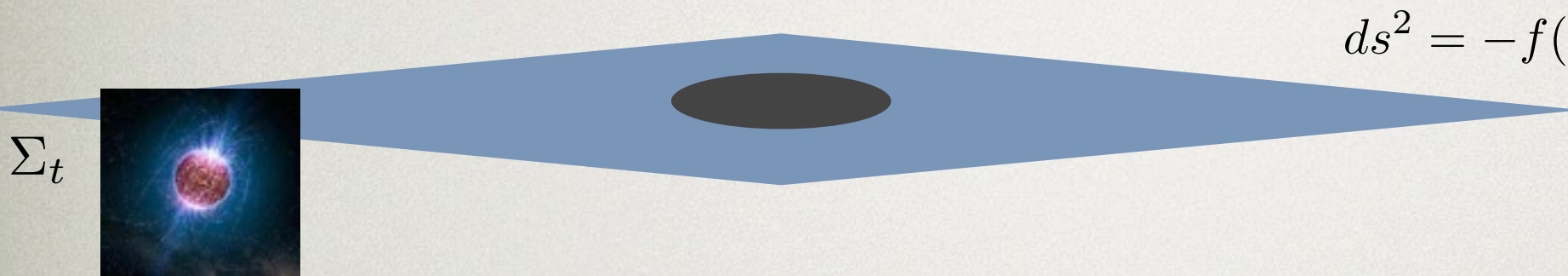
**Liouville equation**



# FROM QUANTUM TO CLASSICAL

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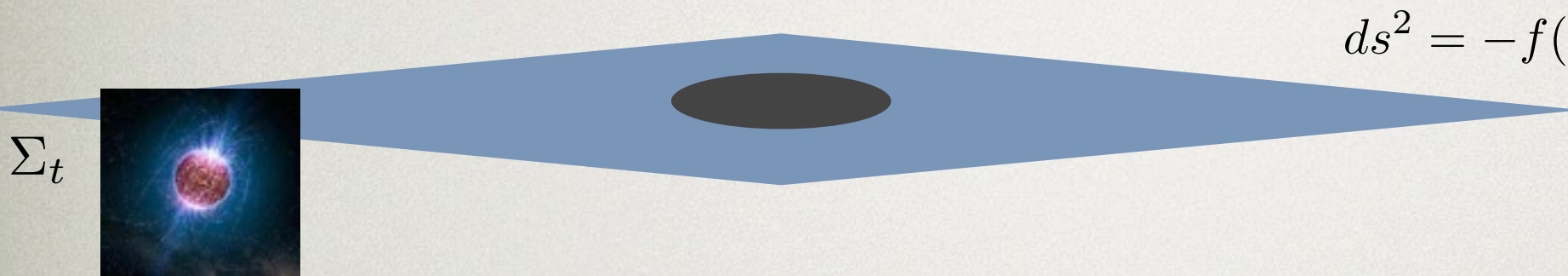
- ❖ For the upside down harmonic oscillator



# FROM QUANTUM TO CLASSICAL

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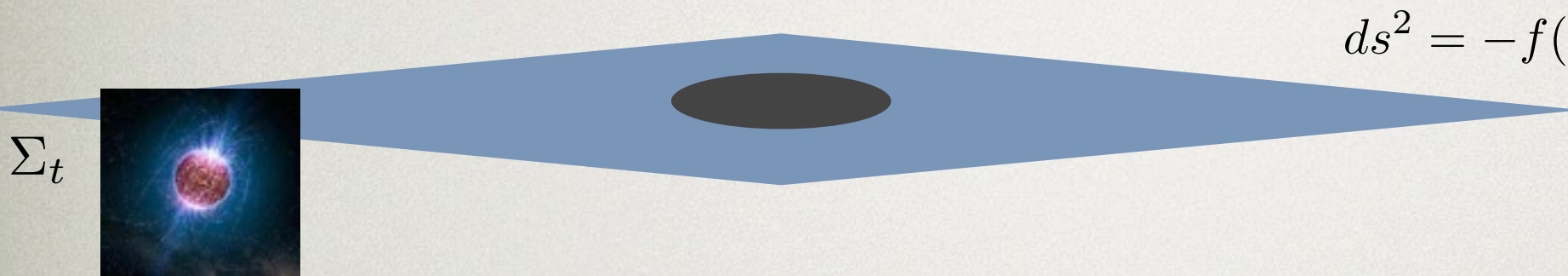
$$H(q, p) \equiv \frac{1}{2}p^2 - \frac{\Omega^2}{2}q^2.$$



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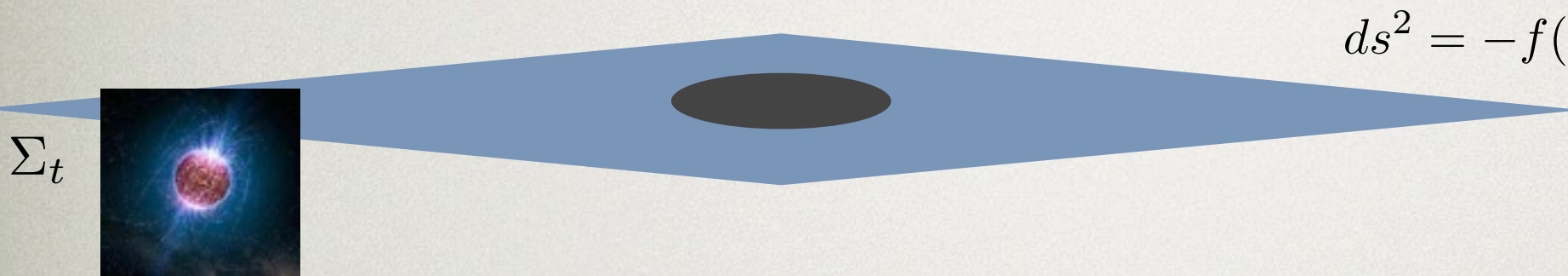
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# FROM QUANTUM TO CLASSICAL

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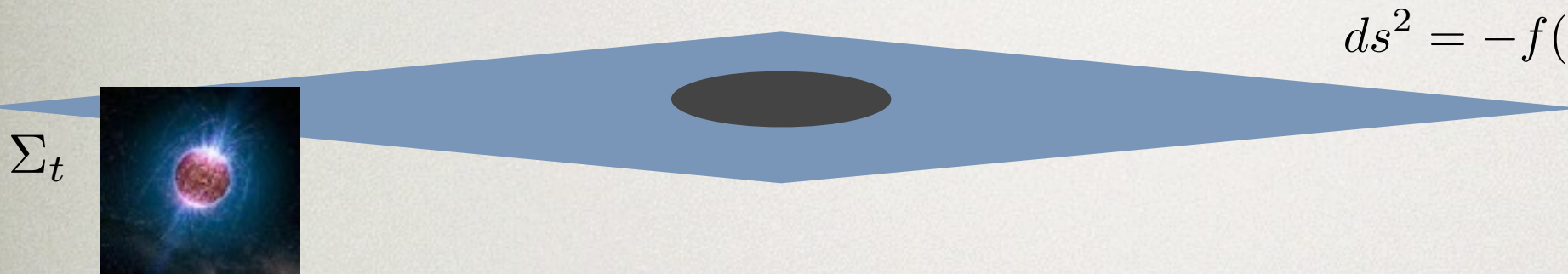
$$W(t, q, p) = \frac{1}{\pi} \exp \left( -a(t)q^2 - \frac{[p + b(t)q]^2}{a(t)} \right).$$



# FROM QUANTUM TO CLASSICAL

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$$W(t, q, p) \stackrel{\Omega t \gg 1}{\approx} |\eta(t, q)|^2 \delta(p - \Omega q),$$



# FROM QUANTUM TO CLASSICAL

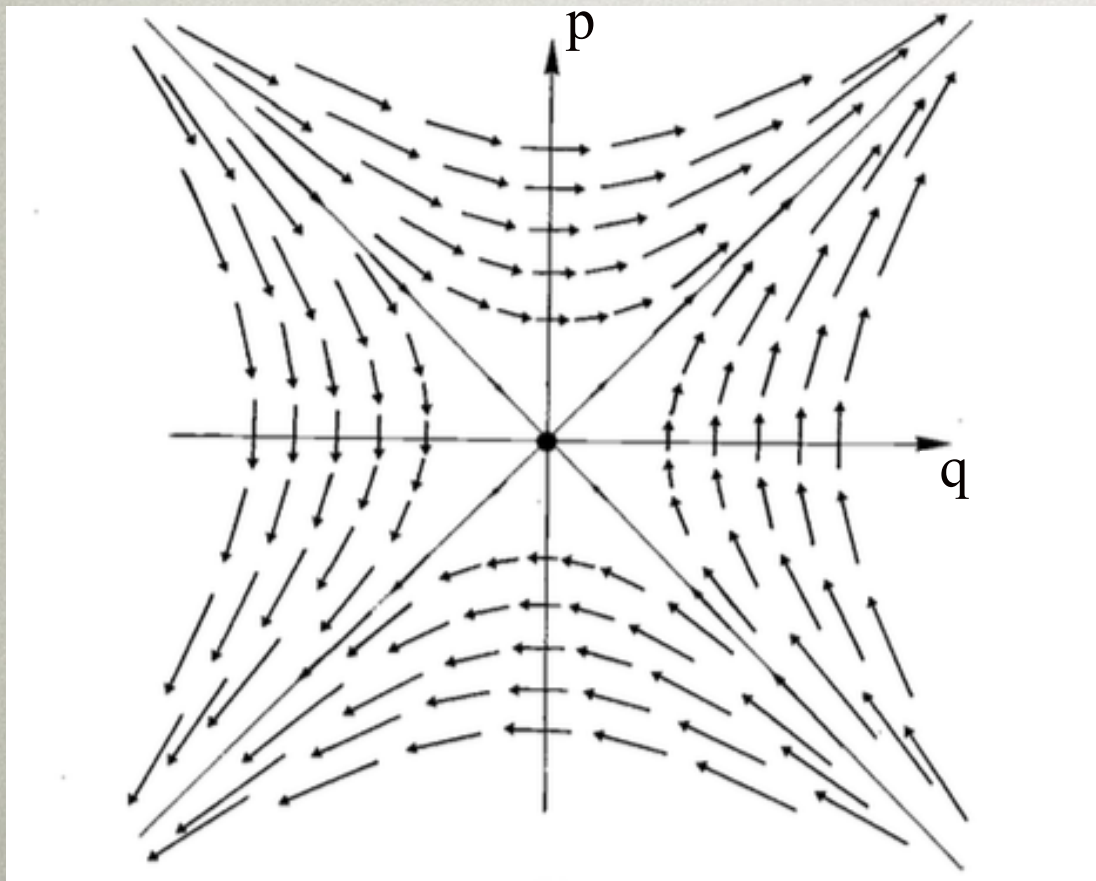
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$\Sigma_t$



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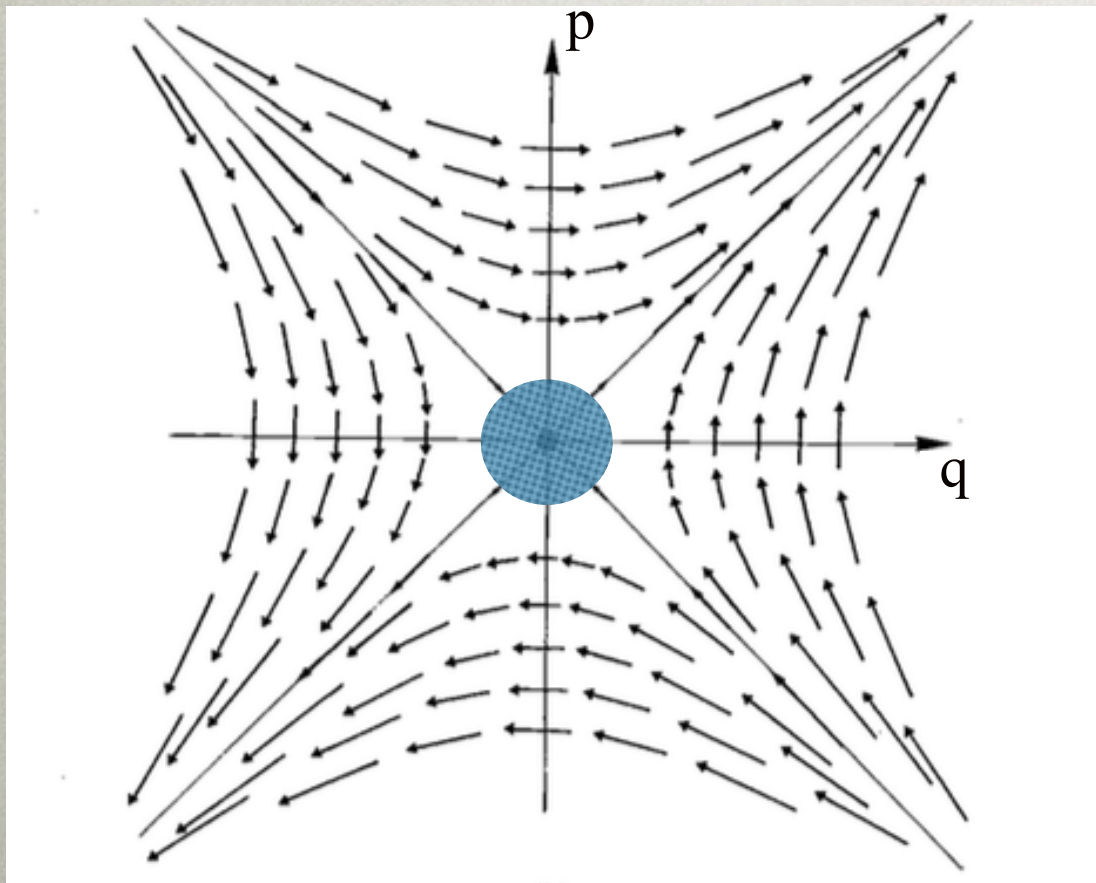
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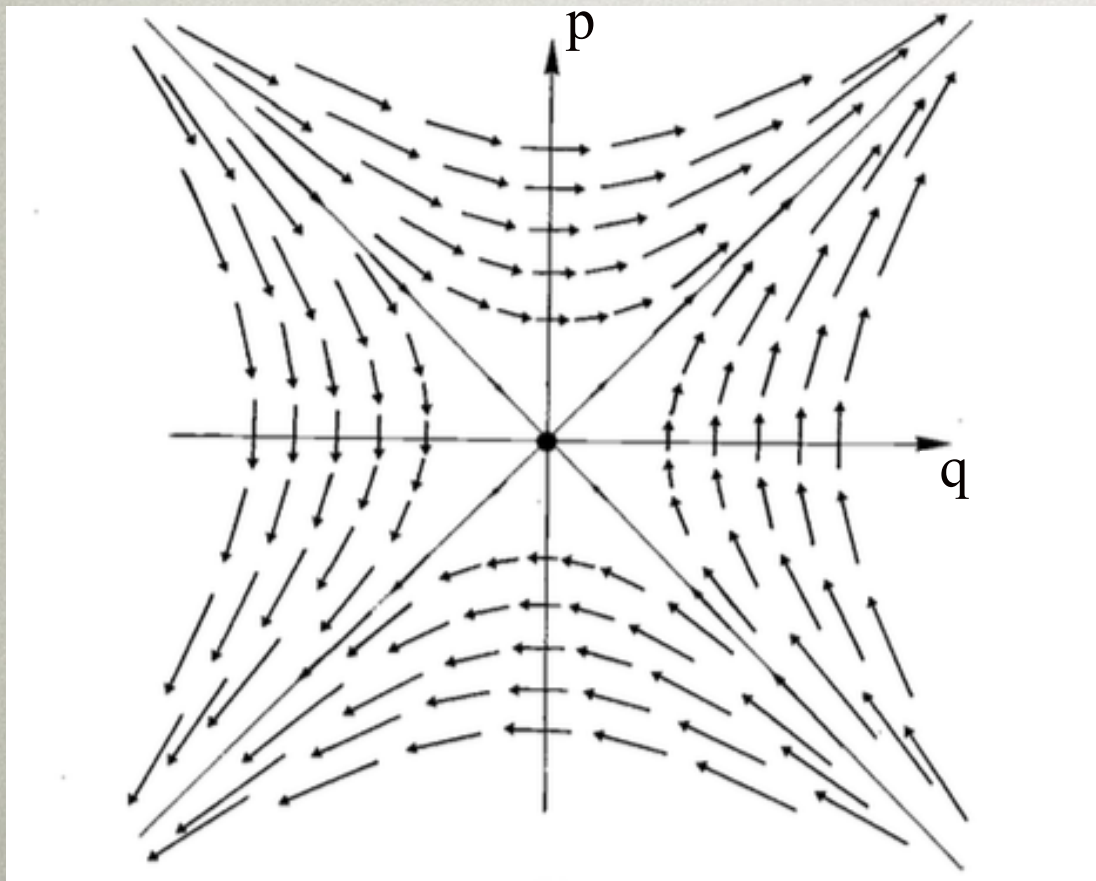
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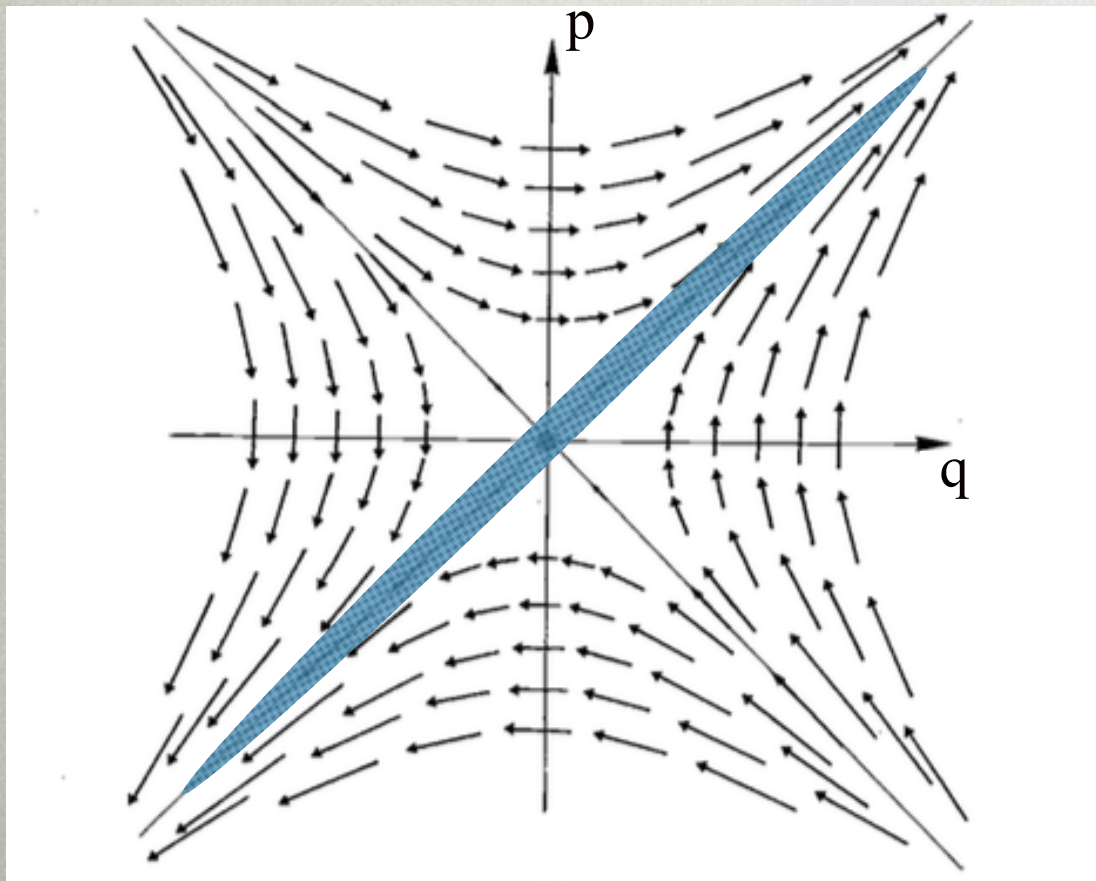
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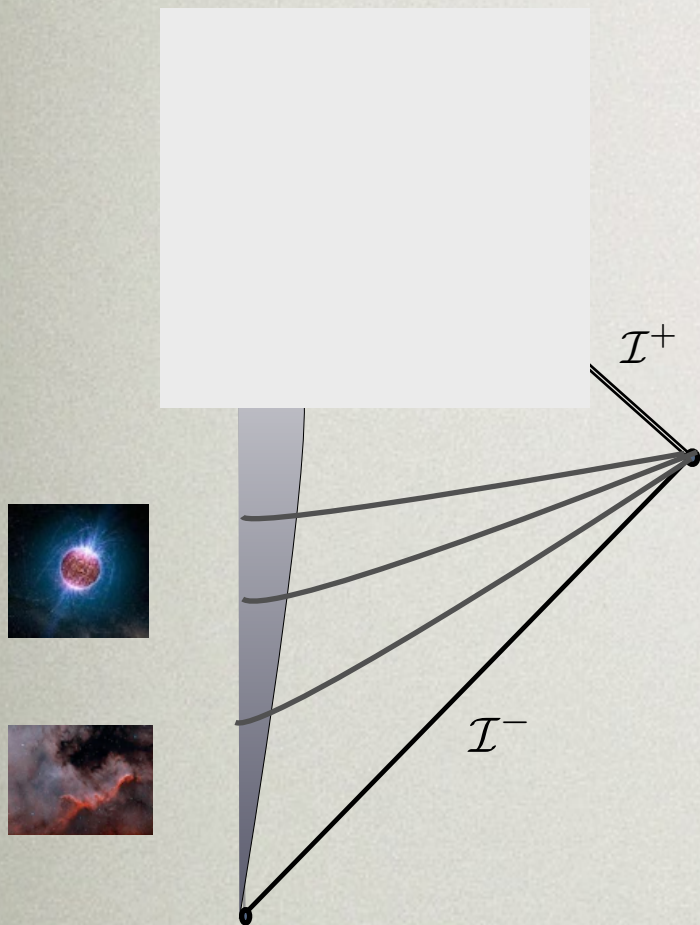
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# FROM QUANTUM TO CLASSICAL

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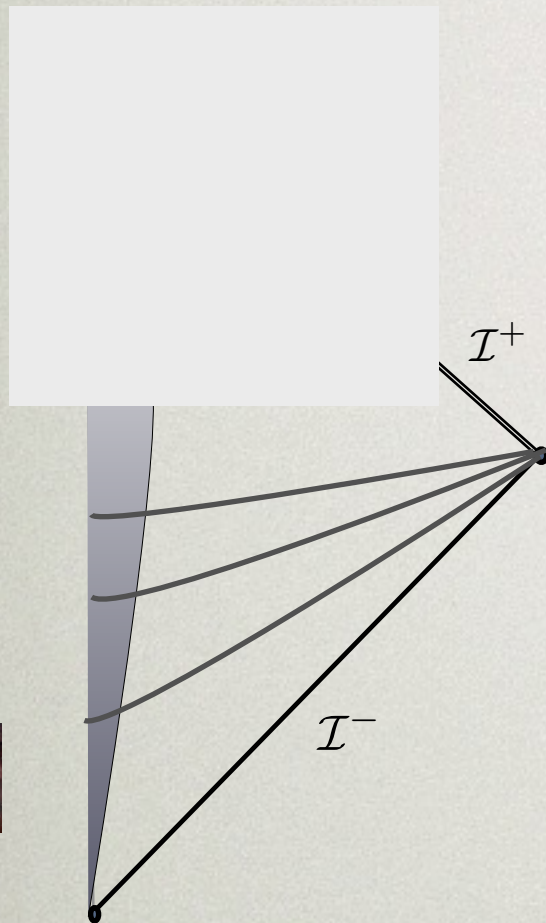




# FROM QUANTUM TO CLASSICAL

**free scalar field**

$$S_\phi[\phi] \equiv -\frac{1}{2} \int_{\mathcal{M}} d^4x \sqrt{-g} \phi P \phi, \quad P \equiv -(\nabla^a \nabla_a + m^2 + \xi R)$$



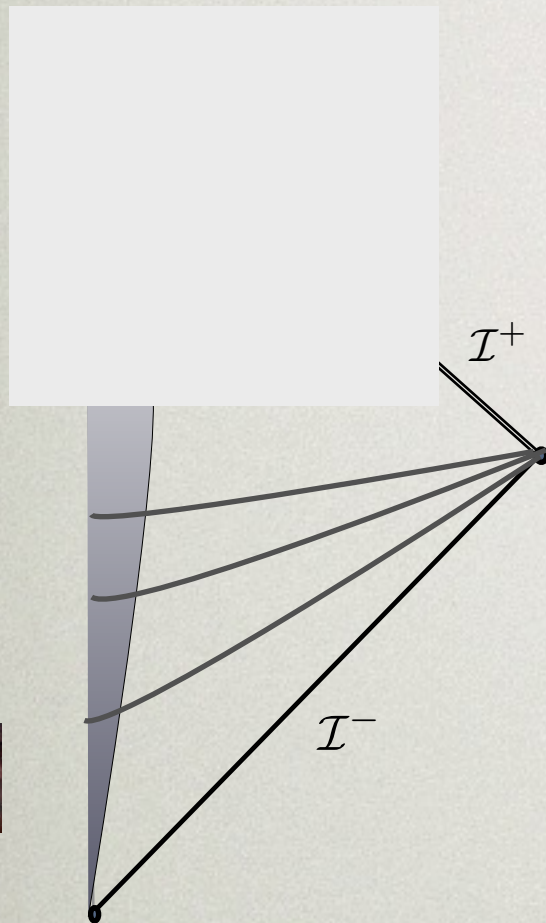


# FROM QUANTUM TO CLASSICAL

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$$g_{ab} \rightarrow g_{ab} + \kappa \gamma_{ab}, \quad \Phi \rightarrow \Phi + \phi \quad (\Phi = 0)$$





# FROM QUANTUM TO CLASSICAL

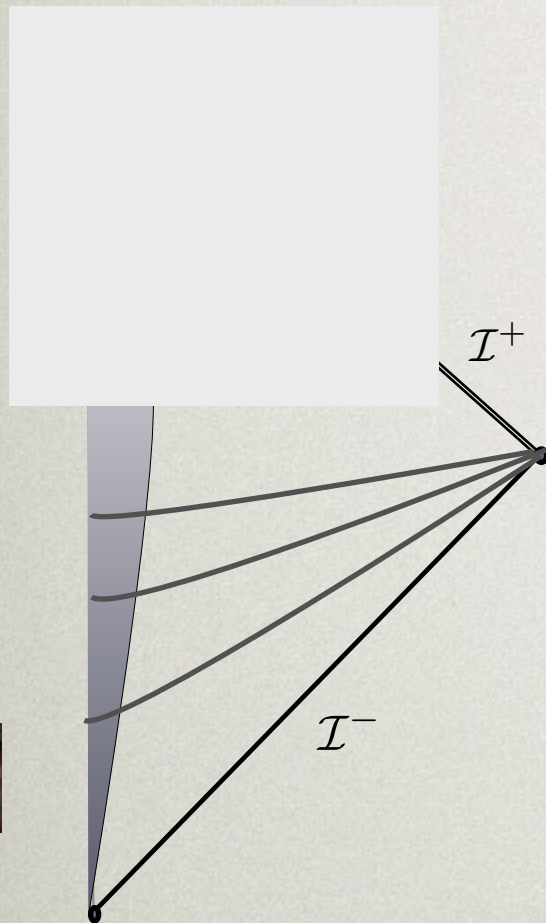
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## free graviton (with gauge fixing term)

$$S_\gamma[\gamma_{ab}] \equiv -\frac{1}{2} \int_{\mathcal{M}} d^4x \sqrt{-g} \gamma^{ab} \mathfrak{D}_{abcd} \gamma^{cd}$$





# FROM QUANTUM TO CLASSICAL

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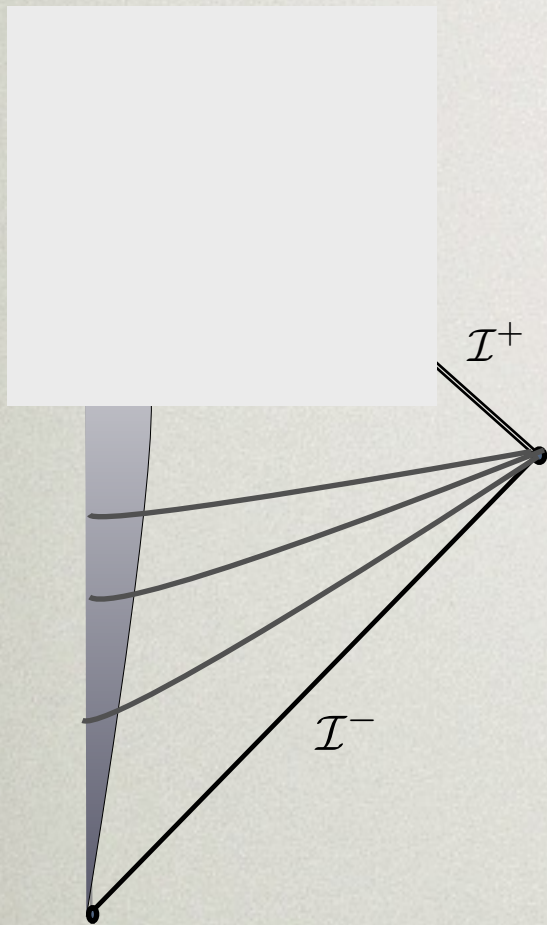
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# FROM QUANTUM TO CLASSICAL

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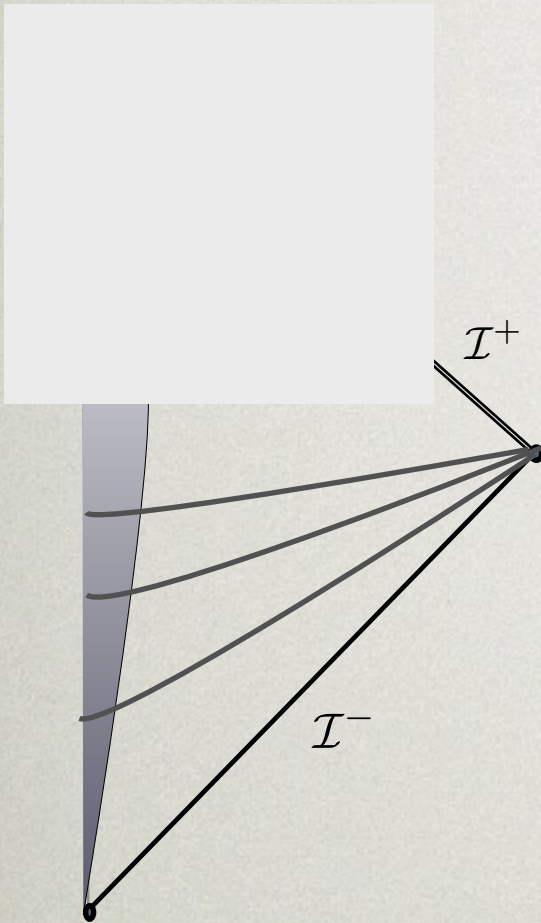
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## Interaction

$$S_{int}[\phi, \gamma_{ab}] \equiv \frac{\kappa}{2} \int_{\mathcal{M}} \sqrt{-g} d^4x T_{ab} \gamma^{ab}$$





# FROM QUANTUM TO CLASSICAL

---

## ❖ Unstable Phase

$\Sigma_t$



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase



**System of interest - Unstable mode (which is the one we expect that "classicalize")**

$\phi_u$

**Environment - Stable modes and graviton**

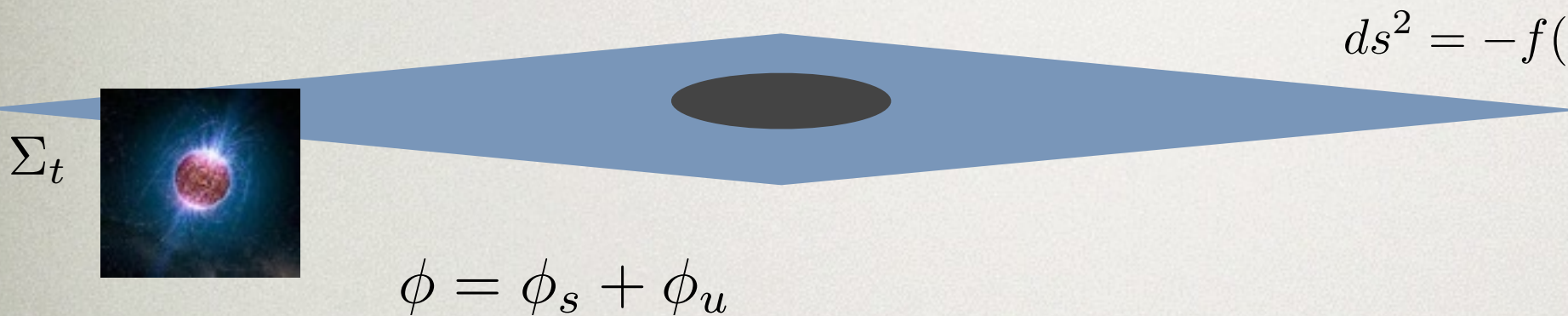
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# FROM QUANTUM TO CLASSICAL

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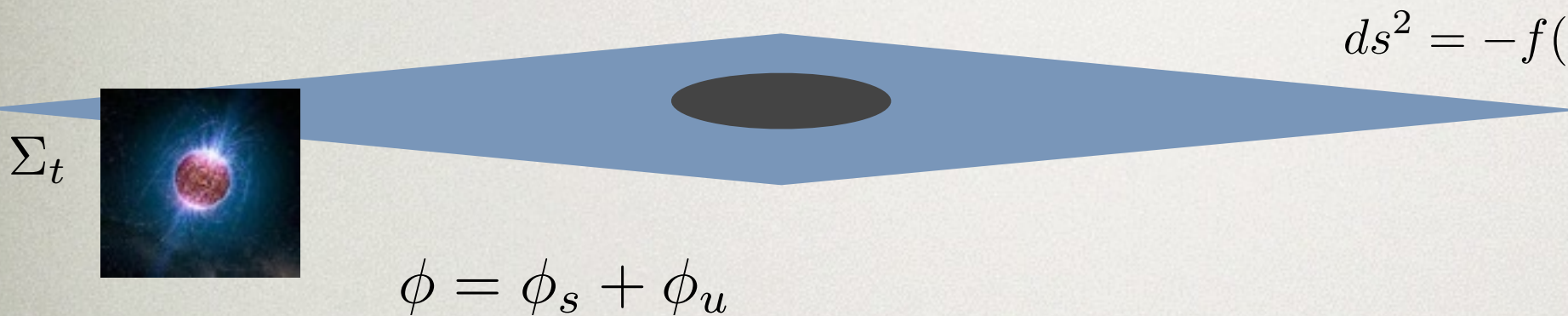
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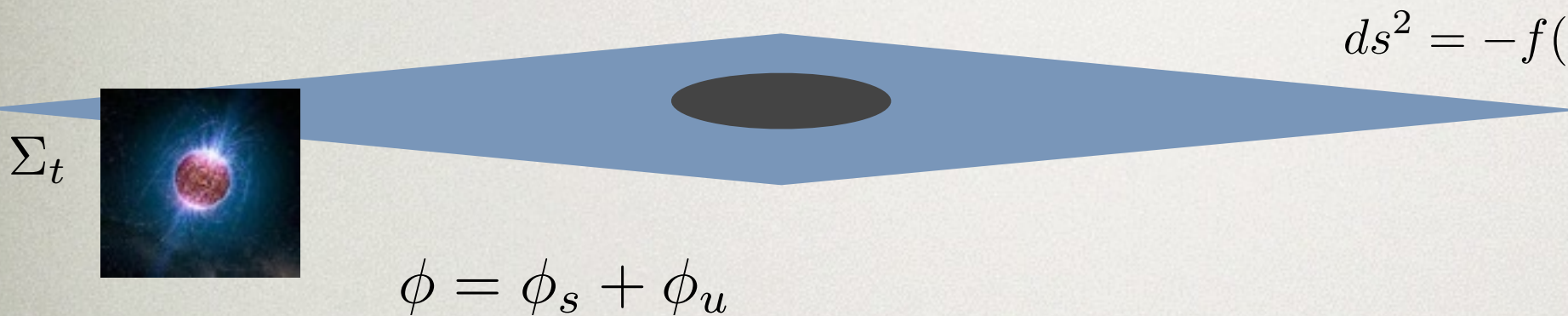
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$$T_{ab} = T_{ab}^{(s)} + T_{ab}^{(u)} + t_{ab}$$



# FROM QUANTUM TO CLASSICAL

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# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase

$\Sigma_t$



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# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase

$\Sigma_t$



**Initial state of the full system**

$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

$$\hat{\rho}(0) = \hat{\rho}_s(0) \otimes \hat{\rho}_u(0) \otimes \hat{\rho}_\gamma(0),$$

$$\hat{\rho}_s(0) = |0_s\rangle\langle 0_s|$$

$$\hat{\rho}_u(0) = |0_u\rangle\langle 0_u|$$

$$\hat{\rho}_\gamma(0) = \text{thermal state with temperature } T$$



# FROM QUANTUM TO CLASSICAL

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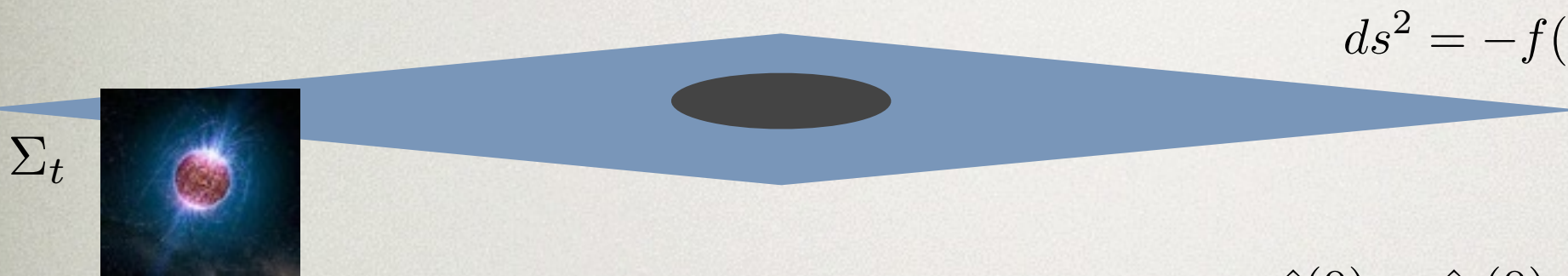
$$\hat{\rho}(0) \xrightarrow{\hat{U}_{u\gamma s}(t)} \hat{\rho}(t)$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase



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# FROM QUANTUM TO CLASSICAL

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$$\hat{\rho}(0) \xrightarrow{\hat{U}_{u\gamma s}(t)} \hat{\rho}(t) \xrightarrow{\text{trace out the environment}} \hat{\rho}_u(t) = \text{tr}_{\gamma s} \hat{\rho}(t)$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase



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$$\hat{\rho}(0) \xrightarrow{\hat{U}_{u\gamma s}(t)} \hat{\rho}(t) \xrightarrow{\text{trace out the environment}}$$

$$\hat{\rho}_u(t) = \text{tr}_{\gamma s} \hat{\rho}(t)$$

$$\varrho_u(0, \psi_u, \psi_u') \equiv \langle \psi_u | \hat{\rho}_u(0) | \psi_u' \rangle$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

**Initial state of the full system**

$$\hat{\rho}(0) = \hat{\rho}_s(0) \otimes \hat{\rho}_u(0) \otimes \hat{\rho}_\gamma(0),$$

$$\hat{\rho}_s(0) = |0_s\rangle\langle 0_s| \quad \hat{\rho}_u(0) = |0_u\rangle\langle 0_u|$$

$$\hat{\rho}_\gamma(0) = \text{thermal state with temperature } T$$

$$\hat{\rho}(0) \xrightarrow{\hat{U}_{u\gamma s}(t)} \hat{\rho}(t) \xrightarrow{\text{trace out the environment}} \hat{\rho}_u(t) = \text{tr}_{\gamma s} \hat{\rho}(t)$$

$$\varrho_u(0, \psi_u, \psi_u') \equiv \langle \psi_u | \hat{\rho}_u(0) | \psi_u' \rangle$$

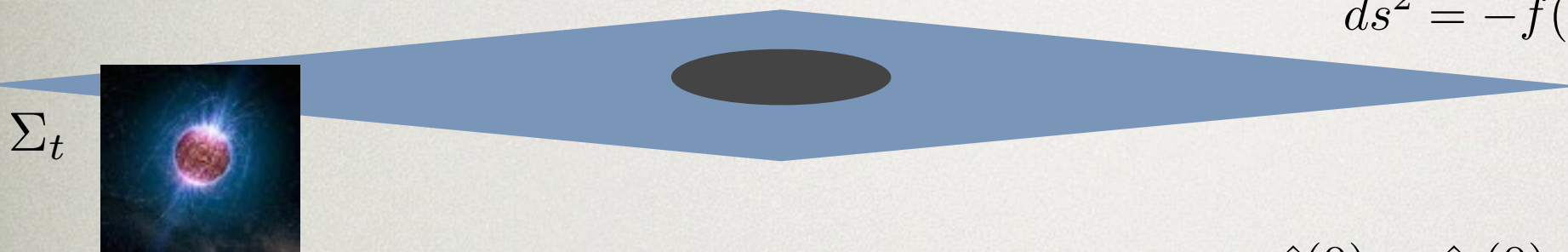
$$\varrho_{\text{red}}(t, \varphi_u, \varphi_u') = \int d\psi_u d\psi_u' \varrho_u(0, \psi_u, \psi_u') \times J_{\text{red}}(t, \varphi_u, \varphi_u'; 0, \psi_u, \psi_u'), \quad \hat{\phi}_u(0, \mathbf{x}) |\psi_u\rangle = \psi_u(\mathbf{x}) |\psi_u\rangle$$

$$J_{\text{red}}(t, \varphi_u, \varphi_u'; 0, \psi_u, \psi_u') \equiv \int_{\psi_u}^{\varphi_u} \mathcal{D}\phi_u \int_{\psi_u'}^{\varphi_u'} \mathcal{D}\phi_u' e^{i\{S_\phi[\phi_u] - S_\phi[\phi_u']\}} F[\phi_u, \phi_u']$$



# FROM QUANTUM TO CLASSICAL

## \* Unstable Phase



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**Feynman-Vernon Influence Functional**

$$\hat{\phi}_u(0, \mathbf{x})|\psi_u\rangle = \psi_u(\mathbf{x})|\psi_u\rangle$$

$$J_{\text{red}}(t, \varphi_u, \varphi_u'; 0, \psi_u, \psi_u') \equiv \int_{\psi_u}^{\varphi_u} \mathcal{D}\phi_u \int_{\psi_u'}^{\varphi_u'} \mathcal{D}\phi_u' e^{i\{S_\phi[\phi_u] - S_\phi[\phi_u']\}} F[\phi_u, \phi_u']$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase

$\Sigma_t$



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase

$\Sigma_t$



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

**Long-time regime and high temperature limit**

$$\Omega t \gg 1$$

$$k_B T \gg \Omega$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase

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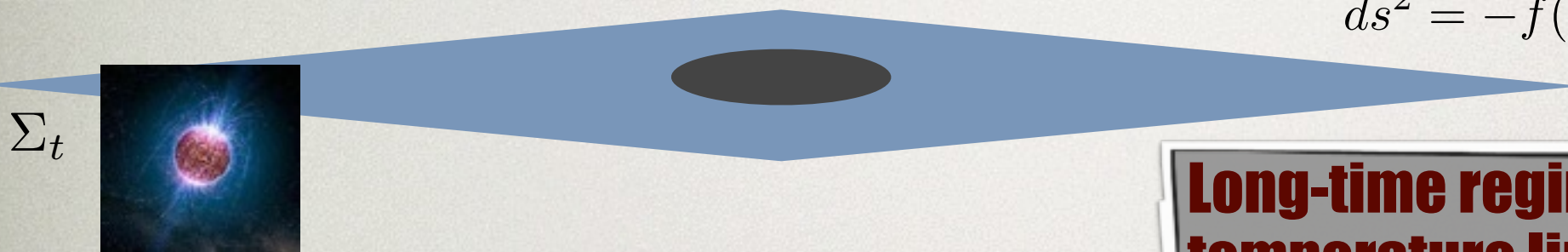
**Master equation for the state of the unstable mode**

$$\partial_t \varrho_{\text{red}} \approx -i \left[ \frac{1}{2} (-\partial_q^2 + \partial_{q'}^2) - \frac{\Omega^2}{2} (q^2 - q'^2) \right] \varrho_{\text{red}} - D(q - q')^2 \varrho_{\text{red}} + \dots$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase



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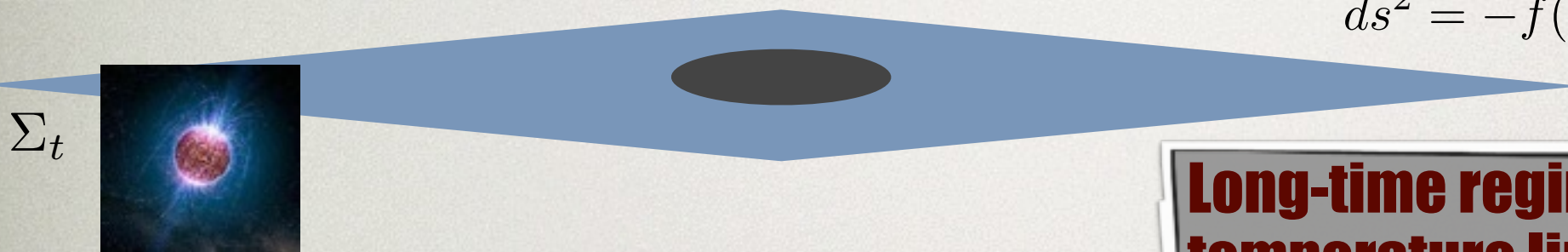
**Wigner function for the unstable mode**

$$\partial_t W_{\text{red}} \approx \{H(q, p), W_{\text{red}}\} + D \partial_p^2 W_{\text{red}} + \dots$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase



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**Wigner function for the unstable mode**

$$D = 8\pi \Delta \left( \frac{\ell_P}{R} \right)^2 \left( \frac{k_B T}{\Omega} \right) \Omega^2$$

$$\partial_t W_{\text{red}} \approx \{H(q, p), W_{\text{red}}\} + D \partial_p^2 W_{\text{red}} + \dots$$



# FROM QUANTUM TO CLASSICAL

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## ❖ Unstable Phase

$\Sigma_t$



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**Wigner function for the  
unstable mode**

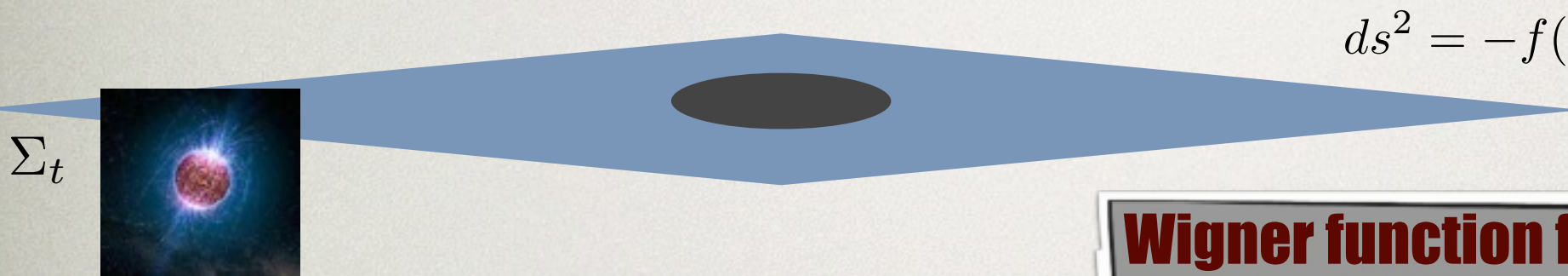
$$\partial_t W_{\text{red}} \approx \{H(q, p), W_{\text{red}}\} + D\partial_p^2 W_{\text{red}} + \dots$$



# FROM QUANTUM TO CLASSICAL

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- ✧ Unstable Phase



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

**Wigner function for the unstable mode**

- ✧ L. Diósi and C. Kiefer J.Phys.A (2002)

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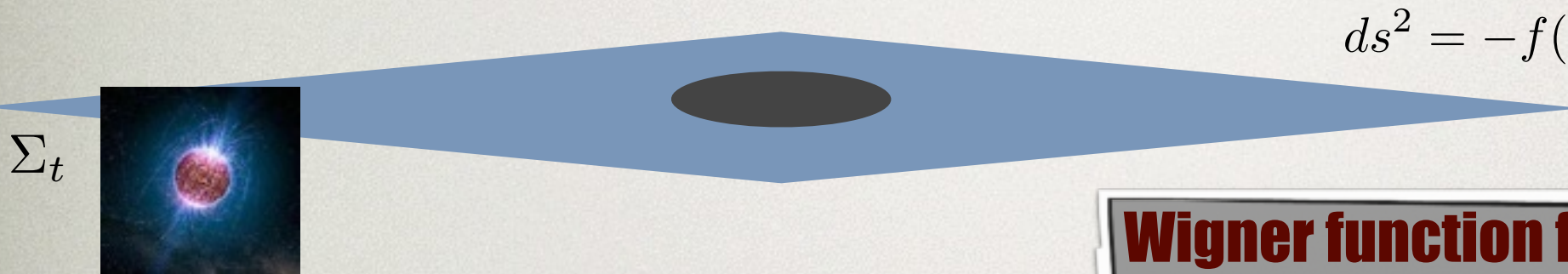
- ✧ O. Brodier and A. Ozorio de Almeida, PRE (2004)



# FROM QUANTUM TO CLASSICAL

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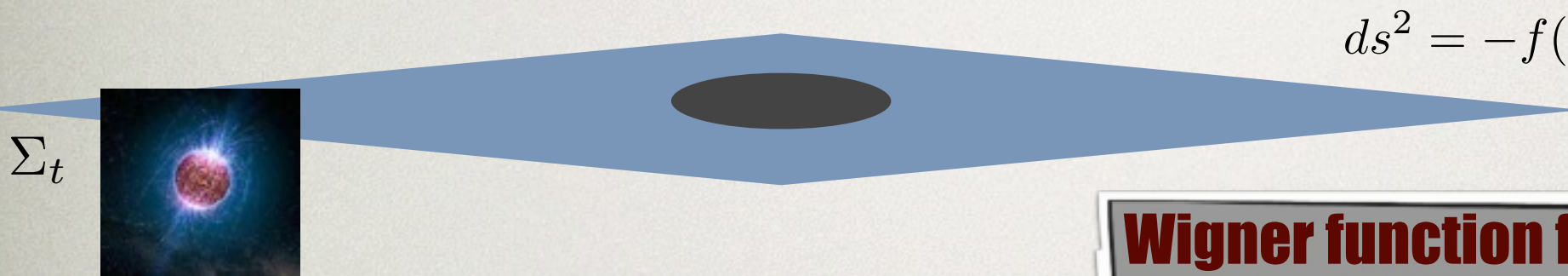
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$$W_{\text{red}}(t > t_d) > 0 \text{ (regardless of the initial state)}$$



# FROM QUANTUM TO CLASSICAL

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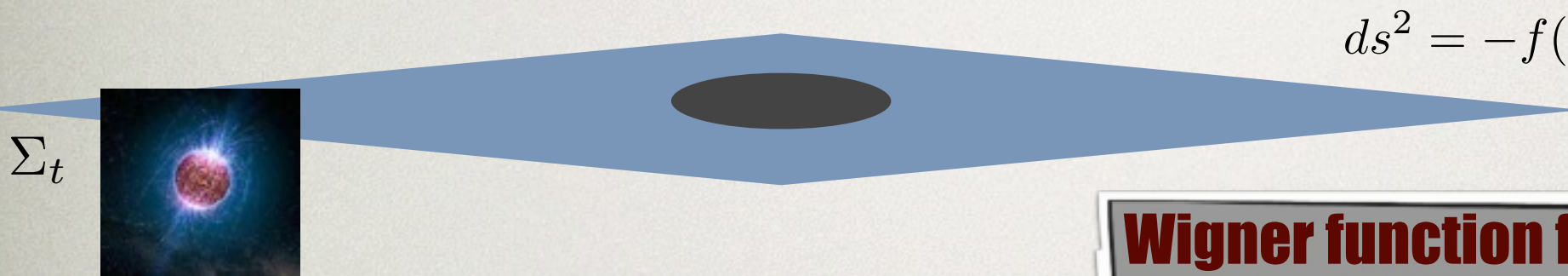
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$$t_d \sim \frac{1}{\Omega} \ln \left[ \frac{1}{8\pi\Delta} \left( \frac{R}{\ell_P} \right)^2 \frac{\Omega}{k_B T} \right]$$



# FROM QUANTUM TO CLASSICAL

## ❖ Unstable Phase



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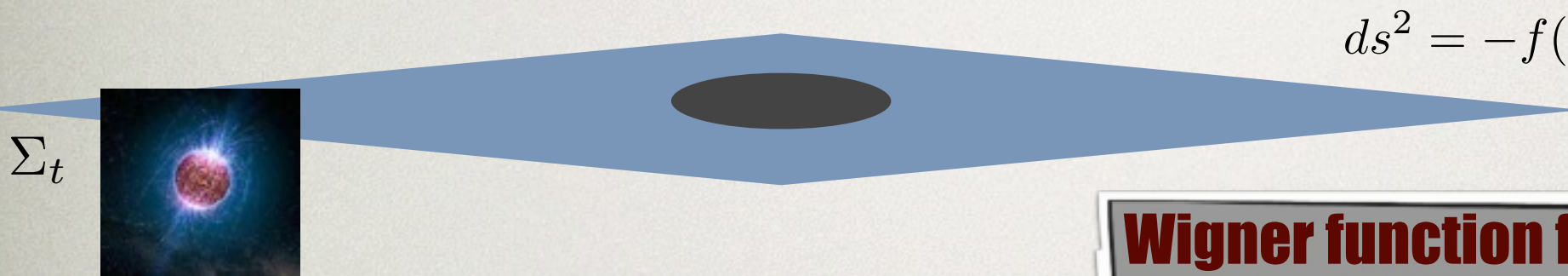
❖ For a  $R \sim 10\text{km}$  neutron star and  $T \sim 1\text{K}$

$$t_d \sim 160 \times \Omega^{-1} \sim 160 \times R \text{ which is of the order of } t_{\text{br}} \sim 10^{-3} \text{ s}$$



# FROM QUANTUM TO CLASSICAL

## \* Unstable Phase



$$ds^2 = -f(dt^2 - d\chi^2) + r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

### Wigner function for the unstable mode

\* L. Diósi and C. Kiefer J.Phys.A (2002)

\* O. Brodier and A. Ozorio de Almeida (2004)

Thus, by the time backreaction becomes ineluctable the (unstable sector of the) initially pure vacuum state has evolved into a statistical mixture of (very) localized states in the field amplitude and momentum representation.

$$t_d \sim \frac{1}{\Omega} \ln \left[ \frac{1}{8\pi\Delta} \left( \frac{R}{\ell_P} \right)^2 \frac{\Omega}{k_B T} \right]$$

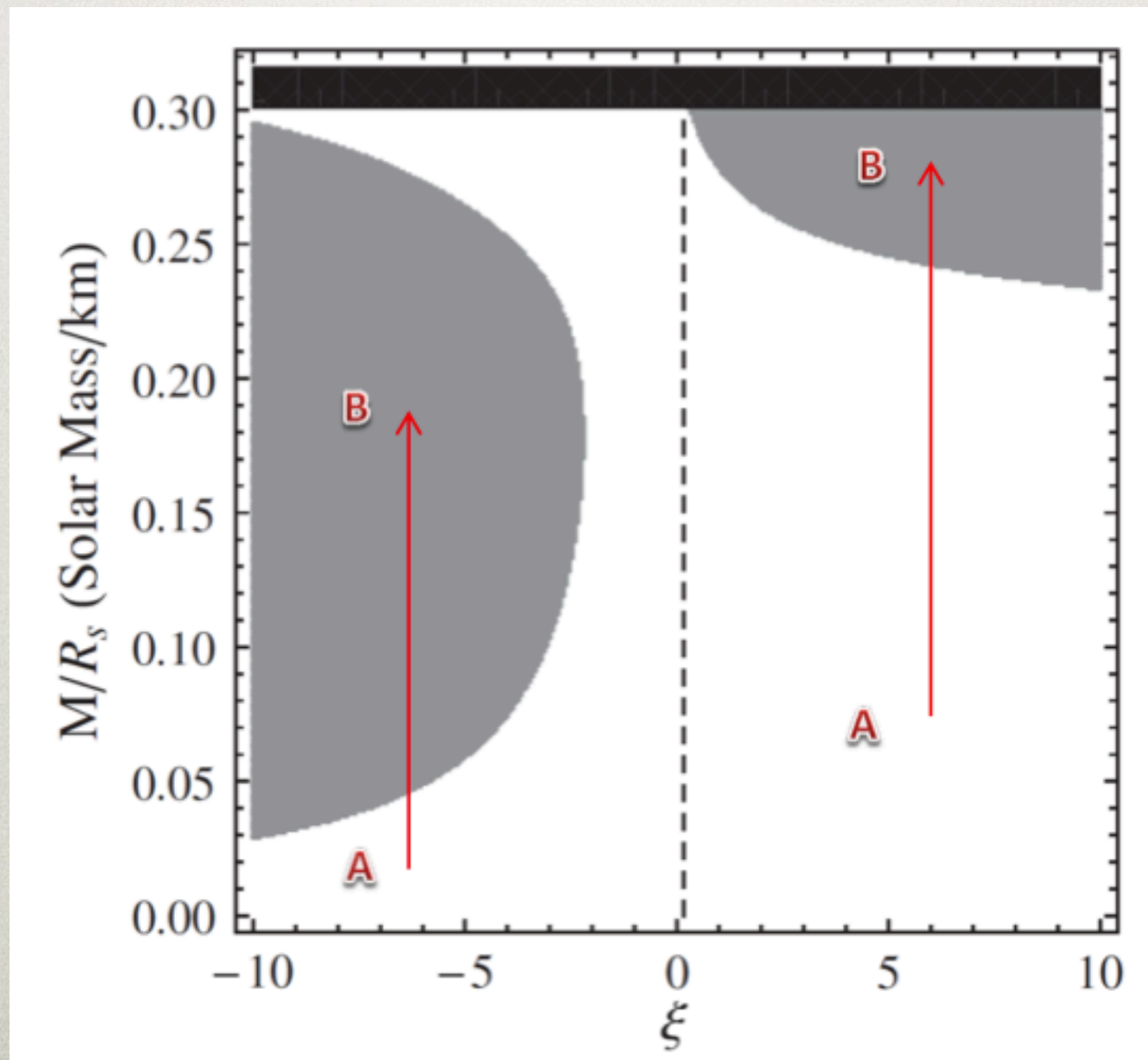
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# BACKREACTION

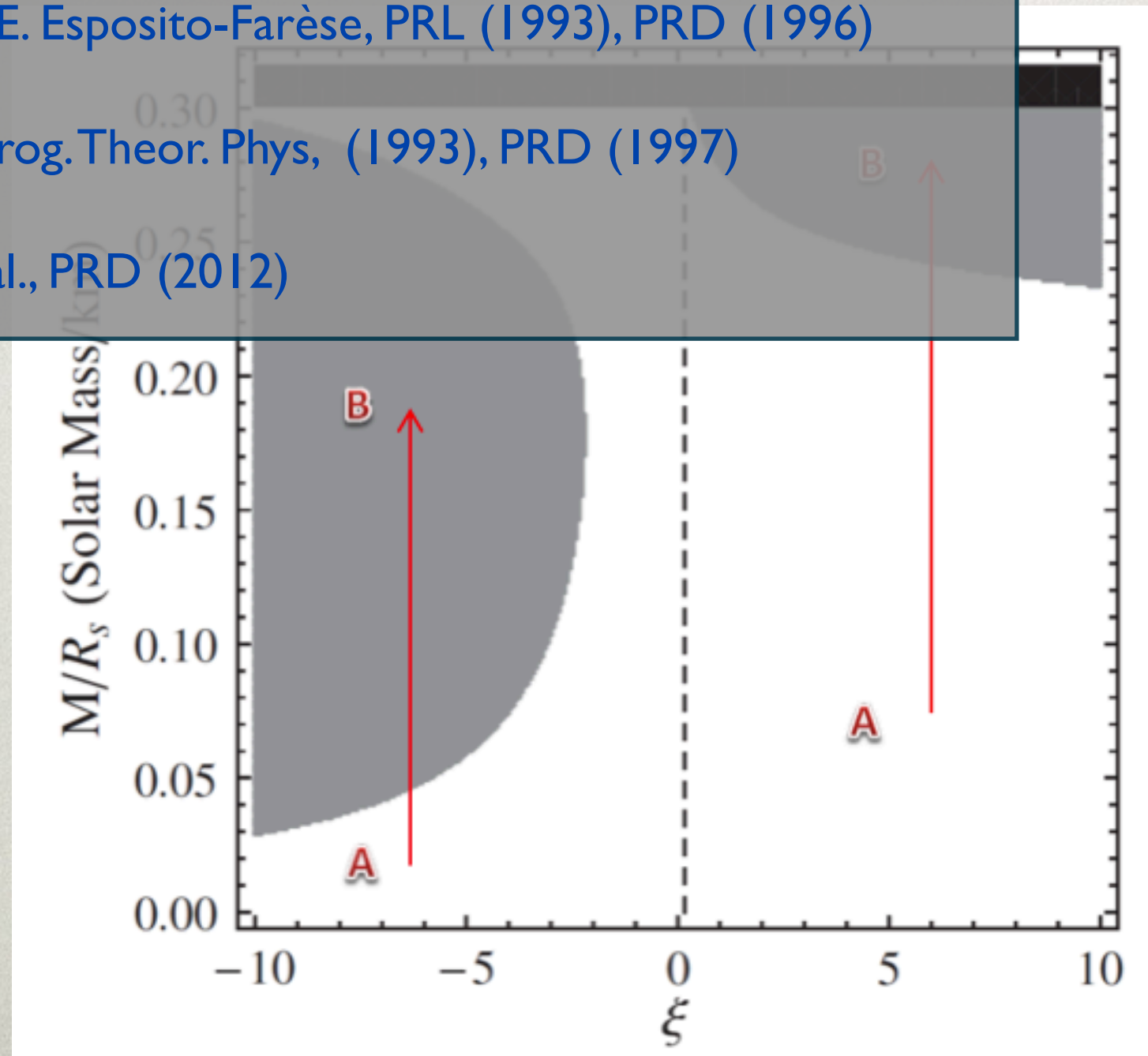
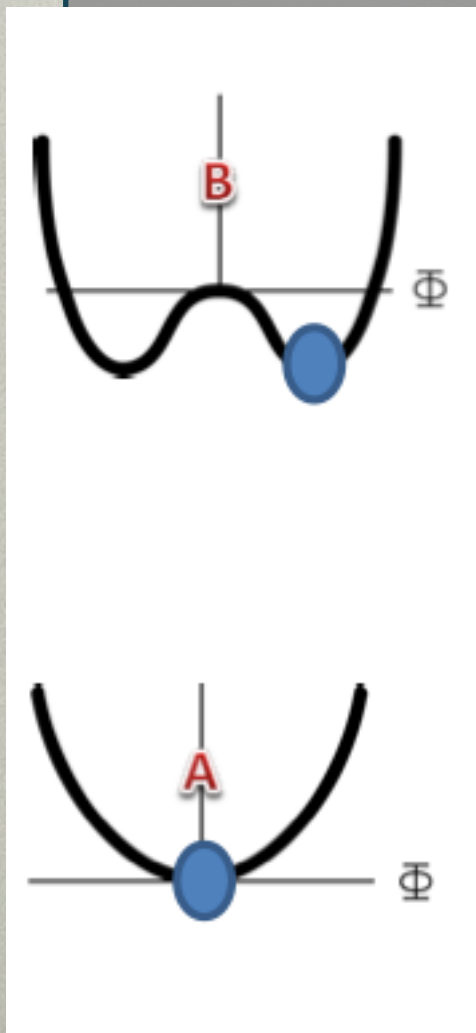
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# BACKREACTION

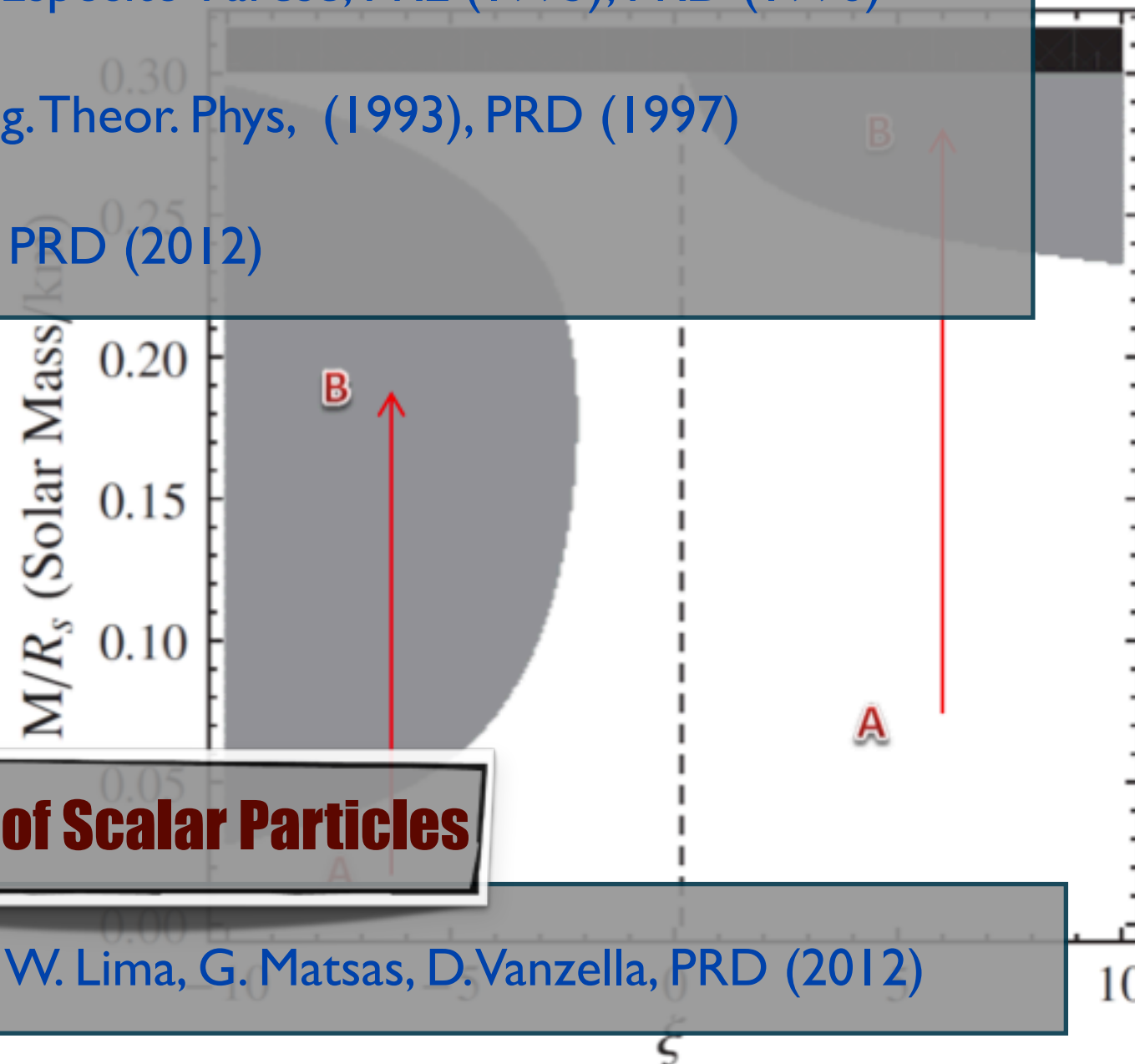
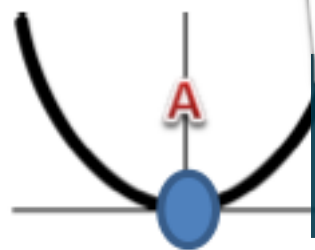
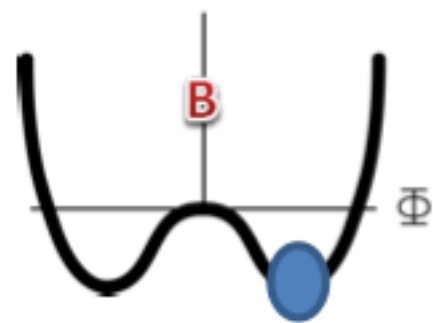
- ❖ P. Pani, V. Cardoso, E. Berti, J. Read, and M. Salgado, PRD (2011)
- ❖ T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)
- ❖ T. Harada, Prog. Theor. Phys, (1993), PRD (1997)
- ❖ M. Ruiz et. al., PRD (2012)





# BACKREACTION

- ✧ P. Pani, V. Cardoso, E. Berti, J. Read, and M. Salgado, PRD (2011)
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## Burst of Scalar Particles

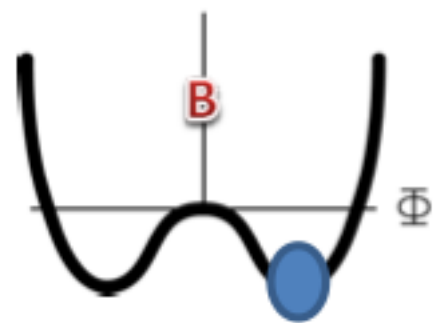
- ✧ AL, W. Lima, G. Matsas, D. Vanzella, PRD (2012)



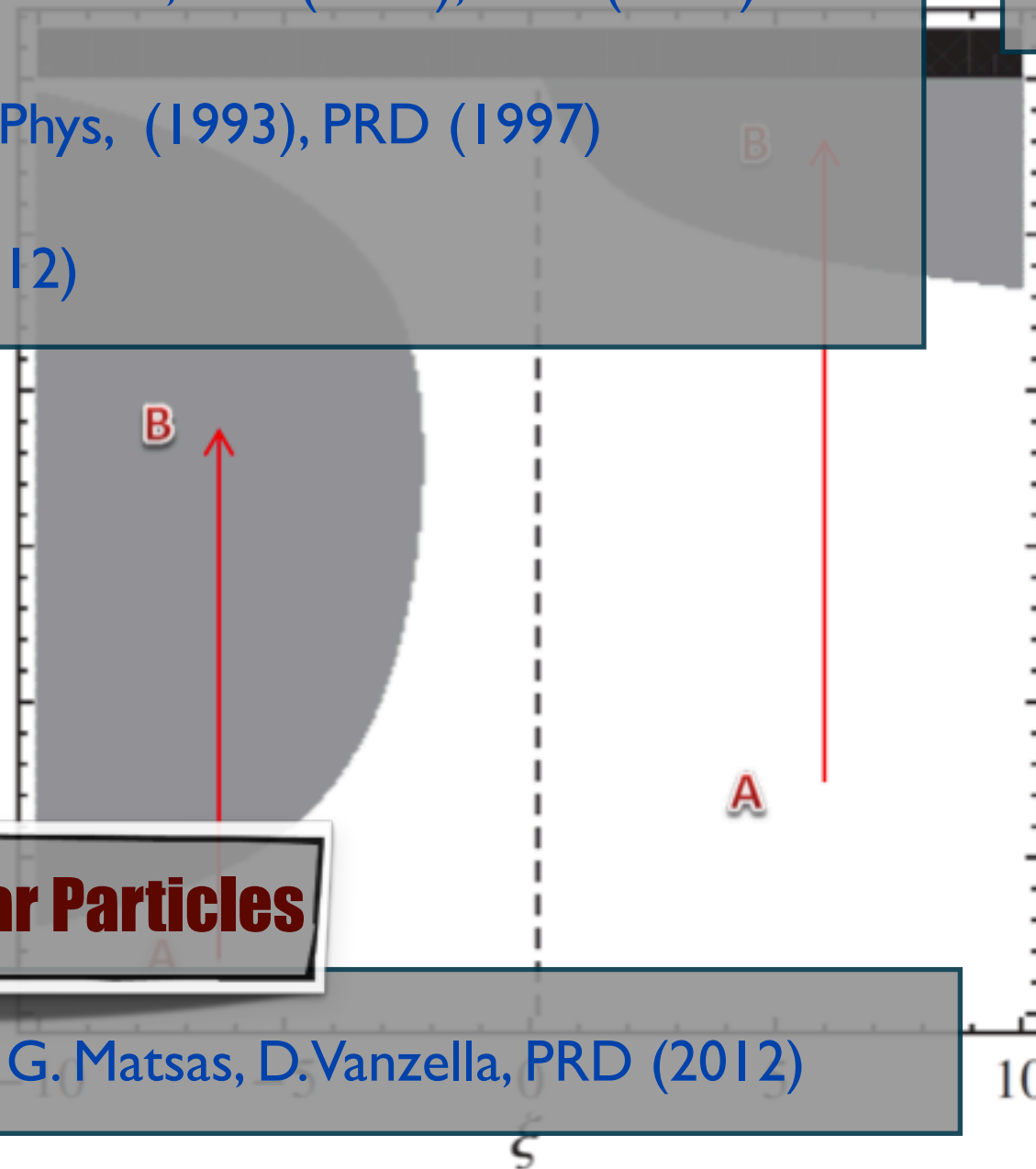
# BACKREACTION

- ✧ P. Pani, V. Cardoso, E. Berti, J. Read, and M. Salgado, PRD (2011)
- ✧ T. Damour, E. Esposito-Farèse, PRL (1993), PRD (1996)
- ✧ T. Harada, Prog. Theor. Phys, (1993), PRD (1997)
- ✧ M. Ruiz et. al., PRD (2012)

- ✧ R. Mendes and N. Ortiz, PRD (2016)



$M/R_s$  (Solar Mass)



## Burst of Scalar Particles

- ✧ AL, W. Lima, G. Matsas, D. Vanzella, PRD (2012)

