Quantum Gravitational Force Between Polarizable Objects

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Outline

1. Motivation
2. The gravitational Casimir-Polder force
3. Applications to neutron stars
4. Summary

Based on :
L. P Ford, M. P. Hertzberg, J. Karouby
"Quantum Gravitational Force Between Polarizable Objects," Phys. Rev. Lett. 116, 15, 151301 (2016)

Motivation

General Relativity $\Longrightarrow$ Theory tested but classical.
February 11, 2016 : LIGO annonce detection of gravitational waves!

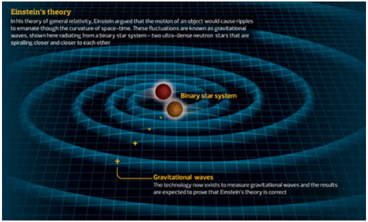


Figure: Binary system of stars producing gravitational waves Predicted by Einstein a century ago! *Illustration: Pete Guest*

- Casimir $\rightarrow$ vacuum energy $\rightarrow$ should exist also for gravity
- Thinking of shift of vacuum energy = easy way to compute Quantum gravity corrections
- Quantum correction $\Longrightarrow$ good test for string theory & loop quantum gravity : should recover our result at low energy

GOAL : Computing quantum correction to newtonian potential !

Van der Waals/ Casimir Polder force

H. B. G. Casimir and D. Polder, "The Influence of retardation on the London-van der Waals forces," Phys. Rev. **73**, 360 (1948).

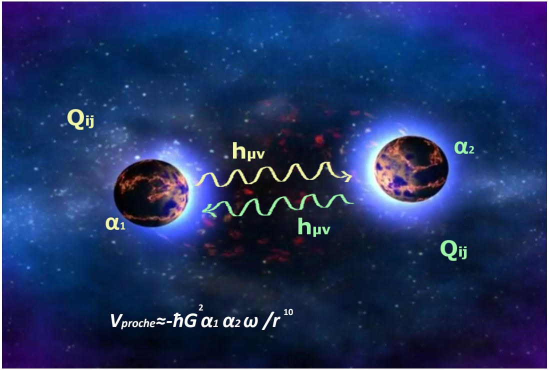


Figure: Neutron stars : Casimir-Polder gravitational force

Computed in Phys. Rev. Lett. 116, 15, 151301 (2016) L. P Ford, M. P. Hertzberg, J. Karouby

Potential

Potential between atoms $=$ shift of vacuum energy between d and ∞
Works for radiating modes : dipole in E&M, quadrupole in Gravity
Using the argument principle the potential is

$$V(r) = \frac{1}{2\pi i}\oint dz \frac{\hbar z}{2}\frac{d}{dz} log[f(z,r)] = \sum_i \hbar \frac{\omega_i|_r}{2} - \sum_j \hbar \frac{\omega_j|_{r\to\infty}}{2} \quad (1)$$

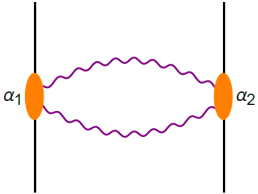
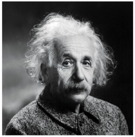


Figure: Loop diagram of gravitons (purple wiggly lines) between polarizable sources (orange disks).

From Electromagnetism to General relativity



Gravitational wave $h_{\mu\nu}$
Massless spherical objects
Gravitational quadrupole Q_{ij}
Quadrupole polarizability $\alpha(\omega)$
Gravito-Electric tidal tensor E_{ij}
Induced $Q_{ij} = \alpha(\omega) E_{ij}$

Electromagnetic wave A_μ
Neutral polarizable atom
Electric dipole $\vec{p}$
Dipole polarizability $\alpha(\omega)$
Electric field $\vec{E}$
Induced $\vec{p} = \alpha(\omega)\vec{E}$

Tidal tensor :$E_{ij} = TracelessR_{0i0j} = C_{0i0j}$

Argument Principle

$$V(r) = \frac{1}{2\pi i} \oint dz \frac{\hbar z}{2} \frac{d}{dz} log[f(z,r)] = \sum_i \frac{1}{2}\hbar\omega_i|_r - \sum_j \frac{1}{2}\hbar\omega_j|_{r\to\infty}$$

Linearized gravity (non static case)

Metric for linearized gravity

$$ds^2 = -(1+2\Phi)dt^2 + 2w_i dx dt + ((1-2\Psi)\delta_{ij} + 2s_{ij})dx^i dx^j$$

Trace-reversed perturbations in Lorenz gauge

$$\bar{h}_{\mu\nu}(t,\vec{x}) = 4G\int \frac{T_{\mu\nu}(t-|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|} d^3y$$

•Spherically symmetric mass distribution with time dependent quadrupole
•Assume localized source of radius R, $R \ll r$ and $r \equiv |\vec{x}|$

Taylor expand the retarded $T\mu\nu$ to 2nd order in $|\vec{y}|/r \ll 1$

$$\frac{T_{\mu\nu}(t-|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|} = T_{\mu\nu}(t-r,\vec{y})\left(\frac{1}{r} + \frac{\vec{x}.\vec{y}}{r^3} + \frac{x^i x^j}{2r^5}[3y_i y_j - \delta_{ij}]\right)$$
$$+\dot{T}_{\mu\nu}(t-r,\vec{y})\left(\frac{\vec{x}.\vec{y}}{r^2} + \frac{x^i x^j}{2r^4}[3y_i y_j - \delta_{ij}]\right) + \ddot{T}_{\mu\nu}(t-r,\vec{y})\frac{x^i x^j}{2r^3} y_i y_j \equiv t_{\mu\nu}(t-r,\vec{x},\vec{y})$$

Exact result for quantum correction to potential

Riemann tensor in vacuum (traceless) for $\omega > 0$

$$R_{0i0j}(\omega, \vec{x}) = \frac{G e^{ir\omega}}{r^5} \Lambda_{ij}^{kl}(\omega\, r, \vec{n})\, Q_{kl}(\omega), \qquad (2)$$

$\Longrightarrow$ Gives Eigenmatrix equation: $M_{ij} \equiv K_{ij}^{kl}(\omega, r, \vec{n}_1, \vec{n}_2)\, Q_{kl}(\omega) = 0$

Quadrupole quantum relativistic correction to the Newtonian potential

$$V(r) = -\frac{\hbar\, G^2}{\pi\, r^{10}} \int_0^\infty d\omega\, S(\omega\, r/c)\, \alpha_1(i\,\omega)\, \alpha_2(i\,\omega), \quad \gamma \equiv \omega\, r/c \qquad (3)$$

$$S(\gamma) \equiv e^{-2\gamma}\left[315 + 630\gamma + 585\gamma^2 + 330\gamma^3 + 129\gamma^4 + 42\gamma^5 + 14\gamma^6 + 4\gamma^7 + \gamma^8\right]$$

Far distance regime $\gamma \gg 1$ (relativistic)

$$V_{\text{far}}(r) = -\frac{3987\, \hbar\, c\, G^2}{4\pi\, r^{11}} \alpha_1(0)\, \alpha_2(0),$$

Near distance regime $\gamma \to 0$ (non relativistic)

$$V_{\text{near}}(r) = -\frac{315\, \hbar\, G^2}{\pi\, r^{10}} \int_0^\infty d\omega\, \alpha_1(i\,\omega)\, \alpha_2(i\,\omega),$$

Application : simple Harmonic model

Polarizability for simple harmonic model: $\alpha_i(\omega) = \frac{\alpha_{iS}}{1-(\omega/\omega_i)^2}$

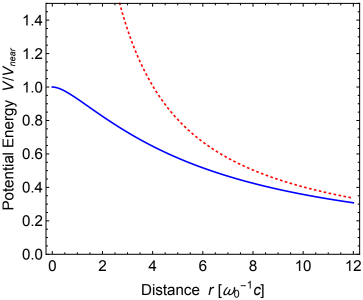
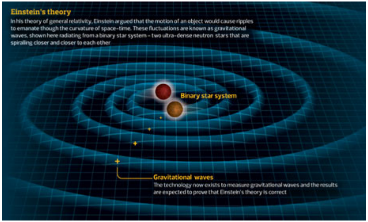


Figure: Potential energy V divided by near result V_{near} as a function of distance r (in units $\omega_0^{-1} c$, with $\omega_1 = \omega_2 = \omega_0$). Blue= exact result $V(r)/V_{\text{near}}(r)$. Red = far result $V_{\text{far}}(r)/V_{\text{near}}(r)$.

Integral over ω in the near regime $\Longrightarrow V_{\text{near}}(r) = -\frac{315\,\hbar\, G^2}{2\, r^{10}} \frac{\omega_1\, \omega_2}{\omega_1+\omega_2}\, \alpha_{1S}\, \alpha_{2S}.$

Application: Neutron stars



"Love number" $k \equiv -3\, G\, \alpha(0)/(2R^5)$

Depends compactness $C \equiv GM/(R\, c^2)$ and equation of state.

For neutron stars and self-bound quark stars, $k = \mathcal{O}(10^{-1})$.

Far regime

$V_{\text{far}}(r) = -\beta\, \hbar\, c\, R_1^5\, R_2^5/r^{11}$, with $\beta \equiv 443\, k_1\, k_2/\pi = \mathcal{O}(1)$.

Consistency requires $R_{1,2} \ll r$, so this gives $|V_{\text{far}}(r)| \ll \hbar\, c/r$: extremely small energy compared to the classical Newtonian potential energy $V_N(r) = -G\, M_1 M_2/r$

Summary and future work

1. Exciting time to study gravitational waves and their quantum aspects!
2. Rare, precise, and definite prediction of quantum gravity, independent of the details of its UV completion
3. Much simpler to compute than the point-particle result
4. Our correction can in principle be larger than the monopole-monopole result for $R \gg R_{Schwartzchild}$ (proved for a simple harmonic model)
5. This new force depends on the intrinsic material properties of the objects through their gravitational quadrupole polarizabilities


QUESTIONS?