Ground state of the Universe and the Cosmological Constant

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VH, arXiv:0906.5562; IJMP D18(2009) 2265;
VH & B. Qureshi, PRL 116, (2016)

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Outline

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Is there a non-perturbative connection between
vacuum energy density, time and the cosmological constant?

If so, what exactly is the relationship?

$$\rho_{vac}(\Lambda, G, t) = ?$$

General Relativity $+ \Lambda +$ Matter

$$S = \int d^4x \sqrt{-g}\,(R(g) - 2\Lambda) + \int d^4x\; L(\phi_i; g_1 \cdots g_N)$$

$$G_{ab} + \Lambda g_{ab} = 8\pi G\; T_{ab}(\phi_i, g_1, g_2 \cdots)$$

- Λ and $g_1, g_2 \cdots g_N$ are coupling constants.
- The "vacuum energy" (density) of any quantum system is given by its ground state:

$$\hat{H}|\psi_0\rangle = E_0|\psi_0\rangle.$$

- For the gravity-matter system

$$\hat{H} = \hat{H}(G, \Lambda, g_1, g_2, \cdots g_N).$$

$\hat{H}$ is the total physical Hamiltonian of the matter+gravity system.

- Expect $E_0 = E_0(G, \Lambda, g_1, g_2, \cdots g_N)$.

But –

what is the physical Hamiltonian of a theory with time reparametrization invariance?

Requires some notion of time ... we will just fix a gauge to illustrate the idea.

Outline

Λ Phenomenology

- Observed Λ (WMAP)

$$\Lambda_{obs} = 1.27 \pm 0.07 \times 10^{-56} cm^{-2}$$

- Context of experimental observation: fit to the metric

$$ds^2 = -dt^2 + a^2(t)(d\vec{x})^2$$

and Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}(8\pi G \rho_M + \Lambda) - \frac{k}{a^2}.$$

ρ_M is energy density of matter (baryons, photons $\cdots$)
k is the curvature constant $(0, \pm 1)$

Microscopic origin of Λ

"crude theory" : assume origin is the vacuum energy of QFT on a fixed background

- $$E_0 = \sum_k \hbar k \longrightarrow \frac{V}{(2\pi)^3} \int d^3k \; \hbar k.$$
- $$\rho_\Lambda := \frac{E_0}{V} = \frac{\hbar}{(2\pi)^3} \int_0^{k_P} 4\pi k^3 dk = \frac{\hbar}{8\pi^2} k_p^4.$$
- $$\implies \Lambda_{theory} \equiv 8\pi G \rho_\Lambda \sim 10^{54} cm^{-2}.$$

so ... $\frac{\Lambda_{theory}}{\Lambda_{obs}} = 10^{110}$: this "theory" is pretty bad $\rightarrow$ the CC problem

The usual discussion/theory compares Λ_{theory} from QFT vacuum on Minkowski space (microscopic) with Λ_{obs} from FRW cosmology (macroscopic) "semiclassical gravity" –

$$G_{ab} + \Lambda g_{ab} = 8\pi G \langle\psi|\hat{T}_{ab}(\hat{\phi}, g)|\psi\rangle$$

hybrid classical-quantum equation: given $\hat{T}_{ab}(\hat{\phi}, g)$, find g and $|\psi\rangle$ self-consistently ....

"semiclassical gravity" –

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plug in

$$|\psi\rangle = |0\rangle + \epsilon|\psi^1\rangle + \epsilon^2|\psi^2\rangle + \cdots$$

$$g_{ab} = \eta_{ab} + \epsilon h^1_{ab} + \epsilon^2 h^2_{ab} + \cdots$$

0th order:

$$\Lambda = \frac{1}{4}\langle 0|\hat{T}_{ab}(\hat{\phi}, g)|0\rangle \eta^{ab}$$

1st order:

$$G_{ab}(\eta, h^1) + \Lambda h^1_{ab} = 8\pi G \left(\langle \psi^1 | \hat{T}_{ab}(\hat{\phi}, \eta) | 0 \rangle + \langle 0 | \hat{T}_{ab}(\hat{\phi}, \eta) | \psi^1 \rangle + \langle 0 | \hat{T}_{ab}(\hat{\phi}, h^1) | 0 \rangle \right)$$

– plug in Λ from 1st order and solve for h^1 and $|\psi^1\rangle \cdots$.

– at next order inner product, which depends on background, changes ...

Examine 0th order: what is

$$\langle 0|\,\hat{T}_{ab}(\hat{\phi}, g_1, g_2\cdots)|0\rangle\eta^{ab}?$$

On dimensional grounds expect this to be

$$M^2 f(g_1, g_2\cdots)$$

where $g_i(k)$, M is some "natural" mass scale, eg. Higgs mass for standard model ...

What type of function is f? Analytic?

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a curious example: in 2D Gross-Neveu a nonperturbative calculation gives

$$\langle T_{ab}\rangle \eta^{ab} \sim M^2 e^{-O(1)/g^2}.$$

(J. Holland, S. Hollands gr-qc/1305.5191)

Outline

A non-perturbative approach

gravity + matter system: phase space (q_{ab}, π^{ab}) (ϕ, P_ϕ).

- notion of vacuum requires a physical Hamiltonian
- physical hamiltonian $H(q, \pi, \phi, P_\phi; \Lambda^0, g_1, g_2, \cdots g_n, t)$ requires a notion of time.
- with these in hand, solve ground state problem

$$\hat{H}|q, \phi\rangle_0 = E_0(t, \Lambda^0, g_i)|q, \phi\rangle_0$$

for the vacuum energy E_0 and state $|q, \phi\rangle_0$.

So ... a non-perturbative view gives the connection

vacuum energy $\leftrightarrow$ true matter-gravity Hamiltonian $\leftrightarrow$ time $\leftrightarrow$ Λ

Expect on general grounds

$$\rho_{vac} = M_P^4 \; f(G, \Lambda, t, g_1, g_2 \cdots).$$

with f a dimensionless function of its arguments.

Outline

FRW cosmology with scalar field

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

- phase space $(a(t), p_a(t))$ and $(\phi(t), p_\phi(t))$, potential $V(\phi)$.
- volume time gauge: $t = a^3 = \sqrt{q}$. (more properly $t = t_p(a/a_0)^3$)

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- volume time gauge: $t = a^3 = \sqrt{q}$. (more properly $t = t_p(a/a_0)^3$)
- corresponding Hamiltonian is

$$H_P = \pm\sqrt{\frac{8}{3}\Lambda + 8\pi G\left(\frac{p_\phi^2}{2t^2} + V(\phi)\right)}$$

This suggest that even in the simplest of cases ("zero mode") the connection between Λ and ρ_{vac} is not expected to be linear.

vacuum energy density

$$\rho = \frac{H_P}{a^3} = \frac{H_P}{t}$$

$$= \sqrt{\frac{8}{3t^2}\Lambda + 8\pi G\left(\frac{p_\phi^2}{2t^4} + \frac{1}{t^2}V(\phi)\right)}$$

This converts to an operator in the quantum theory:

$$\rho \to \hat{\rho}$$

Nonperturbative quantum vacuum for this model with

$$V(\phi) = \frac{1}{2} m^2 \phi^2.$$

Argument of the square root can be diagonalized – S.H.O. with a shift:

$$\left[\frac{8}{3}\Lambda + \left(\frac{\hat{p}_\phi^2}{2t^2} + \frac{1}{2} m^2 \hat{\phi}^2\right)\right] \Psi(\phi) = E^2 \Psi(\phi)$$

$$E_n^2 = \frac{8}{3}\Lambda + \left(n + \frac{1}{2}\right)\omega, \qquad \omega = \frac{m}{t}$$

... therefore lowest energy density eigenvalue “vacuum” is

$$\rho_{vac} = m_p^2 \sqrt{\frac{8}{3t^2}\Lambda + \frac{m}{2t^3}}$$

For large t

$$\rho_{vac} = \frac{\rho_P}{\bar{t}} \sqrt{\frac{8}{3}\Lambda l_P^2}$$

$\bar{t} = t/t_P$.

With inhomogeneities

flat FRW + Λ + scalar field + fluctuations

Put in the ADM 3+1 action

$$q_{ab} = a(t)e_{ab} + \delta h_{ab}(x,t), \qquad \pi^{ab} = \frac{1}{6a}p_a(t)e^{ab} + \delta\pi^{ab}(x,t)$$
$$\phi = \phi(t) + \delta\phi(x,t), \qquad P_\phi = p_\phi(t) + \delta p_\phi(x,t)$$

Fourier transform

$$(\delta h_{ab}(t,k), \delta\pi^{ab}(t,k)), \quad (\delta\phi(t,k), \delta p_\phi(t,k))$$

Fix volume time gauge fix again:

$$S = \int dtd^3x \left(p_\phi \dot{\phi} + \delta\pi^{ab}\delta\dot{h}_{ab} + \delta p_\phi \delta\dot{\phi} - H_P - N^a C_a \right).$$

$$\rho = \frac{\rho_P}{\bar{t}} \sqrt{\frac{8}{3}\left(\Lambda l_P^2 + \frac{1}{8\pi^2 \bar{t}^{4/3}} \right)}$$

Plugging in observed values for Λ and $t =$age of universe

$$\rho_{vac} \sim 10^{-103} \rho_P \sim 10^{-7} Kg/m^3.$$

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Summary

- In a non-perturbative formulation there is a direct connection between the time and vacuum energy and Λ – and no evident "CC problem."

- Exact $\rho_{vac}(G, \Lambda, t, g_1, g_2 \cdots)$ is not a linear function of Λ
- Used observed Λ to predict vacuum energy density of matter + gravity system.

If there remains a CC problem, it will look very different.

Many questions remain:

- More general models?
- time gauge dependence?
- ...

As the problem really involves quantum gravity, ... Assuming that the dynamics gives a unique answer for the vacuum, there will be a unique prediction for the cosmological constant. But that is, at best, a futuristic way of putting things. We are not anywhere near, in practice, to understanding how there would be a unique solution for the dynamics. In fact, with what we presently know, it seems almost impossible for this to be true...

E. Witten, The Cosmological Constant From the Viewpoint of String Theory hep-ph/0002297