Spherically symmetric quantum spacetimes coupled to a thin null-dust shell

Javier Olmedo

Department of physics and astronomy, Louisiana State University

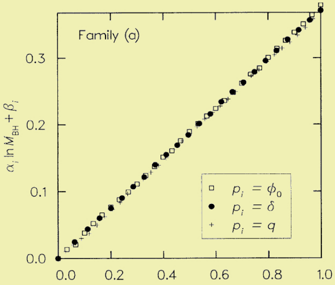
GR21. New York. July, 13th 2016

In collaboration with M. Campiglia, R. Gambini and J. Pullin

Introduction

1) Massless scalar field (Choptuik).

2) Thin null-dust shell (Louko, Whiting and Friedman).
   Quantization: (Hájícek, Kiefer)
   a) Embedding geometrical variables.
   b) Partial quantization in the neighborhood of the shell.
   c) Selfadjointness of the true Hamiltonian prevents eternal black formation: bouncing shells.



Classical system: Ashtekar variables

1) Phase space $\left(K_x(x), E^x(x)\right)$, $\left(K_\varphi(x), E^\varphi(x)\right)$ and (r, p).

2) Spatial metric: $dh^2 = \frac{(E^\varphi)^2}{|E^x|} dx^2 + |E^x| d\Omega^2$

3) The Hamiltonian is a linear combination of constraints

$$H(N) := \int dx\, N \Bigg[\frac{((E^x)')^2}{8\sqrt{|E^x|}E^\varphi} - \frac{E^\varphi}{2\sqrt{|E^x|}} - 2K_\varphi \sqrt{|E^x|} K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{|E^x|}} - \frac{\sqrt{|E^x|}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{|E^x|}(E^x)''}{2E^\varphi} + \frac{\sqrt{|E^x|}}{E^\varphi} \eta p \delta(x-r) \Bigg], \tag{1}$$

$$H_x(N^x) := \int dx\, N^x \left[E^\varphi K_\varphi' - (E^x)' K_x - p\, \delta\,(x-r) \right]. \tag{2}$$

fulfilling the algebra

$$\{H_x(N^x), H_x(\tilde{N}^x)\} = H_x\left(N^x(\tilde{N}^x)' - (N^x)'\tilde{N}^x\right), \ \{H(N), H_r(N^x)\} = H(N^x N'),$$
$$\{H(N), H(\tilde{N})\} = H_x\left(\frac{E^x}{(E^\varphi)^2}\left[N\tilde{N}' - N'\tilde{N}\right]\right). \tag{3}$$

Classical system: new constraint algebra

4) We Abelianize the scalar constraint (as in vacuum)

$$\tilde{H} = \frac{(E^x)'}{E^\varphi} H - 2K_\varphi \frac{\sqrt{|E^x|}}{E^\varphi} H_x, \quad N_{\text{new}} = \frac{E^\varphi}{(E^x)'} N, \quad N^x_{\text{new}} = N^x + 2K_\varphi \frac{\sqrt{|E^x|}}{(E^x)'} N. \quad (4)$$

The total Hamiltonian with boundary terms now reads

$$\begin{aligned} H_T = \int dx \Bigg[&- N'_{\text{new}} \left(-\sqrt{|E^x|} \left(1 + K_\varphi^2\right) + \frac{\left((E^x)'\right)^2 \sqrt{|E^x|}}{4\,(E^\varphi)^2} + F(r) p\, \Theta(x-r) \right) \\ &+ N^x_{\text{new}} \left[-(E^x)' K_x + E^\varphi K'_\varphi - p\,\delta\,(x-r) \right] \Bigg] + N_+ \Big(F(r) p + 2M \Big) + N_- 2M, \end{aligned} \quad (5)$$

where

$$F(r) = \sqrt{E^x} \left(\eta\, (E^x)'\, (E^\varphi)^{-2} + 2K_\varphi\, (E^\varphi)^{-1} \right) |_{x=r}. \quad (6)$$

The constraint algebra is $\{\tilde{H}(N_{\text{new}}), \tilde{H}(\tilde{N}_{\text{new}})\} = 0$ and the usual one with the diffeomorphism constraint.

Classical Dirac observables

2) We can identify two classical observables (in absence of a pre-existing black hole): the mass (total ADM mass)

$$m := F(r)p/2, \tag{7}$$

and its conjugate variable

$$V := \int_r^\infty dy \left(\frac{2}{F(y)} - [\eta\,(1 + 2m/y)] \right) + t - \eta\,[r + 2m \ln(r/(2m))]\,. \tag{8}$$

such that $\{m, V\} = 1$. It represents the Eddington–Finkelstein time of an ingoing/outgoing shell.

Kinematical Hilbert space

1) Sectors:

a) Spin networks

$$\langle K_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(i \frac{k_j}{2} \int_{e_j} dx\, K_x(x) \right) \prod_{v_j \in g} \exp\left(i \frac{\mu_j}{2} K_\varphi(x_j) \right), \tag{9}$$

$k_j \in \mathbb{Z}$ is the valence associated with the edge e_j, and $\mu_j \in \mathbb{R}$ the valence associated with the vertex x_j.

b) Matter $\psi(r) := \langle r | \psi \rangle$.

2) Kinematical Hilbert space:

$$\mathcal{H}^g_{\text{kin}} = \left[\bigotimes_j^n \ell^2_j \otimes L^2_j(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}}) \right] \otimes L^2(\mathbb{R}, dr). \tag{10}$$

The inner product is

$$\langle g, \vec{k}, \vec{\mu}, r | g', \vec{k}', \vec{\mu}', r' \rangle = \delta(r - r') \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'} \delta_{g, g'} \ . \tag{11}$$

Kinematical operators

3) Operator representation: position of the shell and triads

$$\hat{r}|g,\vec{k},\vec{\mu},r\rangle = r|g,\vec{k},\vec{\mu},r\rangle, \quad \hat{p} = -i\partial_r,$$
$$\hat{E}^x(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},r\rangle,$$
$$\hat{E}^\varphi(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 \sum_{v_j\in g}\delta\big(x-x_j\big)\mu_j|g,\vec{k},\vec{\mu},r\rangle, \tag{12}$$

4) Holonomies (of K_φ) of length $\rho(x)$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j\rangle = |\mu_j \pm \rho_j\rangle, \quad x = x_j.$$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j,\mu_{j+1}\rangle = |\mu_j,\pm\rho_j,\mu_{j+1}\rangle, \quad x_j < x < x_{j+1}.$$

Representation of the scalar constraint

The scalar constraint will defined on the lattice

$$\hat{H}(x_j) := \mathbf{H}^{g}_{j} + \frac{1}{2}\sum_i \mathbf{F}_i(\boldsymbol{\theta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\theta}_j), \tag{13}$$

such that the spacing of the vertices is $\epsilon_j = x_{j+1} - x_j$,

$$\mathbf{H}^{g}_{j} = \hat{b}_j\left(-1 - \widehat{K^2_\varphi}(x_j) + \hat{a}^2_j[\widehat{1/E^\varphi}]^2(x_j)\right), \quad \mathbf{F}_j = 2\,\epsilon_j^{-1}\hat{b}_j\left(\hat{a}_j[\widehat{1/E^\varphi}]^2(x_j) + [\widehat{K_\varphi/E^\varphi}](x_j)\right). \tag{14}$$

The quantum algebra close if a) $[\mathbf{H}^{g}_{i}, \mathbf{F}_j] = i\hbar\mathbf{F}^2_i\delta_{i,j}$, which involves

$$[\widehat{K^2_\varphi}(x_i), [\widehat{1/E^\varphi}]^2(x_j)] = -2i\hbar\delta_{ij}\left([\widehat{1/E^\varphi}]^2(x_i)[\widehat{K_\varphi/E^\varphi}](x_i) + [\widehat{K_\varphi/E^\varphi}](x_i)[\widehat{1/E^\varphi}]^2(x_i)\right),$$

$$[\widehat{K^2_\varphi}(x_i), [\widehat{K_\varphi/E^\varphi}](x_j)] = -2i\hbar\delta_{ij}\left([\widehat{K_\varphi/E^\varphi}](x_i)\right)^2,$$

$$[[\widehat{1/E^\varphi}]^2(x_i), [\widehat{K_\varphi/E^\varphi}](x_j)] = -2i\hbar\delta_{ij}\left([\widehat{1/E^\varphi}]^2(x_i)\right)^2, \tag{15}$$

b) $[\boldsymbol{\theta}_i, \mathbf{X}_j] = -i\delta_{ij}\boldsymbol{\delta}_j$, $[\boldsymbol{\theta}_i, \boldsymbol{\theta}_j] = 0 = [\boldsymbol{\theta}_i, \boldsymbol{\delta}_j]$, $\boldsymbol{\delta}_i\boldsymbol{\delta}_j = \delta_{ij}\boldsymbol{\delta}_i$, $(\boldsymbol{\delta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\delta}_j) = 2\delta_{ij}\mathbf{X}_i$.

Representation of the scalar constraint

They can all be written in terms of the elementary operators,

$$\widehat{K_\varphi^2}(x_j) = \frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\hat{E}^\varphi(x_j)\frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\hat{E}^\varphi(x_j)^{-1},$$

$$[\widehat{K_\varphi/E^\varphi}](x_j) = \frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1},$$

$$[\widehat{1/E^\varphi}]^2(x_j) = \cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1}\cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1},$$

$$\hat{a}_j = \frac{\eta}{2\epsilon_j}\left(\hat{E}^x(x_j) - \hat{E}^x(x_{j-1})\right), \quad \hat{b}_n = \sqrt{|\hat{E}^x(x_j)|}. \tag{16}$$

for $\mu_j = 2\rho_j(l_j + \delta_j)$ and $\delta_j \neq 0, 1, 2$. $\boldsymbol{\theta}_j$ and $\mathbf{X}_j$ are operators on $\psi(r)$ defined as

$$\boldsymbol{\theta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\Theta(x_j + \epsilon - r)\ \psi(r),$$

$$\mathbf{X}_j := \frac{1}{2}(\boldsymbol{\delta}_j\hat{p} + \hat{p}\boldsymbol{\delta}_j), \quad \boldsymbol{\delta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\delta(x_j + \epsilon - r)\ \psi(r), \tag{17}$$

Quantum observables and physical inner product

1) In the case of diffeo invariant states, there is an observable (with no analogue classical Dirac obs.) of the form

$$\hat{O}(z)|\Psi_{\rm phys}\rangle = \ell_{\rm Pl}^2 k_{{\rm Int}(zv)}|\Psi_{\rm phys}\rangle, \quad z\in[-1,1], \quad n=2v+1, \tag{18}$$

that allows us to define the parametrized observables

$$\hat{E}^x(x)|\Psi_{\rm phys}\rangle = \ell_{\rm Pl}^2 k_{{\rm Int}\left(z(x)v\right)}|\Psi_{\rm phys}\rangle, \tag{19}$$

$$\left(\hat{E}^x(x)\right)'|\Psi_{\rm phys}\rangle = \ell_{\rm Pl}^2\left(k_{{\rm Int}\left(z(x)v\right)} - k_{{\rm Int}\left(z(x)v-1\right)}\right)|\Psi_{\rm phys}\rangle. \tag{20}$$

after introducing the parameter function $z(x) : x \to [-1,1]$.

2) There are also the mass of the shell $\hat{m} = \widehat{Fp}/2$ and its conjugated momentum $\hat{V}$. We lack a polymer selfadjoint representation, so we adopt a standard one for $[\hat{m},\hat{V}] = i\hbar$.

3) The physical inner product is then $\langle\vec{k},m|\vec{k}',m'\rangle = \delta_{\vec{k},\vec{k}'}\delta(m-m')$.

Classical singularity vs. quantum theory

1) The spacetime metric components can be defined as parametrized observables explicitly in terms of basic Dirac observables and functional parameters.

2) Let us choose a state with radial positions $x_i \in [-L, L]$ where $L = \Delta(v+1)$ such that $\Delta \geq \ell_{\rm Pl}^2/x_r$. We choose $x_i = (i+1)\Delta$ if $i \geq 0$. Then we have that $z(x_i) = x_i/L$ and $k_i = \mathrm{Int}(x_i^2/\ell_{\rm Pl}^2)$. Also that $(E^x(x_i))' \sim (2i+1)\,\Delta$. At $(E^x(x_0))' \sim (k_0 - k_{-1}) = 2\Delta$. If $i < 0$ then $x_i = i\Delta$ and $k_i = -\mathrm{Int}(x_i^2/\ell_{\rm Pl}^2)$.

Classical singularity vs. quantum theory

3) With these assumptions the result of the quantum construction is essentially a discretization of the above classical expressions of the metric on a lattice determined by a given spin network.

4) Away from the high quantum regime we would recover smooth geometries (even more if superpositions of m and $\vec{k}$ are considered). At the deep quantum regime the geometry would not be smooth but regular (the singularity can be avoided).

Conclusions

1) We have provided a quantum scalar constraint compatible with the Dirac quantization approach (formally).
2) We do not know yet the solutions to the constraints in closed form.
3) We are able to construct parametrized Dirac observables,among them the spacetime metric components.
4) We do not know yet a selfadjoint loop representation of some of the basic observables of the model.
5) But, assuming a standard one, we can complete the quantization and explain the way the singularity can be avoided. We can also construct semiclassical geometries with a fundamental discretization where quantum gravity effects emerge at the high curvature regime.