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on the four-dimensional Schwarzschild background

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The stress tensor in thermal equilibrium with conformal anomaly

for 4D Schwarzschild black hole

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Trace anomaly	$$T^\mu_\mu = \alpha(\mathcal{F} + (2/3)\Box R) + \beta\mathcal{G} \neq 0$$ where $\mathcal{F} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + R^2/3$, and $\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$.

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$$T^\mu_\nu = \frac{\pi^2}{90}\left(\frac{1}{8\pi M}\right)^4 \left[\frac{1 - (4 - \frac{6M}{r})^2(\frac{2M}{r})^6}{(1 - \frac{2M}{r})^2}(\delta^\mu_\nu - 4\delta^\mu_0\delta^0_\nu) + 24\left(\frac{2M}{r}\right)^6(3\delta^\mu_0\delta^0_\nu + \delta^\mu_1\delta^1_\nu)\right]$$

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	where $C_0 = \gamma T_{\mathrm{H}}^4$, γ is the Stefan-Boltzmann constant, and $f = -g_{00}(r)$.

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	The ordinary Tolman Temperature	The trace anomaly-induced Tolman Temperature
4D Schwarzschild black hole	$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$	
Proper temperature		
Graph		

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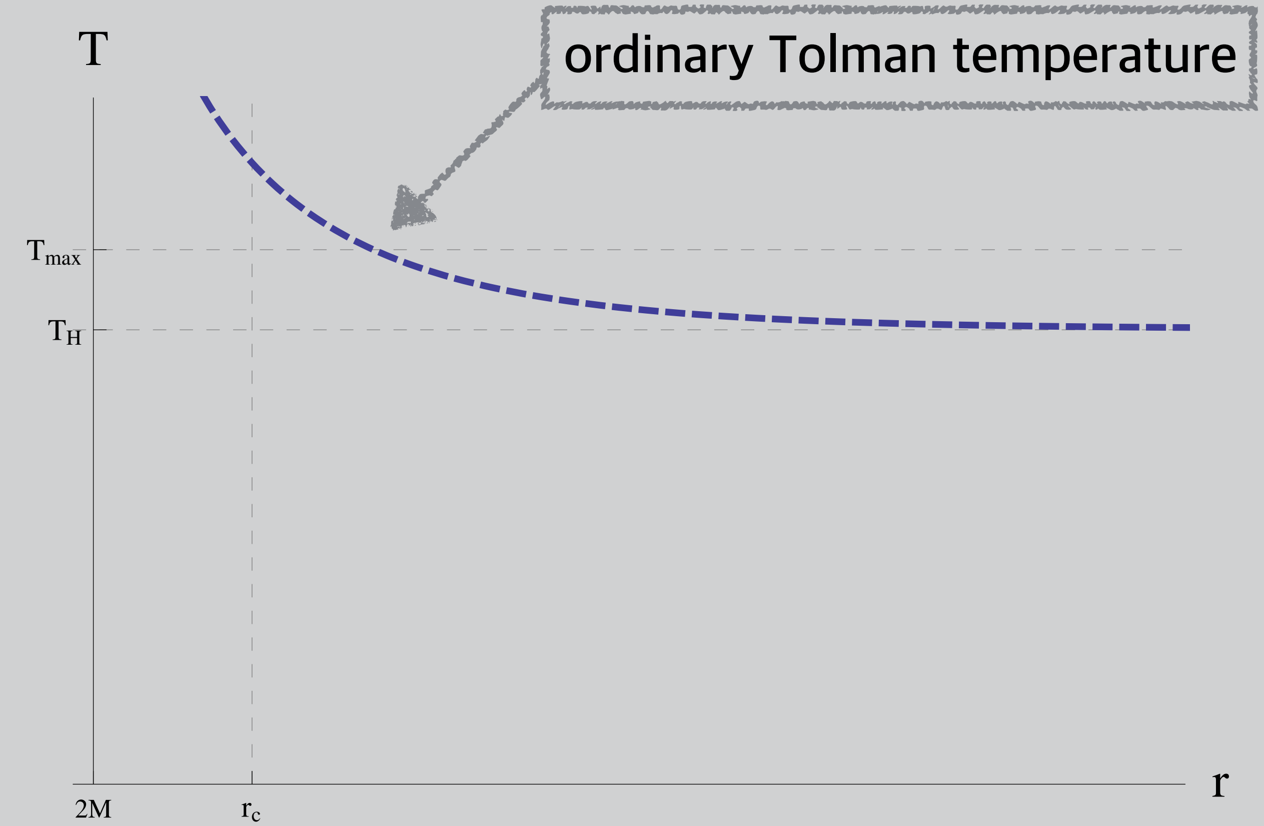
Graph		

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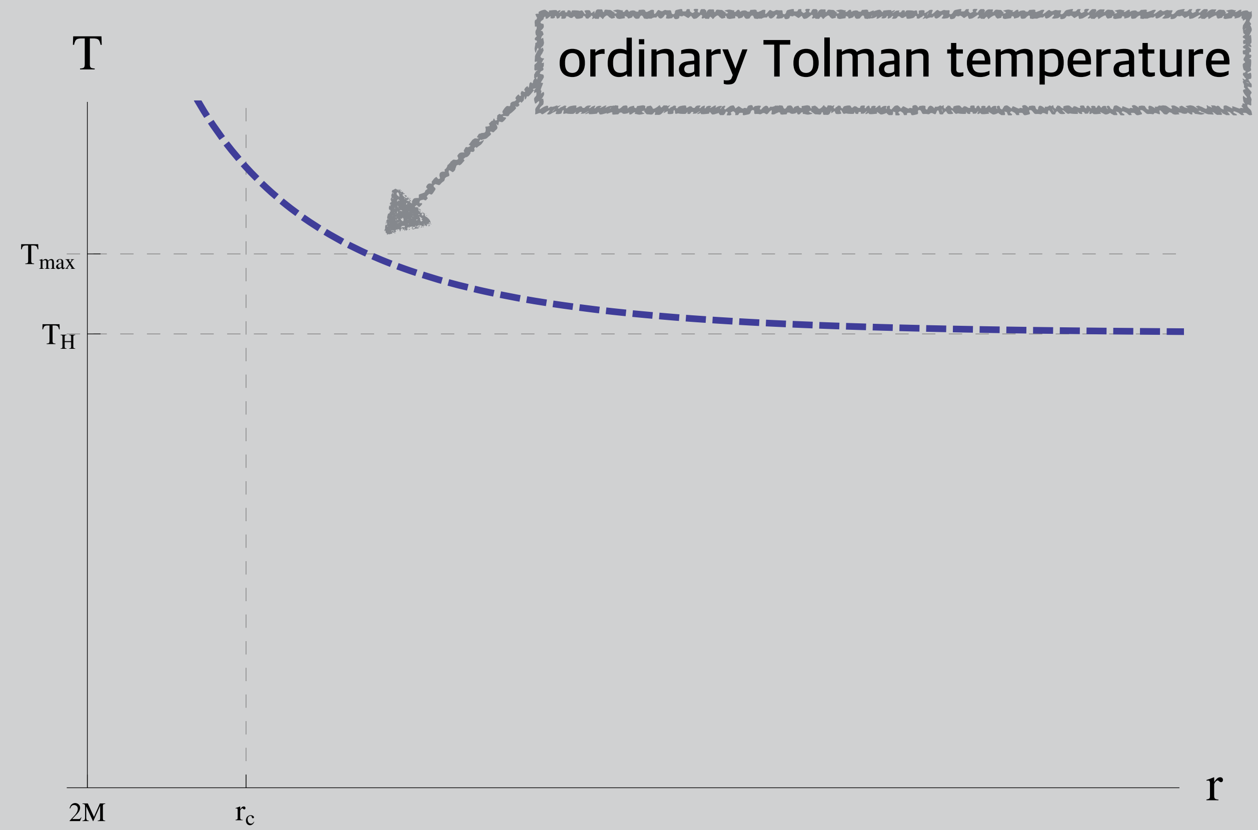
Graph		

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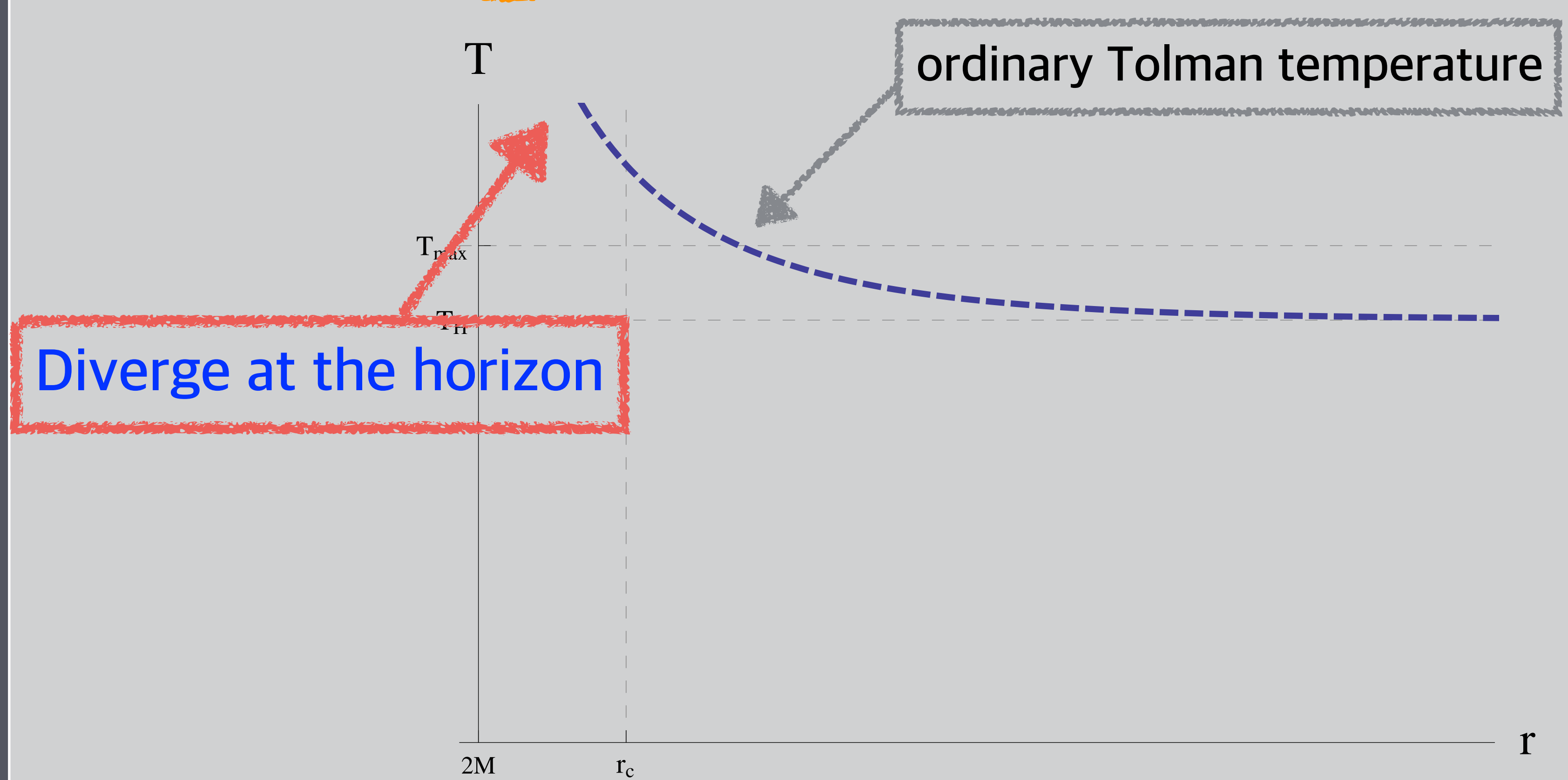
Graph		

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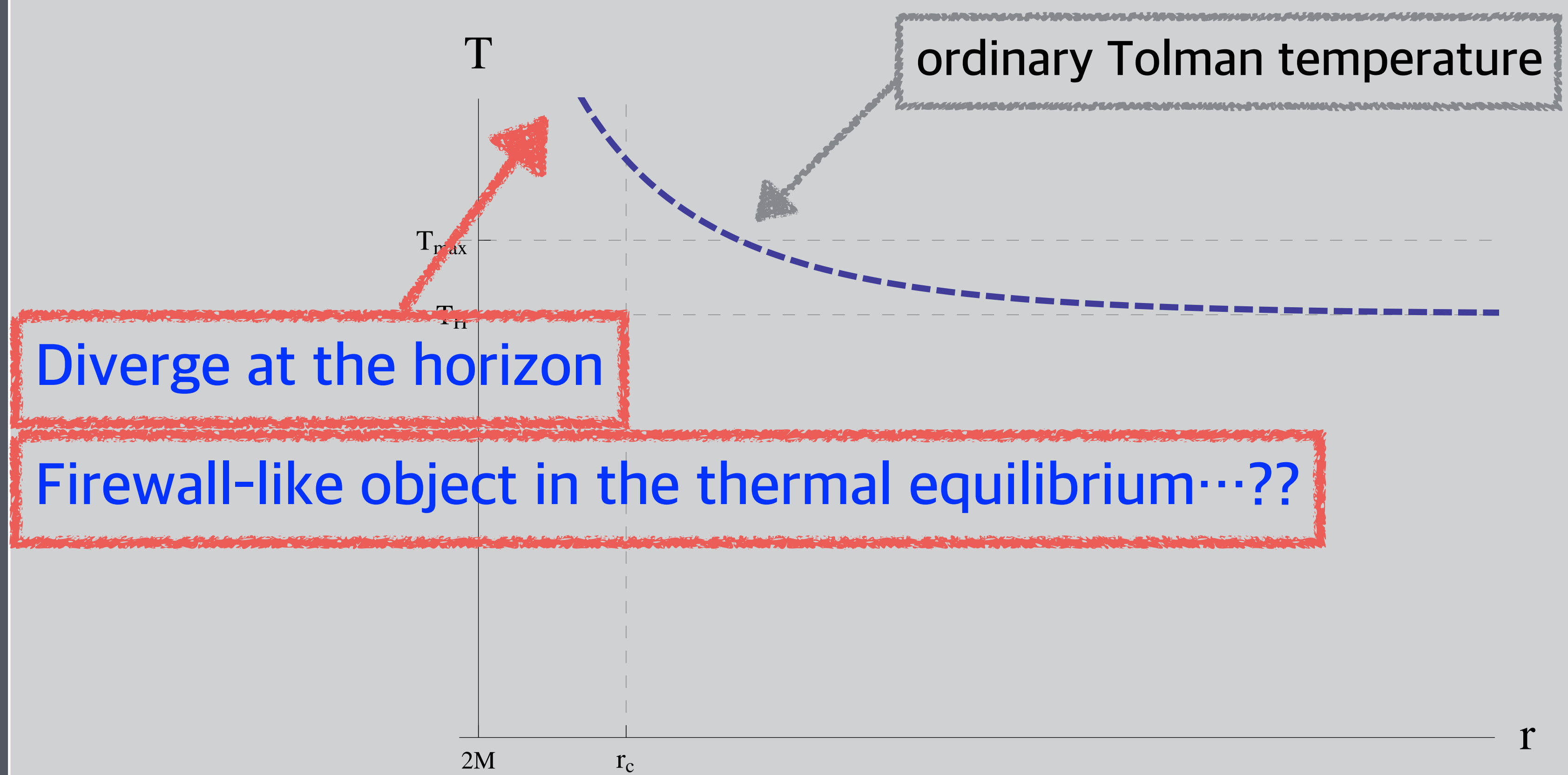
Graph		

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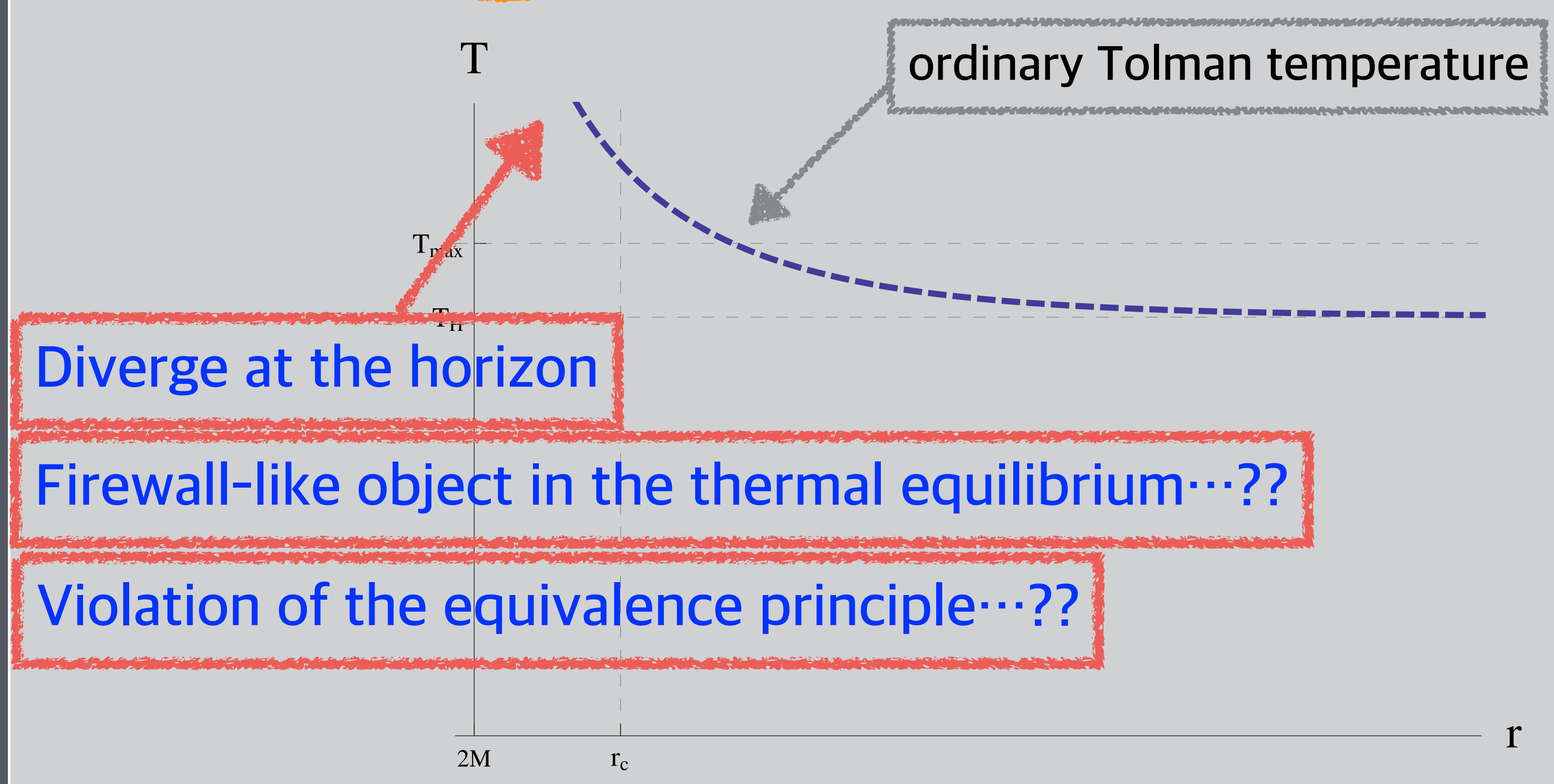

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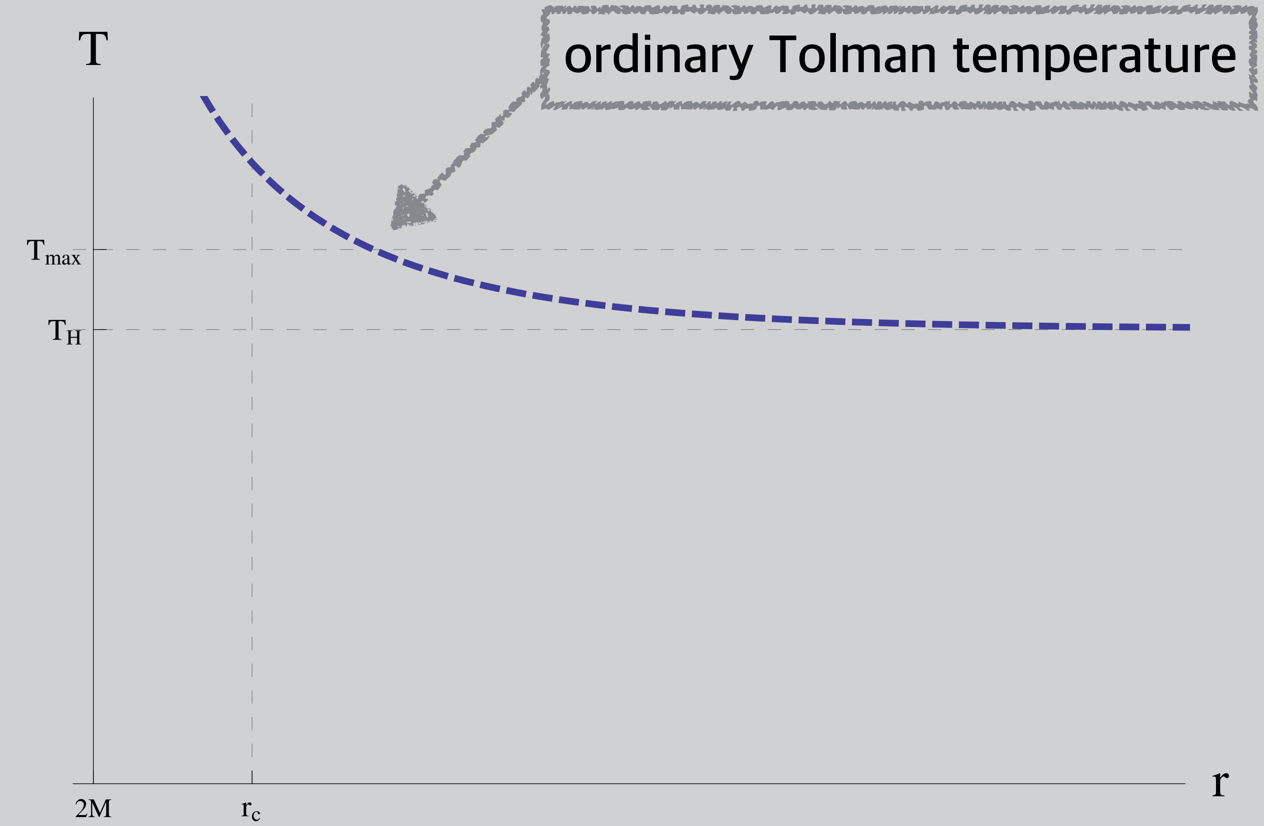
Graph		

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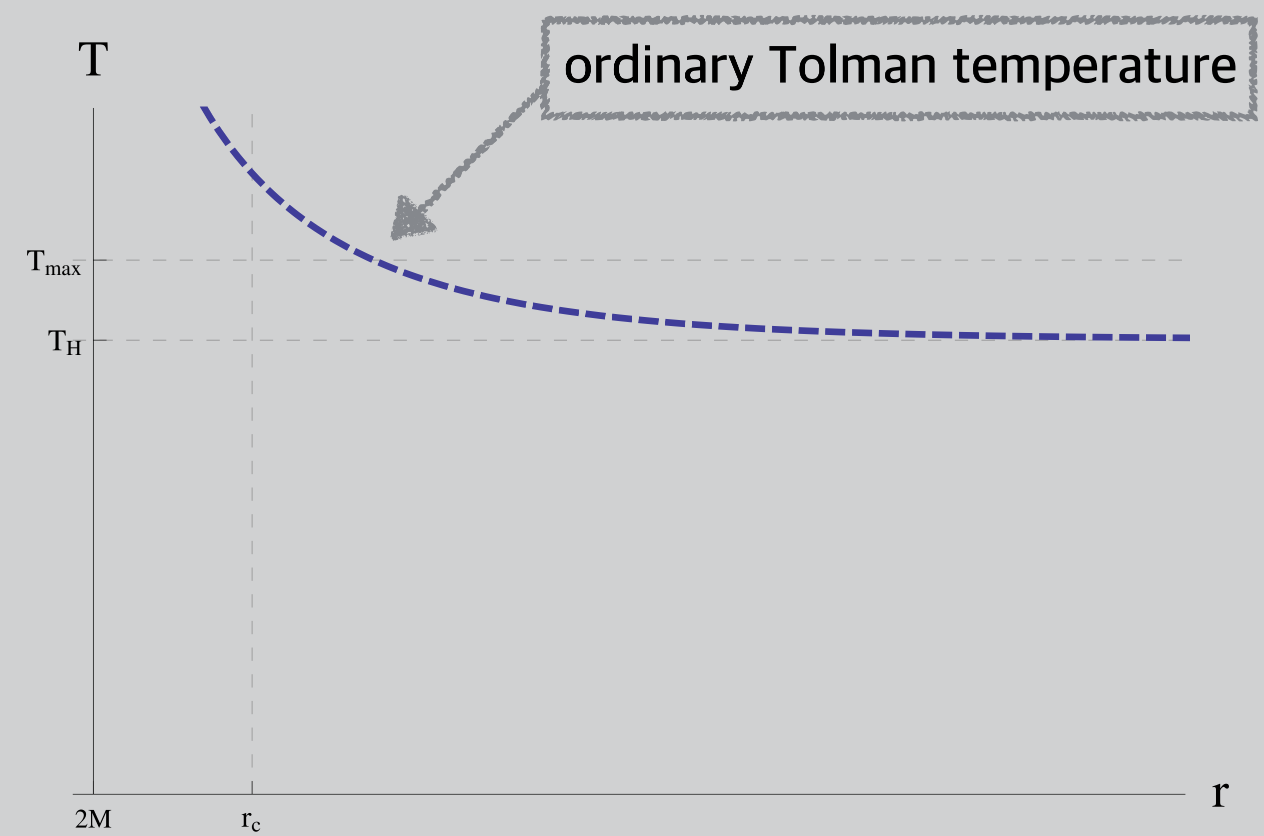
Graph		

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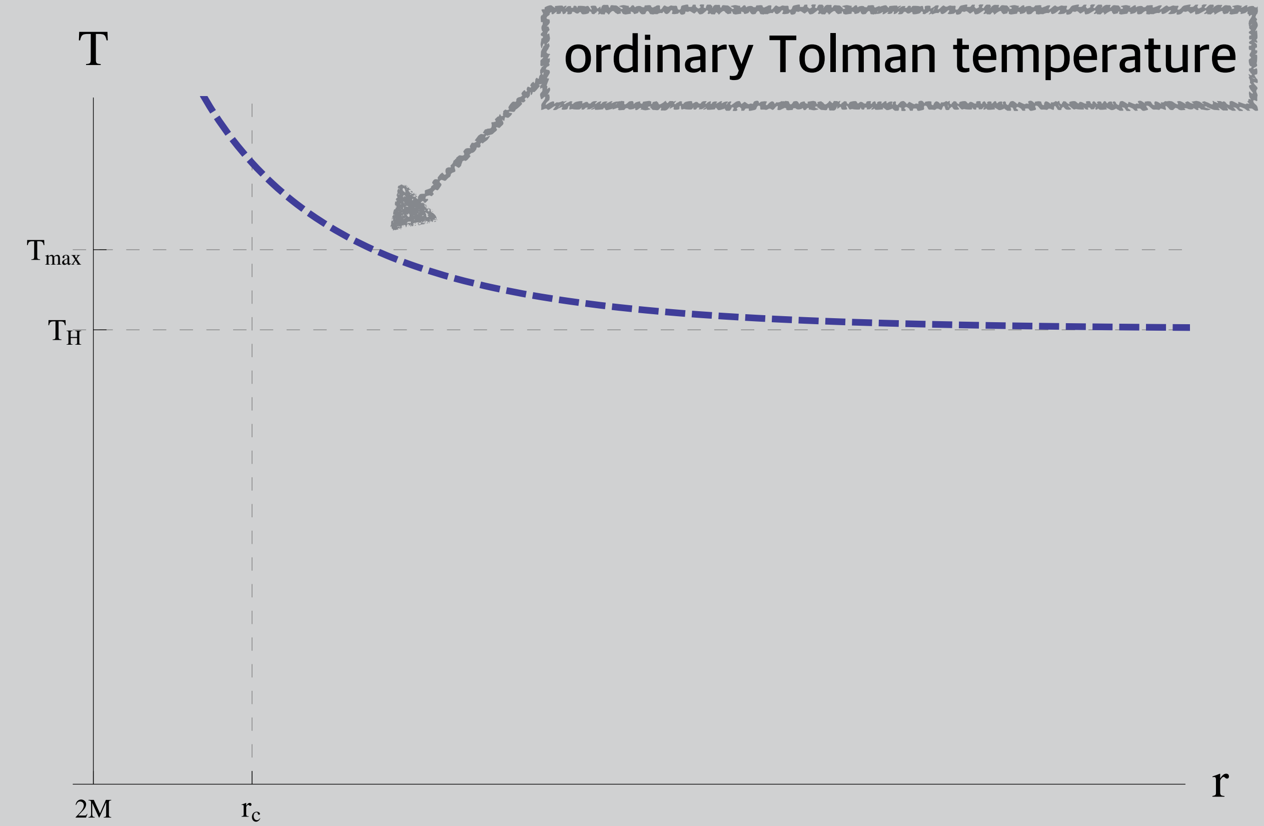
Graph		

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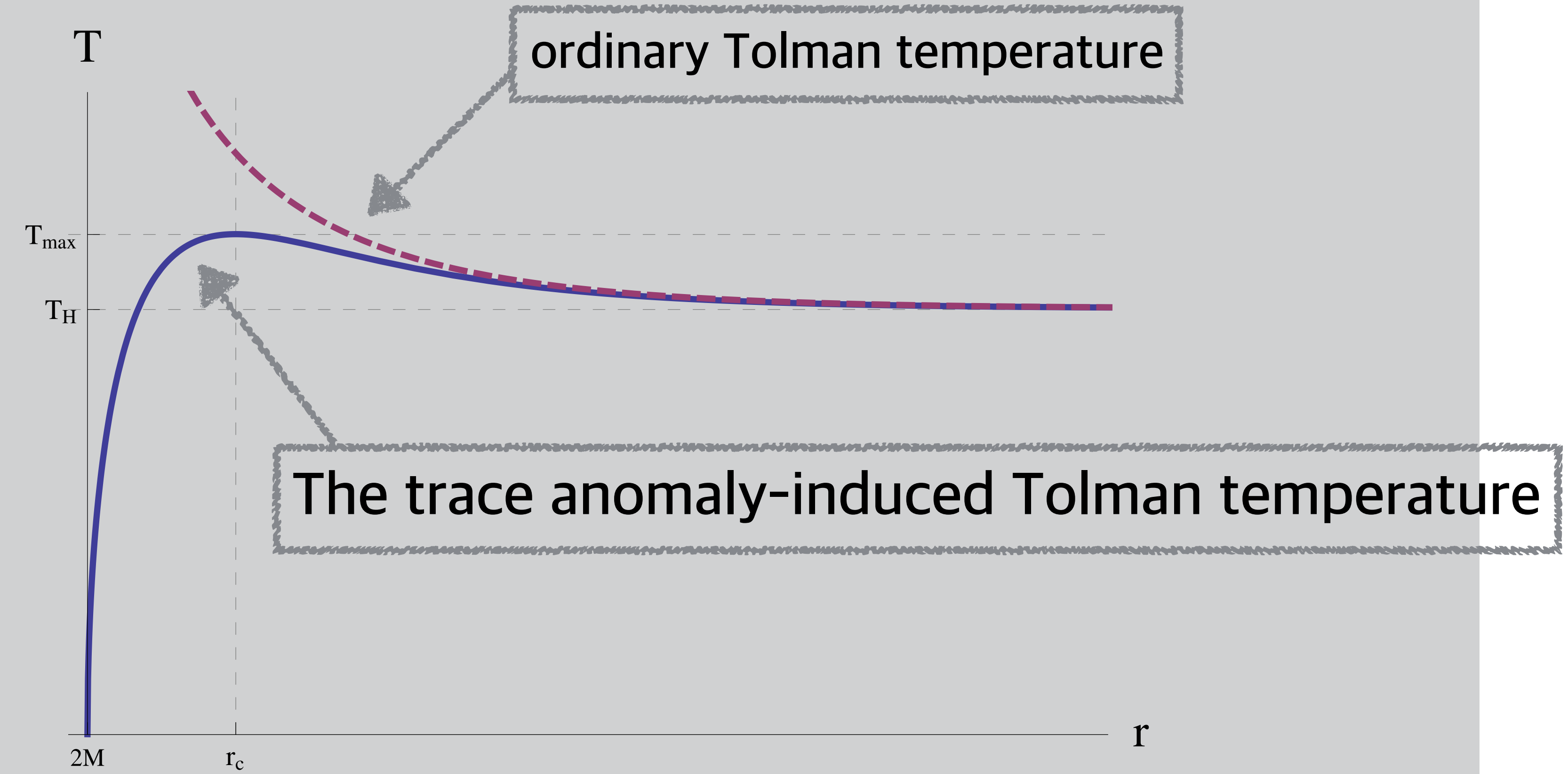
Graph		

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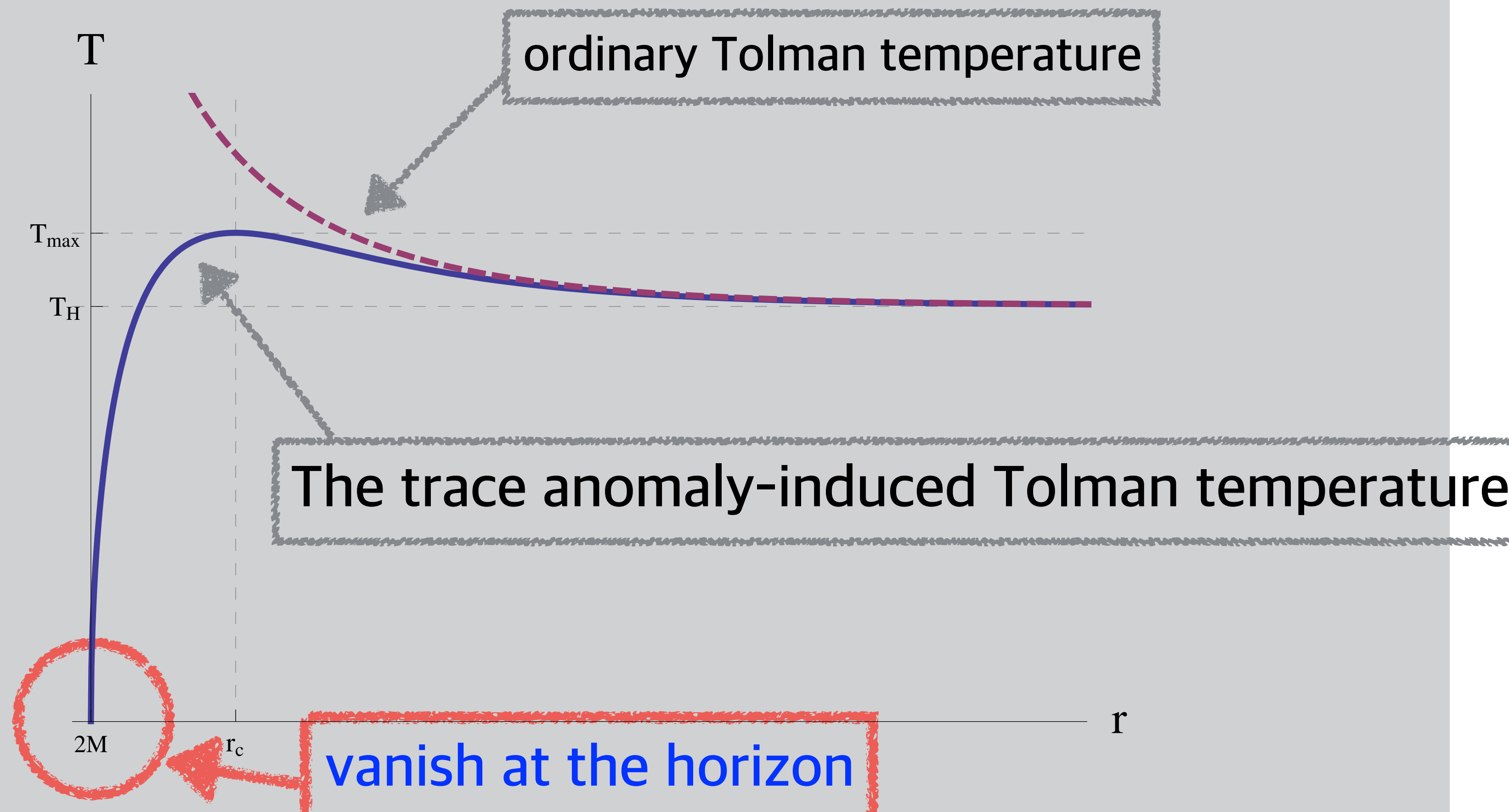
Graph		

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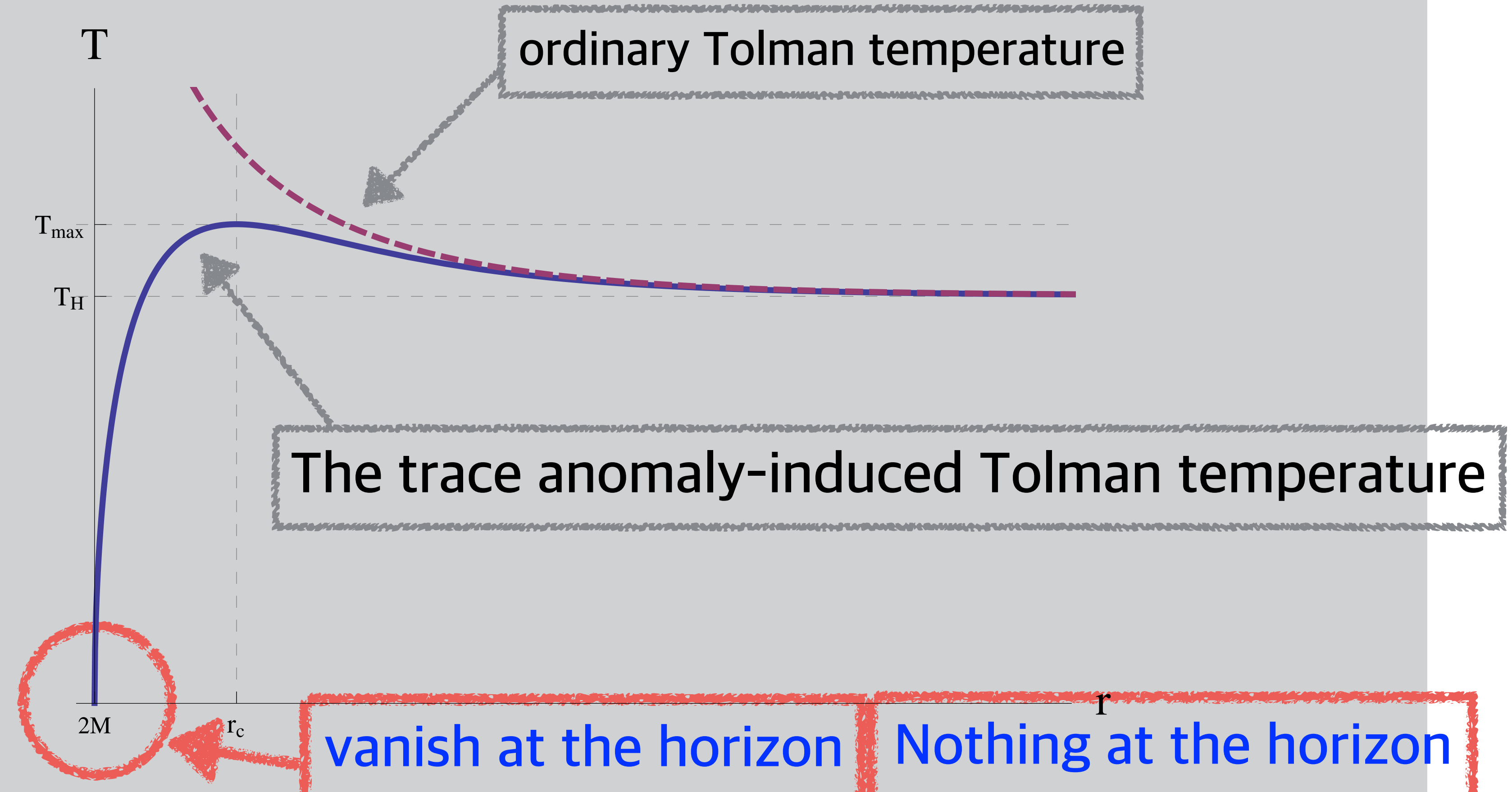

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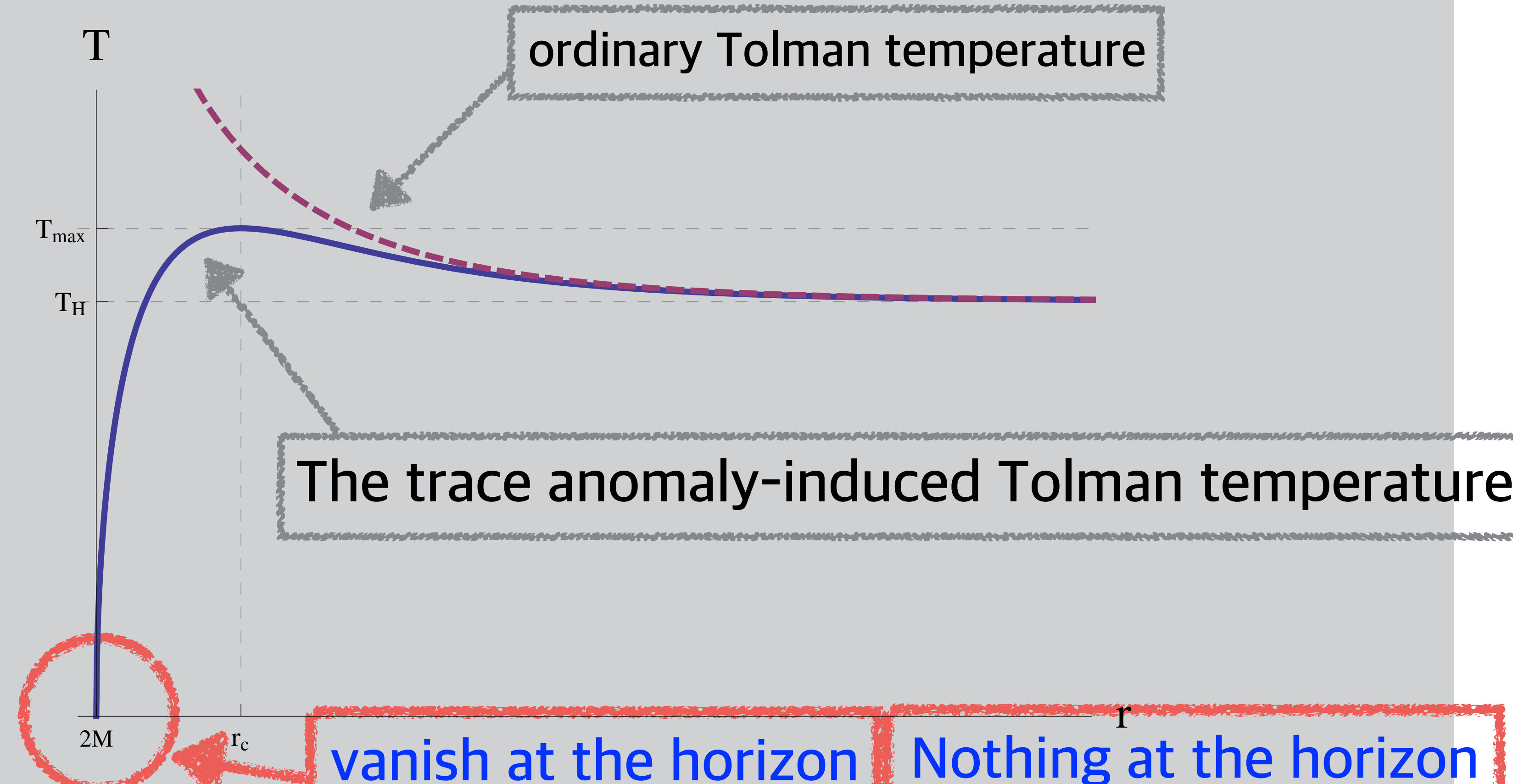
Graph		

vanish at the horizon

Nothing at the horizon

Equivalence principle is recovered.

Conclusions and Discussions

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Thank you for your attention!!