Constraint on ghost-free bigravity from Gravitational Cherenkov radiation

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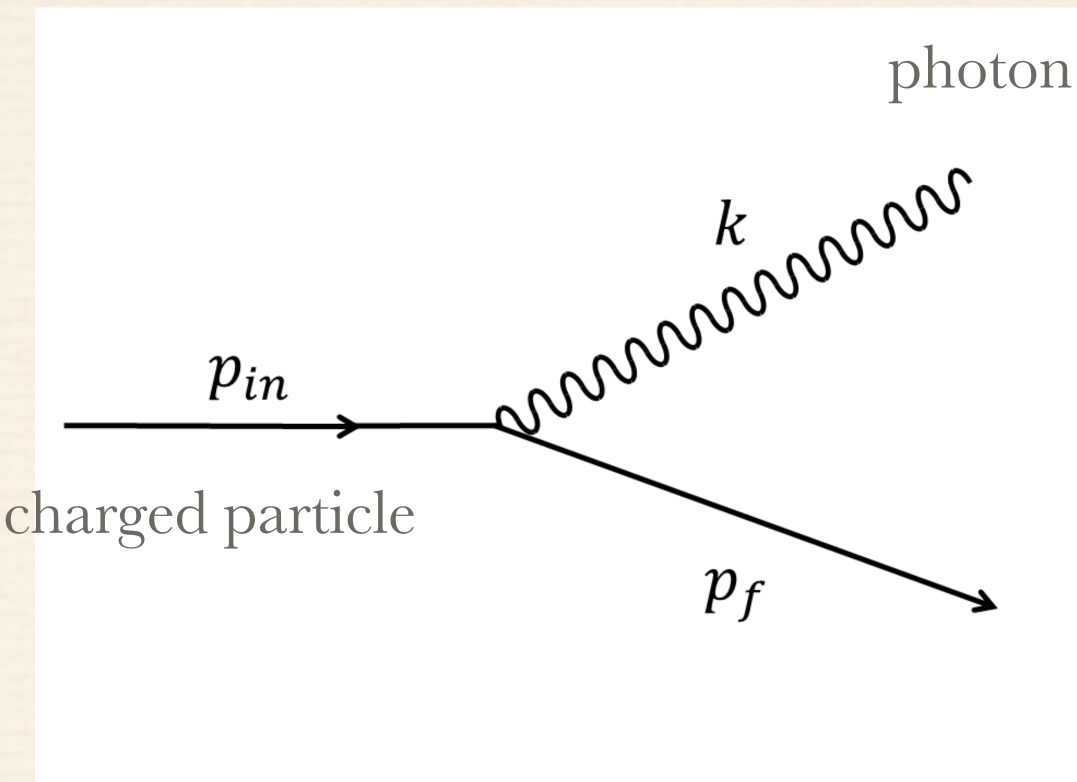
Cherenkov radiation

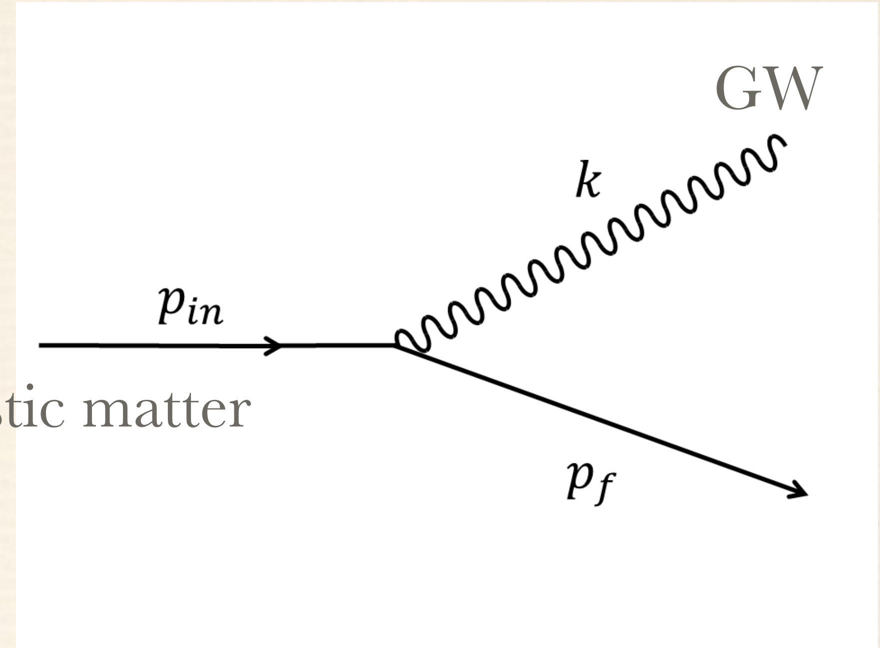


When the phase velocity of photon is smaller than the one of a charged particle, radiation is emitted from the charged particle through the above process.

…**Cherenkov radiation**

Gravitational Cherenkov radiation (GCR)

We can apply this idea to the gravitational field when the graviton's phase velocity is less than *1*.



For example, consider ultra-high energy cosmic rays as a relativistic matter.

The observation of cosmic rays gives a constraint on the phase velocity of the graviton, because the energy of cosmic rays decays through GCR during the propagation

when $\omega^2 := c_s^2 k^2 + m^2 < k^2$ for a graviton with a mass m and sound speed c_s .

Moore and Nelson (2001)

→ small sound speed will be restricted, but a large mass can avoid the restriction?

ghost-free bigravity

bigravity : gravitational theory which contains two gravitons interacting each other

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,[R + V(g,\,\tilde{g})] + \frac{\kappa M_{\rm pl}^2}{2}\int d^4x\sqrt{-\tilde{g}}\,\tilde{R}$$

In general, this theory suffers from ghost.

In order to obtain a healthy bigravity, we have to tune the interaction form as

$$V = m^2\sum_{n=0}^{4} c_n \epsilon^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n}\mathcal{K}^{\nu_1}_{\mu_1}\ldots\mathcal{K}^{\nu_n}_{\mu_n}\,,\;\mathcal{K}^{\nu}_{\mu} = \sqrt{g^{\nu\rho}\tilde{g}_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)
Hassan and Rosen (2012)

two gravitons:

one is massless (helicity-2 mode) and propagates in the light speed,

while the other one is massive (helicity-2 + helicity-1 + helicity-0 modes)
and its propagate speed can be differ from the light speed.

cosmology in ghost-free bigravity

Consider two matter fields: one couples only to $g_{\mu\nu}$ and the other one to $\tilde{g}_{\mu\nu}$

background metric
$$g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + a^2 \delta_{ij} dx^i dx^j \,,$$
$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -n^2 dt^2 + \alpha^2 \delta_{ij} dx^i dx^j \,,$$

Choosing the healthy branch $H = \frac{\alpha}{a}\tilde{H}\,,$

$\frac{\alpha}{a} := \xi \sim \xi_c(c_n)$ at low energies, i.e. $\frac{\rho}{m^2 M_g^2} \ll 1, \quad \frac{\xi^2 \tilde{\rho}}{\kappa m^2 M_g^2} \ll 1\,,$

and the speed of light for the hidden metric $\tilde{c} := na/N\alpha$ is given as

$$\tilde{c} - 1 = \frac{1}{2M_g^2(m_{\text{eff}}^2 - 2H^2)}\left[\rho + P - \frac{\xi_c^2}{\kappa}(\tilde{\rho} + \tilde{P})\right] \sim \frac{H^2}{m_{\text{eff}}^2} \ll 1$$

positive in the branch free from Higuchi ghost

Therefore $\rho \gg \tilde{\rho}$ $\Rightarrow$ $\tilde{c} > 1$ and $\rho \ll \tilde{\rho}$ $\Rightarrow$ $\tilde{c} < 1$

helicity-2 gravitons

The effective action for helicity-2 mode can be written in the form of

$$S = \frac{m_{pl}^2}{8} \int d^4x \sum_{A=1,2} \left[(\dot{h}_{A\,ij})^2 - c_A^2 (\partial_\ell h_{A\,ij})^2 - m_A^2 (h_{A\,ij})^2 \right]$$

$$\begin{aligned} h_{1\,ij} &= \cos\theta_g h_{ij} + \sin\theta_g \tilde{h}_{ij} \\ h_{2\,ij} &= -\sin\theta_g h_{ij} + \cos\theta_g \tilde{h}_{ij} \end{aligned}$$

$$\cot 2\theta_g = \frac{2k^2(\tilde{c}-1)}{m_{\rm eff}^2}$$

The dispersion relation is given as

$$\frac{\omega_{1,2}^2}{k^2} = 1 + \frac{m_{\rm eff}^2}{2k^2} \left[1 - x \mp \sqrt{(1-x)^2 + \frac{4\kappa\xi_c^2}{1+\kappa\xi_c^2} x} \right] + \mathcal{O}((1-\tilde{c})^2)$$

where we define $x := \dfrac{2k^2(1-\tilde{c})}{m_{\rm eff}^2}$

x is negative ($\tilde{c} > 1$) ➡ $\omega_{1,2}^2 > k^2$ … no GCR

x is positive ($\tilde{c} < 1$) ➡ $\omega_1^2 > k^2$ but $\omega_2^2 < k^2$ … 2nd mode emits as GCR

helicity-1 massive graviton

metric perturbation
$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + 2aB_i dt dx^i + a^2\left(\delta_{ij} + \partial_{(i}E_{j)}\right)dx^i dx^j\,,$$
$$\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -n^2 dt^2 + 2n\alpha b_i dt dx^i + \alpha^2\left(\delta_{ij} + \partial_{(i}S_{j)}\right)dx^i dx^j\,,$$

the effective action for helicity-1 mode $\mathcal{E}_i := \sqrt{\dfrac{1+\kappa\xi^2}{\kappa\xi^2}}k(E_i - S_i)$

$$I_{\rm vector} = \frac{\kappa\xi^2}{8(1+\kappa\xi^2)}M_{\rm pl}^2\int dt d^3k \frac{m_{\rm eff}^2}{k^2}\left[\dot{\mathcal{E}}^i\dot{\mathcal{E}}_i^* - \left\{k^2 c_{\rm V}^2 + m_{\rm V}^2\right\}\mathcal{E}^i\mathcal{E}_i^*\right]$$

$1 - c_{\rm V}^2 \sim \dfrac{1+\mathcal{C}}{2}(1-\tilde{c}) \sim \mathcal{O}(1)\,,\quad m_{\rm V}^2 \sim m_{\rm eff}^2$ at low energies

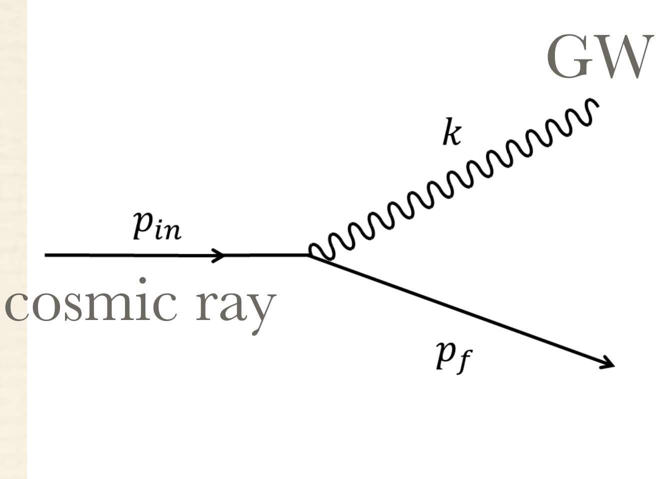
$\because$ $\mathcal{C} = \mathcal{C}(\xi,\, c_n)$ appears in the Vainshtein radius $r_{\rm V} = \mathcal{O}\left(\left(\dfrac{|\mathcal{C}|\,r_g}{m_{\rm eff}^2}\right)^{1/3}\right)$

For large r_V, $\mathcal{C}$ should take a large negative (positive) value

for $\rho \gg \tilde{\rho}$ i.e. $\tilde{c} > 1$ ($\rho \ll \tilde{\rho}$ i.e. $\tilde{c} < 1$)

De Felice et al. (2013)

Energy emission of GCR



The GCR energy emission is given as $E_{\mathrm{A}} = \sum_{\mathbf{k}} \omega_k \langle \hat{a}^\dagger_{\mathrm{A}\mathbf{k}} \hat{a}_{\mathrm{A}\mathbf{k}} \rangle$

A=*T*, *V* represents tensor and vector mode, respectively.

$$\langle \hat{a}^\dagger_{\mathrm{A}\mathbf{k}} \hat{a}_{\mathrm{A}\mathbf{k}} \rangle = 2Re \int_{t_{\mathrm{in}}}^{t} dt_2 \int_{t_{\mathrm{in}}}^{t_2} dt_1 \langle \mathrm{in} | H_{\mathrm{I}}(t_1) \hat{a}^\dagger_{\mathrm{A}\mathbf{k}} \hat{a}_{\mathrm{A}\mathbf{k}} H_{\mathrm{I}}(t_2) | \mathrm{in} \rangle$$

$\hat{a}_{\mathrm{A}\mathbf{k}}$ is the annihilation operator of A-mode graviton

For simplicity, consider a complex scalar field as matter instead of Dirac fermion.

$$S_m = \int d^4x \sqrt{-g} \left[-g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - M^2 \psi^* \psi \right] .$$

The interaction Hamiltonian is given as $H_I = \int d^3x \, \delta g^{ij} \partial_i \psi \partial_j \psi$

where we choose the gauge $\delta g_{0\mu} = 0$

Result

Assuming $\kappa\xi_c^2 \sim 1$,

The energy emission rate of helicity-2 gravitons becomes

$$\frac{dE_{\mathrm{T}}}{dt} \sim \frac{p_{\mathrm{in}}^2 m_{\mathrm{eff}}^2}{M_{\mathrm{pl}}^2}(1-\tilde{c}) \sim \frac{p_{\mathrm{in}}^2 H^2}{M_{\mathrm{pl}}^2} \ll \frac{p_{\mathrm{in}}}{t} \rightarrow$$ **No constraint**

Here we use the relation $1-\tilde{c} \sim H^2/m_{\mathrm{eff}}^2$

The energy emission rate of helicity-1 graviton becomes

$$\frac{dE_{\mathrm{V}}}{dt} \sim \frac{p_{\mathrm{in}}^2 m_{\mathrm{eff}}^2}{M_{\mathrm{pl}}^2}(1-c_{\mathrm{V}}^2)$$

: GCR emission is suppressed for a small mass

because of the coupling to the matter m^2/M_{pl}^2

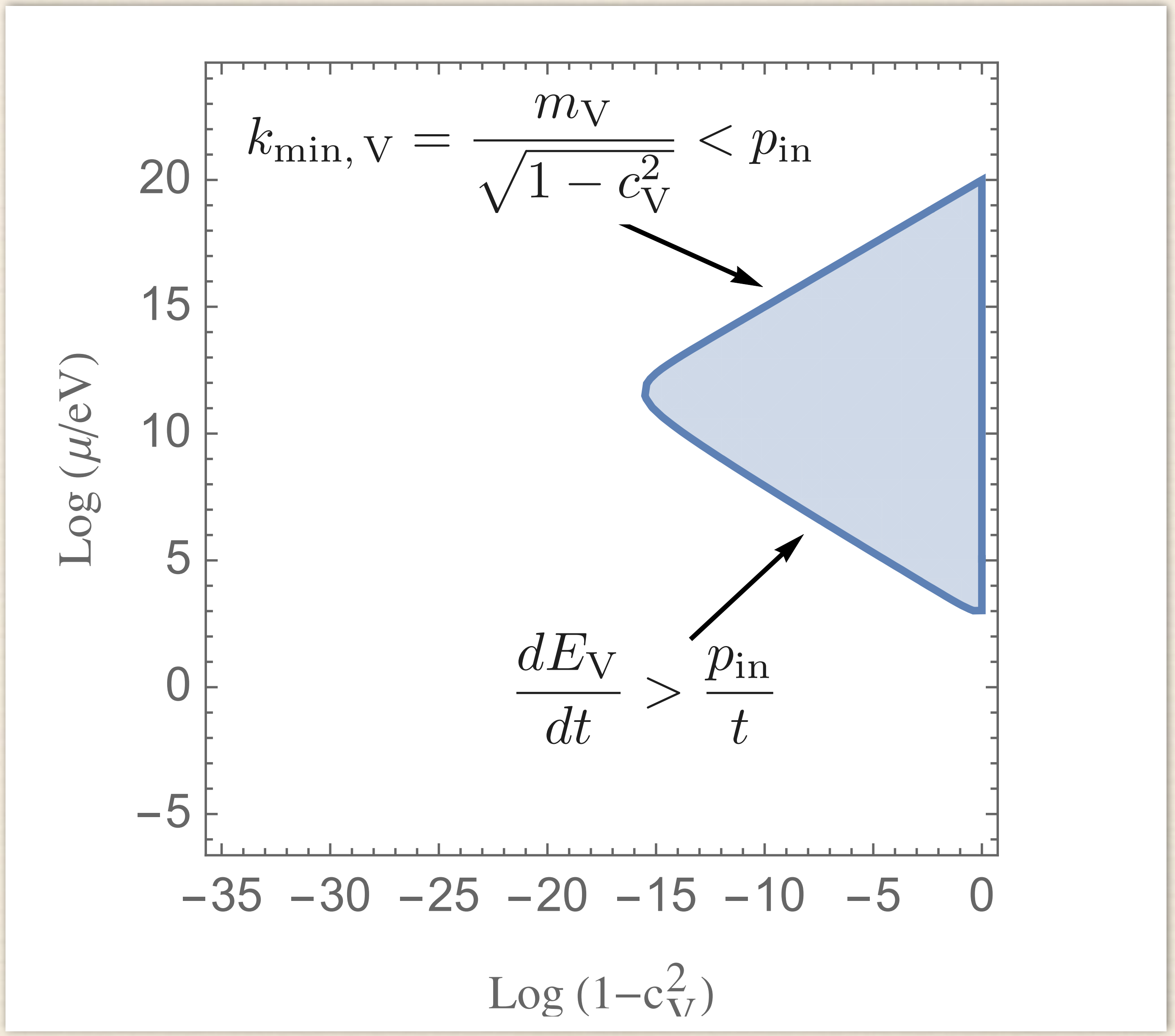
Assuming that the cosmic ray with $p \sim 10^{11}\mathrm{GeV}$ comes from $ct \gtrsim 1\mathrm{Mpc}$,

we obtain a upper bound for the effective mass $m_{\mathrm{eff}} \lesssim 100(1-c_{\mathrm{V}}^2)^{-1/2}\mathrm{eV}$

Constraint on ghost-free bigravity

We present the excluded region in the m_{eff} -$(1-c_{\mathrm{V}}^2)$ plane,

but it is out of the mass range of bigravity as an IR modification of gravity.



GCR of helicity-0 graviton

It is difficult to evaluate the helicity-0 GCR emission, since the dispersion relation is complicated because of the mixing with the scalar matter field, but we estimate it as follows and expect that **the GCR emission is suppressed enough.**

Cosidering conformal scalar field as the matter field,

The coupling squared between the scalar mode in Fierz-Pauli massive graviton and matter in de Sitter background becomes

$$m_{\rm eff}^2(m_{\rm eff}^2 - 2H^2)/k^2 M_{\rm pl}^2 \sim m_{\rm eff}^2/M_{\rm pl}^2$$: same as the vector case

Izumi and Tanaka (2007)

When the matter field is non-conformal,

the coupling is not to be suppressed by the graviton's mass squared,

but to be suppressed by the ratio of mass to momentum squared of the matter field.

Summary

- In the ghost-free bigravity, the phase velocities of gravitons can be subluminal, and hence gravitational Cherenkov radiation gives a constraint on this model.

- In the ghost-free bigravity, GCR of the helicity-2 gravitons is highly suppressed and ignorable, because of the relation $1 - \tilde{c} \sim H^2/m_{\rm eff}^2$.

- The sound speed of helicity-1 mode can be significantly subluminal, but this gives just a weak upper bound for the graviton's mass $m_{\rm eff} \lesssim 100(1 - c_{\rm V}^2)^{-1/2}{\rm eV}$, since the coupling between matter and helicity-1 mode is suppressed by $m_{\rm eff}/M_{\rm pl}$.

- It is hard to evaluate GCR of the helicity-0 graviton because of the complexity of the dispersion relation, but it also seems to be suppressed.

bigravity and Boulware-Deser ghost

bigravity : gravity which contains two interacting gravitons

$$S = \frac{M_g^2}{2}\int d^4x\sqrt{-g}\left[R^{(g)} + \underline{2m^2V(g,\ f)}\right] + \frac{M_f^2}{2}\cancel{\int d^4x\sqrt{-f}\,R^{(f)}}$$

fix f

The interaction term breaks general covariance for g

GR (helicity-2) + 4 gauge breaking (helicity-1, helicity-0, helicity-0)

massive graviton

This mode's kinetic term
has opposite sign!!

Boulware-Deser ghost

Boulware and Deser (1972)

In order to obtain healthy bigravity, we have to tune the interaction form so that the ghost mode is removed by constraints.

ghost-free bigravity

Choosing the form of the interaction as

$$V = \sum_{n=0}^{4} c_n \epsilon^{\mu_1 \dots \mu_n}_{\nu_1 \dots \nu_n} K^{\nu_1}_{\mu_1} \dots K^{\nu_n}_{\mu_n} \qquad K^{\nu}_{\mu} = \sqrt{g^{\nu\rho} f_{\rho\mu}}$$

de Rham, Gabadadze, Tolley (2011)

ADM decomposition

$$N^{-2} = -g^{00}\,, \quad N_i = g_{0i}\,, \quad \gamma_{ij} = g_{ij}\,,$$
$$L^{-2} = -f^{00}\,, \quad L_i = f_{0i}\,, \quad {}^3 f_{ij} = f_{ij}\,.$$

define new shift-like vector n^i
and rewrite N^i with n^i

Then Hamiltonian becomes linear in N, L, L^i.

$$H = N\mathcal{C} + LC^L + L^i C^L_i.$$

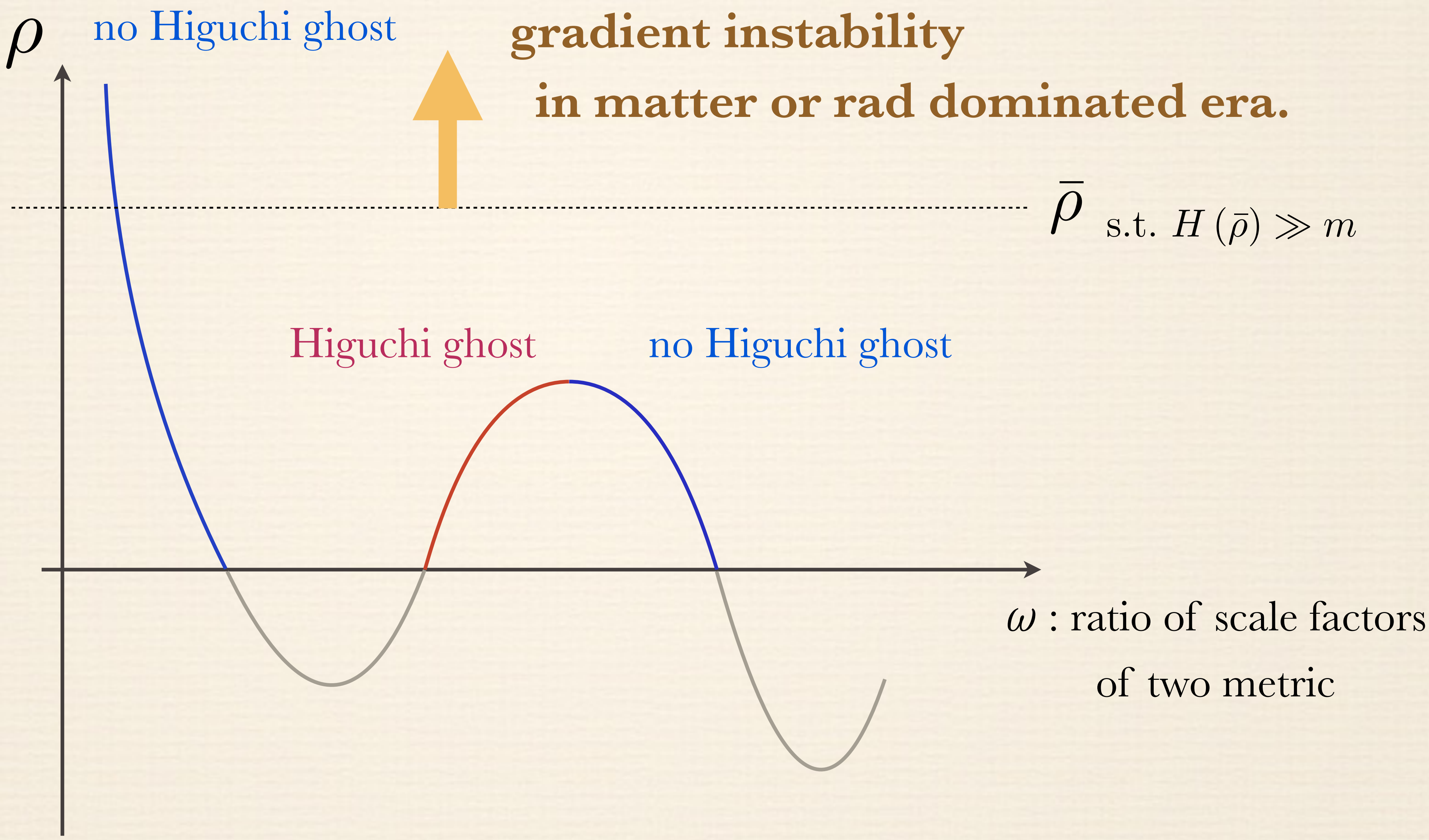
conjugate momentum

$\mathcal{C}$, C^L, C^L_i are functions of $\{\gamma_{ij}, \pi^{ij}, {}^3 f_{ij}, p^{ij}\}$

→ One of the Hamiltonian constraints kills BD ghost.

Hassan and Rosen (2012)

Cosmological solution in ghost-free bigravity



Higuchi ghost in dRGT bigravity

In dRGT model, equation for the de Sitter solution insists

$$\frac{\kappa_4^2}{m^2}\rho_m = \frac{c_1}{\chi\omega} + \left(\frac{6c_2}{\chi} - c_0\right) + \left(\frac{18c_3}{\chi} - 3c_1\right)\omega + \left(\frac{24c_4}{\chi} - 6c_2\right)\omega^2 - 6c_3\omega^3 \equiv f(\omega)$$

ω : ratio of scale factor of two metric

effective mass for massive graviton

$$m_{eff}^2 = m^2(1+(\chi\omega^2)^{-1})\Gamma(\omega) = -\frac{m^2\omega}{3}\underline{f'(\omega)} + 2H^2$$

this sign determines the ghost appearance

$$\Gamma(\omega) \equiv c_1\omega + 4c_2\omega^2 + 6c_3\omega^3$$

For flat vacuum solution, $H \rightarrow 0$ as $\omega \rightarrow \omega_0$ where $\rho_m(\omega_0) \rightarrow 0$,

$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

negative when $\boldsymbol{\Gamma > 0}$ i.e. $\boldsymbol{m_{eff}^2 > 0}$

→ **no Higuchi ghost**

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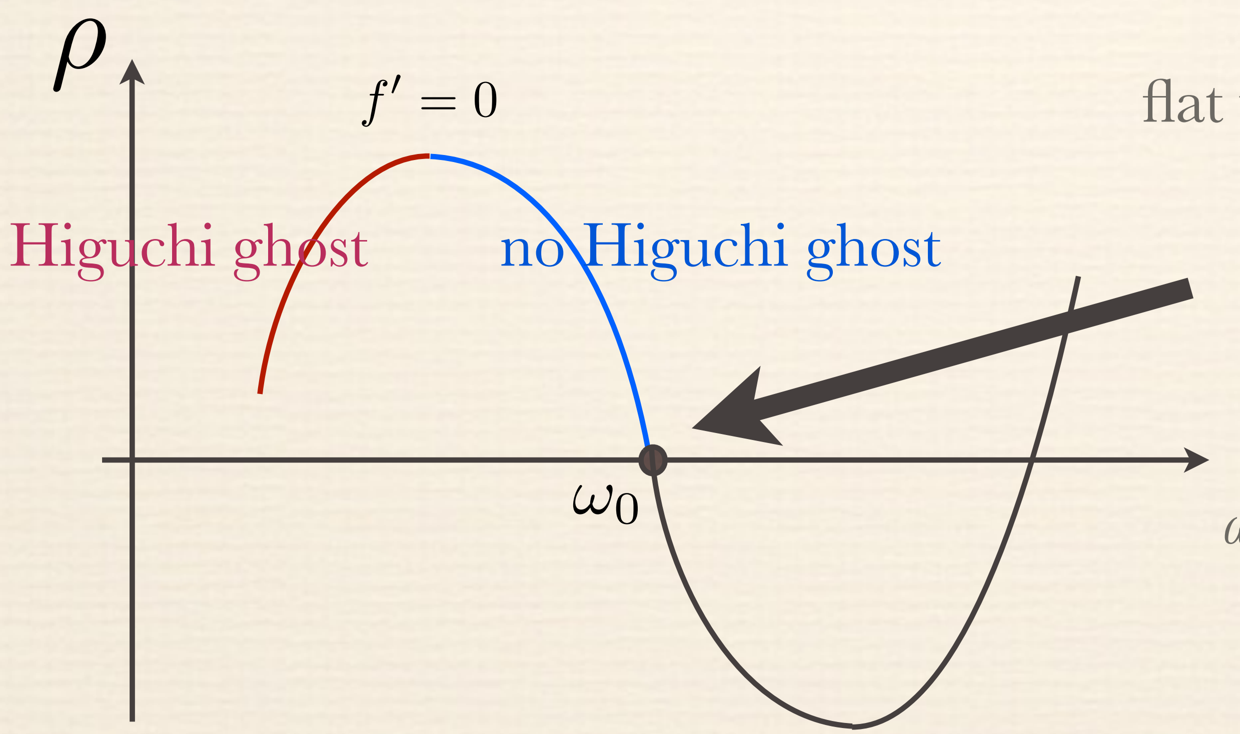


flat vacuum $H = 0, \rho = 0$

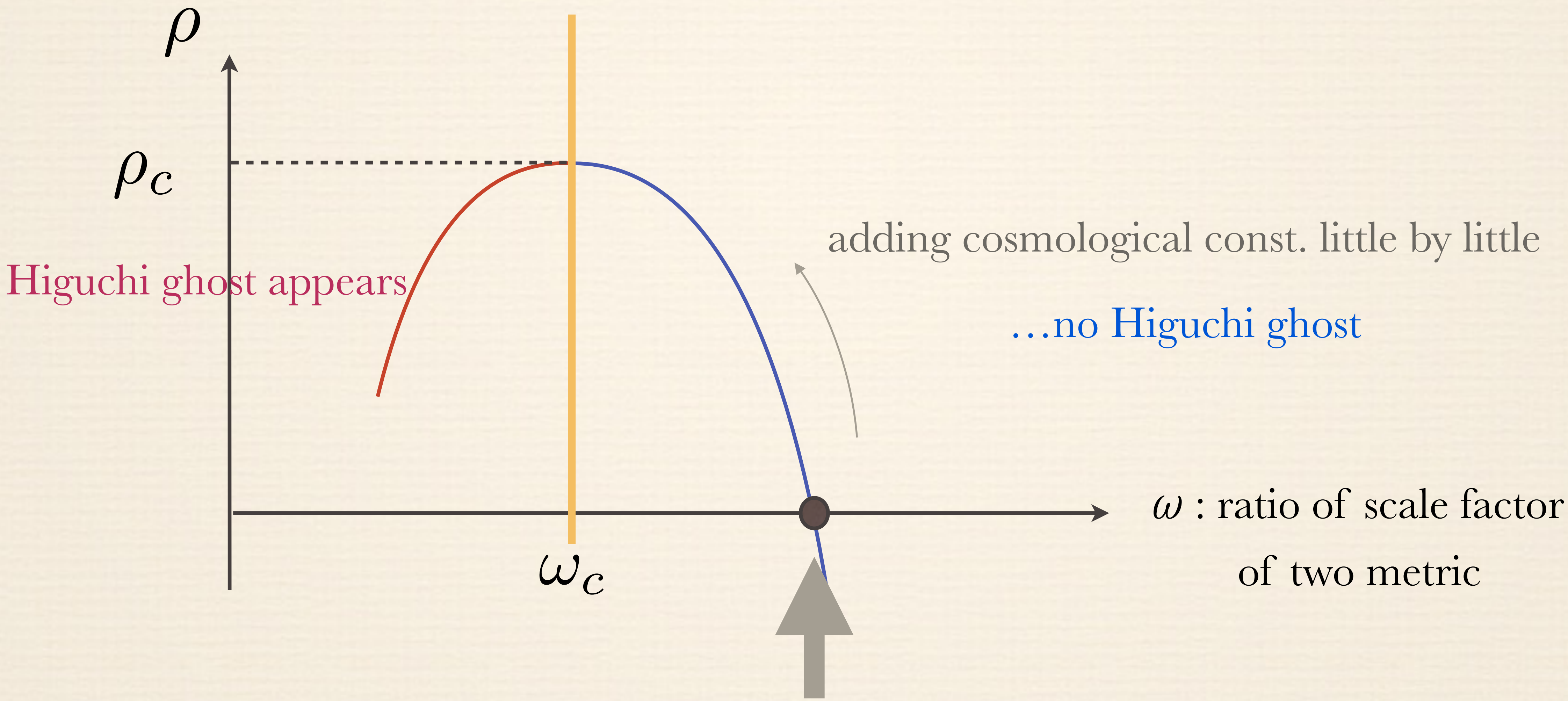
$$f'(\omega_0) = -3\left(1 + \frac{1}{\chi\omega_0^2}\right)\Gamma(\omega_0)$$

...negative

when $\Gamma > 0 \Leftrightarrow m_{eff}^2 > 0$

Higuchi ghost in dRGT bigravity

de Sitter solution does not exist above this critical density,
and Higuchi ghost appears after crossing the critical ω.



choose the branch connected to the vacuum flat spacetime
with positive graviton mass