Modeling the Remnant Black Hole from the Merger of Precessing Black-Hole Binaries

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Modeling Remnant Mass, Spin, and Recoil from Precessing Binaries

- We'd like formulas that depend on the initial spins (magnitude and orientation).
- This is challenging because spins precess.
- Variables like $\vec{S}_{\pm} = \vec{S}_1 \pm \vec{S}_2$ are not conserved.
- We therefore chose to attack the problem in 2 stages:
  - Model the remnant as a function of the spins *near* merger (Well underway).
  - Model the spins near merger as a function of the spins at large separation (Work in progress, PN, full NR, approximate symmetries, etc.).
- In this talk, I'll concentrate on the first stage.

Overview

- First develop an interpolative formula for use in astrophysical simulations $1/10 \lesssim q \leq 1$.
- Start with the known equal-mass terms and incorporate the *particle limit* recoil $\mathcal{O}(q^2)$
- Add terms in a power series in the variables $\delta m, S_\perp, S_\parallel, \Delta_\perp, \Delta_\parallel$ that obey symmetries(Boyle, Kesden, Nissanke & Boyle, Kesden).
- Only low-order expansions are realistic due to the number of terms

Quantity	P	X
$S_\perp/m^2 = (S_1 + S_2)_\perp/m^2$	−	−
$S_\parallel/m^2 = (S_1 + S_2)_\parallel/m^2$	+	+
$\Delta_\perp/m^2 = (S_2/m_2 - S_1/m_1)_\perp/m$	−	+
$\Delta_\parallel/m^2 = (S_2/m_2 - S_1/m_1)_\parallel/m$	+	−
$\hat{n} = \hat{r}_1 - \hat{r}_2$	+	−
$\delta m = (m_1 - m_2)/m$	+	−
$V_\perp$	+	−
$V_\parallel$	−	+
$J_\perp/m^2$	−	−
$J_\parallel/m^2$	+	+
M_{rem}/m	+	+

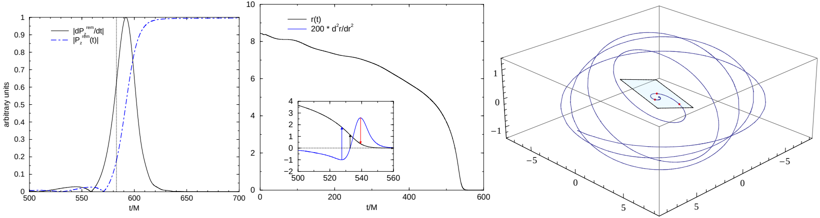
Order	Terms in $V_\parallel$
0th	0
1st	$\Delta_\perp$
2nd	$\Delta_\perp.S_\parallel + \Delta_\parallel.S_\perp + \delta m(S_\perp)$
3rd	$\Delta_\parallel.S_\perp.S_\parallel + \Delta_\perp.S_\parallel^2 + \Delta_\perp.\Delta_\parallel^2 + \Delta_\perp^3 + \Delta_\perp.S_\perp^2$
	$+\delta m\left(\Delta_\perp.\Delta_\parallel + S_\perp.S_\parallel\right) + \delta m^2(\Delta_\perp)$
4th	$S_\perp.\Delta_\parallel^3 + \Delta_\perp.S_\parallel^3 + \Delta_\perp.S_\parallel.\Delta_\parallel^2 + S_\perp.\Delta_\parallel.S_\parallel^2$
	$+\Delta_\perp^3.S_\parallel + S_\perp^3.\Delta_\parallel + \Delta_\perp^2.S_\perp.\Delta_\parallel + \Delta_\perp.S_\perp^2.S_\parallel$
	$+\delta m(S_\perp.\Delta_\parallel^2 + S_\perp.S_\parallel^2 + \Delta_\perp.\Delta_\parallel.S_\parallel + S_\perp.\Delta_\perp^2 + S_\perp^3)$
	$+\delta m^2(\Delta_\perp.S_\parallel + \Delta_\parallel.S_\perp) + \delta m^3(S_\perp)$

The Setup

- Choose configurations that activate specific terms in the expansions, e.g.
  - Symmetric configurations: z-symmetry, π-rotation symmetry, parity symmetry, etc.
  - equal-mass
  - Only 1 BH spinning
- Find these coefficients accurately and then fit additional coefficients with less-symmetric configurations
- Incorporate particle limit results through higher-order terms in the expansion (e.g., if fitting to fourth-order, add particle limit by hand by adding fifth- and higher-order terms that reproduce the PL result.

The Plane

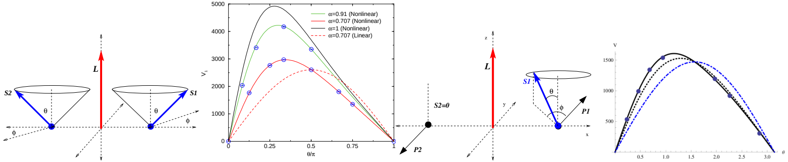
- The remnant's properties depend on the direction and magnitude of the spins in the plane and the components normal to the plane.
- So we need to define *the plane*.
- We define an *average* plane by finding three special points during the final plunge.
- Angles are measured with respect to the ray joining the origin to the first point.
- Polar angle θ defined by the inclination of the spin with respect to the plane.
- The recoil depends strongly on the azimuthal orientations of the spins, the remnant mass and spin only depend weakly on this.
- $V(\theta) \propto \cos\phi + \cos 3\phi + \cdots$ for every θ, we need 6 ϕ configurations to isolate the $\cos\phi$ and $\cos 3\phi$ dependence. α^2 and $\delta M \propto 1 + \cos 2\phi + \cos 4\phi \cdots$.



Some complications

- The orientation of the orbital *plane* can only be approximate due to very fast precession in the plunge.
- We measure spins where the *isolated horizon* formalism is beginning to break down.
- $\Delta_\perp$ and $S_\perp$ in the expansion are actually $\vec{\Delta} \cdot \hat{n}$ and $\vec{S} \cdot \hat{m}$, where $\hat{n}$ and $\hat{m}$ can be different for each $\perp$ term.
- We need to make some simplifying assumptions, e.g.:
  - All $\Delta_\perp$ terms are aligned (i.e., $\Delta_\perp = \vec{\Delta} \cdot \hat{n}$ for a fixed $\hat{n}$).
  - All $S_\perp$ terms are aligned (i.e., $S_\perp = \vec{S} \cdot \hat{m}$ for a fixed $\hat{m}$).
  - There is a fixed relationship between $\hat{n}$ and $\hat{m}$ independent of the parameters of the binary.

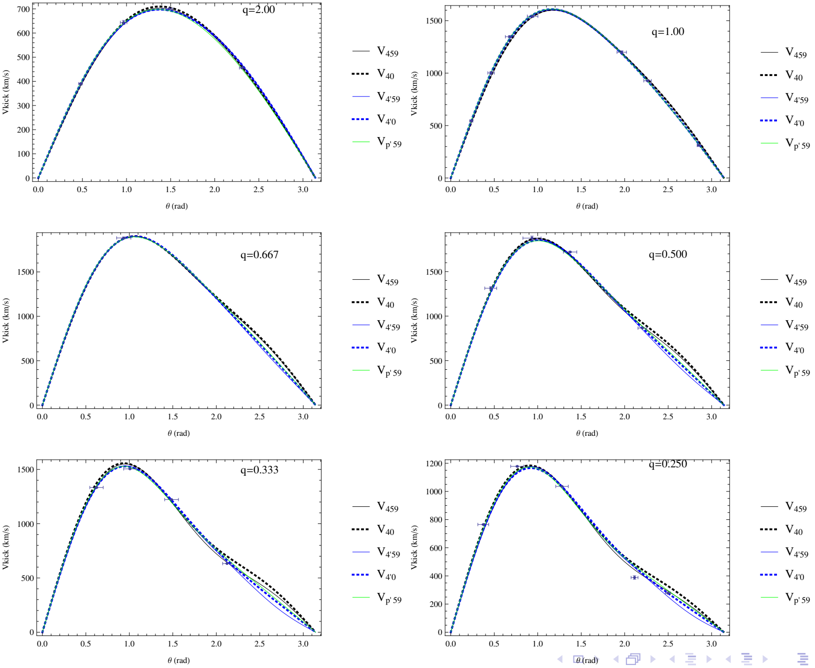
Recent Results

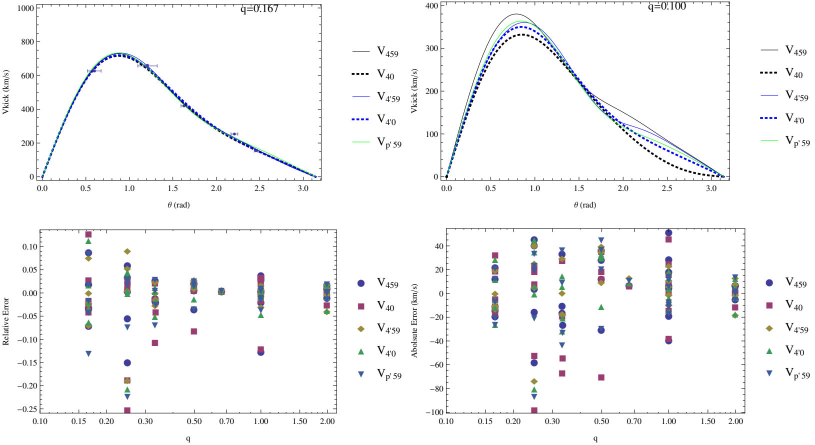


- Now we know much more about recoils.
  - Recoil not maximized when spins in the plane *hangup* kick ($\Delta_\perp(1 + S_z + S_z^2 + \cdots)$ contributions)
  - There are additional non-trivial contributions due to net in-plane spin *cross kick* ($\Delta_z S_\perp(1 + S_z + S_z^2 + \cdots)$ contributions)
- These higher-order contributions are $\sim$ as large as the superkick. But how do they depend on mass ratio?
- Terms proportional to δm can be just as large. Is a simple low-order Taylor expansion realistic?

The starting point for modeling the recoil

$$\begin{aligned}
V_{\mathrm{h}} &= (4\eta)^2 \tilde{\Delta}_{\perp} (3677.76(1 + c_1 \delta m^2) \\
&+ 2481.21(2\tilde{S}_{||})(1 + c_2 \delta m^2) + 1792.45(2\tilde{S}_{||})^2 \\
&+ 1506.52(2\tilde{S}_{||})^3 + c_5 \tilde{\Delta}_{||}^2 + \\
&c_7 \tilde{\Delta}_{||}^2 \tilde{S}_{||} + c_9 \delta m \tilde{\Delta}_{||}), \qquad (1) \\
V_{\mathrm{c0}} &= (4\eta)^2 (2\tilde{S}_{\perp} \tilde{\Delta}_{||})(1199.63 + c_{12} \delta m^2 \\
&+ 2551.46(2\tilde{S}_{||}) + c_{15} \tilde{\Delta}_{||}^2) \\
&+ (2\tilde{S}_{\perp})(c_{16} \delta m + c_{17} \delta m^3 \\
&+ c_{18} \delta m (2\tilde{S}_{||}) + c_{19} \delta m (2\tilde{S}_{||})^2), \qquad (2) \\
V_{\mathrm{c59}} &= (4\eta)^2 (2\tilde{S}_{\perp} \tilde{\Delta}_{||})(2157.59 + c_{12} \delta m^2 \\
&+ 3991.81(2\tilde{S}_{||}) + c_{15} \tilde{\Delta}_{||}^2) \\
&+ (2\tilde{S}_{\perp})(c_{16} \delta m + c_{17} \delta m^3 \\
&+ c_{18} \delta m (2\tilde{S}_{||}) + c_{19} \delta m (2\tilde{S}_{||})^2), \qquad (3) \\
V_{x0} &= V_{\mathrm{h}} + V_{\mathrm{c0}}, \qquad (4) \\
V_{x59} &= \sqrt{V_{\mathrm{h}}^2 + V_{\mathrm{c59}}^2 + 2 V_{\mathrm{h}} V_{\mathrm{c59}} \cos(59\pi/180)}. \qquad (5)
\end{aligned}$$

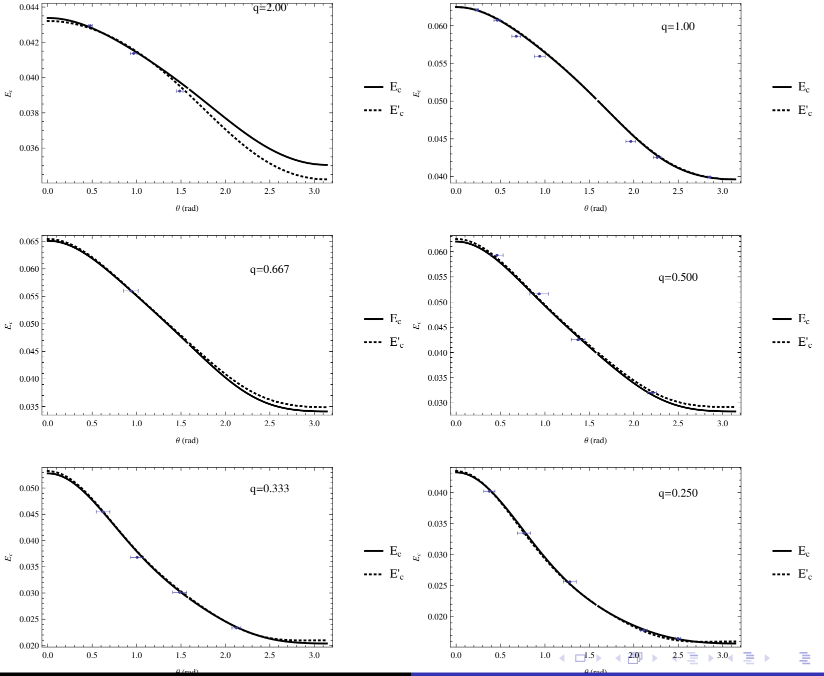

q=2.00
q=1.00
q=0.667
q=0.500
q=0.333
q=0.250
Vkick (km/s)
θ (rad)
V459
V40
V4'59
V4'0
Vp' 59



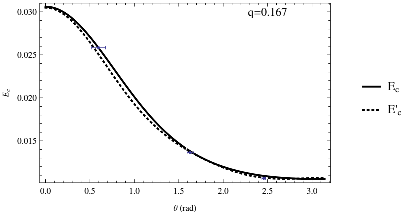
q=0.167
q=0.100
Vkick (km/s)
θ (rad)
Relative Error
Abolsute Error (km/s)
q
V_{459}
V_{40}
$V_{4'59}$
$V_{4'0}$
$V_{p'59}$

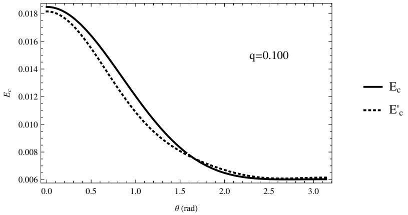
Modeling the Mass loss and Remnant Spin: Overview

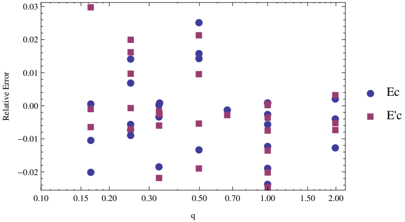
- We start with an expansion by Healy et al. which models the remnant for aligned spin binaries.
- We then add terms, consistent with the symmetries, proportional to $\vec{S}$ and $\vec{\Delta}$ in the plane.
- We also incorporated Hemberger et al.'s model for the remnant spin and mass for equal-mass, non-precessing binaries.
- We looked at different expansion variables.

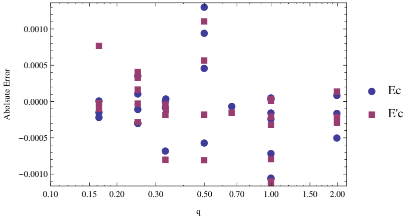


q=2.00
q=1.00
q=0.667
q=0.500
q=0.333
q=0.250
E_c
E'_c
θ (rad)

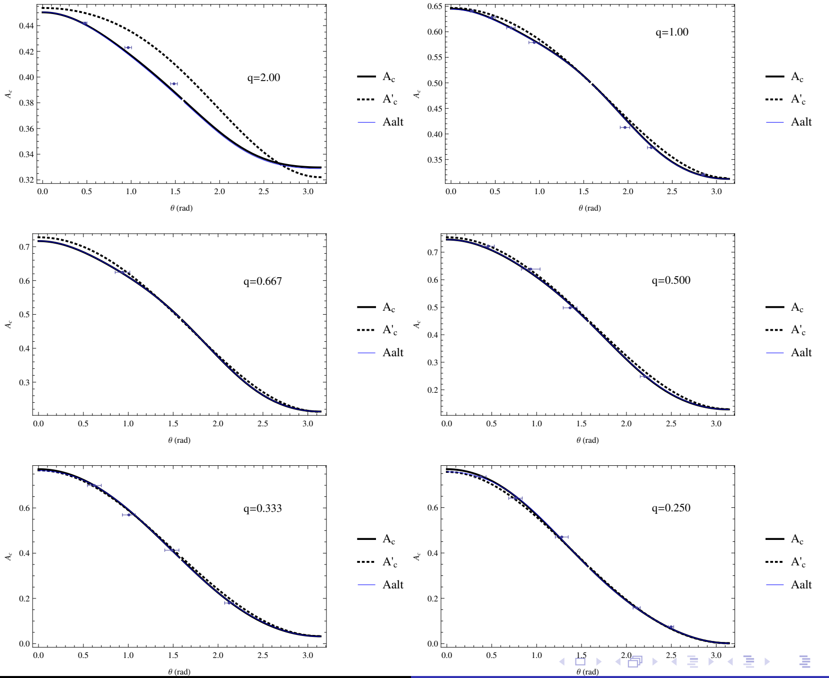




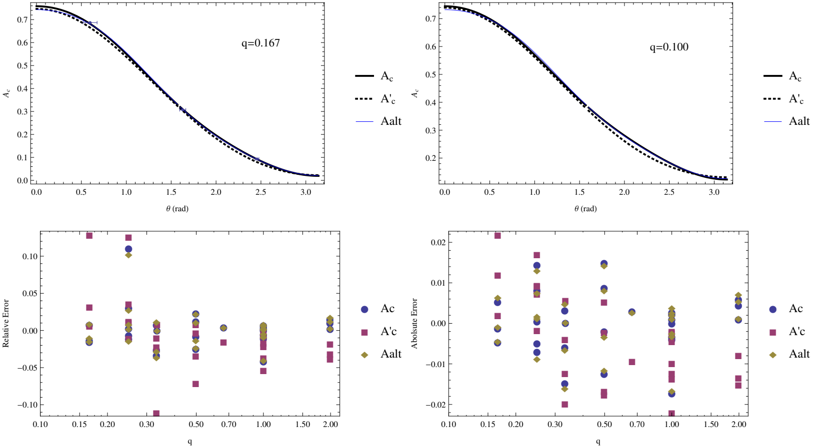




$$\frac{M_1 + M_2 - M_{\mathrm{rem}}}{M_1 + M_2} = E_c + \cdots$$



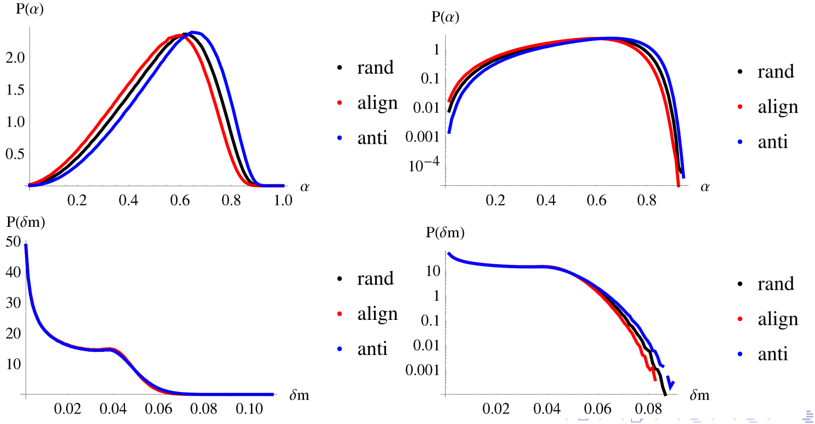
q=2.00
q=1.00
q=0.667
q=0.500
q=0.333
q=0.250
A_c
A'_c
Aalt
θ (rad)
A_c



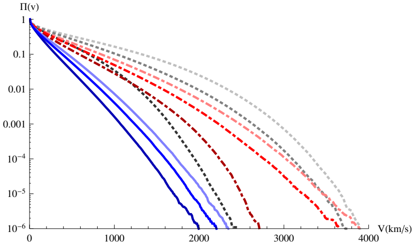
$$\alpha_{\mathrm{rem}} = \sqrt{A_c} + \cdots$$

Universal Dist.

- Start with an arbitrary distribution of spins, and an astrophysical distribution for mass ratios.
- Merge, calculate new spin distributions, merge, etc.
- End up with a universal spin distributions
  - Azimuthal spin orientations are random, aligned, or antialigned
  - Distribution of polar spin orientations is uniform
  - Distribution of mass ratios is fixed



Predictions for Recoils



Model	$\Pi(1000\mathrm{km\,s^{-1}})(\%)$	$\Pi(2000\mathrm{km\,s^{-1}})(\%)$	$\Pi(3000\mathrm{km\,s^{-1}})(\%)$
Hot A	2.875 (2.242)	0.049 (0.028)	0(0)
Hot R	4.965 (4.643)	0.243 (0.223)	0.003 (0.004)
Hot AA	7.075 (7.084)	0.516 (0.535)	0.011 (0.014)
Cold A	0.167 (0.123)	0.0001 (0)	0(0)
Cold R	0.417 (0.408)	0.001 (0.001)	0(0)
Cold AA	0.752 (0.825)	0.002 (0.003)	0(0)
Dry A	3.506 (3.105)	0.007 (0.003)	0(0)
Dry R	10.591 (10.633)	0.949 (0.973)	0.018 (0.018)
Dry AA	16.612 (17.789)	2.522 (2.675)	0.073 (0.79)

What's left to do [A non-exhaustive list] and Outlook

- Need to explore configurations where $\vec{S}$ and $\vec{\Delta}$ are not aligned.
- Need to explore High-spins and small mass ratios (e.g., resonance recoils)
- Need to develop a mapping the binary's initial configuration to the recoil (here we map the binary's configuration during the plunge).
- Need an accurate extrapolation formula to small mass ratios and high spins.
- Modeling behavior at high spins may require different approaches (non-analytic behavior expected near $\alpha = 1$).
- Need to develop ansätze for higher-order fits (e.g., Padé approximations) that reduce the number of fitting parameters if we want more accurate formulas at modest spins and mass ratios.