SpECTRE: A Task-based Discontinuous Galerkin Code for Relativistic Astrophysics

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SIMULATING EXTREME SPACETIMES
Black holes, neutron stars, and beyond...

Outline

1. Motivation and goals
2. Tools
   - DG methods
   - Task-based parallelization
3. Results
   - Status
   - Tests

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Challenges of relativistic hydrodynamics

- >50 coupled nonlinear PDEs with lengthly right-hand-side expressions
- Shocks in the hydrodynamic sector
- Turbulent flows
- Density profiles which quickly vary between unity and zero
- Multiscale and multiphysics
- Matter fields move throughout the computational grid

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Goals

- Higher (than 2^{nd} order) accuracy for relativistic simulations with matter
- Efficient simulations with $\approx 100,000 - 1,000,000$ cores on next generation of supercomputers
- Provide predictions for multi-messenger astronomy

Discontinuous Galerkin (DG) method using task-based parallelism

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Why discontinuous Galerkin methods?

- ✓ High-order where the solution is smooth
- ✓ Shock-capturing, robust
- ✓ Parallelizes well (only communicate surface data)
- ✓ Over 2 decades of experience in engineering and applied math

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Uses in computational relativity and astronomy

- Linearized Einstein equation (Zumbusch 2009, Field+ 2009)
- 1-dimensional Einstein equation (Field+ 2010, 2012)
- 1-dimensional general relativistic hydrodynamics (Radice+ 2011)
- Formulation of method for relativistic astrophysics (Teukolsky 2015)
- 3-dimensional (relativistic) hydrodynamics in an astrophysical context (Schaal+ 2015, Bugner+2015)
- 3-dimensional BSSN system (Miller+ 2016)

Significant opportunity for applications of discontinuous Galerkin methods

Discontinuous Galerkin methods (one slide summary)

1. **Domain decomposition**: Cover the physical domain by local subdomains
2. **Spectral methods**: On each subdomain approximate the solution by a set of basis functions
3. **Finite element**: Substitute the approximate solution into the PDE, integrate the residual against test functions
4. **Finite volume**: Couple neighboring subdomains with a *numerical* flux

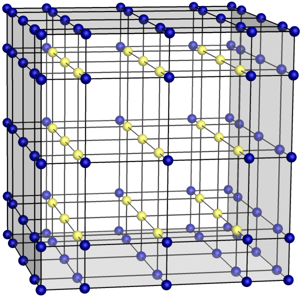

Figure : Typical subdomain with interpolation points. [Fig. from Huismann et al. 2015]

Overview of parallelization strategies

Data-based: Distribute data across cores

- Do exact same thing on each core, pass data with MPI
- Evolution proceeds according to a global simulation time
- Drawbacks: (i) idle cores due to synchronization/waiting and (ii) overlapping computation with communication is non-trivial

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Task-based: Parallelize by which tasks are to be performed

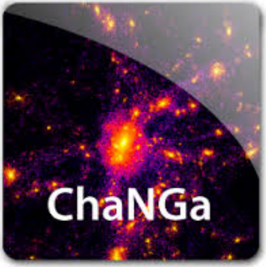
- Computation is naturally broken up into tasks
- As long as task-queues remain non-empty, cores will be hard at work
- No global synchronization
- Overlap of communication and computation is trivial
- Load balancing, task-stealing

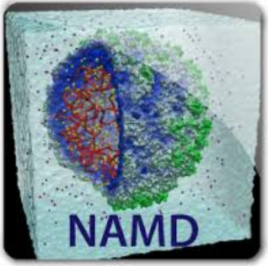
Task-based parallelism with Charm++

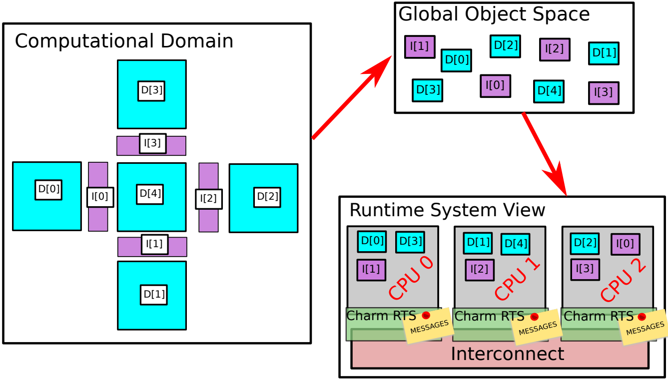
Charm++ (and related tools) is a library maintained by the Parallel Programming Laboratory at UIUC

Project started around 1992

Large community, 10s of code developers and reasonable documentation







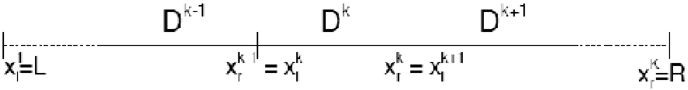
Runtime system maintains a “pool” consisting of tasks to be completed

The runtime system decides which messages from the work-pool to execute

Control switches back-and-forth between the scheduler and SpECTRE’s code

Tasks sent asynchronously (no waiting)

Charm++-ification of a DG scheme



Equation on subdomain D^k (interface between neighbors D^k and $D^{k\pm1}$)

$$\int_{D^k} (\ell_i \partial_t \Psi)\, dx = \int_{D^k} \ell_i \partial_x f(\Psi)\, dx + \text{Numerical Flux}\left(D^{k-1}, D^{k+1}\right)$$

Tasks for each subdomain D^k

- Send data to interfaces
- Compute volume RHS
- Slope limit the solution
- Send data to observers

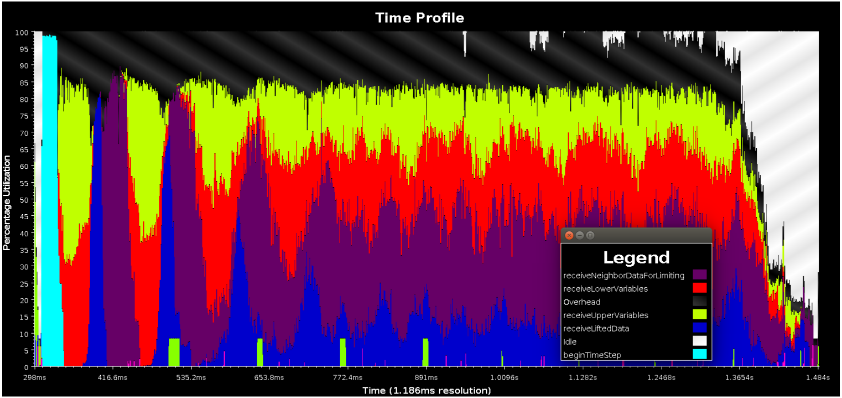
Tasks for each interface

- Compute numerical fluxes

Tasks for Observers

- When data arrives, process
- Write files

Time profile (10 steps of a relativistic MHD system)



Cyan: setup

Black: Charm++

White: idle

Red/Yellow: data to interfaces, local volume RHS

Blue: fluxes to elements

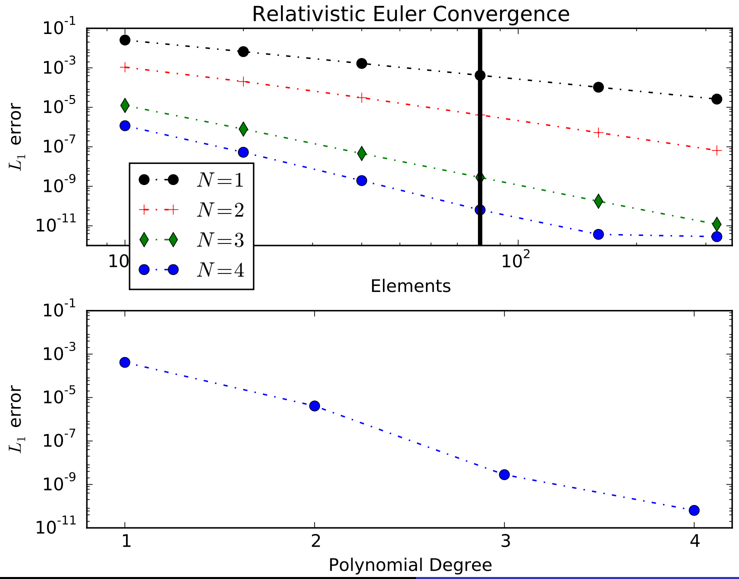
Purple: slope limiting

Outline

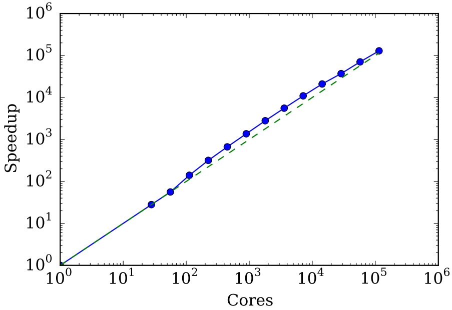
Summary of code

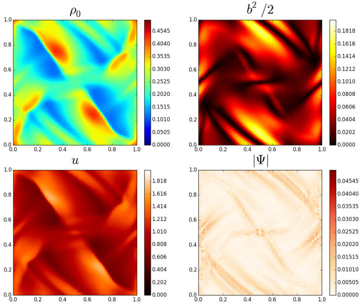
- Discontinuous Galkerkin (DG) method in 1D, 2D and 3D
- Elements are boxes, arbitrary order polynomial basis on each element
- Task-based parallelization carried out with Charm++
- Systems: Scalar wave, (non-)/Relativistic Euler, Relativistic MHD
- Numerical fluxes: Lax-Friedrichs, Roe, Marquina, HLL
- Variety of slope limiters
- Writes to HDF5, simple visualizations

Convergence test – relativistic hydro (smooth solution)



Strong scaling test on Blue Waters





Left: Strong-scaling on Blue Waters for Orszag-Tang vortex test.

Fixed size problem with 660 $\times$ 660 $\times$ 4 second order elements

✓ Similar results on Comet, Stampede

Right: Density, magnetic field strength, velocity

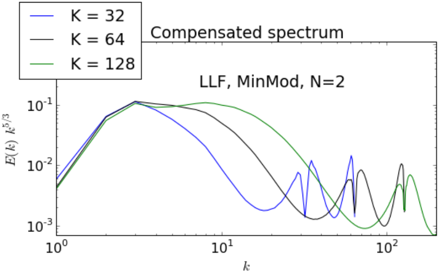
Turbulence

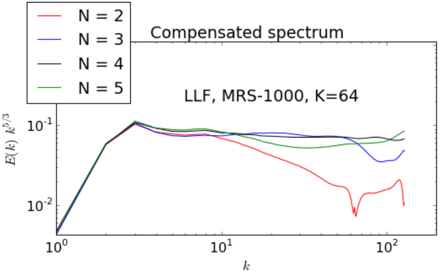
See e.g.
Radice, Couch, Ott (2015).

Kolmogorov inertial range:
$E(k) \sim k^{-5/3}$

- Can we perform high-order turbulence simulations?
- What are good choices for fluxes and limiters?

Additional collaborator:
David Radice (Caltech)





Highlights

- Large-scale simulation code for computational relativity/astronomy using discontinuous Galerkin methods
- First combination of DG methods with task-based parallelization
- Efficient usage up to $\mathcal{O}(100\,,000)$-cores (runs on all of Blue Waters)

Whats next

- h-p adaptivity
- Einstein Equation
- General grids (ie. mappings)
- Load balancing
- Code is almost ready to do science!