Scanning New Horizons: Entanglement, Holography & Gravity

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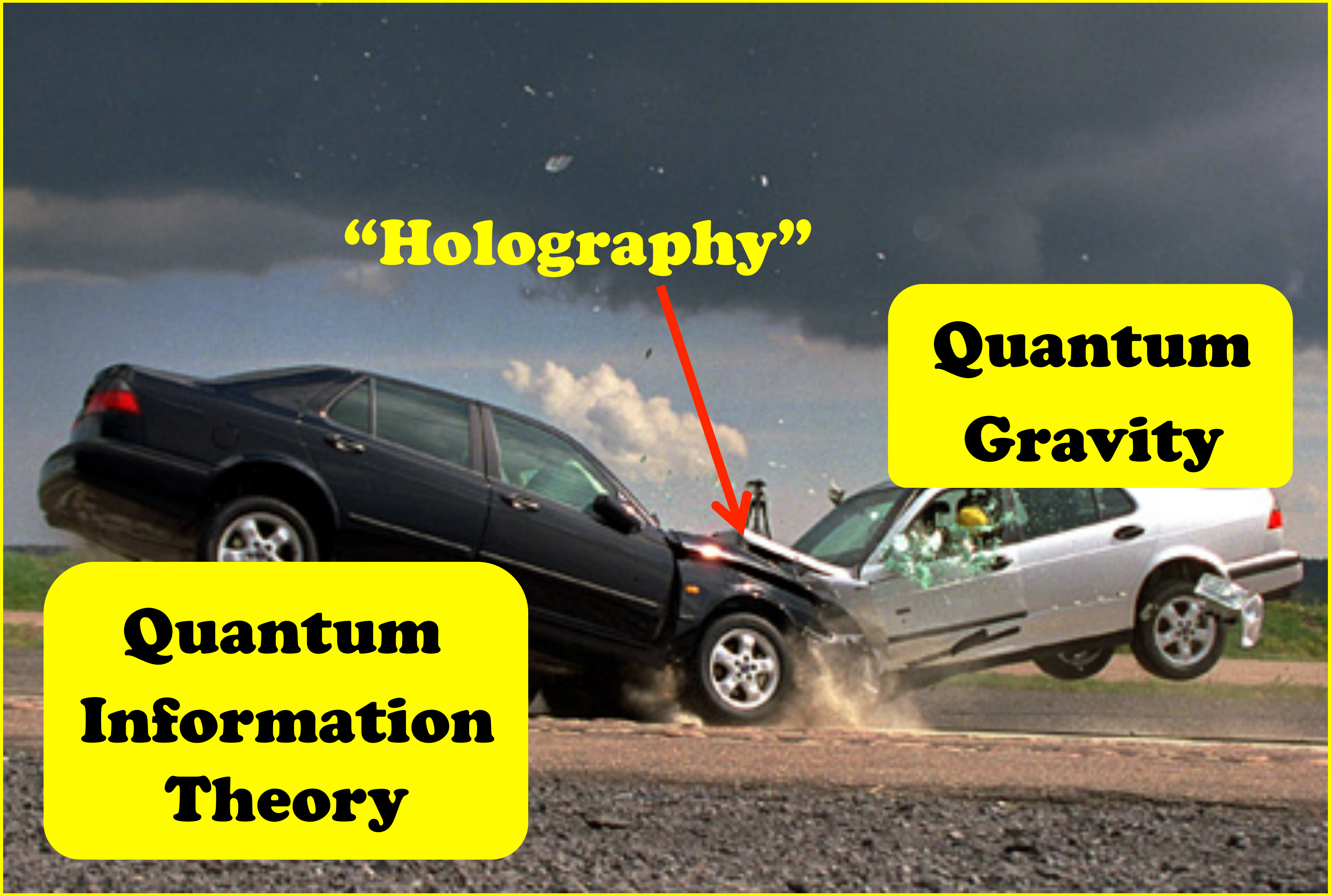
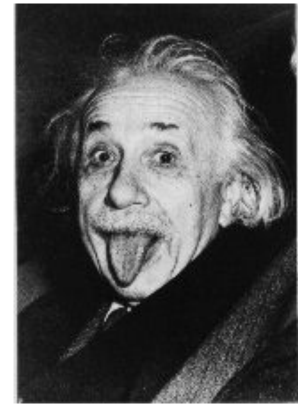
A New ~~Collision~~ Confluence of Ideas:



Einstein:



Gravity? It's all about geometry!

Special Relativity (1905):

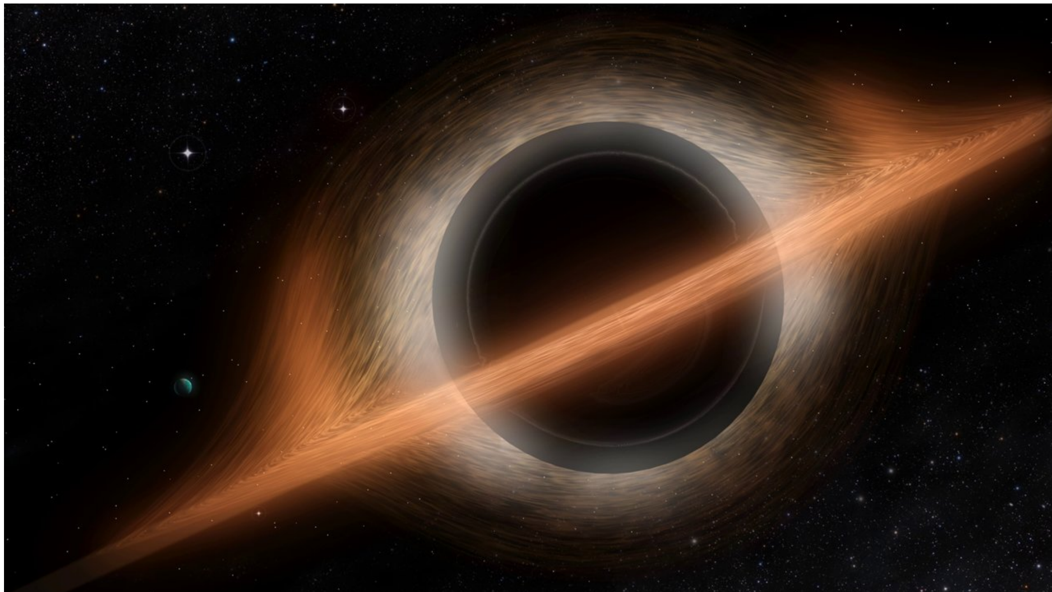
Space and time are inextricably linked → Spacetime

General Relativity (1915):

Gravity is a manifestation of spacetime curvature.

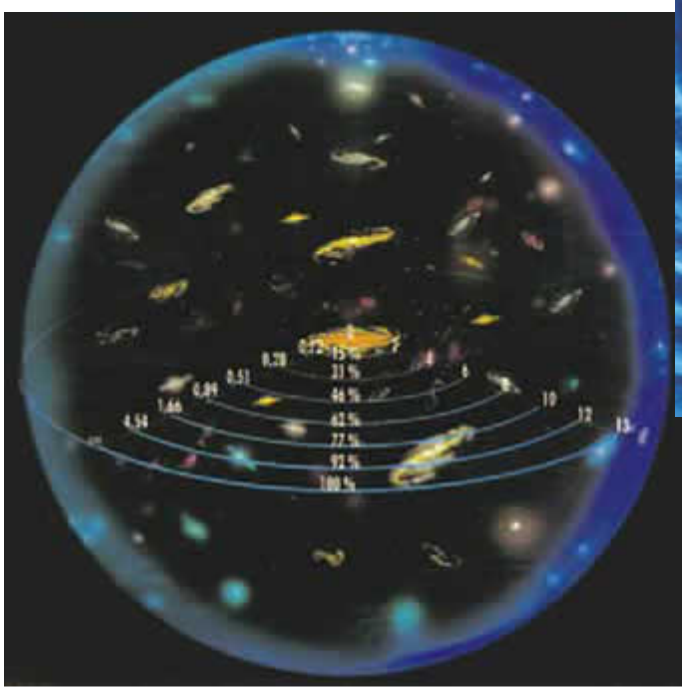
Spacetime moves from providing a stage for physical phenomena, to being both the stage and an active player in the dynamics

General Relativity: a rich source of new ideas



Black holes

Gravitational waves

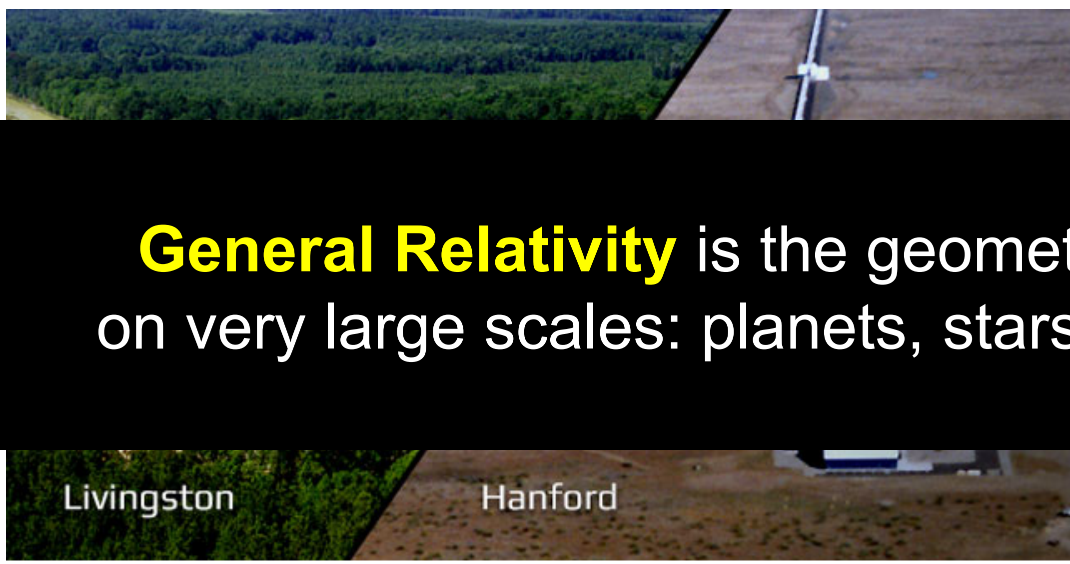


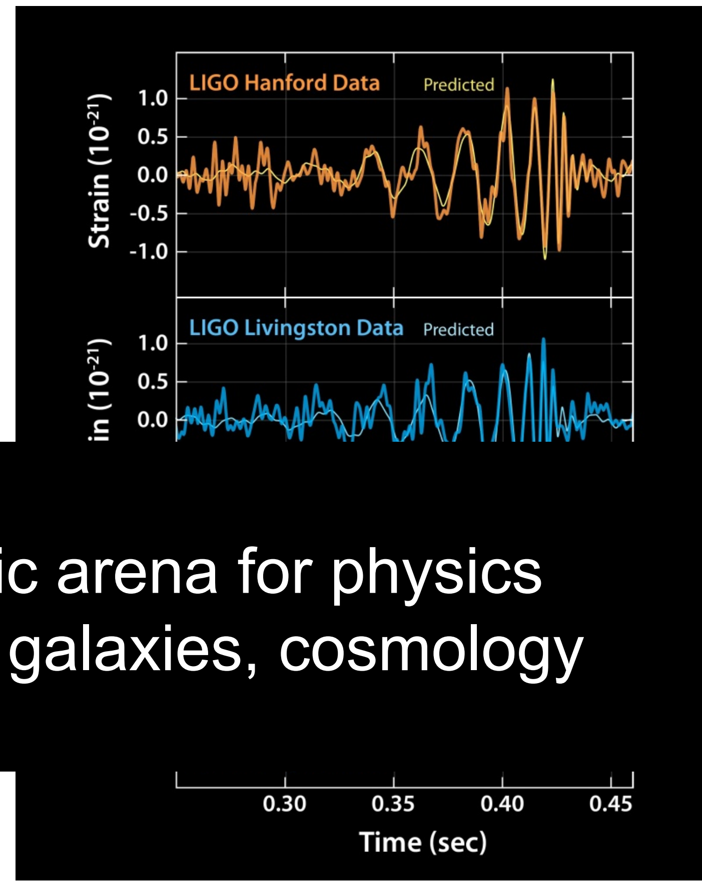
Expanding universe

General Relativity: faced precision tests

- Deflection of Light by the Sun
- Gravitational Redshift
- Precession of the Perihelion of Mercury
- Time Delay of Light, ...

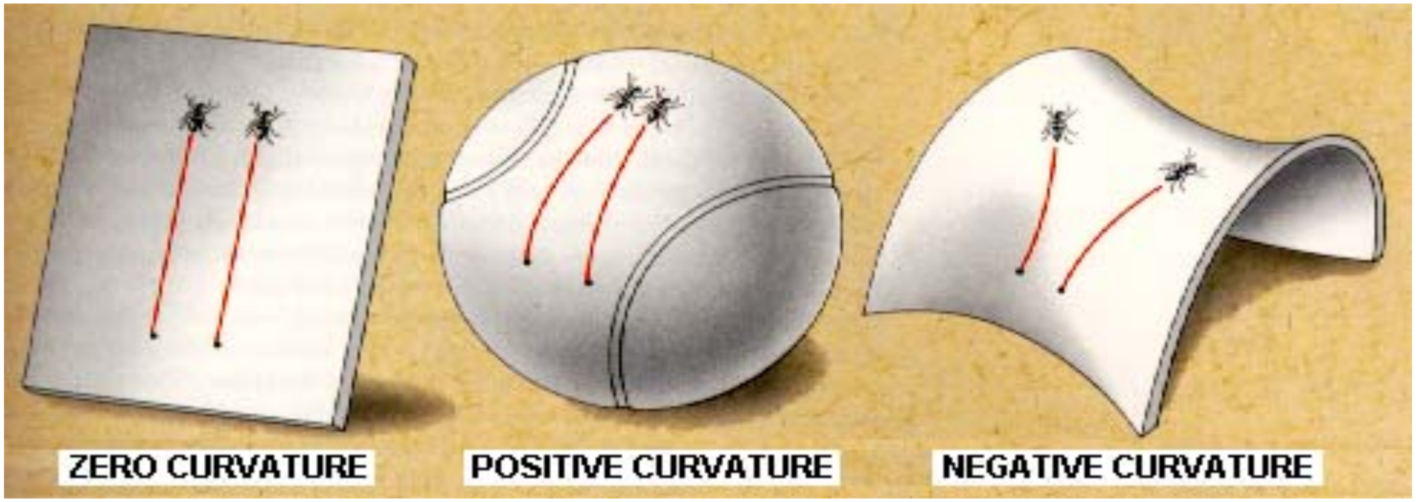
LIGO's direct observation of gravitational waves, 2016!!





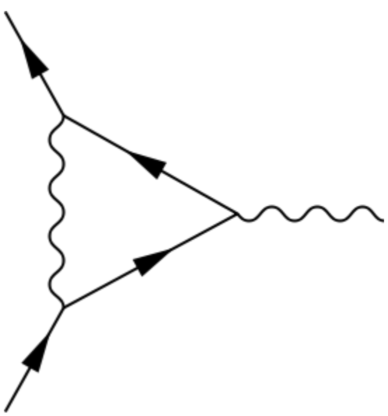
General Relativity is the geometric arena for physics on very large scales: planets, stars, galaxies, cosmology

General Relativity + Quantum Theory = Quantum Gravity?



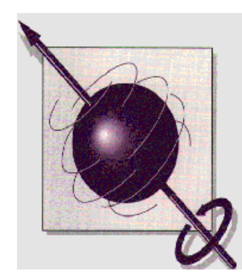
$+\ \hbar$

- quantum fluctuations become manifest at small scales e.g., magnetic moment of the electron, $\mu_e = g e\hbar/4m_e$, with $g \approx 2$ but modified by quantum fluctuations

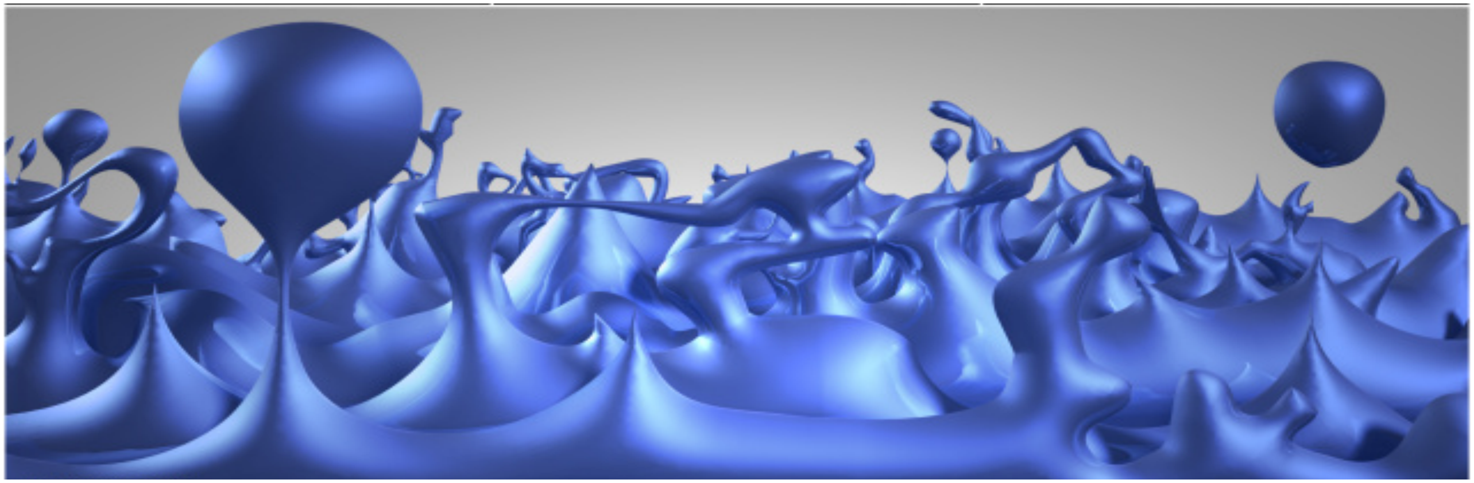


$$g_{\text{theory}} = 2.0023193043\boxed{070}$$

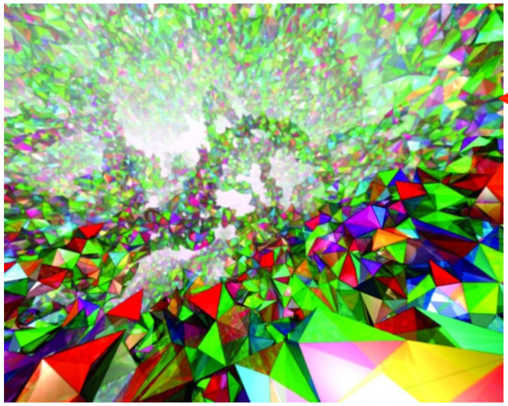
$$\left| \frac{\mu_{\text{theory}} - \mu_{\text{experiment}}}{\mu_{\text{experiment}}} \right| \leq 10^{-10}$$



General Relativity + Quantum Theory = Quantum Gravity?

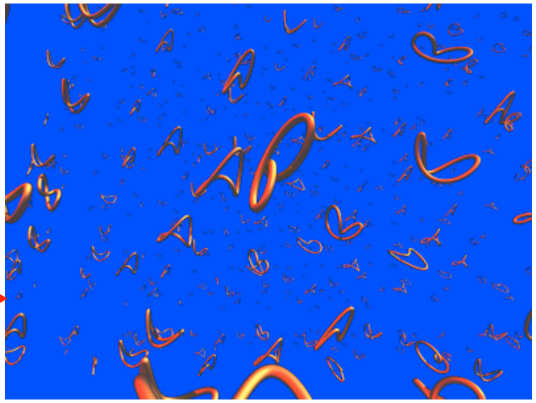


- spacetime geometry exhibits strong fluctuations when examined on very short distance scales
- how do we make sense of spacetime framework?

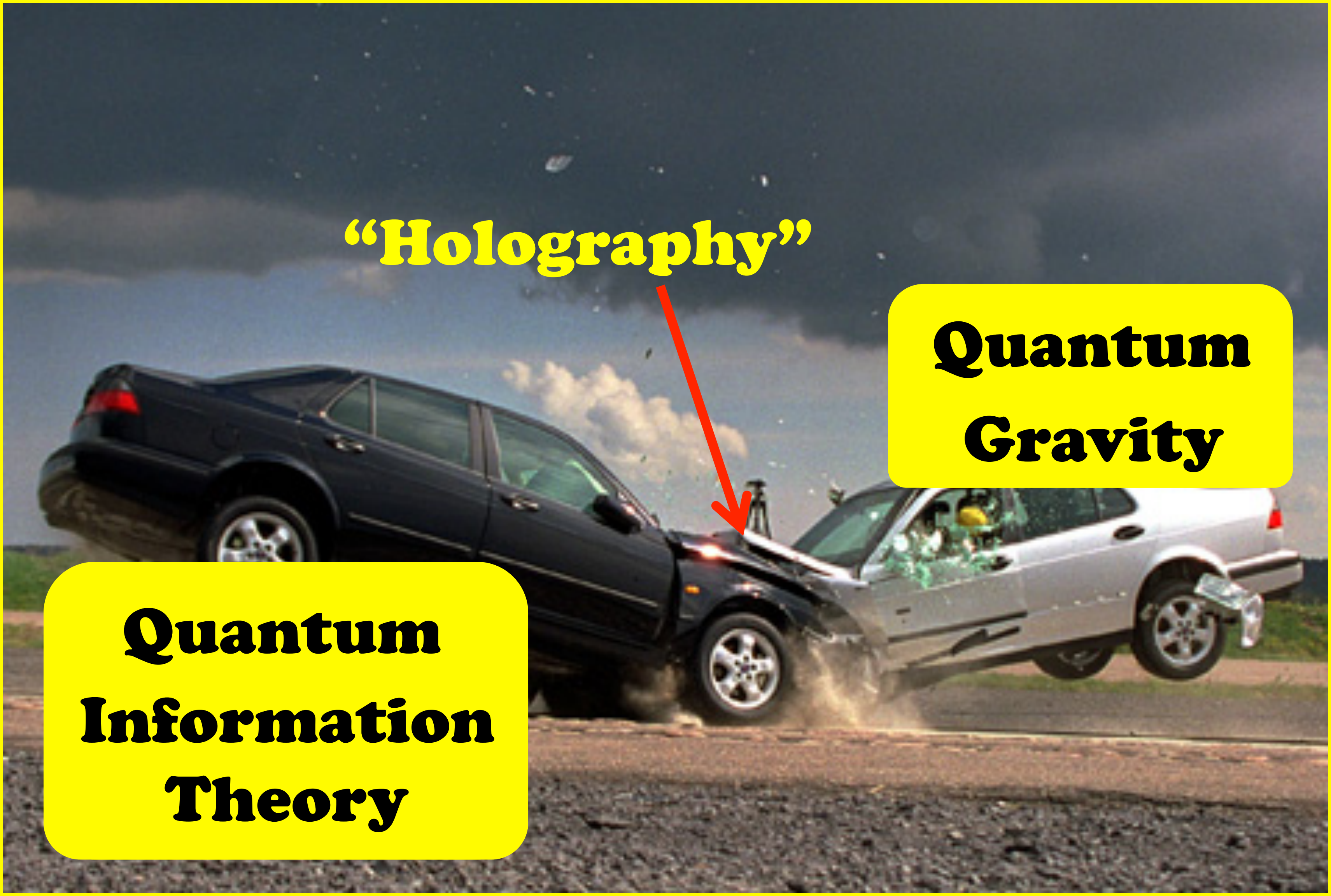


modify geometry
at short distances

modify spectrum
at short distances

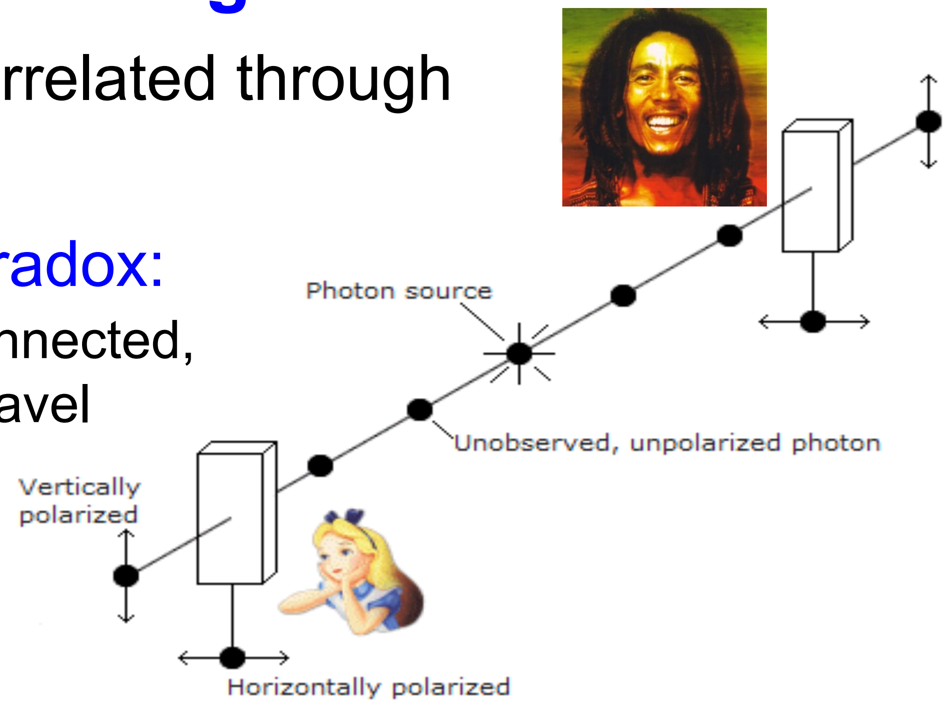


A New Collision of Ideas:



Quantum Entanglement

- different subsystems are correlated through global state of full system



Einstein-Podolsky-Rosen Paradox:

- polarizations of pair of photons connected, no matter how far apart they travel

"*spukhafte Fernwirkung*" = spooky action at a distance

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\,\Big)$$

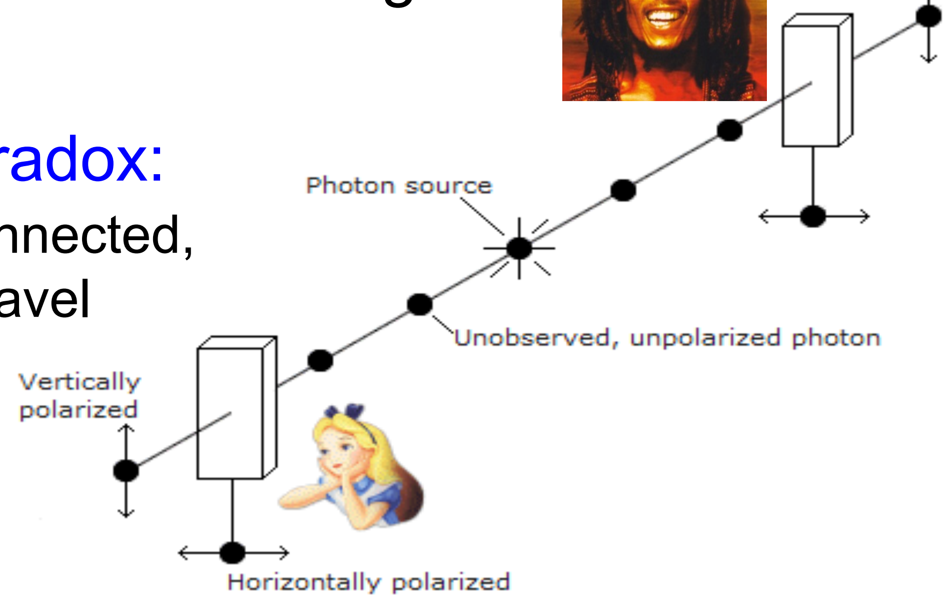
Quantum Information: entanglement becomes a resource for (ultra)fast computations and (ultra)secure communications

Condensed Matter: key to "exotic" phases and phenomena, e.g., quantum Hall fluids, unconventional superconductors, quantum spin fluids, ...

Quantum Field Theory, Quantum Gravity, ...

Quantum Entanglement

- different subsystems are correlated through global state of full system



Einstein-Podolsky-Rosen Paradox:

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$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big)$$

compare: $$|\psi'\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big)$$

$$= \frac{1}{2}\Big(|\uparrow\rangle + |\downarrow\rangle\Big) \otimes \Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$ **No Entanglement!!**

$$|\psi''\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle - |\downarrow\downarrow\rangle\Big)$$ **Entangled!!**

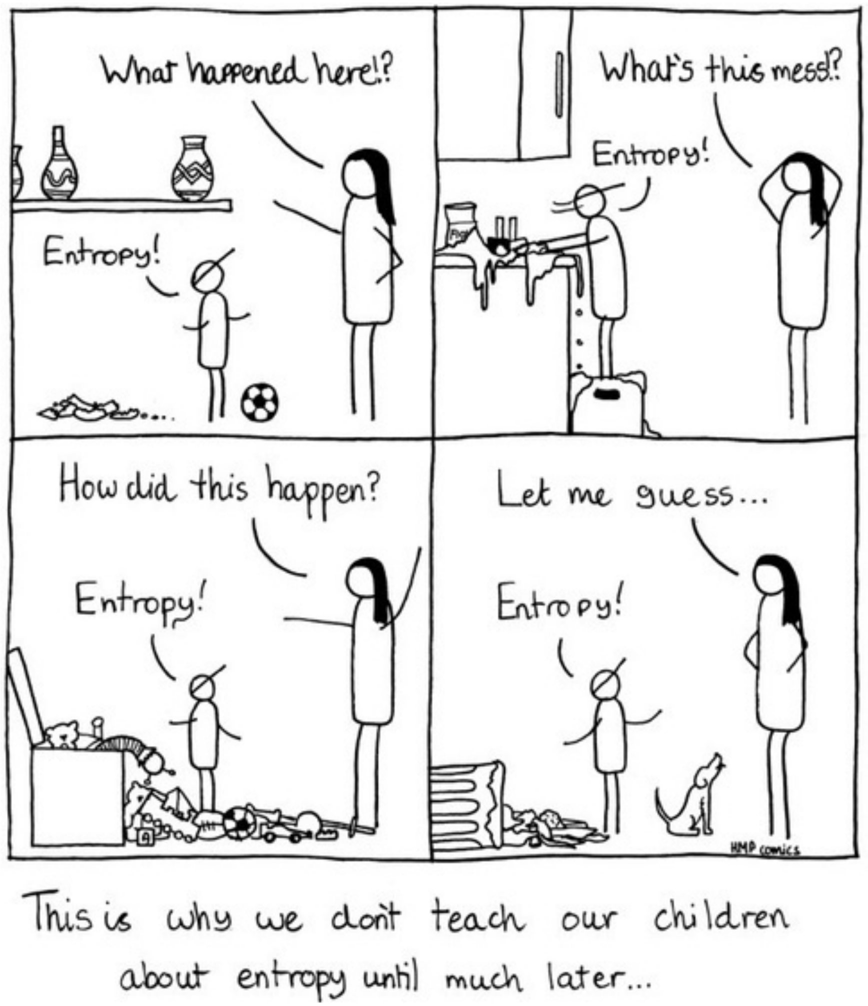
Entanglement Entropy : Entanglement What? Whose Entropy?

Thermodynamics: **entropy** quantifies heat losses

Stat. Mechanics: **entropy** quantifies uncertainty in microstate

Information theory: **entropy** quantifies information in a message

Quantum information theory: general diagnostic giving quantitative measure of entanglement using **entropy** to detect correlations between two subsystems; *one of many diagnostics*



Entanglement Entropy :

- procedure:
  - divide system into two subsystems, eg, A and B
  - trace over degrees of freedom in subsystem B
  - remaining dof in A are described by a density matrix ρ_A
  - calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A\ \log \rho_A\right]$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\!\uparrow\downarrow\rangle + |\!\downarrow\uparrow\rangle\Big) \longrightarrow \rho = \mathrm{Tr}_2\left(|\psi\rangle\langle\psi|\right) = \frac{1}{2}\left(|\!\downarrow\rangle\langle\downarrow\!| + |\!\uparrow\rangle\langle\uparrow\!|\right)$$

$$\longrightarrow S_{EE} = \log 2$$

compare: $|\psi'\rangle = \frac{1}{2}\Big(\,|\!\uparrow\uparrow\rangle + |\!\uparrow\downarrow\rangle + |\!\downarrow\uparrow\rangle + |\!\downarrow\downarrow\rangle\Big)$

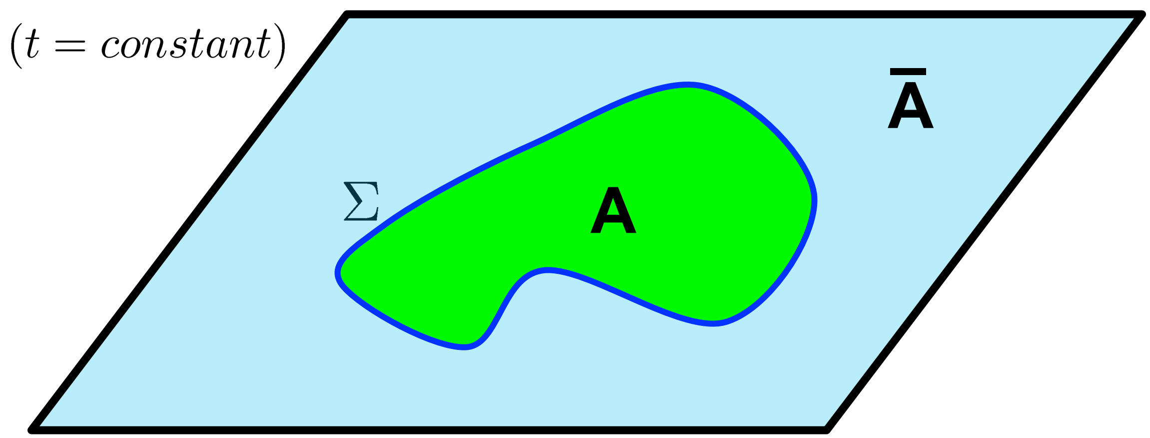
$$= \frac{1}{2}\Big(\,|\!\uparrow\rangle + |\!\downarrow\rangle\Big)\otimes\Big(\,|\!\uparrow\rangle + |\!\downarrow\rangle\Big) \Longrightarrow$$ No Entar $S_{EE} = 0$ t!!

$$|\psi''\rangle = \frac{1}{2}\Big(\,|\!\uparrow\uparrow\rangle + |\!\uparrow\downarrow\rangle + |\!\downarrow\uparrow\rangle - |\!\downarrow\downarrow\rangle\Big) \Longrightarrow$$ Entai $S_{EE} = \log 2$

Entanglement Entropy in Quantum Field Theory

- general diagnostic to give a quantitative measure of entanglement using entropy to detect correlations between two subsystems
- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- integrate out degrees of freedom in "outside" region
- remaining dof are described by a density matrix ρ_A

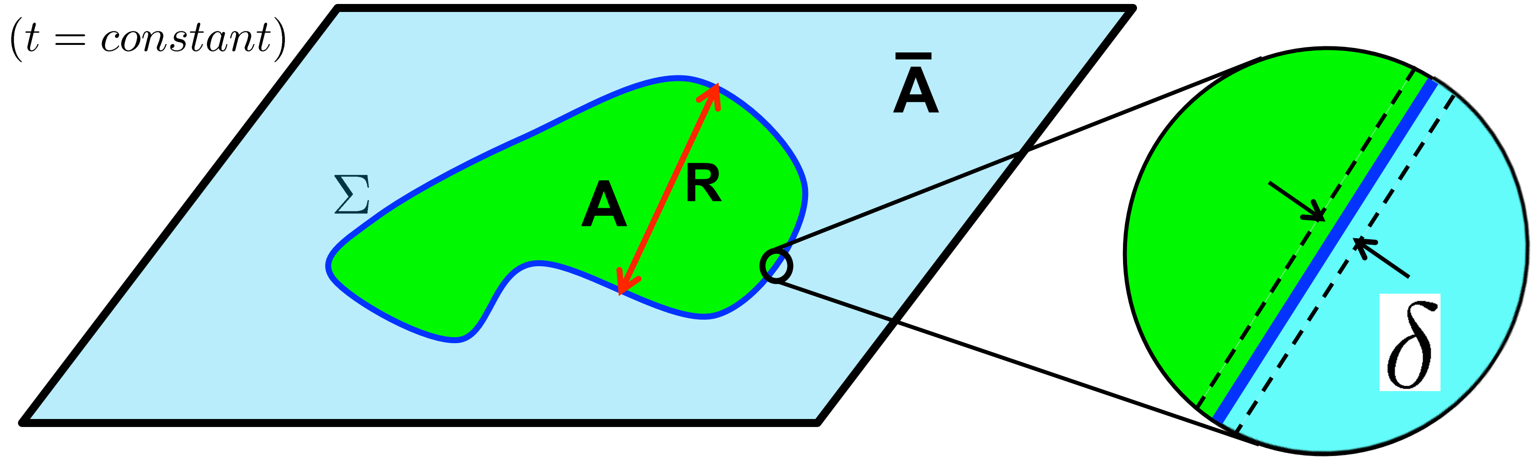
→ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \,\log \rho_A\right]$



Entanglement Entropy in QFT

- remaining dof are described by a density matrix ρ_A

→ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$

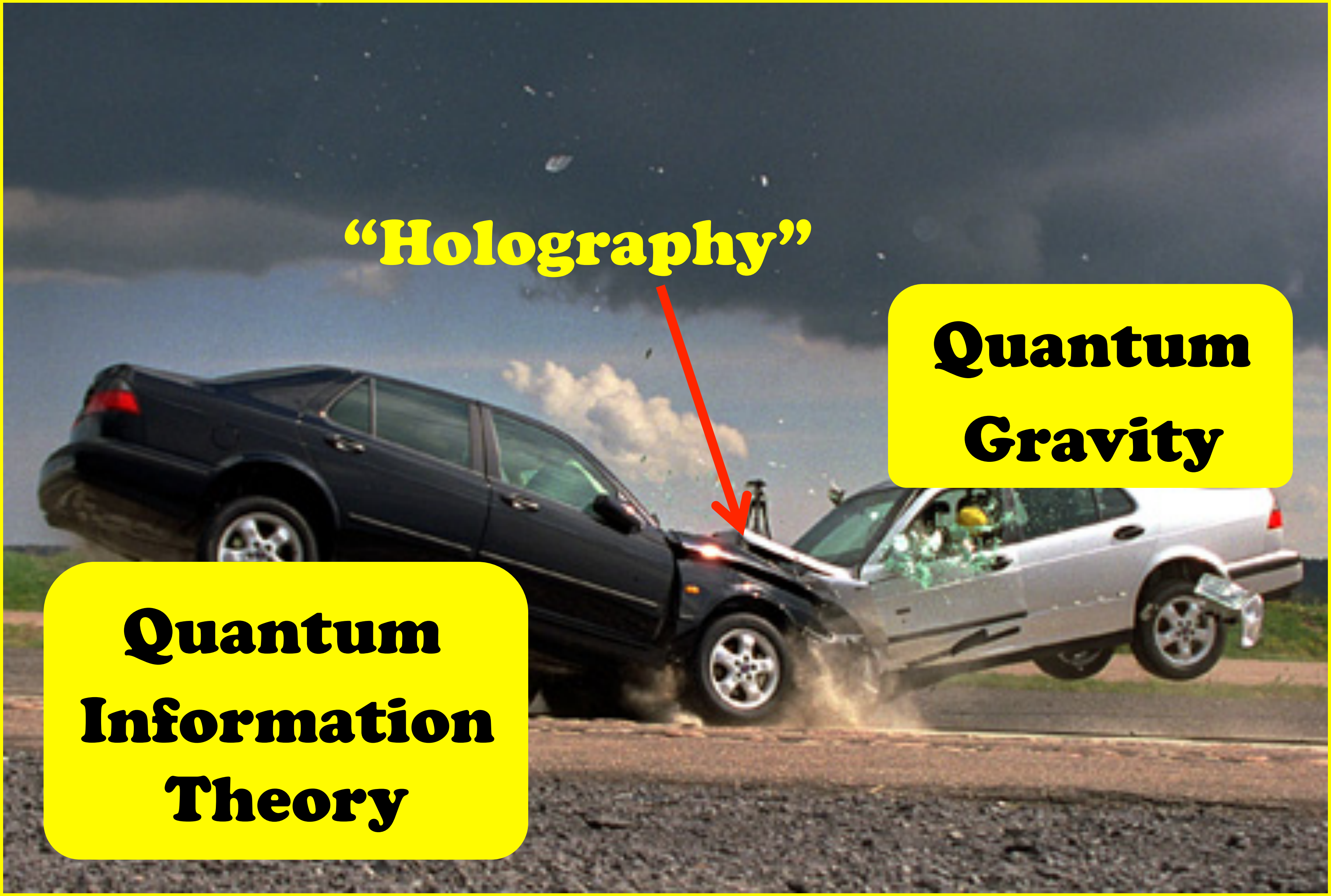


- result is UV divergent!
- must regulate calculation: δ = short-distance cut-off

$$S = c_0 \frac{R^{d-2}}{\delta^{d-2}} + c_2 \frac{R^{d-4}}{\delta^{d-4}} + \cdots$$ d = spacetime dimension

→ geometric structure, eg, $\boxed{S = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}}} + \tilde{c}_2 \frac{\oint_\Sigma \text{"curvature"}}{\delta^{d-4}} + \cdots$

A New Collision of Ideas:

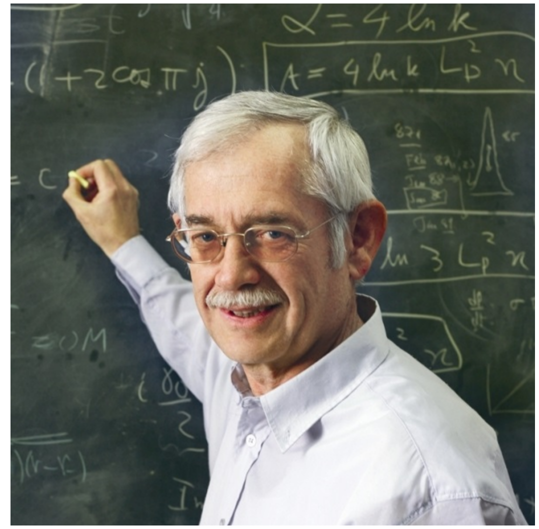


Roots of the collision: 1972

Gravity meets Information:

- Bekenstein: “black holes have entropy!”

$$S_{BH} = ´ \frac{k_B\ c^3}{\hbar} \frac{A_{horizon}}{G}$$

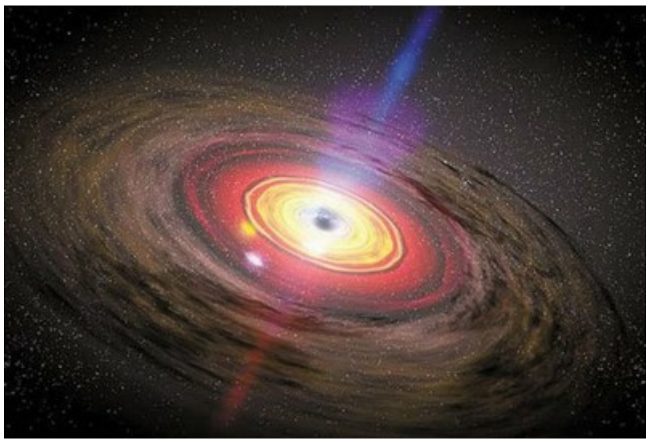


“we discuss black-hole physics from the point
of view of information theory”

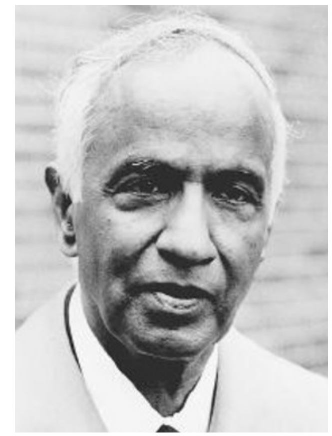
“Black Holes and Entropy”, PRD, April 1973

Black Holes in 1972:

- **black holes** remain largely a theoretical playground for mathematical physics



Chandrasekhar:

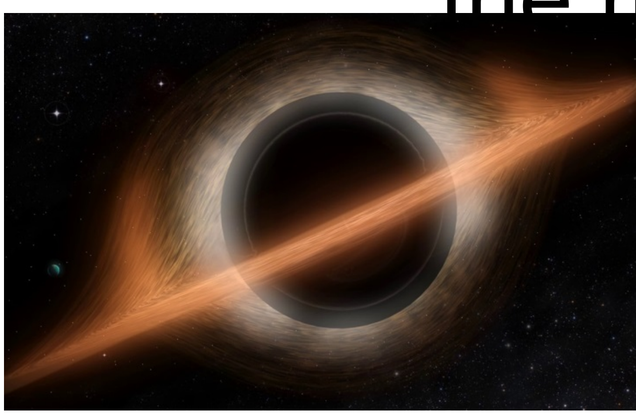


“the most perfect macroscopic obje

re are in the universe: the onl

ents in their construction are

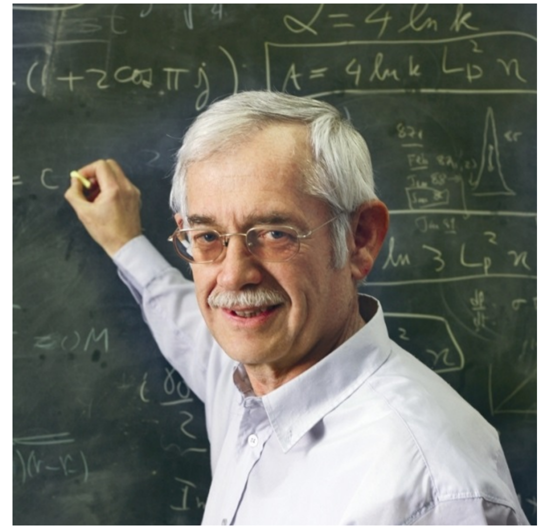
concepts of space and time”



Black Holes:

- **Bekenstein**: “black holes have entropy!”
- **Bekenstein and Hawking**: “black holes have entropy!”

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{A_{horizon}}{4G}$$

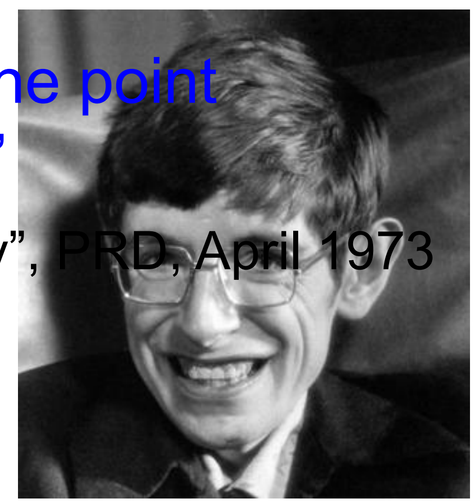


- Hawking radiation: quantum fluctuations allow black holes to leak emitting (almost pure) blackbody radiation with

“we discuss black-hole physics from the point of view of information theory”

$$T_H = \frac{\hbar}{k_B c} \frac{\kappa}{2\pi}$$

“Black Holes and Entropy”, PRD, April 1973



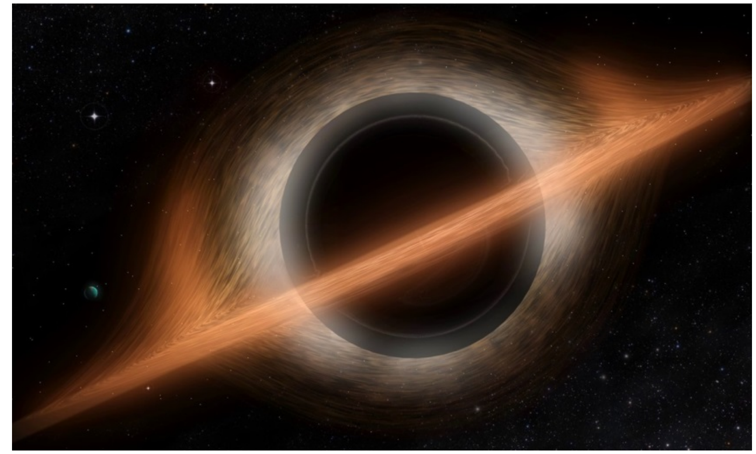
Black Hole Entropy:

• Bekenstein and Hawking: "black holes have entropy!"

thermodynamics relativity geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\,\frac{\mathcal{A}_{horizon}}{4G}$$

quantum **quantum gravity** gravity



• Hawking radiation: quantum fluctuations allow black holes to leak emitting (almost pure) blackbody radiation with $T_H = \frac{\hbar}{k_B\ c}\,\frac{\cdot}{2¼}$

• window into the quantum theory of gravity?!?

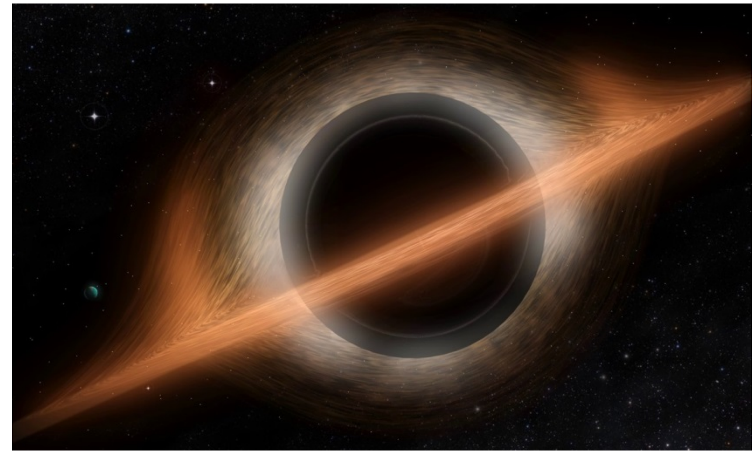
• quantum gravity provides a fundamental scale: $\ell_P^2 = 8\pi\, G\ \hbar/c^3$

Black Hole Entropy:

- Bekenstein and Hawking: “black holes have entropy!”

microstates geometry

$$\underbrace{S_{BH}}_{\text{microstates}} = 2\pi \frac{\overbrace{A_{horizon}}^{\text{geometry}}}{\ell_P^2} k_B$$



- quantum gravity provides a fundamental scale: $\ell_P^2 = 8\pi\, G\, \hbar/c^3$

- k_B reminds us that entropy is associated with “heat”

$\longrightarrow$ black hole thermodynamics

- statistical mechanics also says: $S = -Tr\,[\rho_A \log \rho$

$k_B = 1.3806$

$\longrightarrow$ black hole microstates

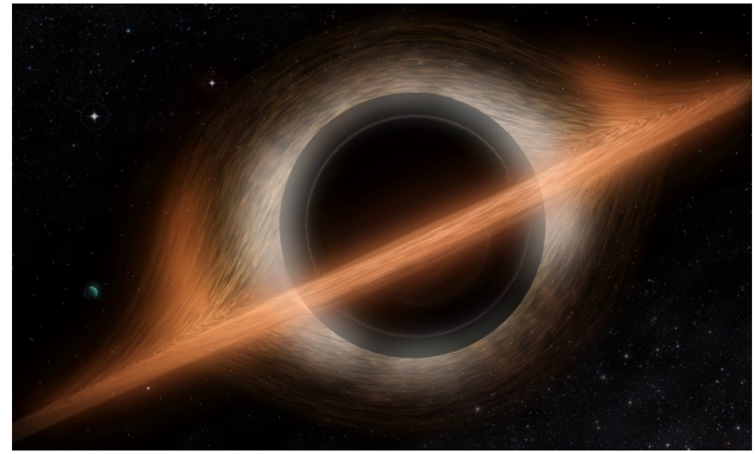
Black Hole Entropy:

- Bekenstein and Hawking: "black holes have entropy!"

microstates

geometry

$$S_{BH} = 2\pi \frac{A_{horizon}}{\ell_P^2}$$



- quantum gravity provides a fundamental scale: $\ell_P^2 = 8\pi\, G\; \hbar/c^3$

- entropy is not extensive; grows with **area** rather than volume

$\longrightarrow$ Sorkin '84: black hole entropy ≈ **"entanglement entropy"**

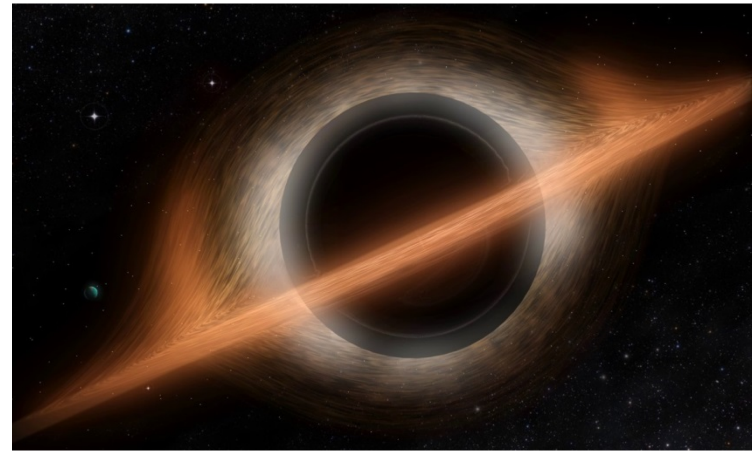
Black Hole Entropy:

- Bekenstein and Hawking: “black holes have entropy!”

microstates

geometry

$$S_{BH} = 2\pi \frac{A_{horizon}}{\ell_P^{d-2}}$$



- quantum gravity provides a fundamental scale: $\ell_P^{d-2} = 8\pi G\hbar/c^3$
- Sorkin ‘84: black hole entropy ≈ **“entanglement entropy”**

$$S_{EE} = \tilde{c}_0 \frac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \cdots \longrightarrow$$

“area law” suggestive of BH formula if $\delta^{d-2} \simeq \ell_P^{d-2}$

(Sorkin `84; Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; . . .)

- entanglement entropy of QF’s contributes to gravitational entropy

(Susskind & Uglum; ...)

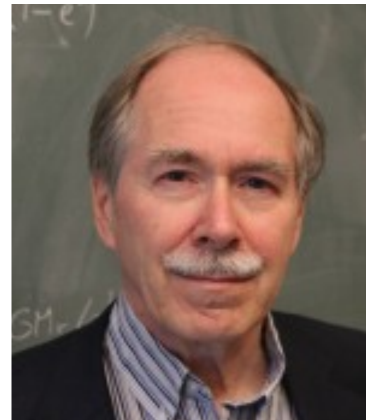
- microscopic gravitational dof? approaches differ (eg, Bianchi, …)

Holography v1.0: Black Holes

- Bekenstein and Hawking: “black holes have entropy!”

microstates geometry

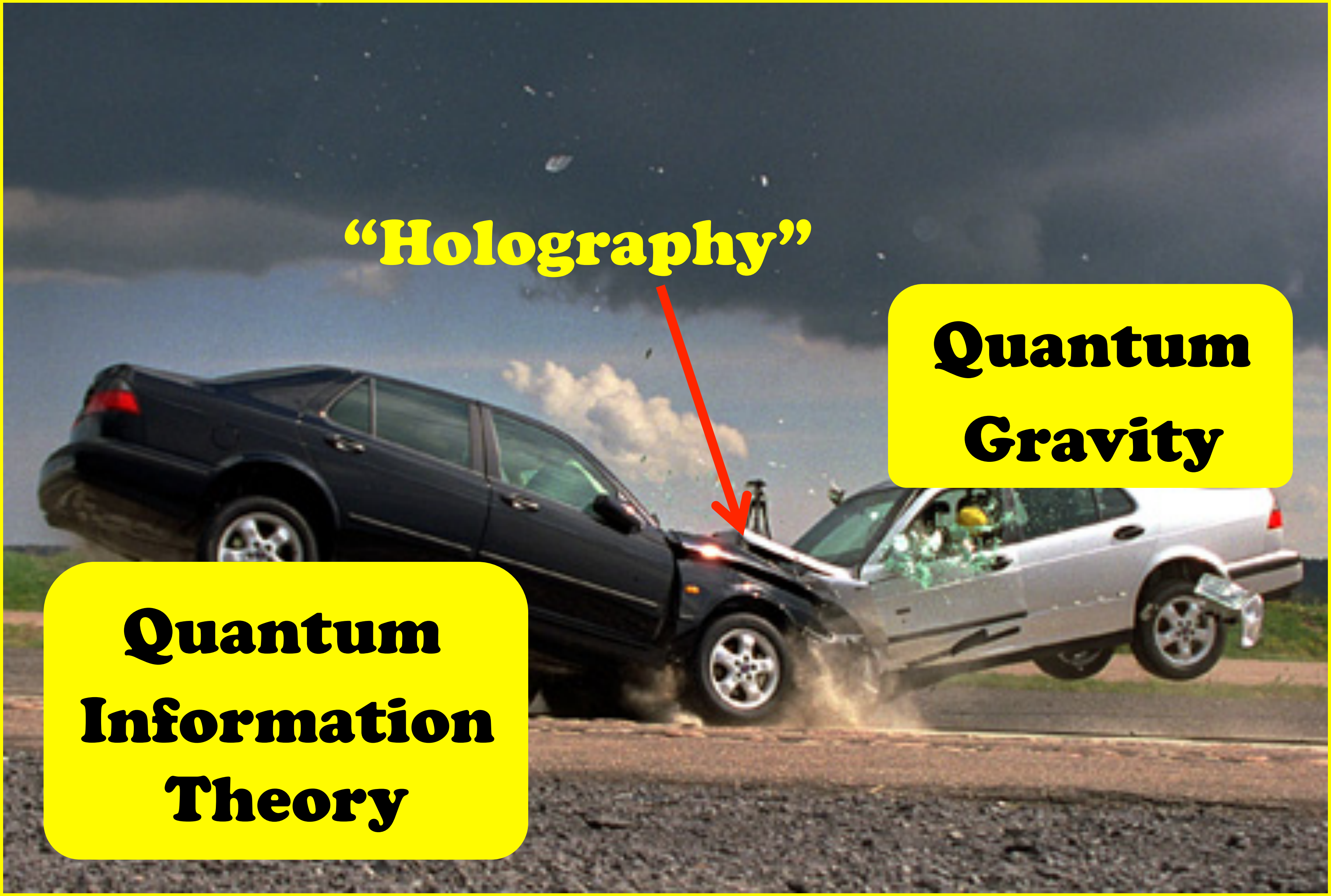
$$S_{BH} = 2\pi \frac{A_{horizon}}{(\ell_P)^2}$$



- quantum gravity provides a fundamental scale: $(\ell_P)^2 = 8\pi G\,\hbar/c^3$
- BH formula is only part of the realization that black holes behave like thermal systems emitting (almost pure) blackbody radiation
- “holography” suggests that not only does the horizon encode information about microstates but also that the evolution and dynamics of the black hole can be described in terms of a “dual” theory living on the horizon (in one less dimension)

(‘t Hooft; Susskind)

A New Collision of Ideas:



Holography v2.0: AdS/CFT correspondence

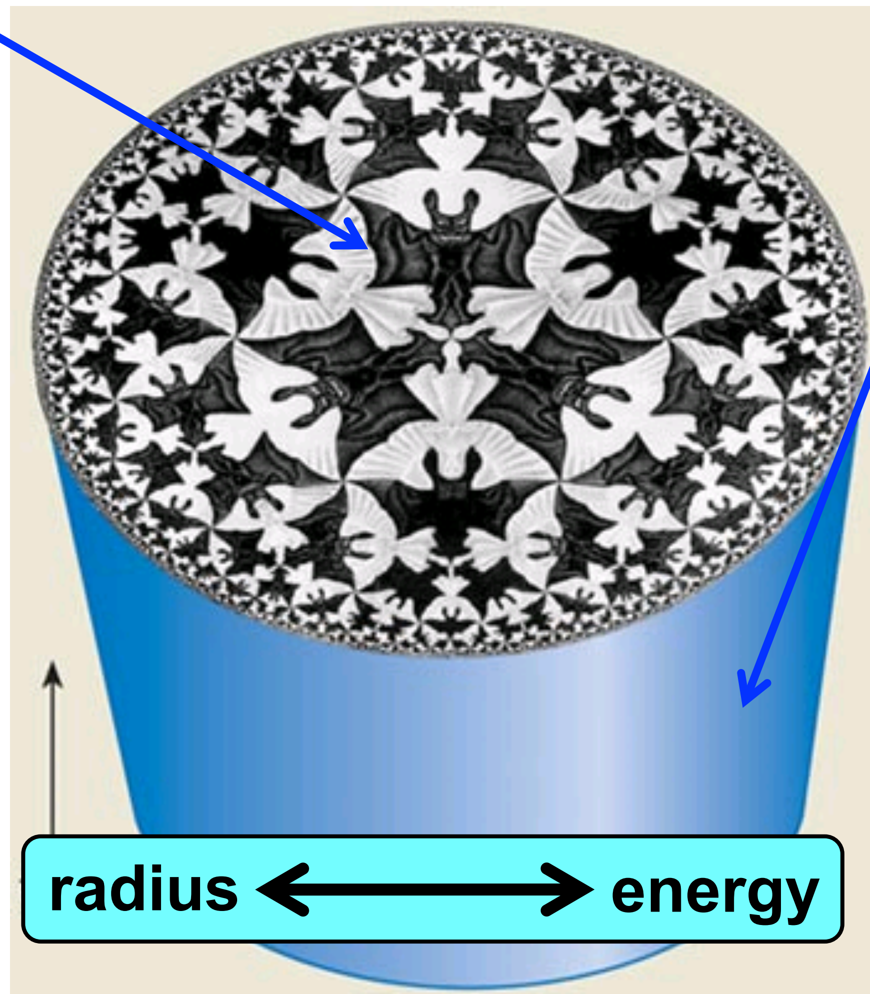
Bulk: gravity with negative Λ
in **d+1** dimensions

Boundary: quantum field theory
without intrinsic scales
in **d** dimensions

"holography"

anti-de Sitter space

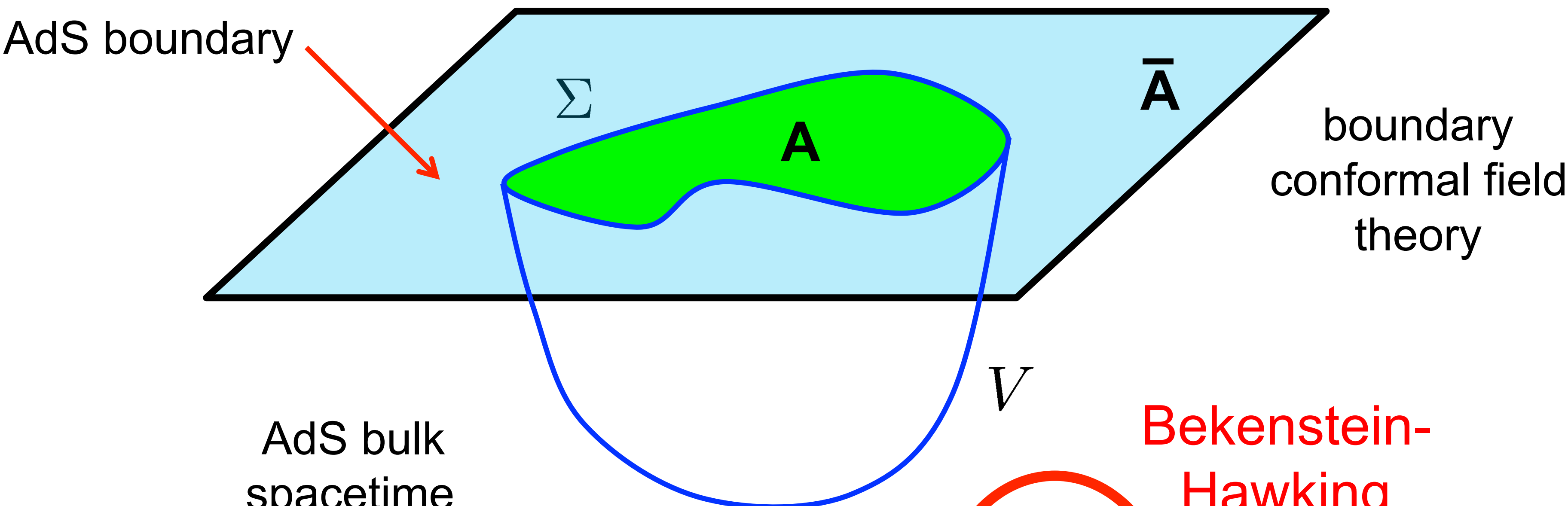
conformal field theory



(Maldacena '97)

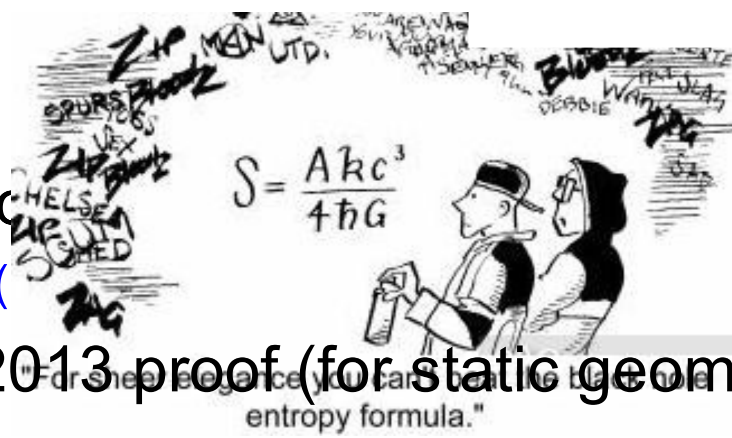
(Ryu & Takayanagi `06)

Holographic Entanglement Entropy:



Bekenstein-Hawking formula

$$S(A) = \min_{V \sim A} \frac{A_V}{4G_N}$$



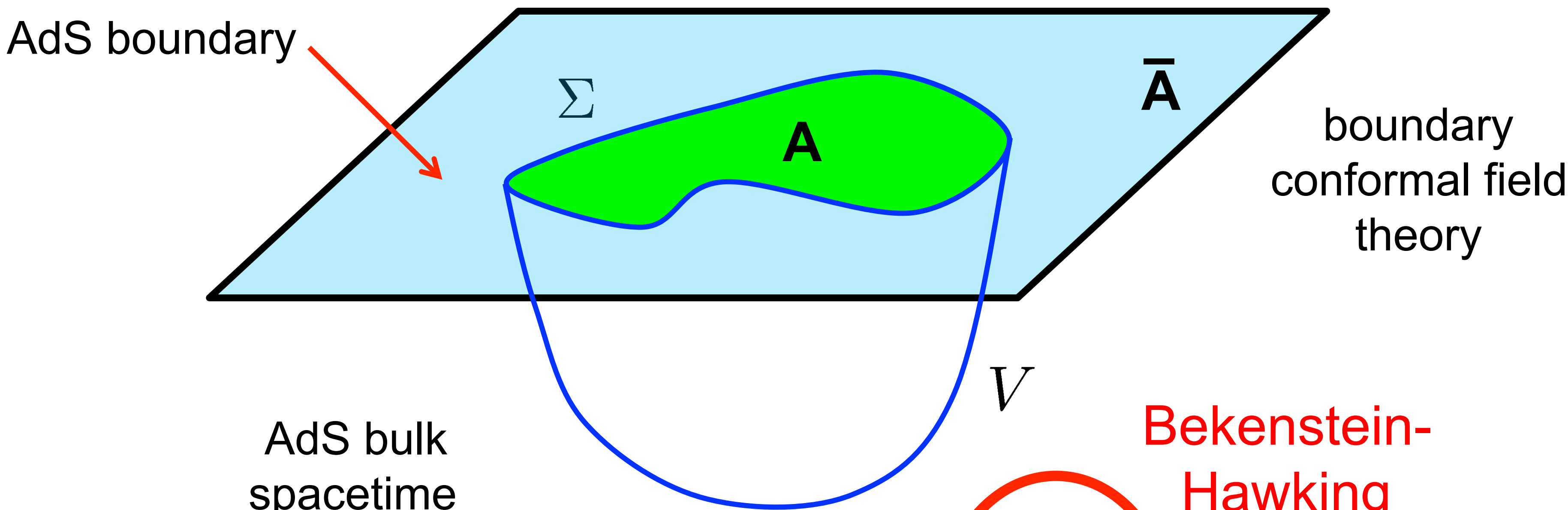
• c … detailed consistency tests

(… nani, Headrick, Hung, Smolkin, RM, Faulkner, . . .)

• 2013 proof (for static geometries) (Maldacena & Lewkowycz)

(Ryu & Takayanagi `06)

Holographic Entanglement Entropy:

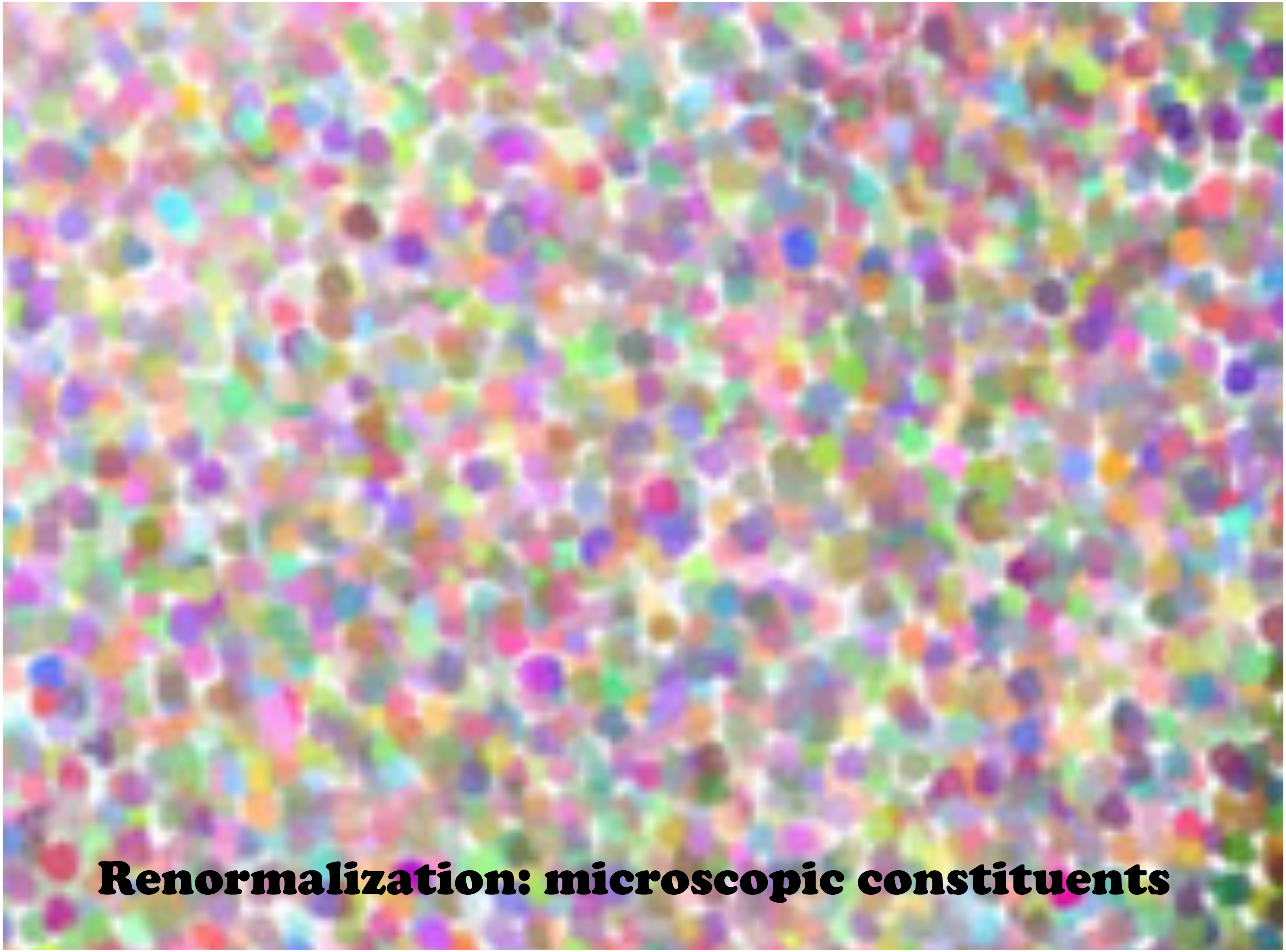


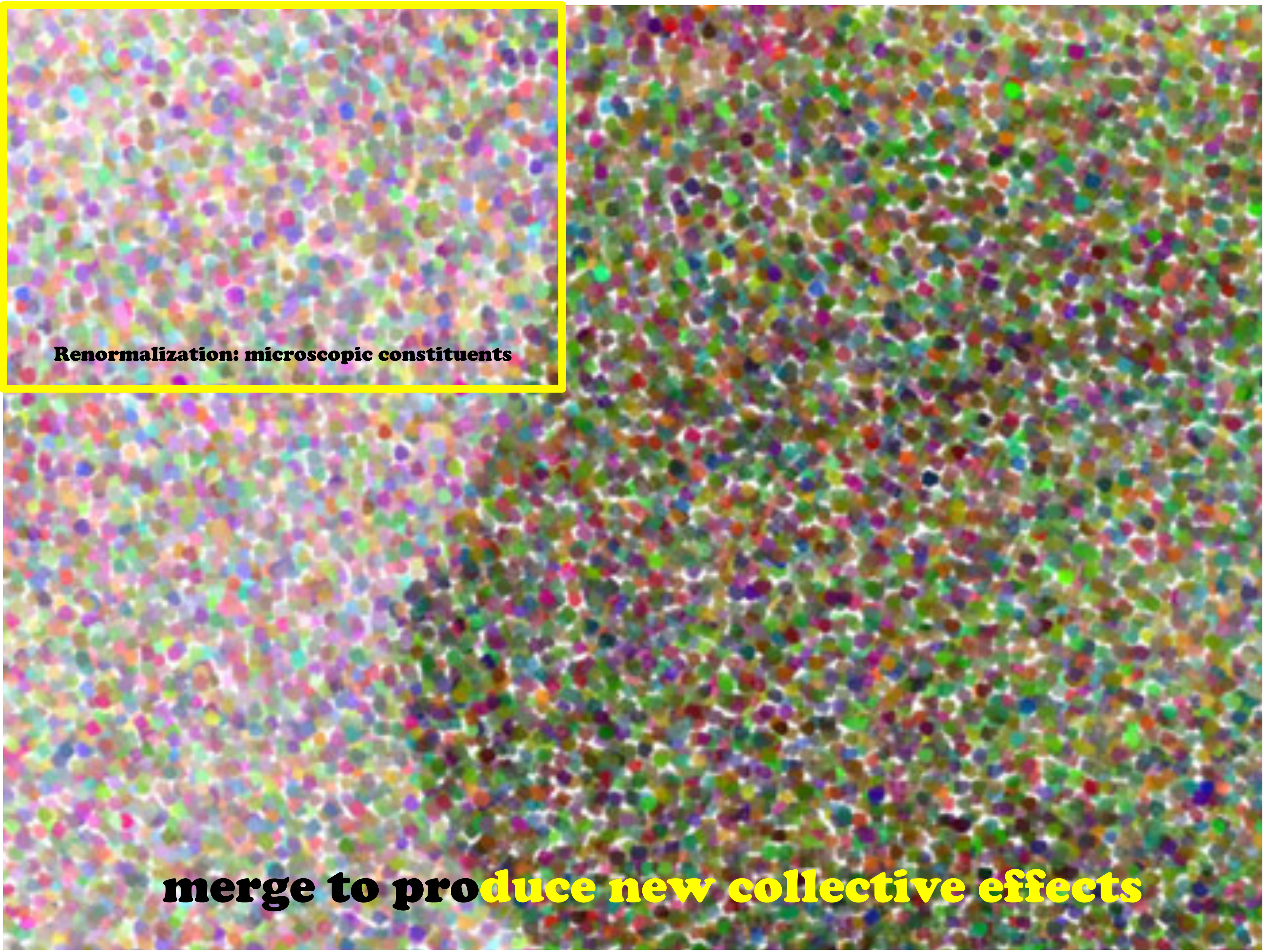
Bekenstein-Hawking formula

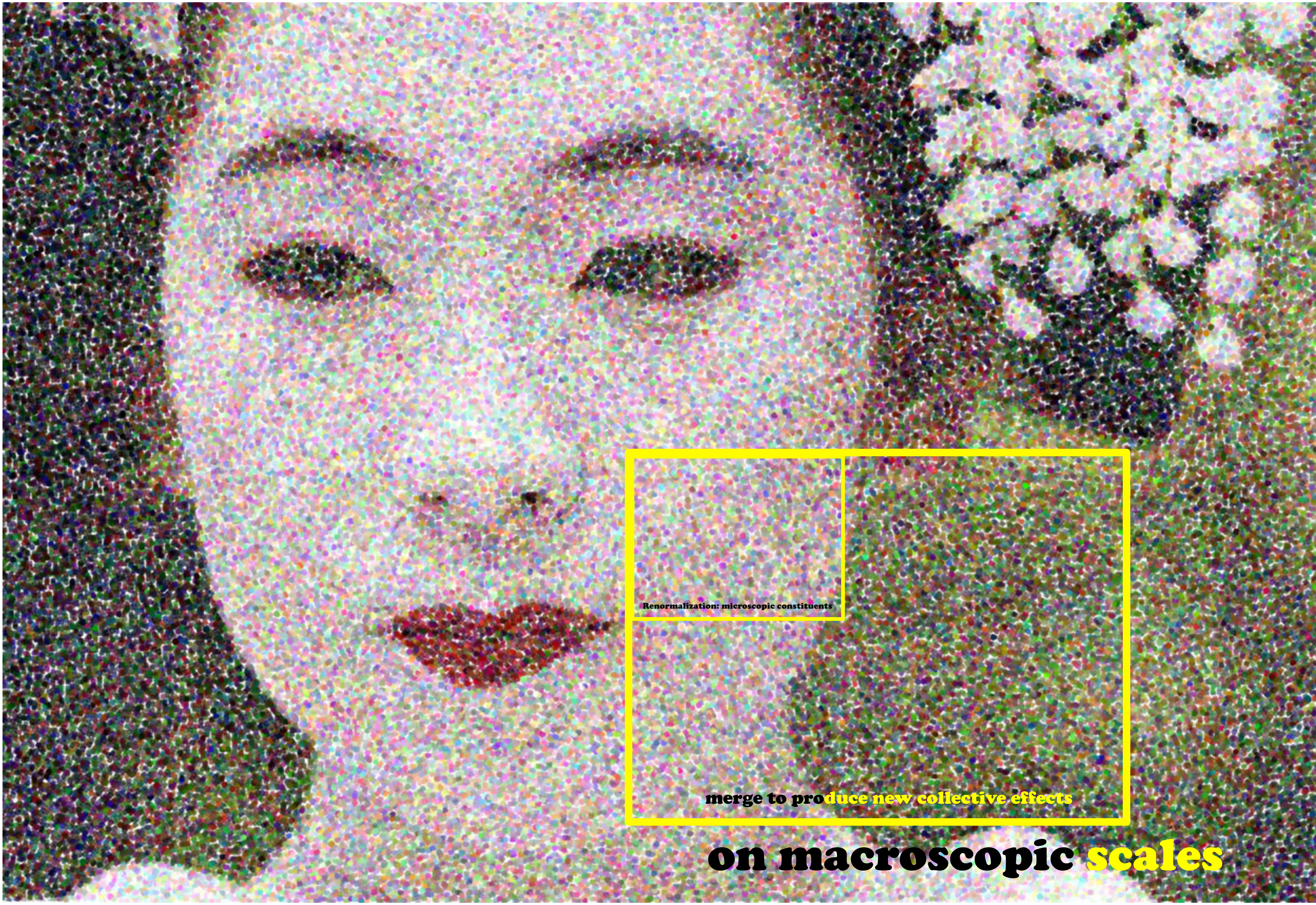
$$S(A) = \min_{V \sim A} \frac{A_V}{4G_N}$$

• holographic EE: interesting forum for bulk-boundary dialogue

$\longrightarrow$ new properties of QFTs, eg, RG flows and c-theorems


Renormalization: microscopic constituents


Renormalization: microscopic constituents
merge to produce new collective effects


Renormalization: microscopic constituents
merge to produce new collective effects
on macroscopic scales

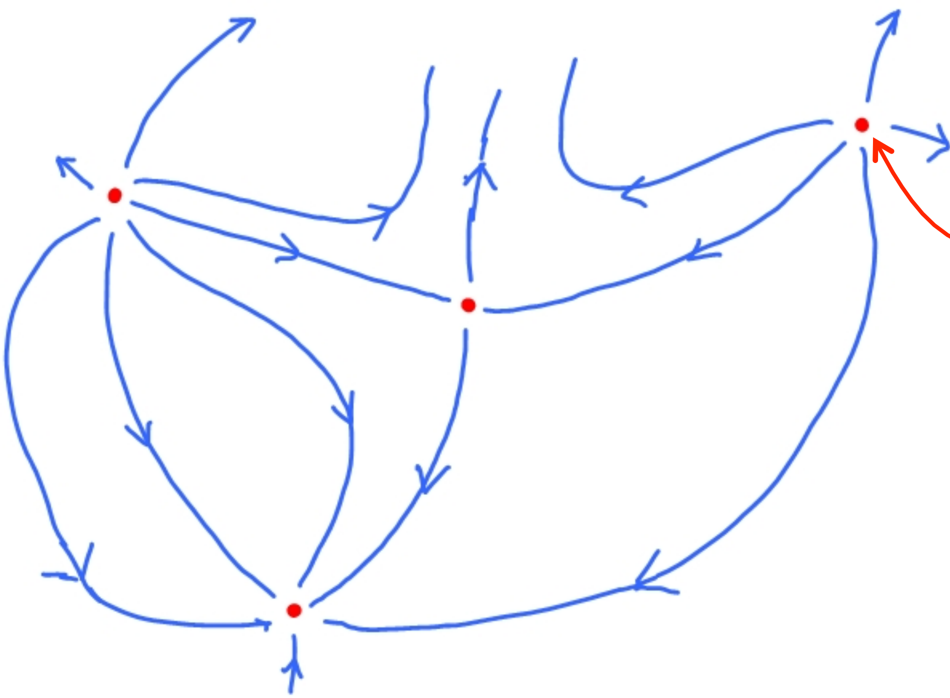
Renormalization Group

Wikipedia:

"mathematical apparatus that allows systematic investigation of the changes of a physics system as viewed at different *distance scales*"

➡ Effects of short-distance physics is absorbed in values of a few parameters of an effective theory

RG Flows describe how parameters of a quantum field theory change as more and more degrees of freedom at different scales methodically "integrated out"



fixed points: flow is stationary

(conformal field theory or "critical phenomena")

Zamolodchikov's c-theorem (1986):

- renormalization-group (RG) flows can seen as one-parameter motion

$$\frac{d}{dt} \equiv -\beta^i(g)\,\frac{\partial}{\partial g^i}$$

in the space of (renormalized) coupling constants $\{g^i,\ i = 1, 2, 3, \cdots\}$ with beta-functions as "velocities"

- for unitary, Lorentz-inv. QFT's in two dimensions, there exists a positive-definite real function of the coupling constants $C(g)$:

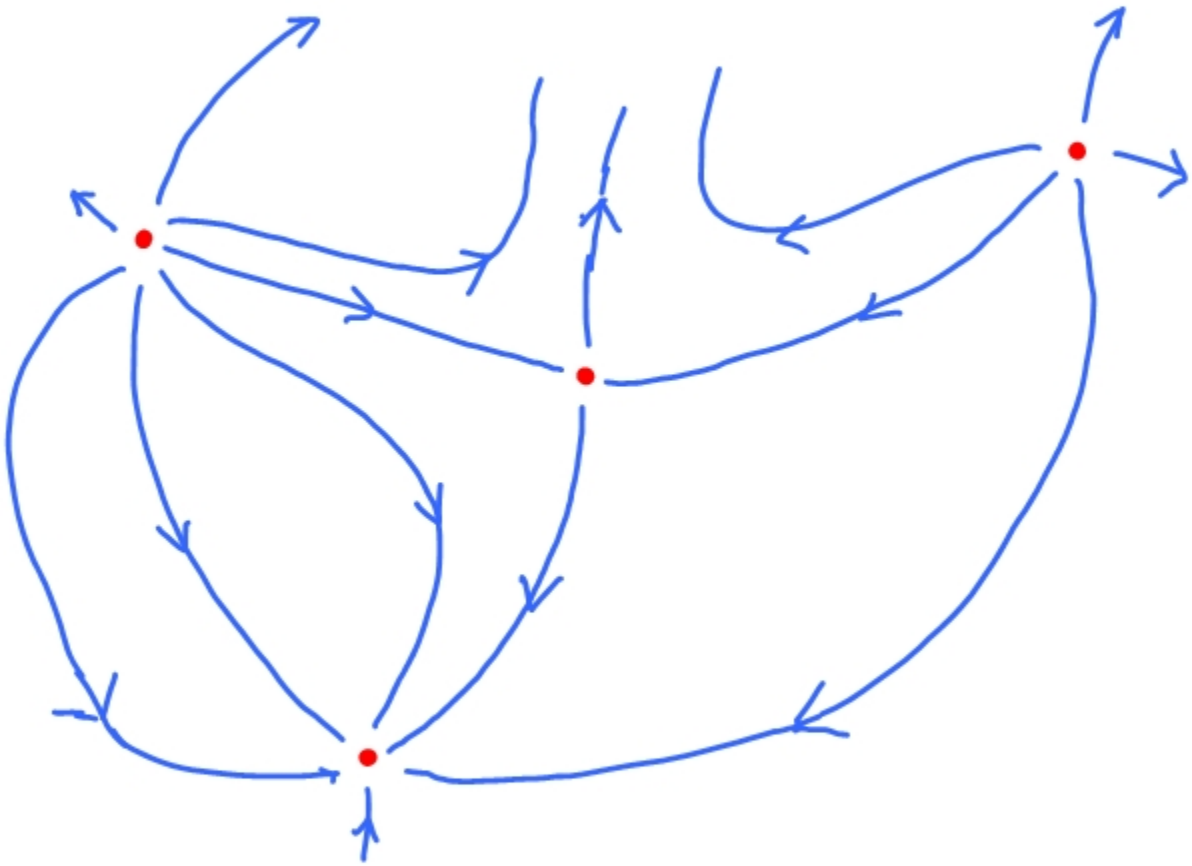
1. monotonically decreasing along flows: $\frac{d}{dt}C(g) \leq 0$
2. "stationary" at fixed points $g^i = (g^*)^i$:

$$\beta^i(g^*) = 0 \longleftrightarrow \frac{\partial}{\partial g^i}C(g) = 0$$

3. at fixed points, it equals central charge of corresponding CFT

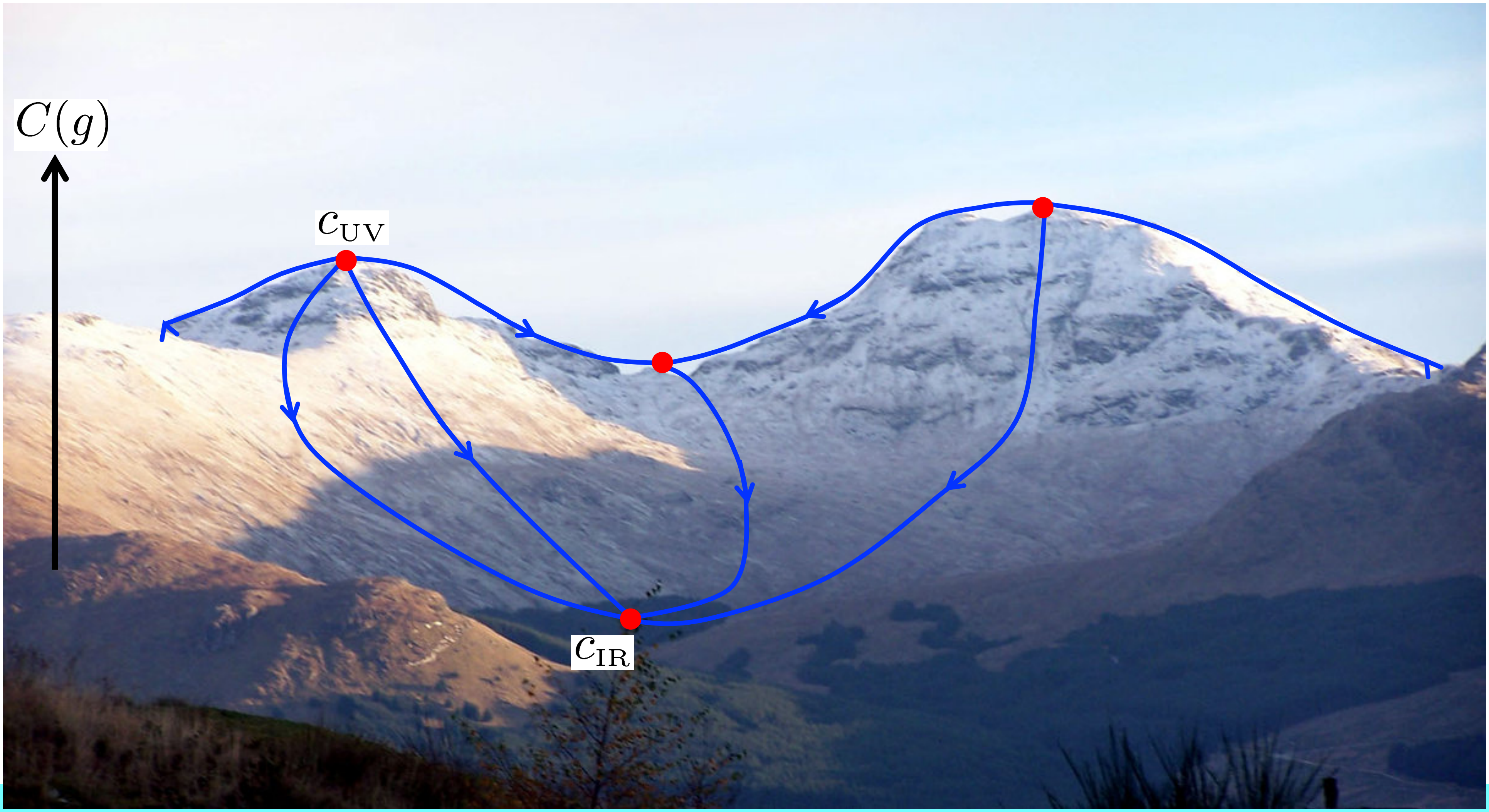
$$C(g^*) = c$$

Zamolodchikov's C-function adds a dimension to RG flows:



BECOMES

Zamolodchikov's C-function adds a dimension to RG flows:



Simple consequence for any RG flow in d=2: $c_{UV} > c_{IR}$

C-theorems in higher dimensions??

d=2: $$\langle T_\mu{}^\mu \rangle = -\frac{c}{12} R$$

d=4: $$\langle T_\mu{}^\mu \rangle = \cancel{\frac{c}{16\pi^2} I_4} - \frac{a}{16\pi^2} E_4 - \cancel{\frac{a'}{16\pi^2} \nabla^2 R}$$

where $I_4 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$ and $E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$

- in 4 dimensions, have three central charges: $c,\ a,\ a'$
- do any of these obey a similar "c-theorem" under RG flows? $[\,??\,]_{\mathrm{UV}} > [\,??\,]_{\mathrm{IR}}$

<u>a -theorem</u>: proposed by Cardy (1988)

- numerous nontrivial examples, eg, perturbative fixed points (Osborn '89), SUSY gauge theories (Anselmi et al '98; Intriligator & Wecht '03)
- holographic field theories with Einstein gravity dual (Freedman et al '99; Giradello et al '98)
- progress stalled; no proof found; ~~(counter example '98);~~ . . . .

- past few years have seen a resurgence of interest and rapid progress

(RM & Sinha `10)

C-theorems in higher dimensions:

- RG flows in generalized holographic models with higher curvatures

→ found new holographic c-theorem: $[a_d^*]_{UV} \geq [a_d^*]_{IR}$

$$a_d^* = \frac{\pi^{(d-2)/2} L^{d-1}}{8\Gamma(d/2) G_N f_\infty^{(d-1)/2}} \left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

where $\alpha^2 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0$

gravitational couplings

d = spacetime dimension of boundary theory

eg, for d=5: $I = \frac{1}{16\pi G_N}\int d^5x\,\sqrt{-g}\left[\frac{12}{L^2}\alpha^2 + R + L^2\frac{\lambda}{2}\chi_4 + L^4\frac{7\mu}{4}\mathcal{Z}_5\right]$

with
$$\chi_4 = R^{abcd}R_{abcd} - 4R_{ab}R^{ab} + R^2$$

$$\mathcal{Z}_5 = R_{a\ b}^{\ c\ d}R_{d\ c}^{\ e\ f}R_{e\ f}^{\ a\ b} + \frac{1}{56}\left(21R_{abcd}R^{abcd}R - 72R_{abcd}R^{abc}{}_{e}R^{de}\right.$$
$$\left. +120R_{abcd}R^{ac}R^{bd} + 144R_a{}^bR_b{}^cR_c{}^a - 132R_a{}^bR_b{}^aR + 15R^3\right)$$

(RM & Robinsion)

(RM & Sinha)

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gravitational couplings

d = spacetime dimension of boundary theory

- compare trace anomaly for CFT's in even dimensions (Deser & Schwimmer)

$$\langle T^\mu{}_\mu \rangle = \sum B_i\, (\text{Weyl invariant})_i - 2(-)^{d/2} A\, (\text{Euler density})_d + \nabla_\mu K^\mu$$

- precisely reproduces coefficient of A-type anomaly: $\boxed{a_d^* = A}$

(Henningson & Skenderis; Nojiri & Odintsov; Blau, Narain & Gava; Imbimbo, Schwimmer, Theisen & Yankielowicz)

→ agrees with Cardy's general conjecture!!

What about odd dimensions??

(RM & Sinha)

C-theorems in higher dimensions:

- identify central charge with universal contribution in entanglement entropy of ground state of CFT across sphere S^{d-2} of radius R:

$$S_{univ} = \begin{cases} (-)^{\frac{d}{2}-1}\, 4\, a_d^* \log(2R/\delta) & \textbf{for even } \boldsymbol{d} \\ (-)^{\frac{d-1}{2}}\; 2\pi\, a_d^* & \textbf{for odd } \boldsymbol{d} \end{cases}$$

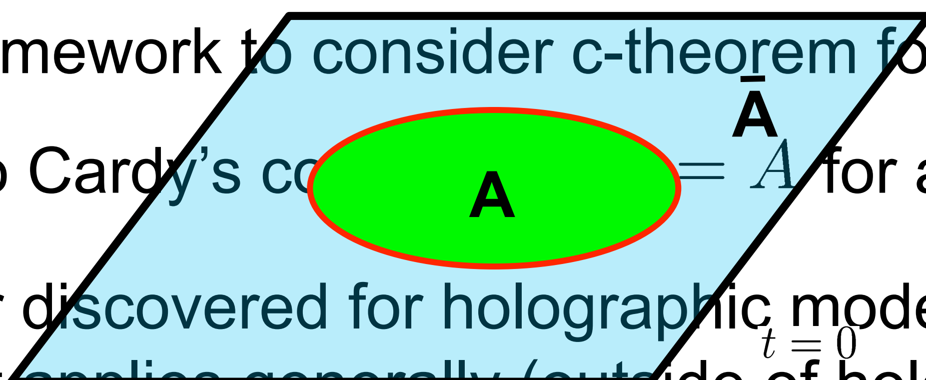
- for RG flows connecting two fixed points

$$(a_d^*)_{UV} \;\geq\; (a_d^*)_{IR}$$

⟶ unified framework to consider c-theorem for **odd** or even *d*

⟶ connect to Cardy's co $= A$ for any CFT in even *d*

⟶ behaviour discovered for holographic model but conjectured that result applies generally (outside of holography)



C-theorems in higher dimensions:

(RM & Sinha)

- RG flows in generalized holographic models with higher curvatures

  → identify C-theorem for universal coefficient in entanglement entropy for spherical surface in any spacetime dimension d

  → conjecture new c-theorems for odd dimensions beyond holography

- entanglement entropy and free energy approaches same (Casini, Huerta & RM)

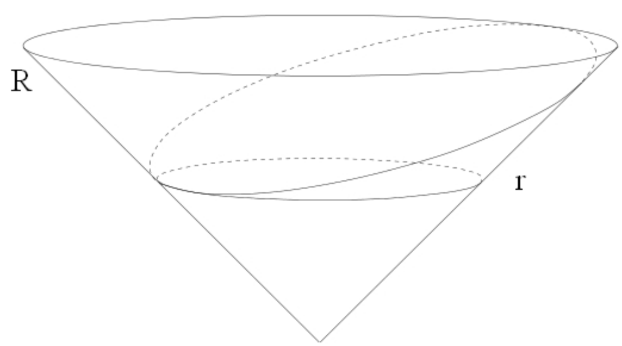
- F-theorem: d=3 (or any odd d) c-function = “free energy” = log(Z on d-sphere);

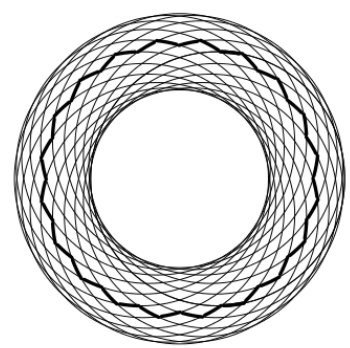
  → evidence from SUSY and nonSUSY QFT’s

  (Jafferis, Klebanov, Pufu & Safdi)

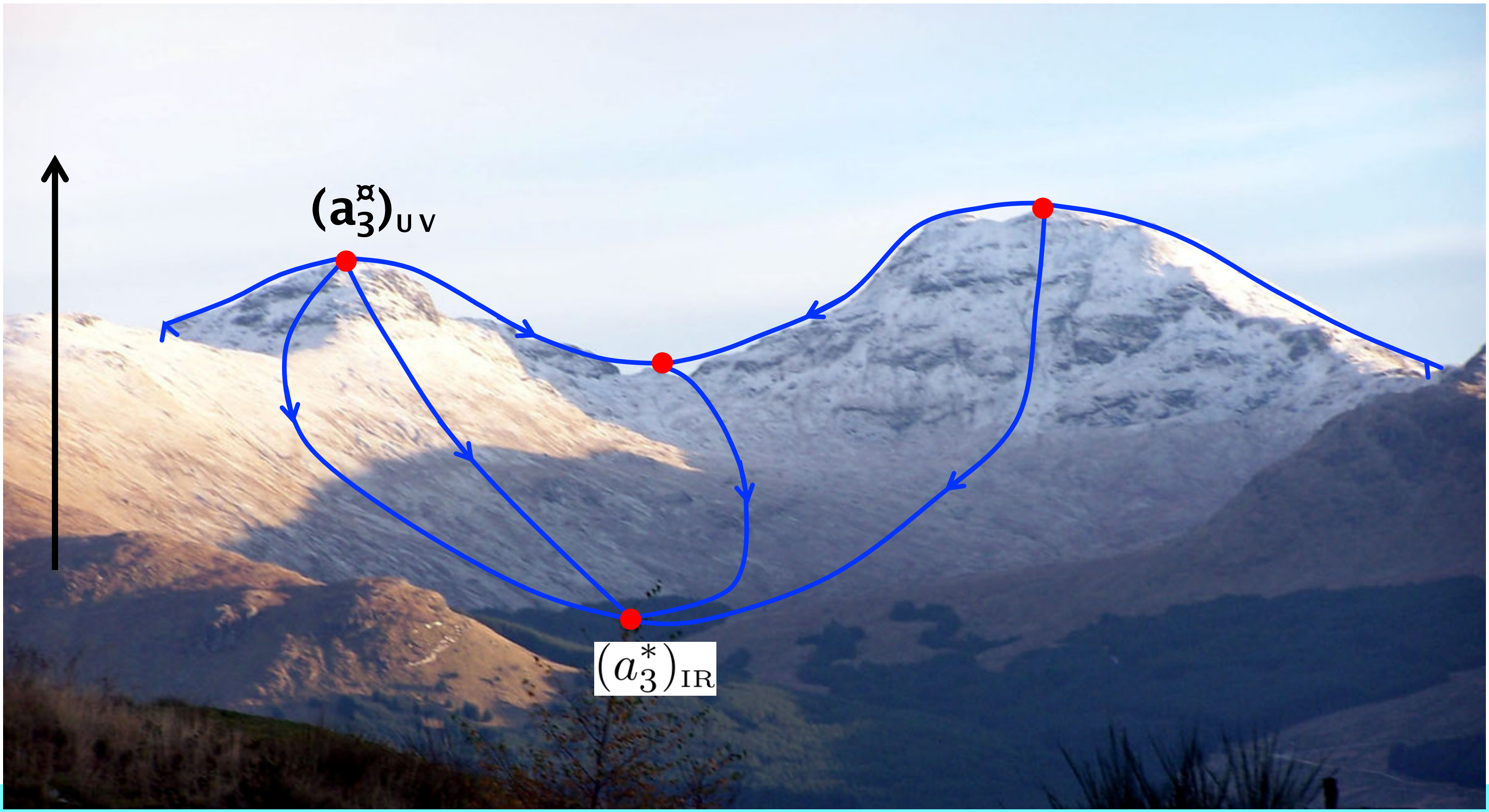
- **d=3 F-theorem proved** with entanglement entropy**!**

  → unitarity, Lorentz invariance & subaddivity (Casini & Huerta)





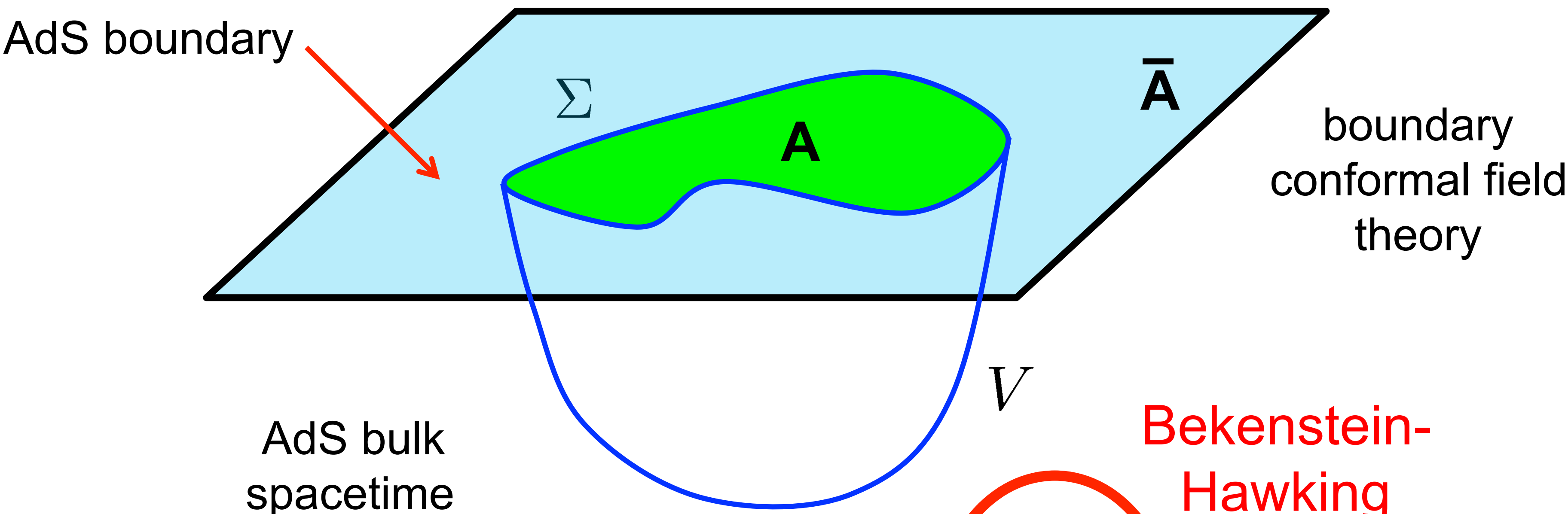
gravity/holography + EE → RG flows in (2+1)-dimensions



F-theorem: $(a_3^*)_{UV} > (a_3^*)_{IR}$

(Ryu & Takayanagi `06)

Holographic Entanglement Entropy:



Bekenstein-Hawking formula

$$S(A) = \min_{V \sim A} \frac{A_V}{4G_N}$$

• holographic EE: interesting forum for bulk-boundary dialogue

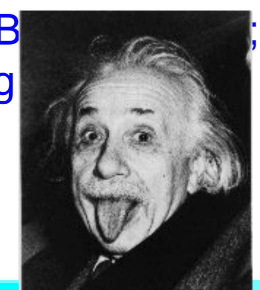
⟶ new features of quantum gravity

⟶ new properties of QFTs, eg, RG flows and c-theorems

• van Raamsdonk: entanglement essential to emergent spacetime

Spacetime Geometry = Entanglement

- → Bekenstein-Hawking formula: spacetime geometry encodes S_{BH}
- → black hole entropy is entanglement entropy (Sorkin, ….)
- → use BH formula for holographic entanglement entropy (Ryu & Takayanagi; ….)
- → connectivity of spacetime requires entanglement (van Raamsdonk)
- → spacetime entanglement conjecture (Bianchi & RM)
- → AdS spacetime as a tensor network (MERA) (Swingle, Vidal, ….)
- → "ER = EPR" conjecture (Maldacena & Susskind)
- → hole-ographic spacetime (Balasubramanian, Chowdhury, Czech, de B ; RM, Rao & Sugishita; Czech, Dong



spacetime provides both the stage for physical phenomena and the agent which manifests gravitational dynamics

Gravitation from Entanglement:

- relative entropy: $S(\rho_1|\rho_0) = \mathrm{tr}(\rho_1 \log \rho_1) - \mathrm{tr}(\rho_1 \log \rho_0)$
- let: $\rho_0 =$ reference state; $\rho_1 =$ perturbed state

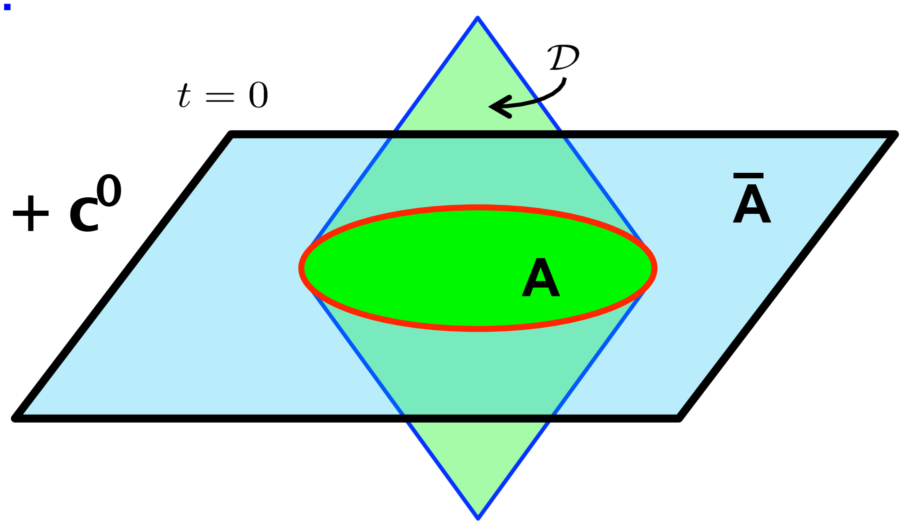
  $= \exp(-H_A)$ ⟵ modular Hamiltonian

⟶ "1st law" of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$

- this is **the** 1st law for thermal state: $\rho_A = \exp(-H/T)$

- generally H_A is "nonlocal mess" but H_A has simple form for CFT and spherical entangling surface:

$$H_A = 2\pi \int_{|\vec{y}|<R} d^{d-1}y \, \frac{R^2 - |\vec{y}|^2}{2R} \, T_{tt}(\vec{y}) + c'$$



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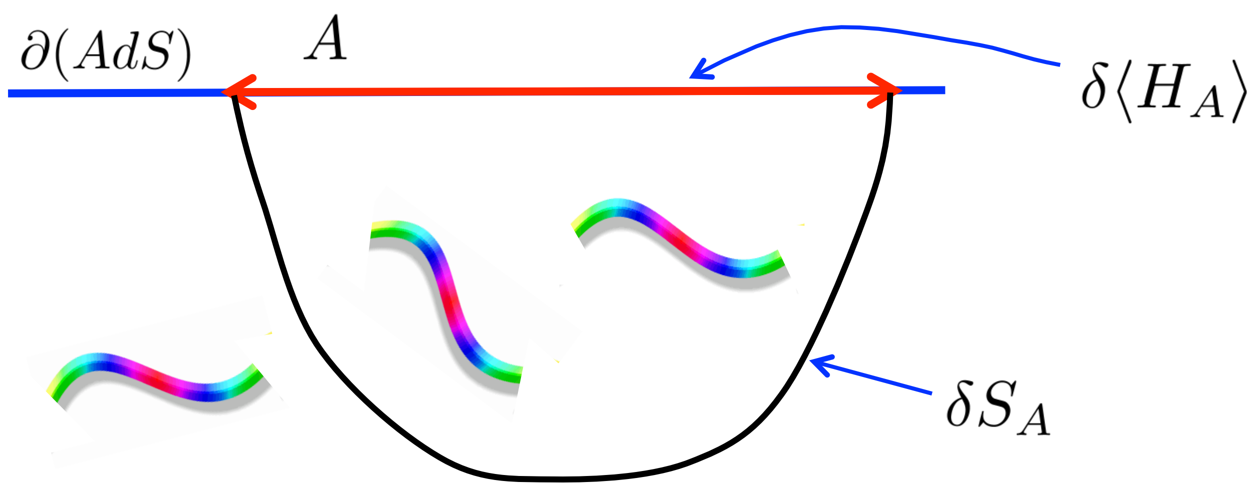
- holographic realization:



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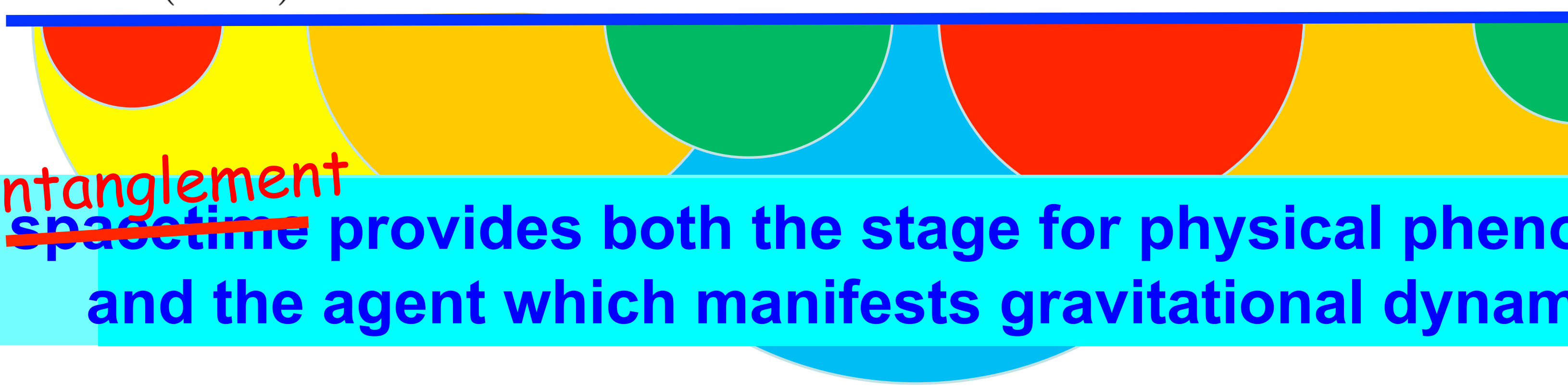
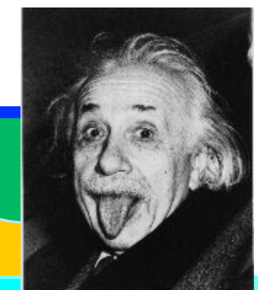
- holographic realization:

$\partial(AdS)$





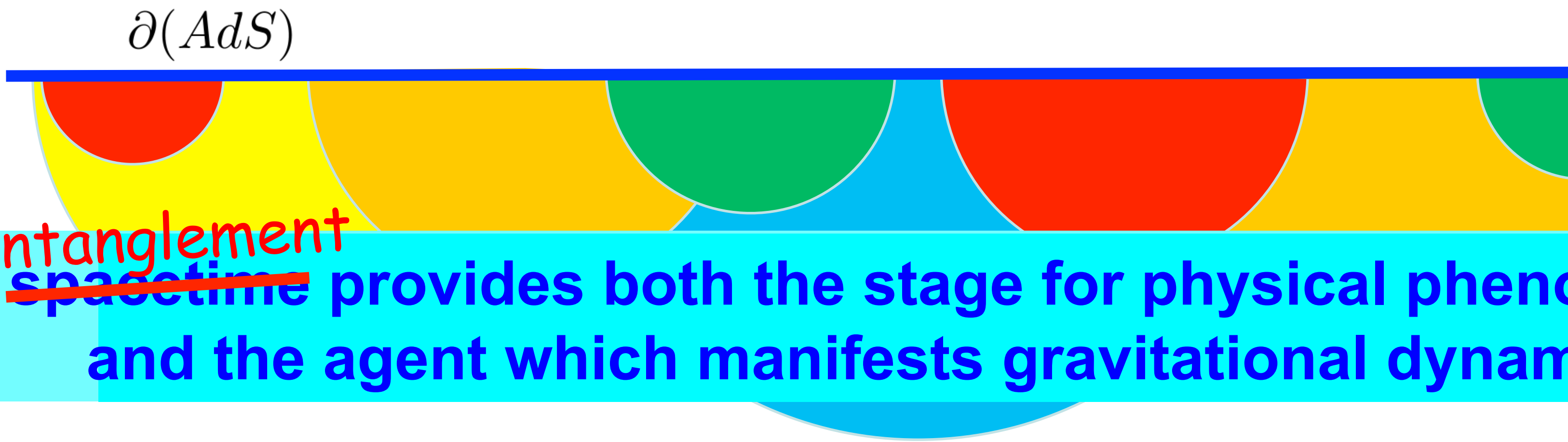
entanglement ~~spacetime~~ provides both the stage for physical phenomena and the agent which manifests gravitational dynamics

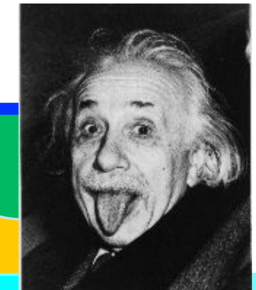
- apply 1st law for spheres of all sizes, positions and in all frames:

1st law of S_{EE} ⟺ bulk geometry satisfies linearized Einstein eq’s

(Lashkari, McDermott & Van Raamsdonk; Swingle & Van Raamsdonk; Faulkner, Guica, Hartman, RM & Van Raamsdonk)

- holographic realization:





entanglement
~~spacetime~~ provides both the stage for physical phenomena and the agent which manifests gravitational dynamics

- apply 1^{st} law for spheres of all sizes, positions and in all frames:

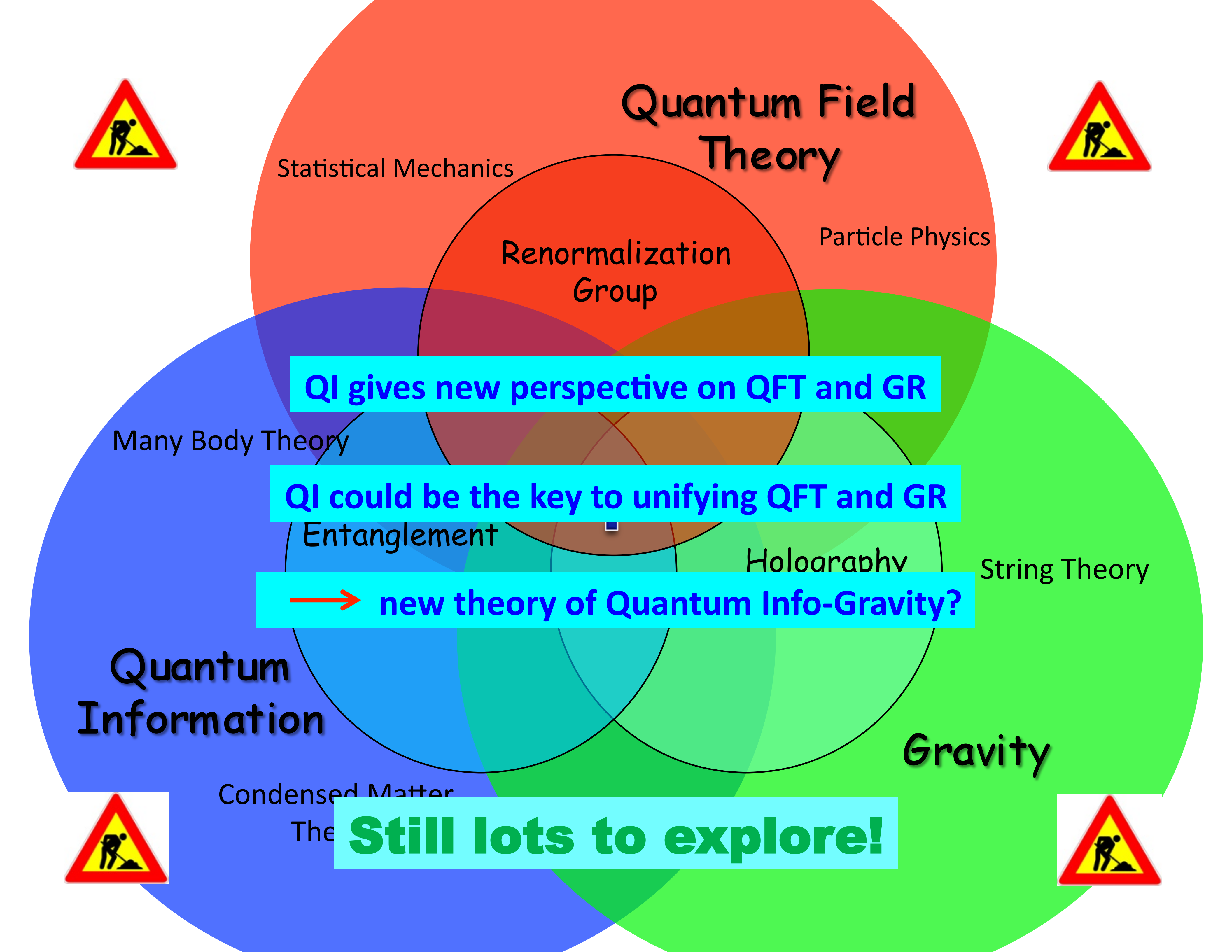
1^{st} law of S_{EE} ⟺ bulk geometry satisfies linearized Einstein eq's

(Lashkari, McDermott & Van Raamsdonk; Swingle & Van Raamsdonk; Faulkner, Guica, Hartman, RM & Van Raamsdonk)

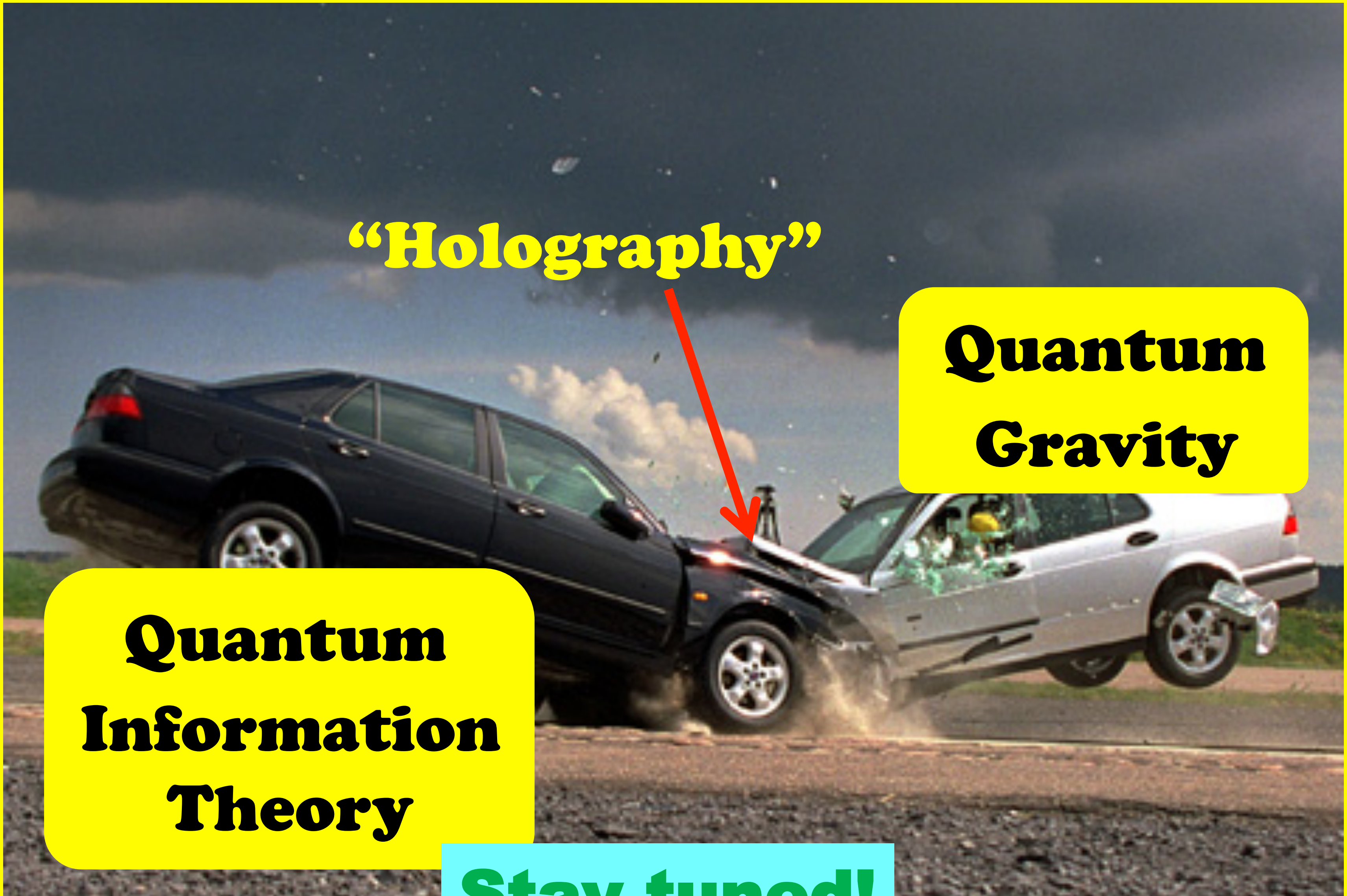
- Jacobson: Einstein equation from entanglement

No AdS/CFT needed here!!

see also: Smolin (session D1, Thursday)


Quantum Field Theory
Statistical Mechanics
Particle Physics
Renormalization Group
QI gives new perspective on QFT and GR
Many Body Theory
QI could be the key to unifying QFT and GR
Entanglement
Holography
String Theory
new theory of Quantum Info-Gravity?
Quantum Information
Gravity
Condensed Matter
Still lots to explore!

A New Collision of Ideas:



Stay tuned!

[end slides]