Relativistic cosmological modelling for non-linear structure formation

Viraj A A Sanghai

Queen Mary University of London

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VAAS and T. Clifton

Based on **Arxiv:** *1503.08747 , Phys. Rev. D 91, 103532*

Based on **Arxiv:** *1604.06345 , Phys. Rev. D 94, 023505*

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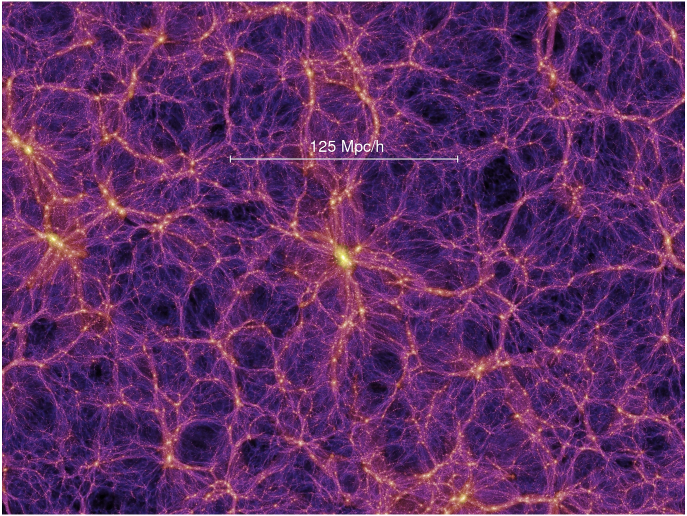
Motivation



Millenium simulation from Volker Springel et al.

Motivation

- Theoretical: The standard FLRW top down approach with cosmological perturbation theory is not ideal.
- Observables: This could be important to interpret data from large-scale surveys such as *Euclid and SKA (Square Kilometre Array).*

Our approach

Arxiv:1503.08747

- Bottom-up approach
- We patch together weak field regions together
- Post-Newtonian approximation to gravity
- We construct a periodic model

Building a post-Newtonian cosmology

Arxiv:1503.08747

- We put a large numbers of cells next to each other to form a periodic lattice structure.
- Cell shape - regular polyhedra.

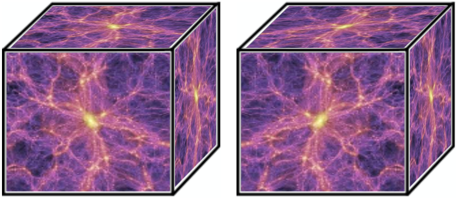


Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

Building a post-Newtonian cosmology

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Lattice Structure	**Lattice Curvature**	**Cell Shape**	**Cells per Lattice**
{333}	+	Tetrahedron	5
{433}	+	Cube	8
{334}	+	Tetrahedron	16
{343}	+	Octahedron	24
{533}	+	Dodecahedron	120
{335}	+	Tetrahedron	600
{434}	0	Cube	∞
{435}	-	Cube	∞
{534}	-	Dodecahedron	∞
{535}	-	Dodecahedron	∞
{353}	-	Icosahedron	∞

Building a post-Newtonian cosmology

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- We put a large numbers of cells next to each other to form a periodic lattice structure.
- Cell shape - regular polyhedra.
- The geometry of each cell is given by a perturbed Minkowski metric.
- Interior of each cell satisfies the post-Newtonian formalism.
- Cell size $<< H_0^{-1}$.
- We assume reflective symmetry across the boundary of the cells.

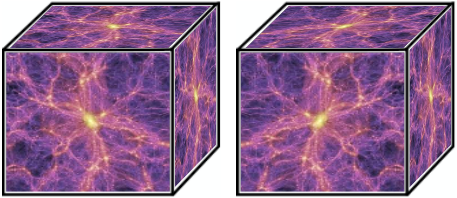


Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

- We join these perturbed Minkowski patches together to construct a global spacetime.

Building a post-Newtonian cosmology **Arxiv:**1503.08747

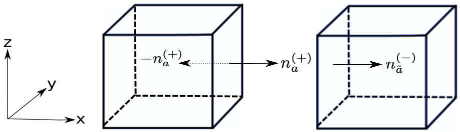
We match these cells together using Israel junction conditions,

$$\gamma_{ij}^{(+)} = \gamma_{ij}^{(-)},$$
$$K_{ij}^{(+)} = K_{ij}^{(-)},$$

where γ_{ij} is the induced metric, and K_{ij} is the extrinsic curvature of the boundary, defined by

$$K_{ij} \equiv \frac{\partial x^a}{\partial \xi^i}\frac{\partial x^b}{\partial \xi^j} n_{a;b} \ ,$$

where ξ^i denotes the coordinates on the boundary, and n^a is the space-like unit vector normal to the boundary.



Now, mirror symmetry implies that

$$K_{ij} = 0.$$

Post-Newtonian formalism

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Einstein's field equations are given by

$$R_{ab} = 8\pi G \left(T_{ab} - \frac{1}{2} T g_{ab} \right)$$

We treat this equation perturbatively, with an expansion parameter

$$\epsilon \equiv \frac{|\boldsymbol{v}|}{c} \ll 1,$$

where $\boldsymbol{v} = v^{\alpha}$ is the 3-velocity associated with the matter fields.

Extended Post-Newtonian formalism

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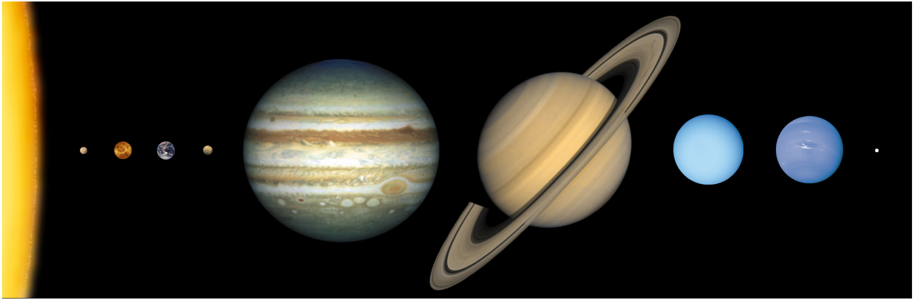

Image Credit: NASA

- Add a fluid that has $\rho \sim P$
- Include Λ at background order

Green's Functions

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- We do not assume asymptotic flatness, as one may do in the case of isolated systems. For example,

$$\Phi(\mathbf{x}, t) \neq -\frac{1}{4\pi G} \int_\Omega \frac{\rho(\mathbf{x}, t)}{|\mathbf{x} - \mathbf{x}'|} d^3x'$$

- We use a Green's function formalism

$$\Phi = \bar{\Phi} + 4\pi G \int_\Omega \mathcal{G}\rho \; dV + \int_{\partial\Omega} \mathcal{G}\boldsymbol{n} \cdot \nabla\Phi \; dA \,,$$

  where Ω is the spatial volume, $\mathcal{G}$ is the Green's function and $\boldsymbol{n}$ is the normal to the centre of the cell face.

- We can derive this equation for any potential that satisfies a Poisson-like equation using Gauss' theorem.

Post-Newtonian Order

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Constraint equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_M^{(2)} + \rho_r^{(2)}) - \frac{k}{a^2} + \frac{\Lambda}{3} + \mathcal{B}_2 + O(\epsilon^6),$$

where $\mathcal{B}_2$, is given by

$$\mathcal{B}_2 \simeq -\left(2\pi G \rho_M^{(2)} aL\right)^2 \left(1.50 - 0.80\frac{\Omega_r}{\Omega_M} + 1.76\frac{\Omega_k}{\Omega_M}\right)$$

$$\simeq B_m a^{-4} + B_r a^{-5} + B_k a^{-3}\,.$$

where B_m, B_r and B_k are constants and L is the length of the edge of a cell.

Summary and Future Work

- We have constructed a perturbative framework that consistently tracks non-linear effects of small-scale structure on the large-scale expansion.
- Calculate observables in these type of models.
- Generalize this model.
- With future large-scale surveys, such as Euclid and SKA (Square Kilometre Array), we will have more data to help understand the large-scale expansion of the universe.
- Inhomogeneous models may help us include any non-linear effects that we have not already considered.
- Thank you to the RAS, IOP and STFC for their support.

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