Gravitationally Induced Quantum Transitions

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also related Graviton Laser, (to be published IJMP)

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Gravitation and Quantum Mechanics

- There has not been very much work on the interplay of gravity and quantum mechanics.
- Here I do not mean quantum gravity, but the effect of gravity on a quantum mechanical system.
- The reason is probably that the effects are very weak. The gravitational coupling constant is $\frac{G}{c^4}$
- This combination has units of inverse Newtons, and in the MKS system it is numerically of the order $\sim 10^{-45}$
- However the gravitational potential is also proportional to the product of the two masses involved.
- The interaction of gravitation with a quantum mechanical system in the lab has only recently been observed.

Q-bounce

- The Q-bounce experiment, which stands for quantum bouncer, was proposed and carried out in the last ten years
- Here a system of ultra cold neutrons were observed.
- Ultra cold neutrons are normally defined to have a kinetic energy of less that 300 neV, and they are unable to penetrate into the solid material walls of a vessel, they bounce off the walls, and are in fact contained.
- If they are further distilled in energy so that the kinetic energy is in the few peV range, then they start to feel the gravitational potential due to the earth.
- The energy levels of the Schrödinger equation are easily found.

- The Schrödinger equation is:

$$-\frac{\hbar^2}{2m_N}\frac{d^2}{dz^2}\psi_E(z)+m_N g z\psi_E(z)=E\psi_E(z)$$

- The energy eigenfunctions are:

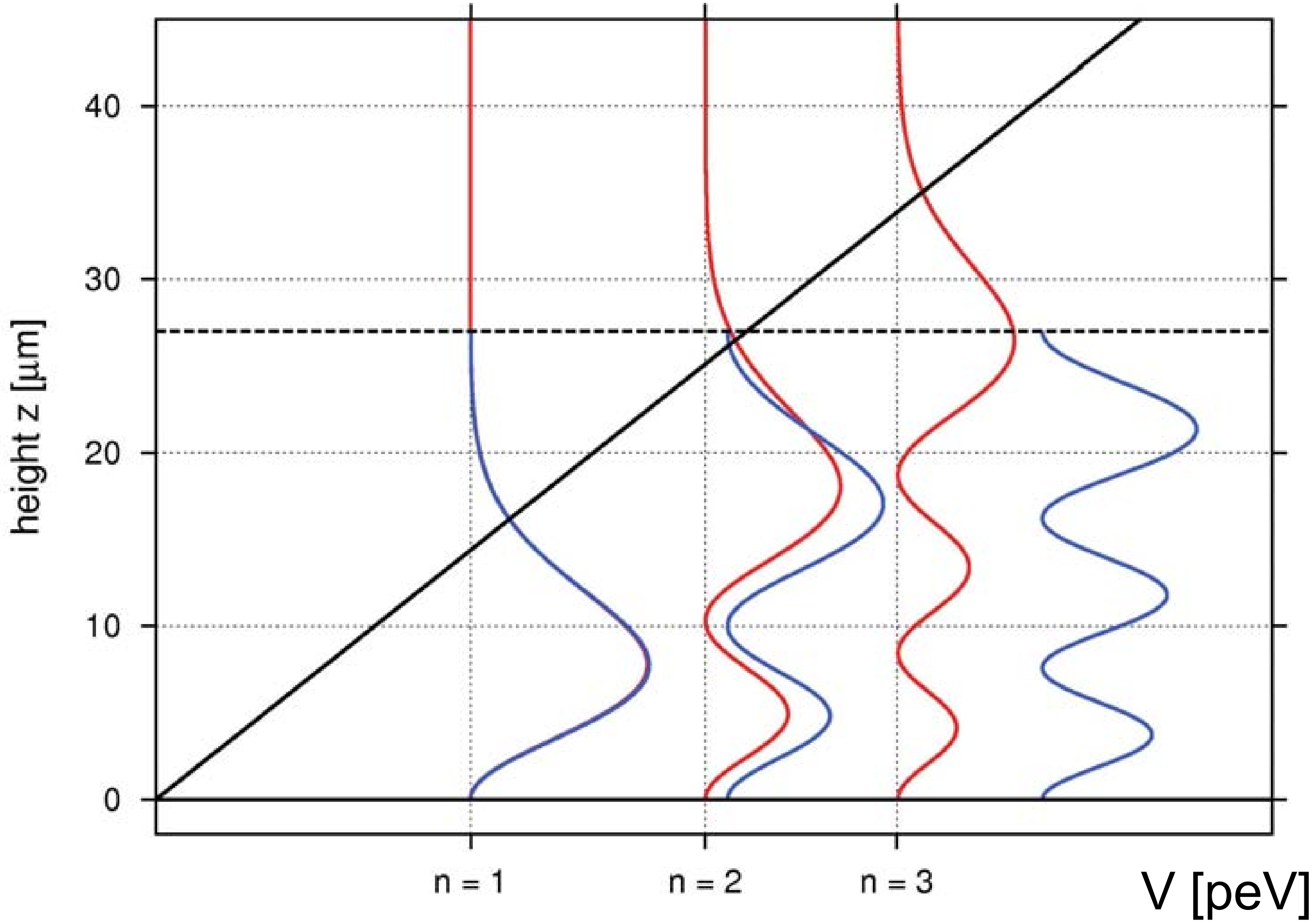
$$\psi_n(z)=\mathcal{N}_n Ai(\tfrac{z}{z_0}-\alpha_n) \qquad \mathcal{N}_n=\frac{1}{\sqrt{z_0}Ai'(-\alpha_n)}$$

$$E_n=m_N g z_0\alpha_n \qquad z_0^3=\frac{\hbar^2}{2gm_N^2}$$

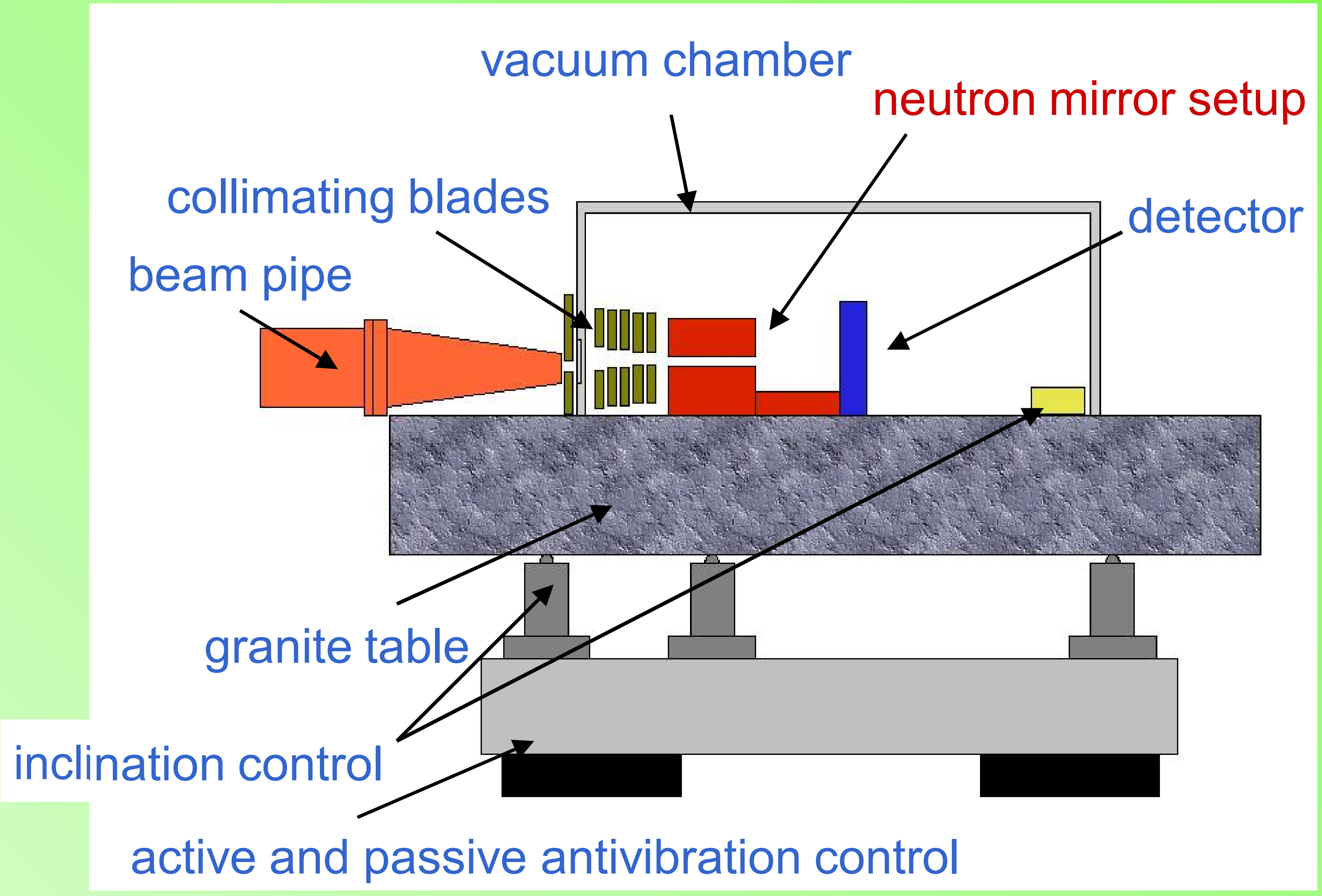
- In numbers, the energy levels are approximately given by:

$$E_n=\left(\frac{9m_N h^2 g^2}{32}\left(n-\frac{1}{4}\right)^2\right)^{1/3}\times 10^{-12}eV=1,69\left(n-\frac{1}{4}\right)^{2/3}peV$$

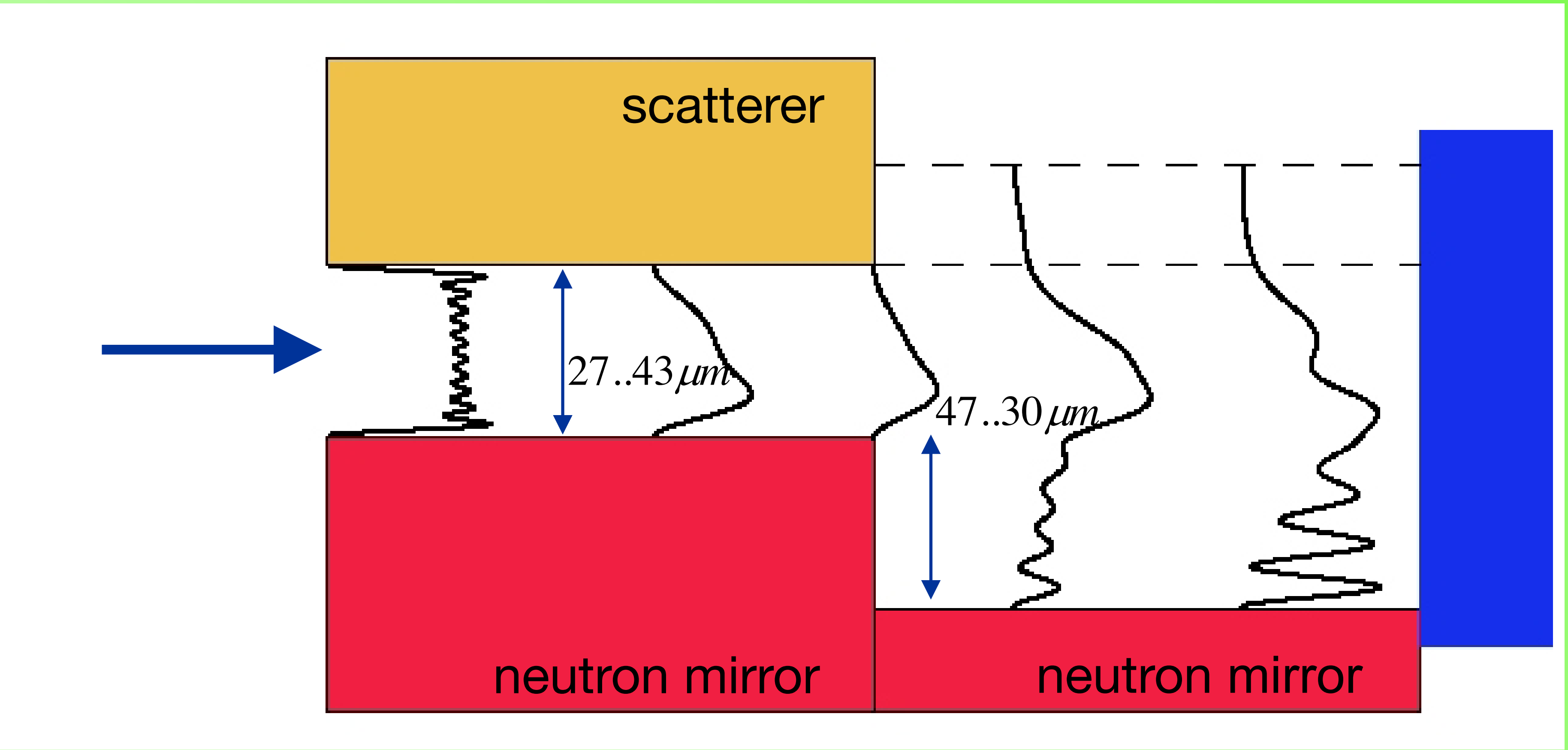
	E_n	E_n
1st state	1.41peV	1.41peV
2nd state	2.46peV	2.56peV
3rd state	3.32peV	3.97peV



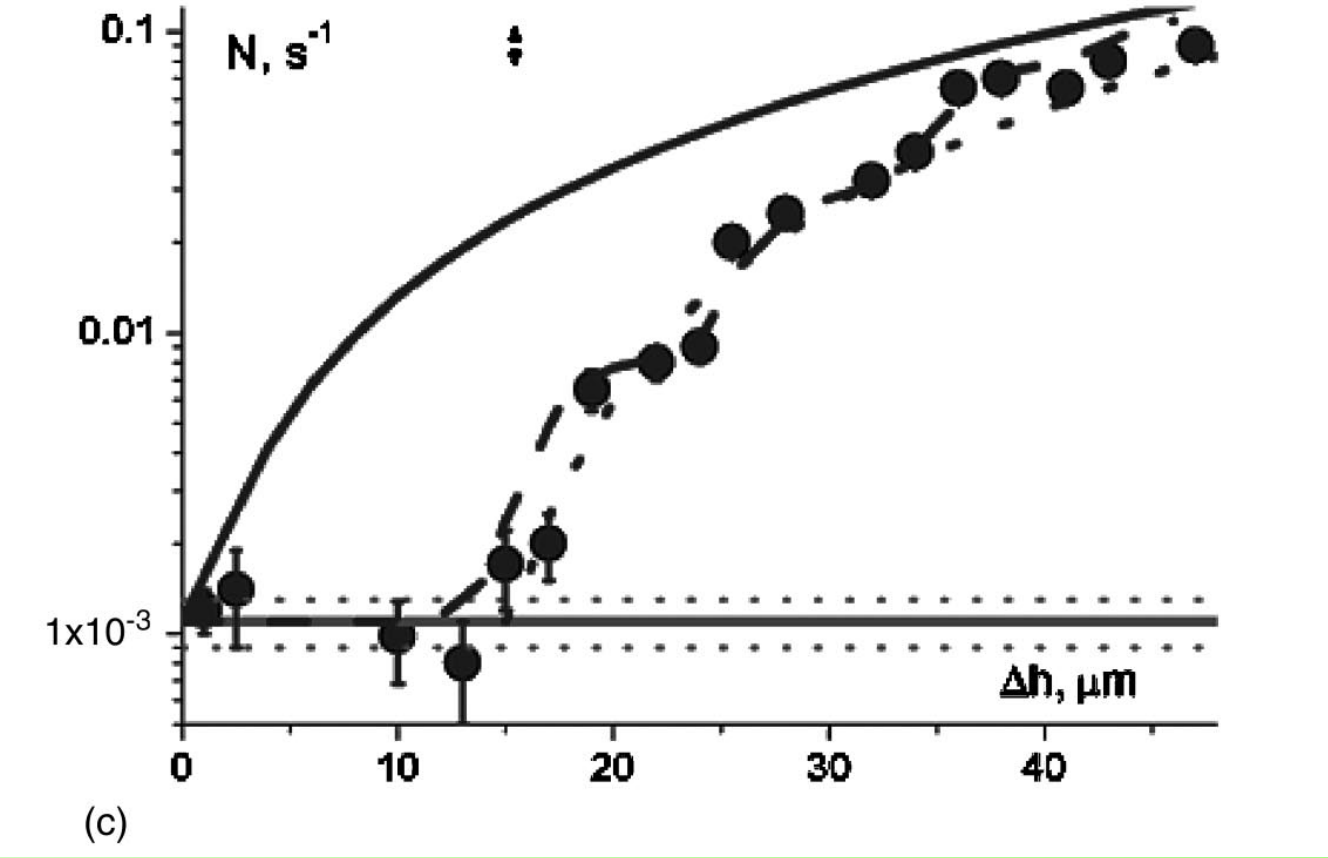
Experimental overview of Q-bounce



Neutron mirror setup



- The neutron scatterer serves as a filter, removing all neutrons that can rise to a specified height above the base.
- If the height of the scatterer is less than the mean height of the neutrons in the lowest quantum state, essentially no neutrons can pass. As the height is raised, first only those in the first level pass, subsequently those in the second level and so on.



- One can induce transitions between the quantum levels by driving the system with an external perturbation.
- Neutrons notoriously have no interactions.
- Q-bounce realized that by vibrating the base at the resonant frequency corresponding to the difference in energy levels, about 600Hz, they could induce transitions.

$$H = \begin{pmatrix} \frac{\hbar\omega_{pq}}{2} & \frac{1}{2}\hbar\Omega_R e^{-i\omega t} \\ \frac{1}{2}\hbar\Omega_R e^{i\omega t} & \frac{-\hbar\omega_{pq}}{2} \end{pmatrix}.$$

$$P(t) = \left(\frac{\Omega_R}{\Omega_R'}\right)^2 \sin^2\!\left(\frac{\Omega_R'}{2}t\right)$$

$$\Omega_R' = \sqrt{\Omega_R^2 + (\omega_{pq} - \omega)^2} = \sqrt{\Omega_R^2 + \delta^2},$$

Experimental setup: Ramsey oscillations

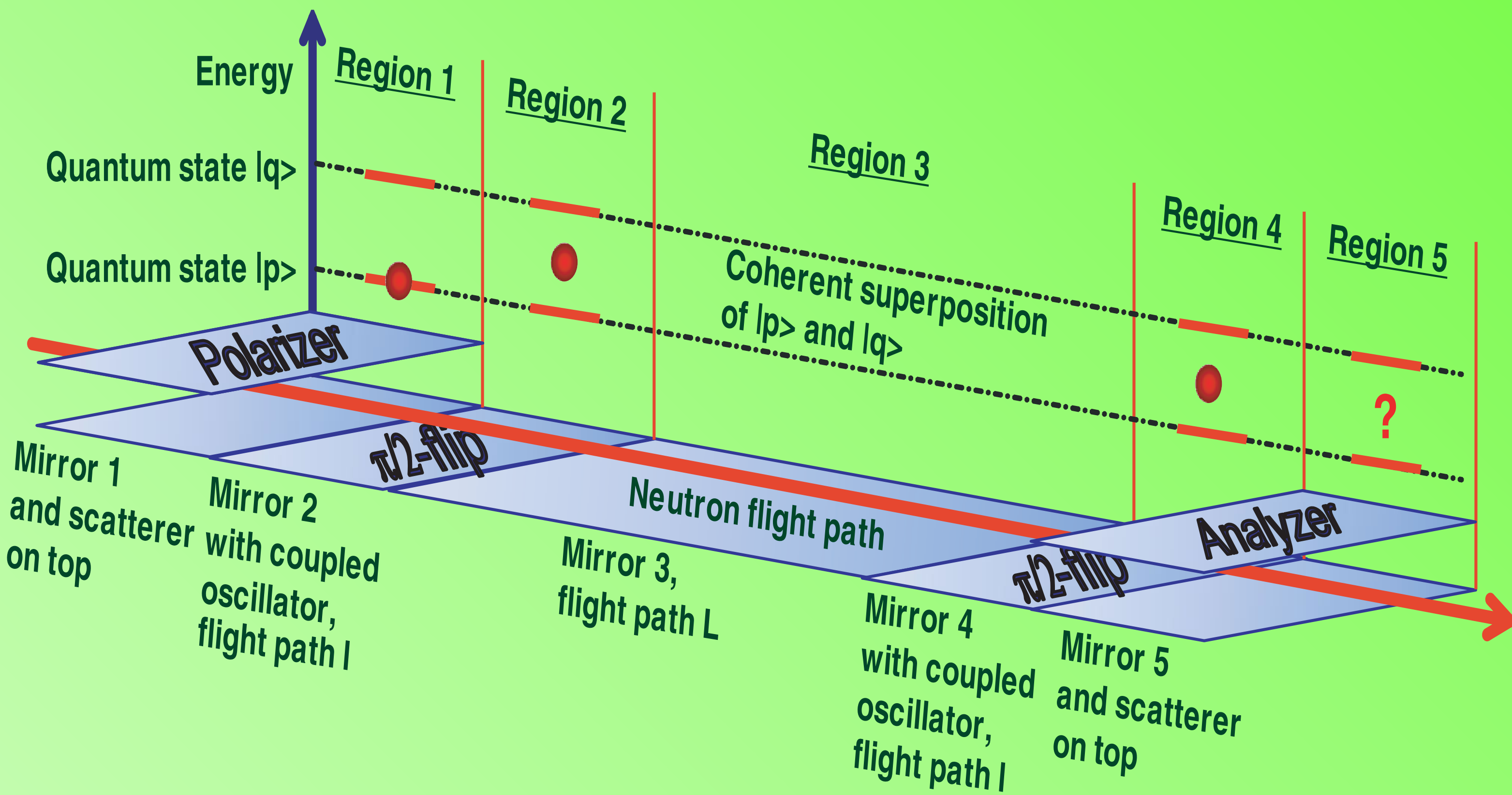


FIG. 2 (color online). Sketch of the proposal. Region 1: Preparation in a specific quantum state, e.g. state one with polarizer. Region 2: Application of first $\pi/2$-flip. Region 3: Flight path with length L. Region 4: Application of second $\pi/2$ flip. Region 5: State analyzer.

- In region 1, the neutrons are filtered so that only those in ground state may pass.
- In region 2, the system is driven by vibrating the base at the resonant frequency, with $\Omega\tau = \pi/2$ which creates a superposition with equal amplitude.
- In region 3, the neutrons are in a coherent superposition of two states over a flight path of length L.
- In region four, another $\Omega\tau = \pi/2$ resonant pulse by vibrating the base, is applied which exactly reverses the coherent superposition.
- In region 5, the neutron state is analyzed via an identical state selector as in region 1 with a neutron counter at the end.

Gravitationally Induced Quantum Transitions

- The neutrons are non-interacting except for the interaction with the base (phonons) and the interaction with the earth’s gravitational field.
- In the previous experiments, transitions were induced by modulating the base, using the interactions with phonons.
- We asked is it also possible to induce transitions by modulating the gravitational field.
- At first thought, it seemed that this would be impossible, that the gravitational interaction is of almost negligible strength.
- Surprisingly, this is not true, and we find that it is in principle possible to induce transitions using gravity.

Relative strengths of gravitational fields

- A quick comparison with the earth's gravitational field and that of a local mass source is easy:

$$g = \frac{GM_\oplus}{R_\oplus^2} \approx \frac{10m}{sec^2}$$

- while a 10kg gold sphere has a radius of 5 cm, gives and acceleration:

$$\begin{aligned} g_M = \frac{GM}{r^2} &\approx \frac{6.67 \times 10^{-11} M}{r^2} \frac{m}{sec^2} \\ &\sim \frac{6.67 \times 10^{-11} 10}{(.05)^2} \frac{m}{sec^2} \\ &\approx 2.7 \times 10^{-7} \frac{m}{sec^2} \end{aligned}$$

- which is much smaller, but not absurdly so!

Perturbing Hamiltonian

- We imagine a gold ball of 10kgs brought 5cm above a system of ultra cold neutrons and oscillated with the frequency corresponding to resonance between two states of the neutrons. Its height above the base is given by:

$$\zeta(t) = \zeta_0 + \Delta\zeta + \Delta\zeta cos(\omega t)$$

- The perturbation then is:

$$W(t,z) = \frac{Gm_N M}{\zeta(t) - z} \approx W_1(z) - \Delta\zeta W_2(z) cos(\omega t)$$

- where: $W_1(z) = \frac{Gm_N M}{(\zeta_0 + \Delta\zeta - z)}$ $W_2(z) = \frac{Gm_N M}{(\zeta_0 - z)^2}$

Transition Probability

- The simple formula for transitions to first order is:

$$P_{nm}(\omega, t) = \frac{\Delta\zeta^2}{\hbar^2} \left| \int_0^t dt' \ \langle \psi_m(z, t') \left| W_2(z) cos(\omega t') \right| \psi_n(z, t') \rangle \right|^2$$

$$P_{nm}(\omega, t) = \left| \frac{G m_N M \Delta\zeta}{\hbar} \left[\int_0^t dt' exp\left(i\omega_{mn} t' \right) cos(\omega t') \right] \left[\int_0^\infty dz \frac{\psi_m(z)\psi_n(z)}{(\zeta_0 - z)^2} \right] \right|^2$$

- The time integral is elementary:

$$\frac{1}{4} \left[\left(\frac{\sin\left(\omega_{mn} t\right)}{\omega_{mn}} \right)^2 + t \left(\frac{\sin\left(2\omega_{mn} t\right)}{\omega_{mn}} \right) + t^2 \right] \to t^2/4$$

- While the spatial integral is obtained via:

$$I_2\left(\alpha_m,\alpha_n\right)=\int_0^\infty dz\frac{1}{\left(\zeta_0-z\right)^2}\psi_m(z)\psi_n(z)$$
$$\approx\frac{1}{\zeta_0^2}\delta_{mn}+\frac{2}{\zeta_0}\mathcal{N}_m\mathcal{N}_n\left(\frac{z_0}{\zeta_0}\right)^2\int_0^\infty dyyAi(y-\alpha_m)Ai(y-\alpha_n)$$

- With

$$\int_0^\infty dyyAi(y-\alpha_m)Ai(y-\alpha_n)=\frac{-2}{(\alpha_m-\alpha_n)^2}Ai'(-\alpha_m)Ai'(-\alpha_n)$$

- Which gives:

$$I_2\left(\alpha_m,\alpha_n\right)\approx\frac{1}{\zeta_0^2}\delta_{nm}-\frac{4z_0}{\zeta_0^3}\frac{1}{(\alpha_m-\alpha_n)^2}$$

Transition probability

- This gives the transition probability:

$$P_{nm}(\omega, t) = \left(\frac{G m_N M \Delta\zeta}{2\zeta_0^2 \hbar}\right)^2 \left(\frac{4z_0}{\zeta_0}\frac{1}{(\alpha_m - \alpha_n)^2}\right)^2 \; t^2$$

- Using the density and radius of the ball we get:

$$P_{nm}(\omega, t) = \left(\frac{8\pi G m_N \rho z_0 \Delta\zeta}{3\hbar}\right)^2 \frac{1}{(\alpha_m - \alpha_n)^4} \; t^2$$

- The same calculation can be done for a cylinder rather than a sphere, which gives

$$P_{nm}(\omega, t) \;\; = \left(\frac{G\rho\pi m_N z_0 \Delta\zeta}{\hbar}\right)^2 \frac{1}{(\alpha_n - \alpha_m)^4} \; t^2$$

Numerical values

- The probability increases quadratically with time. However the pre-factor is very small, so that we will not build up appreciable excited neutrons. for the following parameters:

$$M = 10 \text{ kg} \quad \zeta_0 = 5 \text{ cm} \ \ \Delta\zeta = .5 \text{ cm}$$

$$G = 6.67 \times 10^{-11} \text{ Nt m}^2/\text{kg}^2, \ \hbar = 1.054 \times 10^{-34} \text{ Joule sec}$$

$$m_N = 1,67 \times 10^{-27} \text{kg}, \ z_0 = 5,874 \times 10^{-6} \text{ m}$$

- and for transitions between the first two levels

$$\Delta E = .493 \ peV \qquad (\alpha_2 - \alpha_1) = 1.64$$

- We find the pre-factor:

$$\left(\frac{G m_N M \Delta\zeta}{2\zeta_0^2 \hbar}\right)^2 \left(\frac{4z_0}{\zeta_0} \frac{1}{(\alpha_m - \alpha_n)^2}\right)^2 = 3.43 \times 10^{-12} \text{sec}^{-2}$$

- Thus the system can be driven for a long time before we build up an appreciable number of excited neutrons.

Ideal pumping time

- The neutrons are unstable, they decay with a mean lifetime of 880 sec.

$$N = N_0 e^{-t/\tau}$$

- But the transition probability increases quadratically in time, thus the function to maximize is:

$$t^2 e^{-t/\tau}$$

- This occurs exactly in two lifetimes: $t = 2\tau$
- Then the time dependent terms becomes:

$$(2 \times 880)^2 = 3.10 \times 10^6 \mathrm{sec}^2$$

- Which gives a probability of transitions:

$$P_{12}(t) = 1.06 \times 10^{-5}$$

Experimental feasibility

- Ultra-cold neutrons are defined to those that are totally reflected out of material containers, their energies are of the order of $100neV$
- These neutrons can now be produced with densities of the order of $50/cc$ in containers of size of the order of a cubic meter to a tenth of a cubic meter.
- This gives of the order of 10 to 100 million UCNs.
- However we need very much colder UCN, with energies of the order of a few peV
- The Q-bounce experiments were done with about 4500, very cold UCN.
- Thus the notion of obtaining around a million very cold UCN is not absurd!

Experimental feasibility

- However, another difficulty is the long time required to get an appreciable number of transitions.
- The UCN's are easily up-scattered due to impurities on the surface of the container and imperfections.
- They also have a transmission probability of actually entering the walls of the container. The loss rate from the wall is around 10^{-5} per collision. However, this is very sensitive to the neutron kinetic energy, and is the value that is relevant for $100neV$ UCN's.
- This yields a storage time of only about 100-200 sec.
- However, for the very cold UCN's that we require, this storage time will be substantially longer.