The Anomaly-Free Quantum Einstein Constraints and the Minkowski Theorem

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Canonical LQG

Kinematical quantum states:
spin-network states

$$|\gamma, j, \iota >$$

$\mathrm{SU}(2)$

Σ - underlying 3-manifold

γ - graph in Σ

j - labeling of edges by irreducible representations

ι - labeling of vertices by intertwines

Hilbert spaces:
kinematical, solutions to Diff, solutions to all the constraints, solutions to partial Diff, …

$$\mathcal{H}_{\mathrm{kin}}, \mathcal{H}_{\mathrm{Diff}}, \mathcal{H}_{\mathrm{vtx}}, \ldots$$

Quantum constraint operators of Ashtekar's GR:

$\hat{H}(N)$ - scalar constraint

$\hat{D}(\vec{N})$ - vector constraint

$\hat{G}(\Lambda)$ - Gauss constraint

The problem

Classically:

$$\{H(N), H(M)\} = D(\vec{L})$$

$$L_i^a = E_i^a E_i^b (M N_{,b} - N M_{,b})$$

In LQG:

$$[\hat{H}(N), \hat{H}(M)] = 0$$

Excuse: $$\hat{E}\hat{E}\hat{D}_a\Psi \;=\; 0$$

Criticism: $$[\hat{H}_{\mathrm{gr}}(N), \hat{H}_{\mathrm{gr}}(M)]\Psi(\mathrm{gr}, \mathrm{matter}) \;=\; 0$$

Excuse: $$\hat{E}\hat{E}\hat{D}_{\mathrm{gr}}\Psi(\mathrm{gr}, \mathrm{KG}) \;=\; 0$$

JL, Sahlmann - 2015

Laddha-Tomlin-Varadarajan model

Starting point

The Smolin limit of Ashtekar's Euclidean gravity:

$$G_{\mathrm{Newton}} \to 0$$

$$\mathrm{SU}(3) \mapsto \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{U}(1)$$

The same Poisson algebra of the constraints

$$H(N), D(\vec{N}), G(\Lambda)$$

Laddha-Tomlin-Varadarajan model

The second step:

Suitable reformulation of the constraints

$$H(N), D(\vec{N}) \;\mapsto\; C(N'), D(\vec{N}_i)$$

$$i = 1, 2, 3$$

They satisfy:

$$\{C(N), C(M)\} \;=\; -\{D(\vec{N}_i, D(\vec{N}_i\}$$

Laddha-Tomlin-Varadarajan model

The third step

Quantum states: “charge networks”

$$|\gamma, q^1, q^2, q^3 >$$

Σ - 3-manifold

γ - a graph in Σ

q^1, q^2, q^3 - labelling of the edges by 3 integers

at each vertex: $\sum_e \pm q^i_e = 0$

$$[\hat{C}(N), \hat{C}(M)] = -[\hat{D}(\vec{N}_i, \hat{D}(\vec{N}_i]$$

Our results

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The action: $\hat{C}(N)\Psi, \;\; \hat{D}(N)\Psi$

Solutions to:

$$\hat{C}(N)\Psi \;=\; 0$$

$$\hat{D}(N_i)\Psi \;=\; 0$$

The result: at each vertex

$$\sum_e q^i_e \dot{e} \;=\; 0$$