CHARGED ROTATING BLACK HOLES AT LARGE D

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based on arXiv:1605.08854
“Charged rotating black holes at large D”

PURPOSE

- **Study charged rotating black holes in Einstein-Maxwell theory**
  - We have no analytic solutions in higher dimensions

    Kerr-Newman solution in 4D, other asymptotics or theories in higher dimensions

    [Newman et.al. (1965), Breckenridge et.al. (1997), and so on]
  - Some perturbation analysis to find stationary solutions

    by a charge on MP-BHs or rotations on RN-BHs

    [Navarro-Larida (2010), Krtous (2007), Aliev (2007), and so on]
  - Numerical analysis or blackfold method is also useful

    to investigate phase diagram of stationary solutions

    [Kunz-Navarro-Larida-Petersen (2005), Caldarelli-Emparan-Pol (2011), and so on]
  - How about dynamical properties such as (in)stabilities ?

BH INSTABILITIES

- **There are two (kinds of) black hole instabilities in higher dimensions**

  Instability of rotating black holes in D>5 (ultraspinning instabilities)

  Myers-Perry black holes become unstable at larger rotations

  [Emparan-Myers (2003), Dias et.al. (2010,2011,…), and so on]

  Instability of dS Reissner-Nordstrom black holes in D>6 or 5

  RN-BHs become unstable at larger charge in de Sitter

  [Konoplya-Zhidenko (2008), KT (2016)]

- What is the relation between these instabilities ?

Instability of MP-BHs ⇦ **Instability of charged rotating BHs (in dS)** ⇨ **Instability of RN-BHs**

LARGE D METHOD

□ 1/D expansion of Einstein equations

[Emparan-Suzuki-KT (2013), Roberto's talk on Wednesday in D2 session]

- Gravity is confined in very near region of the BH at large D

$$r_0 \text{ (horizon scale)} \gg r_0/D \text{ (confinment scale)}$$

Effective theory of black hole horizon dynamics in terms of "hydrodynamics" (membrane paradigm)

[Bhattacharyya et.al. (2015,2016), Emparan-Izumi-Luna-Suzuki-KT (2016)]

- Equations become much simpler at $D = \infty$

Solvable analytically (QNMs, (dynamical) black hole solutions,…)

[Emparan-Suzuki-KT (2013,2015), KT (2016), and so on]

Due to the appearance of the conformal symmetry ?

[Emparan-Grumiller-KT (2013)]

LARGE D BH

☐ One simple example: dynamical black branes

[Emparan-Suzuki-KT (2015)]

- Dynamical black brane solution of the Einstein equation

$$ds^2 = -\left(1 - \frac{\boldsymbol{\rho(t,\sigma^a)}}{\boldsymbol{r^D}}\right)dt^2 + 2dtdr + \frac{d\sigma^2}{D} + \frac{\boldsymbol{2p_a(t,\sigma^a)}}{\boldsymbol{r^D}}\frac{dtd\sigma^a}{D\sqrt{D}} + r^2 d\Omega_{D-3}^2$$

- Effective equations by hydrodynamics from Einstein equations

$$\boldsymbol{\partial_t\rho + \partial_a(\rho v^a) = 0} \qquad \boldsymbol{\partial_t(\rho v^a) + \partial_b\left(\rho v^a v^b + \tau^{ab}\right) = 0}$$

[continuity eq for mass] **[momentum-stress eq]**

$$p_a = \rho v_a + \partial_a m \qquad \tau_{ab} = -\rho\delta_{ab} - 2\rho\partial_{(a}v_{b)} - \rho\partial_a\partial_b \log\rho$$

These equations can be solved easily (final state of GL instabilities)

EQUAL ROTATIONS

☐ How about charged rotating black holes?

- Consider equally rotating solutions at large D

Equally rotating Myers-Perry black hole is cohomogeneity-1 solution

$$ds^2 = \frac{2r}{\sqrt{r^2 - a^2 + a^2 r^2 \hat{\Lambda}}}(dt - a\boldsymbol{\Psi})\,dr - \left(1 - r^2\hat{\Lambda} - \left(\frac{r_0}{r}\right)^n\right)dt^2$$
$$- a\left(\frac{r_0}{r}\right)^n \boldsymbol{\Psi} dt + \left(r^2 + a^2\left(\frac{r_0}{r}\right)^n\right)\boldsymbol{\Psi}^2 + r^2 d\Sigma_N^2$$

$D = 2N + 3 = n + 3$ $\quad \boldsymbol{\Psi} = d\psi + \boldsymbol{A}_{(N)}$ $\quad d\Sigma_N$: Fubini-Study metric on CP^N

- The solution is parametrized by the mass $\sim r_0^{2N}$ and rotation a
- The solution has N angular momenta with same value
- We consider dynamical deformations of this solution with a (not small) charge by 1/D expansion in Einstein-Maxwell theory

LARGE D METRIC

- Large D solution of Einstein-Maxwell eq

$$
\begin{aligned}
ds^2 = \frac{2r}{\sqrt{r^2 - a^2 + a^2 r^2 \hat{\Lambda}}} (dt - a\boldsymbol{\Psi})\, dr - \left(1 - r^2\hat{\Lambda} - \left(\frac{r_0}{r}\right)^n\right) dt^2 \\
- a \left(\frac{r_0}{r}\right)^n \boldsymbol{\Psi} dt + \left(r^2 + a^2 \left(\frac{r_0}{r}\right)^n\right) \boldsymbol{\Psi}^2 + r^2 d\Sigma_N^2 \\
-\frac{1}{D}\left(\frac{\boldsymbol{p_A}}{\boldsymbol{r^n}} d\boldsymbol{A} dt + \frac{\boldsymbol{p_\psi}}{\boldsymbol{r^n}} d\psi dt + \frac{\boldsymbol{p_\theta}}{\boldsymbol{r^n}} d\theta dt\right)
\end{aligned}
$$

with $\left(\frac{r_0}{r}\right)^n = \frac{\boldsymbol{\rho}}{\boldsymbol{r^n}} + \frac{\boldsymbol{q}}{\boldsymbol{2r^{2n}}}$

$$
A = \frac{\boldsymbol{q}}{\boldsymbol{r^n}} dt + a \frac{\boldsymbol{q}}{\boldsymbol{r^n}} d\boldsymbol{\Psi} + \mathrm{O}(1/\mathrm{D})
$$

- Solution has five dynamical fields : mass ρ, momentum p_A, p_Ψ, p_θ and charge q

EFFECTIVE EQUATIONS

◻ Effective equations for dynamical fields

- Five equations for five dynamical fields

  - Nonlinear dynamical equations

  - These equations can be cast into "hydrodynamic" form

  - Stationary solutions can be found analytically

    not only charged rotating BHs, but also **bumpy (deformed) charged rotating BHs** are obtained

$$\cosh\alpha\,\hat{\partial}_t q - \sinh\alpha\,\partial_\psi q - V^{-1}\cot\theta\,\partial_\theta q + \frac{p_2 q}{\rho}\cot\theta = 0,$$

$$\cosh\alpha\,\hat{\partial}_t \rho - \sinh\alpha\,\partial_\psi \rho - V^{-1}\cot\theta\,\partial_\theta \rho + p_2\cot\theta = 0,$$

$$\begin{aligned}
&\cosh\alpha\,\hat{\partial}_t p_1 - \sinh\alpha\,\partial_\psi p_1 - V^{-1}\cot\theta\,\partial_\theta p_1 - V^{-2}\sinh^3\alpha\,\hat{\partial}_t(\rho_+ - \rho_-) \\
&\quad + V^{-2}\cosh\alpha\sinh^2\alpha\,\partial_\psi(\rho_+ - \rho_-) + 2\cosh^2\alpha\,\frac{\rho_+ p_3}{V\rho} + \frac{p_1 p_2}{\rho}\cot\theta \\
&\quad - 2\cosh\alpha\sinh\alpha\cot\theta\,\frac{\rho_+ p_2}{V^2\rho} + \cosh\alpha\,\partial_\psi\rho - \sinh\alpha\,\hat{\partial}_t\rho \\
&\quad + V^{-1}\rho_-\cot\theta\left[-\sinh\alpha\,\hat{\partial}_t\left(\frac{p_2}{\rho}\right) + \cosh\alpha\,\partial_\psi\left(\frac{p_2}{\rho}\right) + \partial_\theta\left(\frac{p_1}{\rho}\right)\right] = 0,
\end{aligned}$$

$$\begin{aligned}
&\cosh\alpha\,\hat{\partial}_t p_2 - \sinh\alpha\,\partial_\psi p_2 - V^{-1}\cot\theta\,\partial_\theta p_2 + 2V^{-1}\cot\theta\rho_-\partial_\theta\left(\frac{p_2}{\rho}\right) \\
&\quad + 2\cosh\alpha\sinh\alpha\tan\theta\,p_3 + \left(1 - V^{-2}\sinh^2\alpha\right)\partial_\theta\rho + 2V^{-1}\sinh\alpha\tan\theta\partial_\psi\rho \\
&\quad - p_2\left[V^{-1} - \frac{p_2}{\rho}\cot\theta - \frac{\rho_+ - \rho_-}{V\rho}\cot^2\theta\right] = 0,
\end{aligned}$$

$$\begin{aligned}
&\cosh\alpha\,\hat{\partial}_t p_3 - \sinh\alpha\,\partial_\psi p_3 - V^{-1}\cot\theta\,\partial_\theta p_3 - \operatorname{sech}\alpha\left(1 - V^{-2}\sinh^2\alpha\right)\partial_\psi\rho \\
&\quad + V^{-1}\cot\theta\,\rho_-\left[-\operatorname{sech}\alpha\partial_\psi\left(\frac{p_2}{\rho}\right) + \partial_\theta\left(\frac{p_3}{\rho}\right)\right] + p_2\cot\theta\left(\frac{p_3}{\rho} - 2\tanh\alpha\right) \\
&\quad + 2V^{-1}\tanh\alpha\cot\theta\partial_\theta\rho - \frac{2p_3\rho_+}{V\rho} = 0.
\end{aligned}$$

QNM

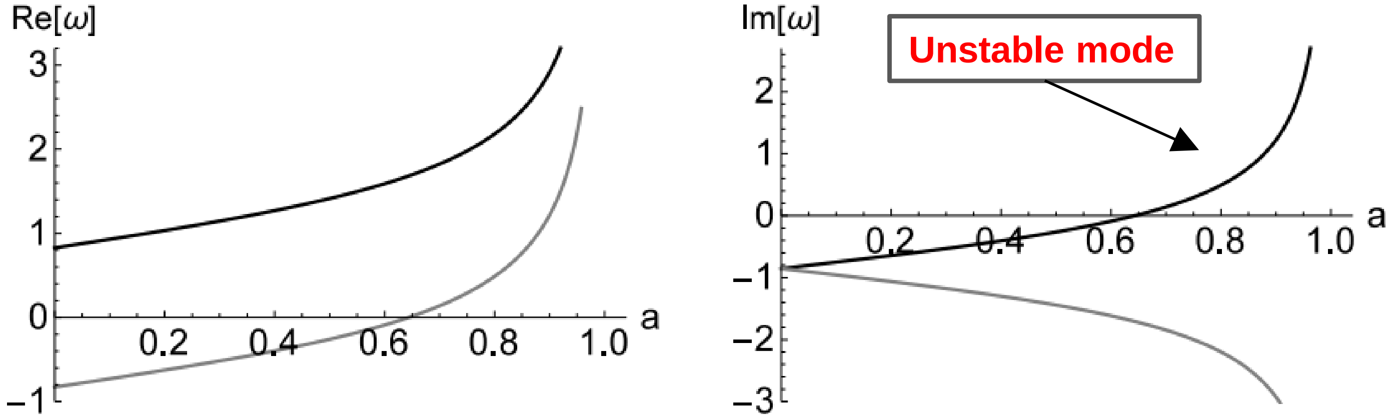
- Perturbation analysis of effective equations

$$\rho = 1 + \delta\rho\, \mathbf{Y}_\ell\, e^{-i\omega t} e^{im\psi} \qquad q = q + \delta q \mathbf{Y}_\ell\, e^{-i\omega t} e^{im\psi}$$

$$p_a = \delta p_a \mathbf{Y}_\ell\, e^{-i\omega t} e^{im\psi}$$

$\mathbf{Y}_\ell$: scalar harmonics on CP^N

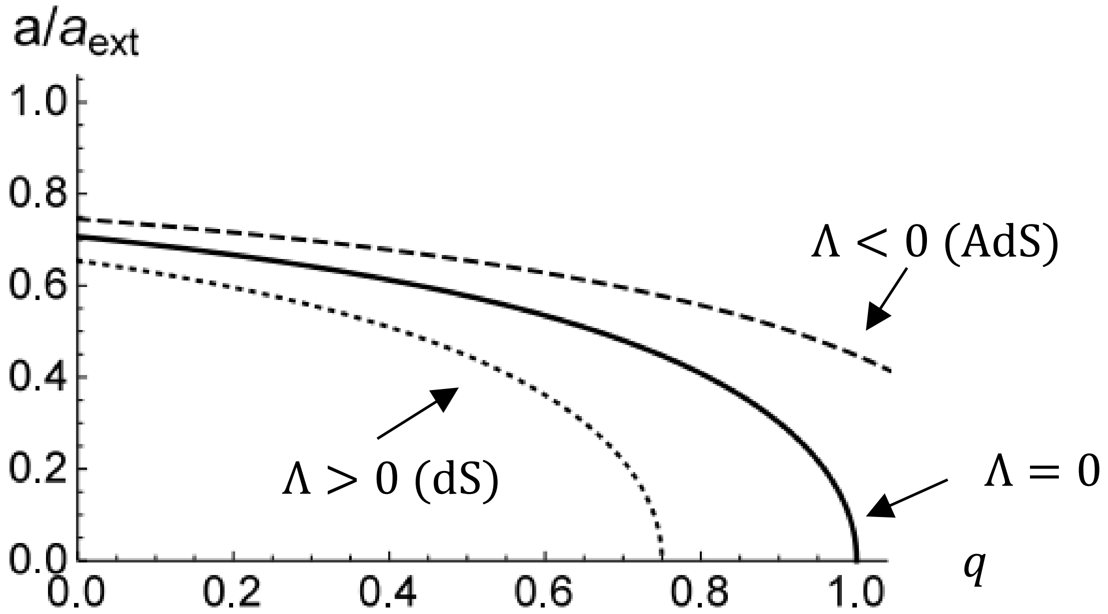
QNMs of $\ell = 2, m = 2$ $(q = 0.5, \Lambda = 0)$



- Threshold angular momentum of instability is $a \sim 0.6$

INSTABILITIES

- Threshold angular momentum with q dependence



- Static solution (a=0) can be unstable at larger charge in dS

  - This instability corresponds to one of dS Reissner-Nordstrom

  - Ultraspinning instability = instability of RN-BHs in charged rotating BHs

SUMMARY AND OUTLOOK

- We constructed **dynamical charged equally rotating black hole solutions** in Einstein-Maxwell theory by using **1/D expansion method**

  - Perturbation analysis gives quasinormal mode formula in analytic way

  - QNM formula reproduces ultraspinning instability and instability of RN-BHs at q=0 or a=0

  - QNM formula shows ultraspinning instability and instability of RN-BHs has same dynamical origin

- Extension to more general solutions

  - Singly rotating charged solution

    Does the solution has the Kerr-Schild form or not?

  - Inclusion of other terms such as dilaton or Chern-Simons term