Gauge Inflation with Chern-Simons term

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Inflation

- Inflation: a theory of accelerating expansion of space in the early universe.
- Motivation: solve the horizon problem, flatness problem and so on.
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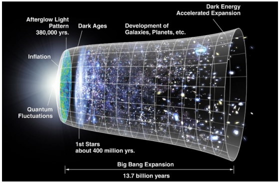
  - Effective models with scalar fields
    - Standard Slow-Roll Inflation
  - Models inspired by standard model of particle physics



Model

- Motivation:an inflation model inspired by standard model of particle physics.
- FLRW Metric:
  $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$
- The Lagrangian for the model:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(A_\mu - \frac{1}{m}\partial_\mu\theta)(A^\mu - \frac{1}{m}\partial^\mu\theta) - qA_\mu\mathcal{J}^\mu \\ &- \frac{\theta}{4M_*}F_{\alpha\beta}\tilde{F}^{\alpha\beta} + i\bar{\psi}\gamma^\mu\nabla_\mu\psi - M\bar{\psi}\psi,\end{aligned} \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\tilde{F}^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}$ and $\nabla_\mu A^\mu = -m\theta$.

Fermionic Dynamics

- The fermion propagator is

$$iG^{ab}(x,y) = a(\eta_x)(i\gamma^\mu\nabla_\mu + M)\frac{H^2}{\sqrt{a(\eta_x)a(\eta_y)}} \left[iG_+(x,\,y)\frac{1+\gamma^0}{2} + iG_-(x,\,y)\frac{1-\gamma^0}{2}\right] \tag{2}$$

see "J.F.Koksma and T.Prokopec, Class.Quant.Grav.26(2009)125003,arXiv:0901.4674"

- The current is defined as

$$\langle J^I\rangle \simeq \lim_{y\to x} G^{ab}(x,\,y)\gamma^I_{ba}\,. \tag{3}$$

- By dimensional regularization, we get

$$G^{ab}(x,y) \simeq \frac{H^2}{16\pi^2}\left[\frac{M^2}{H}\left(\frac{1+\gamma^0}{2}\right)^{ab} - \frac{M^2}{H}\left(\frac{1-\gamma^0}{2}\right)^{ab}\right], \tag{4}$$

- Currents:

$$\langle\mathcal{J}^0\rangle = \langle J^0\rangle \simeq \frac{M^2H}{4\pi^2}\,, \qquad \langle\mathcal{J}^i\rangle = 0\,. \tag{5}$$

Dynamic of the Background of Gauge Fields

- We treat $\frac{\theta}{4M_*}$ small, and impose the gauge $\nabla_\mu A^\mu = -m\theta$.

$$\ddot{A}^0 + 3\dot{H}A^0 + 3H\dot{A}^0 + m^2 A^0 = q\mathcal{J}^0 \tag{6}$$

with $\mathcal{J}^0 \simeq J = const$ and small constant $\epsilon = -\frac{\dot{H}}{H}$. We get the solution $A^0 = \frac{q\mathcal{J}^0}{m^2 - 3\epsilon H^2} = const$.

$$\ddot{A}^i + 3H\dot{A}^i + m^2 A^i = 0. \tag{7}$$

because of $\mathcal{J}^i = 0$. In the limit $m \ll H$, the solution is $A^i = c^i \exp^{-3Ht} = \frac{c}{a^3}$, where we assume $c^i = c$ is a constant.

- The constant $A_\mu \mathcal{J}^\mu$ drives inflation.
- Gauge $\nabla_\mu A^\mu = -m\theta$ imposes that θ is a constant.

Perturbations of Gauge Fields

- Gauge of the perturbations: $\nabla_\mu \delta A^\mu = 0$
- Temporal components

$$\delta \ddot{A}^0 - \frac{1}{a^2}\nabla^2 \delta A^0 + 3\dot{H}\delta A^0 + 3H\delta \dot{A}^0 + m^2 \delta A^0 = 0. \tag{8}$$

In the conformal coordinate, it is

$$\partial_\eta^2 \delta A^0 - \frac{2}{\eta}\partial_\eta \delta A^0 + k^2 \delta A^0 - \frac{3\epsilon}{\eta^2}\delta A^0 + \frac{m^2}{H^2\eta^2}\delta A^0 = 0. \tag{9}$$

The solution is

$$\delta A^0 = -ik^{-4}\sqrt{-\frac{\pi\eta^3}{2}}H^{(2)}_{\frac{1}{2}\sqrt{9-\frac{4m^2}{H^2}+12\epsilon}}(-k\eta) \tag{10}$$

In the super horizon limit $-k\eta \ll 1$ and the the limit $m \ll H$,

$$\delta A^0 \simeq \frac{1}{k^{\frac{11}{2}-\frac{m^2}{3H^2}+\epsilon}} \tag{11}$$

Perturbations of Gauge Fields

- Spatial components

$$\delta \ddot{A}^i - \frac{1}{a^2}\nabla^2 \delta A^i + 3H\delta \dot{A}^i + m^2 \delta A^i = 0. \tag{12}$$

In the conformal coordinate, it is

$$\partial_\eta^2 \delta A^i - \frac{2}{\eta}\partial_\eta \delta A^i + k^2 \delta A^i + \frac{m^2}{H^2\eta^2}\delta A^i = 0. \tag{13}$$

The solution is

$$\delta A^i = -ik^{-3}\sqrt{-\frac{\pi\eta^3}{2}}H^{(2)}_{\frac{1}{2}\sqrt{9-\frac{4m^2}{H^2}}}(-k\eta) \tag{14}$$

- In the superhorizon limit $-k\eta \ll 1$ and the limit $m \ll H$,

$$\delta A^i \simeq \frac{1}{k^{\frac{9}{2}-\frac{m^2}{3H^2}}}. \tag{15}$$

Curvature Perturbation in Spatially-flat gauge

- Curvature perturbation in spatially-flat gauge $\zeta = \frac{\delta\rho}{3(\rho+p)}$

$$\zeta \simeq -\frac{a^3}{6c^i}\delta A^i + \frac{(m^2 A_0 - q\mathcal{J}_0)a^4}{18H^2c^2}\delta A^0 \tag{16}$$

- Power spectrum of curvature perturbation at horizon crossing $aH = k$ is

$$\begin{aligned}\mathcal{P}_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta|^2 &= \frac{1}{8\pi^2 H^6 c^2} k^{\frac{2m^2}{3H^2}} \\ &+ \frac{(m^2 A_0 - q\mathcal{J}_0)^2}{648\pi^2 H^{12} c^4} k^{\frac{2m^2}{3H^2}-2\epsilon} - \frac{(m^2 A_0 - q\mathcal{J}_0)}{36\pi^2 H^9 c^3} k^{\frac{2m^2}{3H^2}-\epsilon}\end{aligned} \tag{17}$$

Curvature Perturbation in Spatially-flat gauge

- The spectral index is

$$n_\zeta - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} = \frac{2(9cm^2H^3k^\epsilon - (m^2A_0 - q\mathcal{J}_0)(m^2 - 3H^2\epsilon))}{3H^2(9cH^3k^\epsilon - m^2A_0 + q\mathcal{J}_0)} \tag{18}$$

- Running of the tilt

$$\alpha \equiv \frac{dn_\zeta}{d \ln k} = -\frac{18cH^3(m^2A_0 - q\mathcal{J}_0)k^\epsilon\epsilon^2}{(9cH^3k^\epsilon - m^2A_0 + q\mathcal{J}_0)^2} \tag{19}$$

which is positive with positive c

Tensor Perturbations

- Metric
$$ds^2 = a^2(\eta)\left(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j\right), \tag{20}$$
in which h_{ij} is a traceless and transverse vector $\partial_i h^i_j = h^i_i = 0$.
- Equation for h_{ij}
$$\delta^{ik}\left(h''_{jk} + 2\frac{a'}{a}h'_{jk} - \Delta h_{jk}\right) = -16\pi G a^2 \pi^i_j, \tag{21}$$
where π^i_j is the traceless and transverse part of the perturbation of energy momentum tensor δT^i_j.
- $\pi^i_j = 0$ in our model.
- Power spectrum of tensor perturbation in superhorizon limit
$$\mathcal{P}_h(k) \equiv (k^3/2\pi^2)|h|^2 \simeq (k^3/2\pi^2)\frac{2H^2}{k^3} = \frac{H^2}{\pi^2} \tag{22}$$
It's the same as standard slow-roll inflation.

Conclusion and Work in Progress

- Our inflation model is inspired by the framework of standard model of particle physics.
- The interaction part $A_\mu \mathcal{J}^\mu$ drives inflation.
- Nearly scale invariant power spectrum of curvature perturbation.
- We can get the positive running of tilt.
- Tensor perturbation $\mathcal{P}_h(k) = \frac{H^2}{\pi^2}$.
- In spite of having Chern-Simons term, we do not get chirality violating gravitational waves and this is an interesting finding.
- We are working in progress with Maresuke Shiraishi and Michele Liguori about the phenomenological analysis of the model (constraining parameters of the theory).