Fourier-domain modulation and delay of gravitational wave signals: application to the response of LISA-type detectors and to precessing binaries

Sylvain Marsat (AEI Golm)
J. Baker (GSFC), P. Graff (APL), A. Buonanno (AEI)

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Motivation

(e)LISA prospective parameter estimation

- Bayesian sampling for parameter extraction is expensive: several 10^6 samples - limitation so far to inspiral and/or Fisher matrix

- Impact of merger/ringdown, higher modes, spins, precession, instrument design ?

- Available fast FD IMR waveform models: aligned spins (SEOBNRv2 ROM, PhenomD), precession (PhenomP, NR surrogate), compact FD amplitude/phase representation

- Fourier-domain response of the instrument for IMR waveforms ?

Modeling waveforms from precessing binaries

- Precessing waveform as a frame rotation of non-precessing waveform

- Fourier-domain transposition: beyond the inspiral, extension to merger-ringdown ?

Delayed and modulated signals

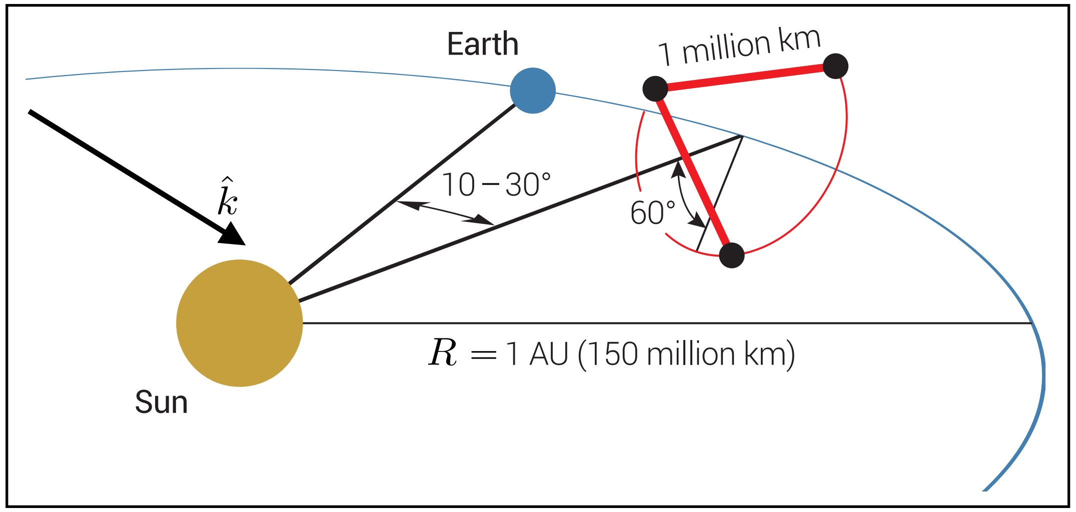
(e)LISA response

Frequency observables:

$$y = \Delta\nu/\nu$$

$$y_{slr} = \Phi_l(t_s - \hat{k}\cdot p_s) - \Phi_l(t_r - \hat{k}\cdot p_r)$$

$$\Phi_l = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot h^{\mathrm{TT}} \cdot n_l$$



$$\mathrm{FT}[F(t)h(t+d(t))] \leftrightarrow \tilde{h}(f),\ F(t),\ d(t)$$

F , d Instrument timescale (1yr) ⟷ h Waveform timescale

Frame rotation for precessing waveforms

$$h^{\mathrm{I}}_{\ell m} = \sum_{m'} D^{\ell\ *}_{m' m}(\alpha,\beta,\gamma)\ h^{\mathrm{P}}_{\ell m'}$$

- Inertial-frame hI obtained as a rotation of a precessing-frame hP
- Approximate hP as a non-precessing waveform (SpinTaylorF2, PhenomP, SEOBNR inspiral)

$$\mathrm{FT}[F(t)h(t)] \leftrightarrow \tilde{h}(f),\ F(t)$$

F Precessional timescale ⟷ h Waveform timescale

Delays and modulations in Fourier domain

A general view

$$\tilde{h}(f) = A(f)e^{-i\Psi(f)}$$

$$s(t) = F(t)h(t + d(t))$$

$$\tilde{s}(f) = \int df' \, \tilde{h}(f - f')\tilde{G}(f - f', f')$$

$$\tilde{G}(f, f') = \int dt \, e^{2i\pi f' t} e^{-2i\pi f d(t)} F(t)$$

Convolution with frequency-dependent kernel

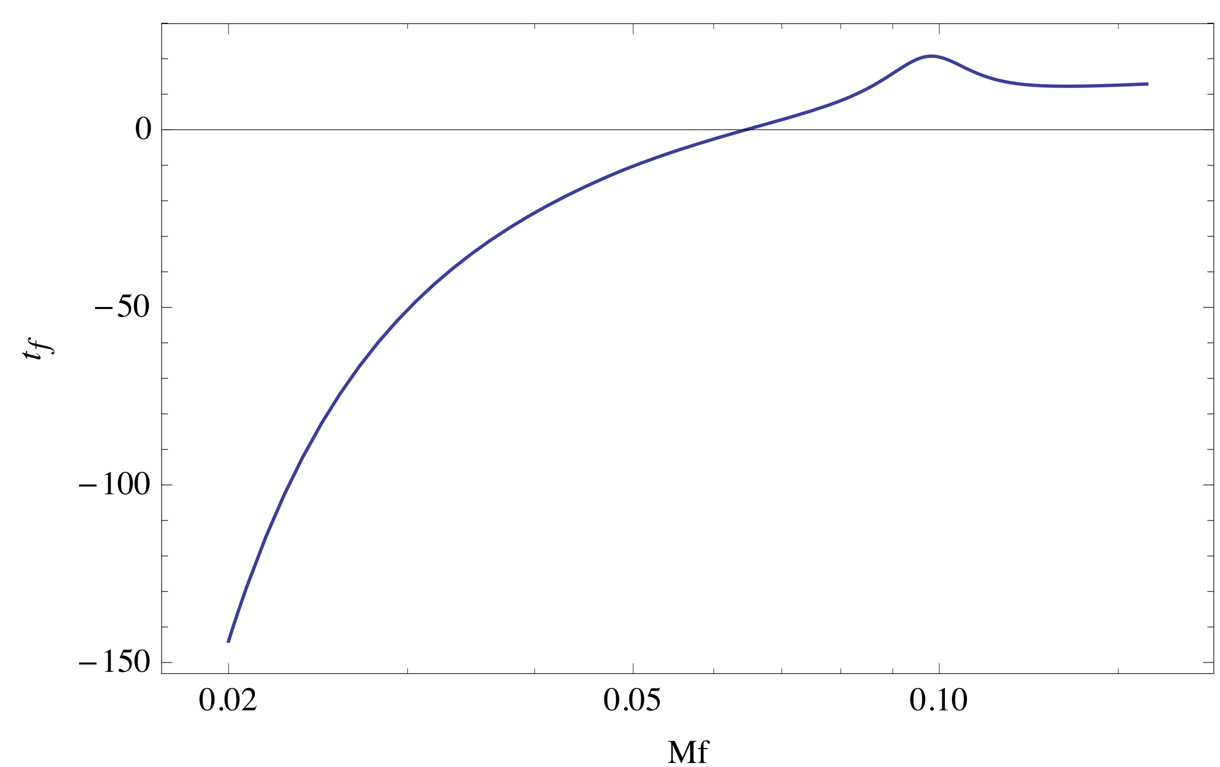
$\rightarrow$ Separation of timescales: if F, d have only frequencies <<f, local convolution - expand h(f-f') in f'

The leading order approximation

Keeping linear term in the phase:

$$t_f \equiv -\frac{1}{2\pi}\frac{\mathrm{d}\Psi}{\mathrm{d}f}$$

$$\tilde{s}(f) = H(f)\tilde{h}(f)$$
$$H(f) = G(f, t_f)$$



Delays and modulations in Fourier domain

Higher-order corrections

$\tilde{s}(f) = H(f)\tilde{h}(f)$ Leading order: $H(f) = G(f, t_f)$

Phase (quadratic term): $$H(f) = \sum \frac{1}{p!}\left(\frac{i}{8\pi^2}\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}\right)^p \partial_t^{2p} G(f, t_f)$$

Amplitude: $$H(f) = \sum \frac{1}{(2i\pi)^p p!}\frac{1}{A}\frac{\mathrm{d}^p A}{\mathrm{d}f^p}\partial_t^p G(f, t_f)$$

Frequency-dependence: $$H(f) = \sum \frac{1}{(2i\pi)^p p!}\partial_f^p \partial_t^p G(f, t_f)$$

Separation of timescales

(e)LISA:

$$\partial_t G \sim 2\pi f_0 G$$

$$f_0 = 1/\mathrm{yr} = 3.10^{-8}\mathrm{Hz} \ll f$$

Precessing binaries: $G = F(t)$

Inspiral: $\partial_t^2 F \sim \Omega_{\mathrm{prec}}^2 \sim 2\mathrm{PN}$

+ Merger-ringdown ?

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \sim T_{\mathrm{RR}}^2 \sim -2.5\mathrm{PN}$$

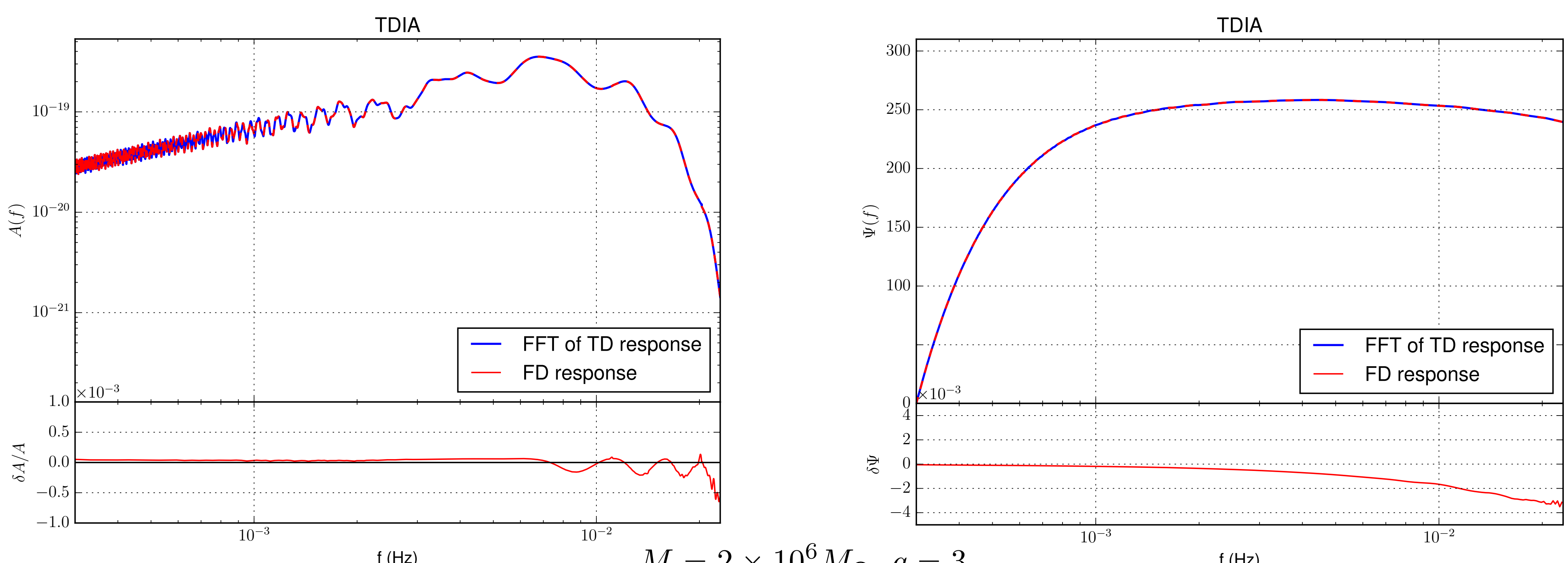
Application to the (e)LISA response

Magnitude of corrections

- Amplitude corrections $< 10^{-3}$
- Phase corrections $< 10^{-3}$
- High frequency: $\partial_t \partial_f G \sim R^2 f f_0 / c^2 \sim 10^{-1}$ at $1\mathrm{Hz}$

Example of errors

- TD response tested against SynthLISA
- Typical errors 10^{-3}



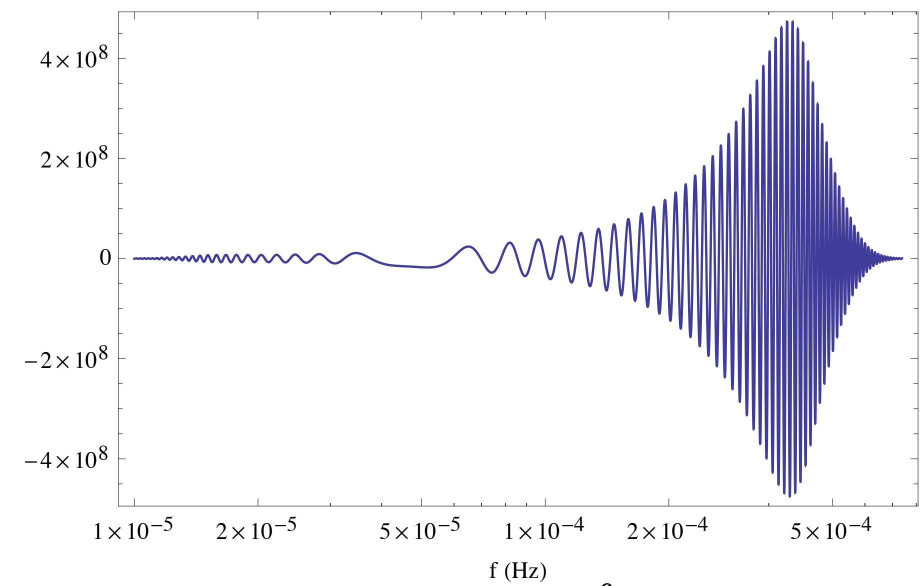
$M = 2 \times 10^6 M_\odot$, $q = 3$

Implementation

Accelerated no-noise overlaps

- Amplitude/phase: splines on ~200 intervals
- Cost increases when including HM

Example of oscillatory integrand



$$(h_1|h_2) = 4\mathrm{Re}\int df\, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} \longrightarrow \int_{f_i}^{f_{i+1}} P(f)e^{i[af+bf^2]} \longrightarrow \int_{f_i}^{f_{i+1}} e^{i[af+bf^2]}$$

Performances

- Fourier-domain IMR sparse amplitude/phase waveforms (ROM) (w/o 21,33,44,55 modes)
- Accelerated overlaps for amplitude/phase (no noise)

Likelihood cost	Single mode	5 modes
Lin. overlaps	~10ms	~50ms
Acc. overlaps	~1ms	~10ms

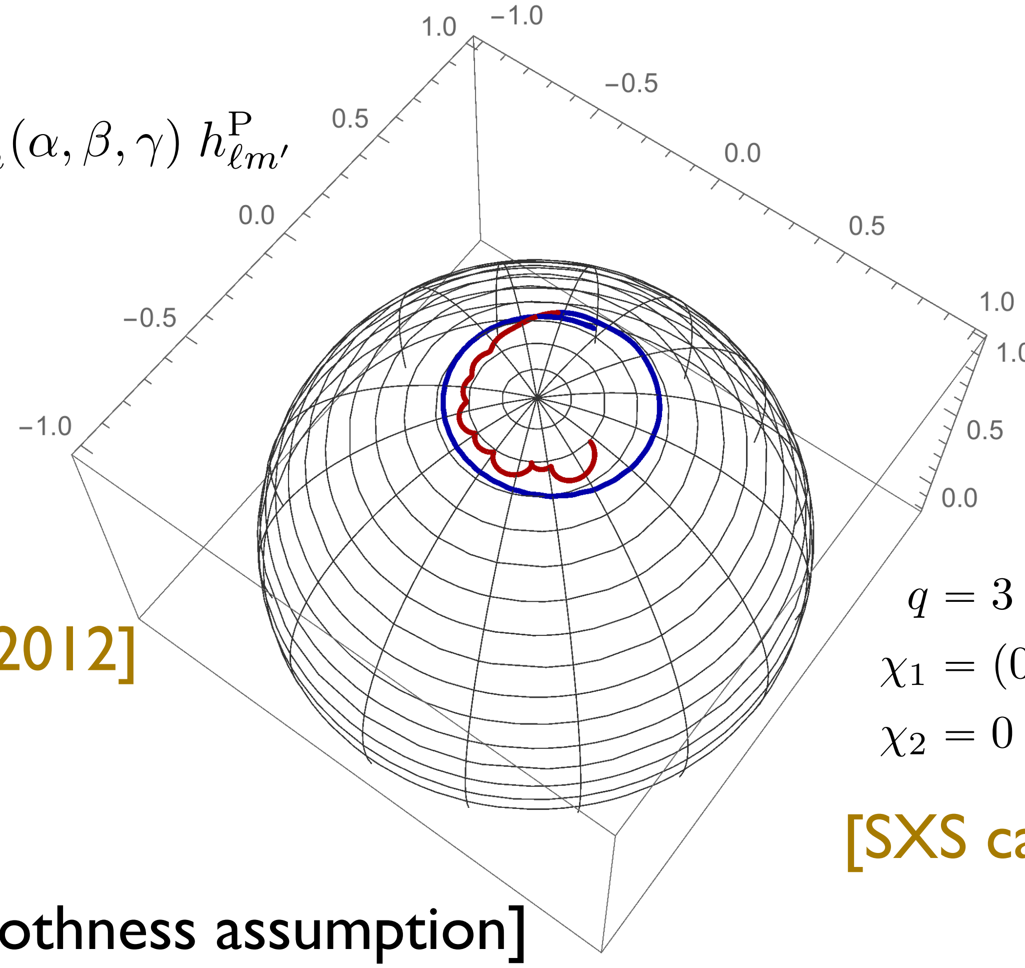
Precessing waveforms: frame evolution

Frame trajectory

$$h^{\mathrm{I}}_{\ell m} = \sum_{m'} D^{\ell\,*}_{m'm}(\alpha,\beta,\gamma)\; h^{\mathrm{P}}_{\ell m'}$$

- PN dynamics: $Z_{\mathrm{frame}} = \hat{L}$
- Extracting frame from the waveform (IMR) [O'Shaughnessy&al 2011]
- Approximate behaviour post-merger:

$\Omega_{\mathrm{frame}} \sim \omega^{\mathrm{QNM}}_{220} - \omega^{\mathrm{QNM}}_{210}$ [O'Shaughnessy&al 2012]

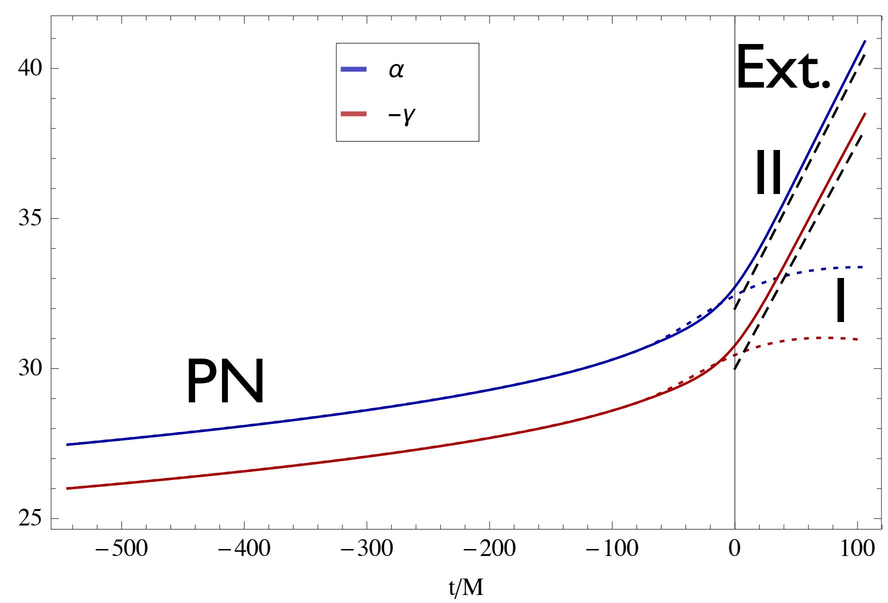


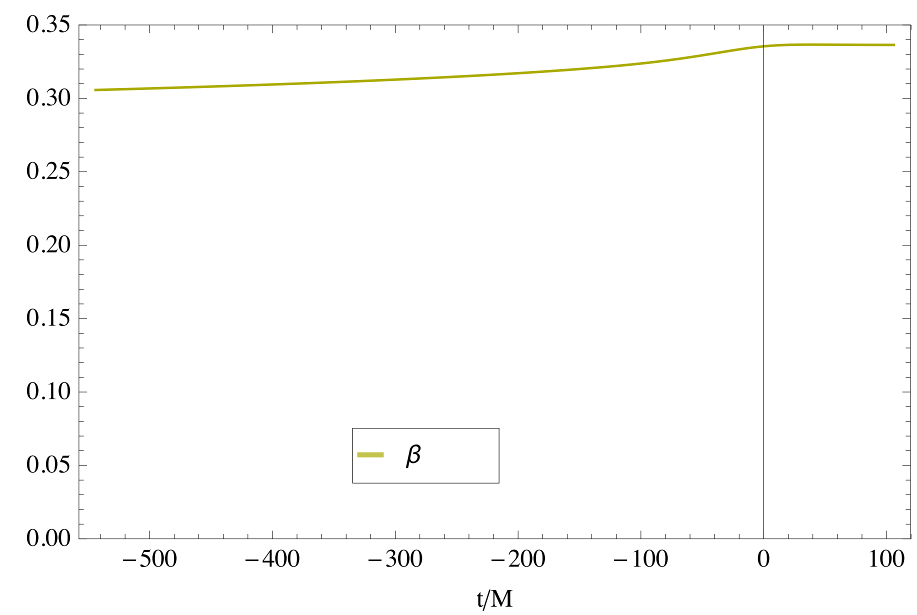
$q = 3$
$\chi_1 = (0.5, 0, 0)$
$\chi_2 = 0$

[SXS catalog]

Pre- and post-merger frame: toy model

[Smoothness assumption]





Relation to previous works

Previous works:
- Leading order (different MR) [SpinTaylorF2, PhenomP]
- Quadratic phase (SUA) [Klein&al 2014]

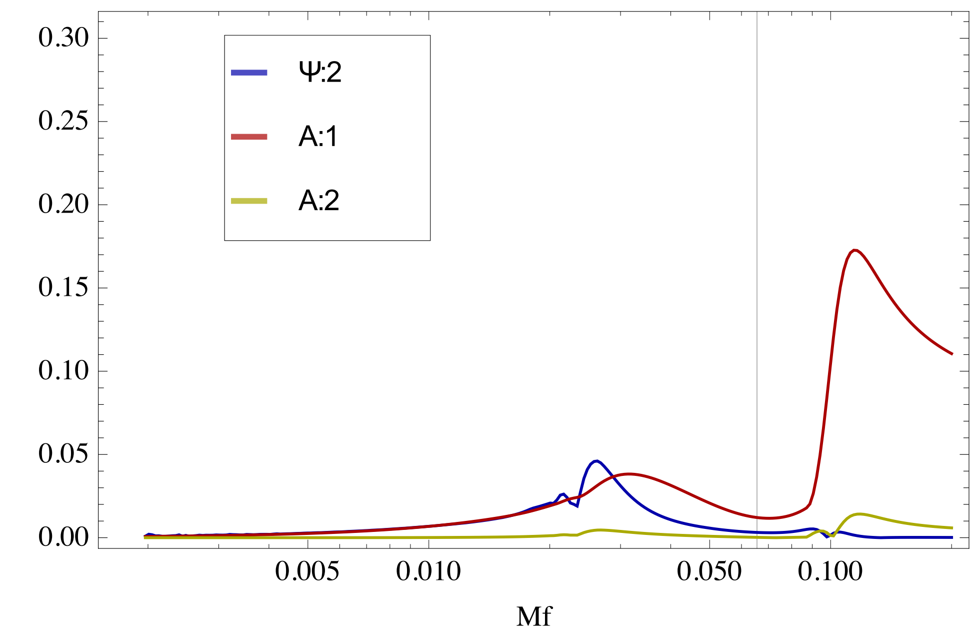
SPA/SUA	Fourier domain approach
$t_f \; : \; \omega(t_f) = \pi f$ (SPA)	$t_f = -\frac{1}{2\pi}\frac{\mathrm{d}\Psi}{\mathrm{d}f}$ (IMR)
$T_f = \frac{1}{\sqrt{2\dot{\omega}(t_f)}}$ Rad. Reac. (SUA)	$T_f^2 = \frac{1}{4\pi^2}\left\| \frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2} \right\|$ (IMR)
$\tilde{s}(f) = \tilde{h}(f) \sum \frac{(-i)^p}{2^p p!} T_f^{2p} \partial_t^{2p} F$ (SUA)	$\tilde{s}(f) = \tilde{h}(f) \sum \frac{(-i)^p}{2^p p!} T_f^{2p} \partial_t^{2p} F$ Taylor FD Quad. phase
$\tilde{s}(f) = \tilde{h}(f) \sum a_k F(t_f \pm k T_f)$ (Resum.)	$\tilde{s}(f) = \tilde{h}(f) \int dt \, \exp\left[-\frac{i}{2} \left(\frac{t - t_f}{T_f} \right)^2 \right] F(t)$

New corrections:
- Higher-order amplitude corrections $\mathrm{d}^p A / \mathrm{d} f^p$
- Local convolution approach for post-merger

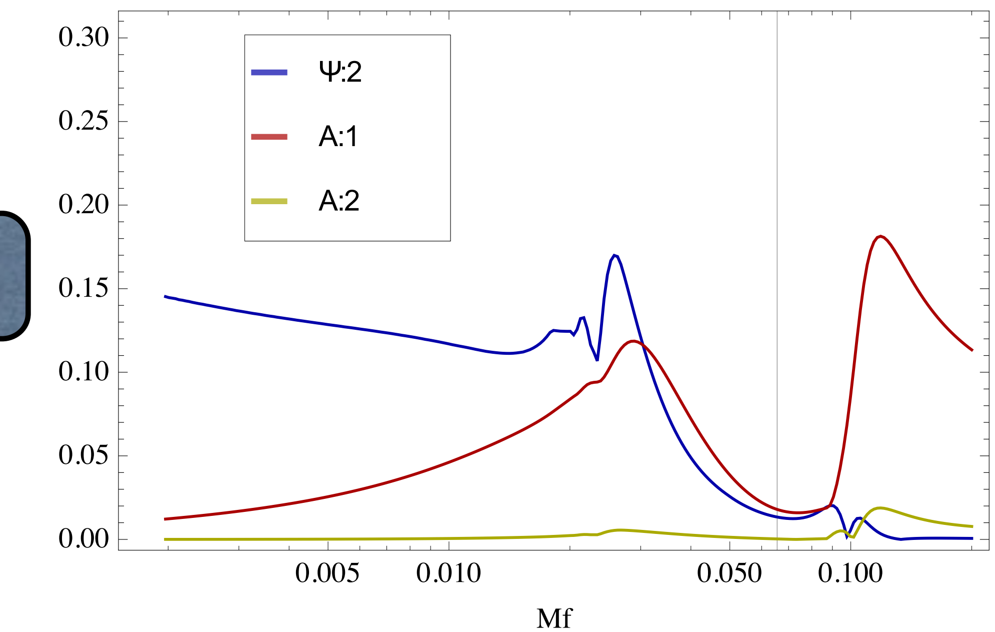
Precession: magnitude of corrections

Example: $q = 3,\ \chi_1 = (-0.3, 0.5, 0.7),\ \chi_2 = (0.3, -0.2, -0.5)$

$h^{\mathrm{P}}_{22} \to h^{\mathrm{I}}_{22}$

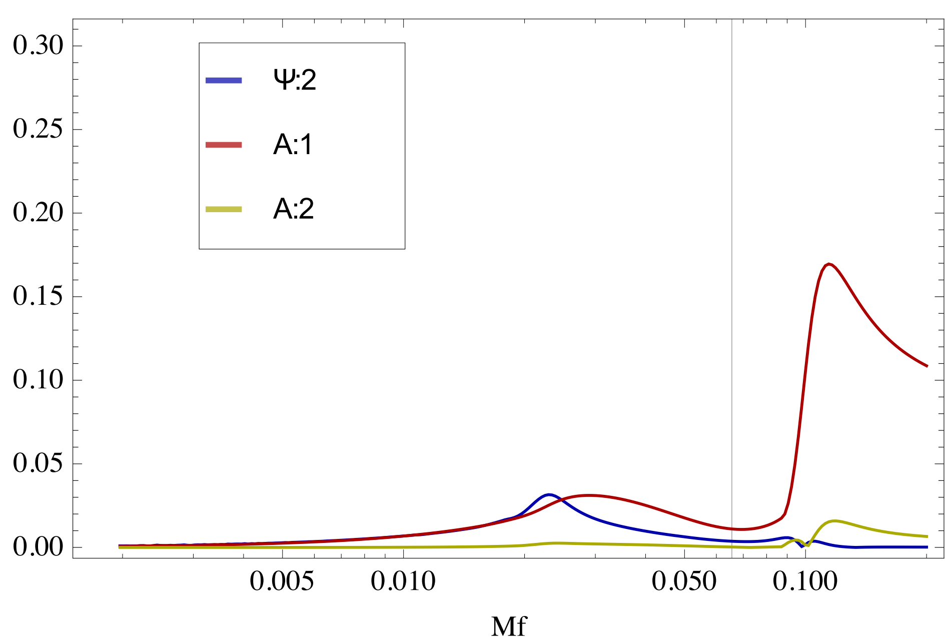


$h^{\mathrm{P}}_{22} \to h^{\mathrm{I}}_{21}$

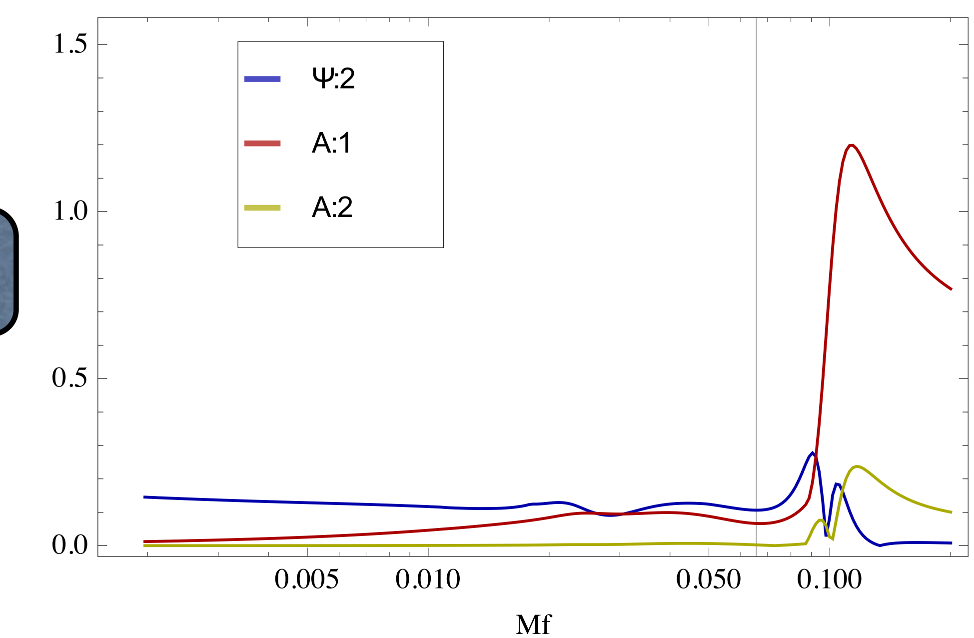


Case I

$h^{\mathrm{P}}_{22} \to h^{\mathrm{I}}_{22}$



$h^{\mathrm{P}}_{22} \to h^{\mathrm{I}}_{21}$

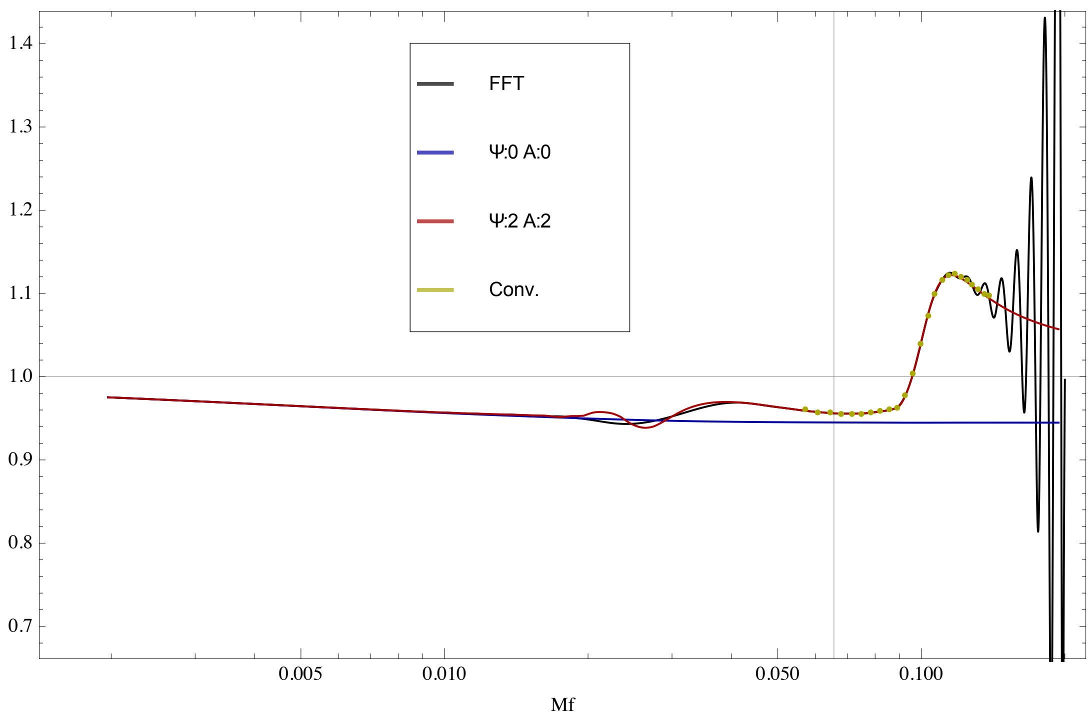


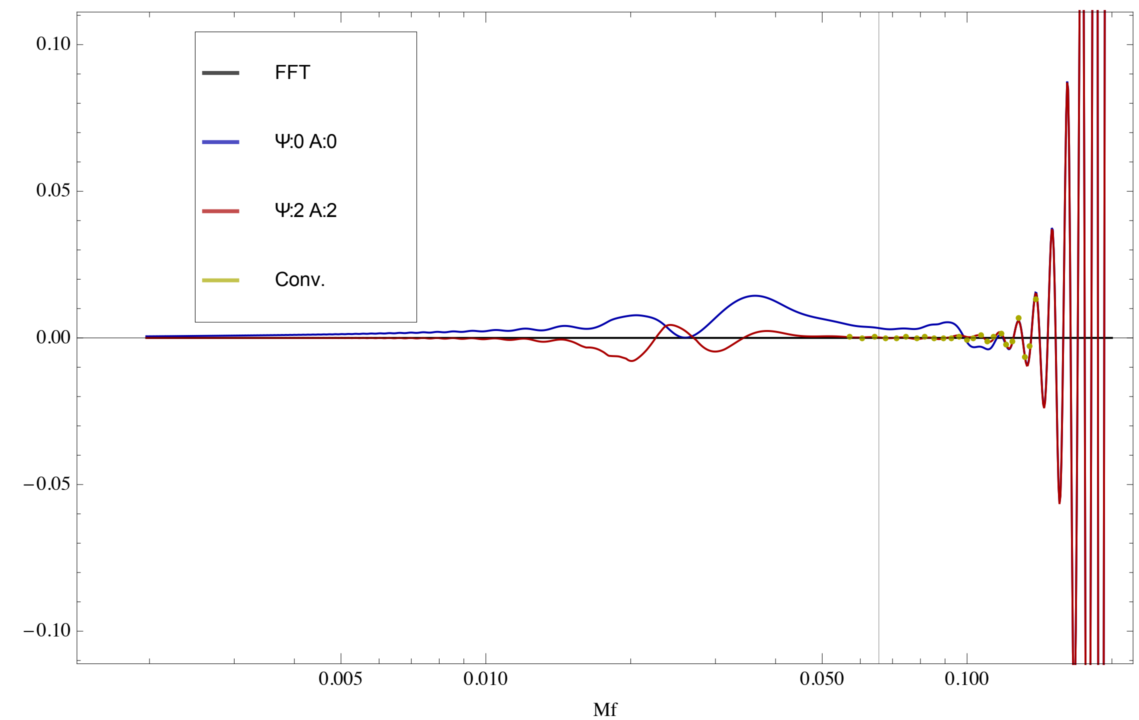
Case II

Precession: errors - case I

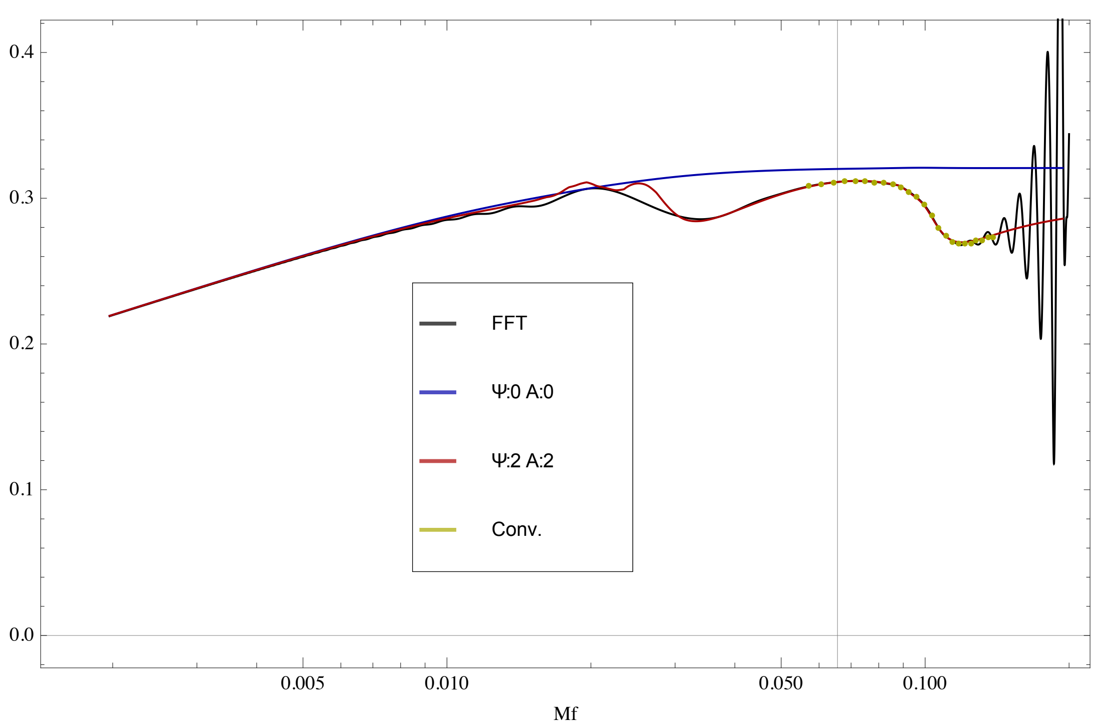
Amplitude relative to h22P | **Phase difference**

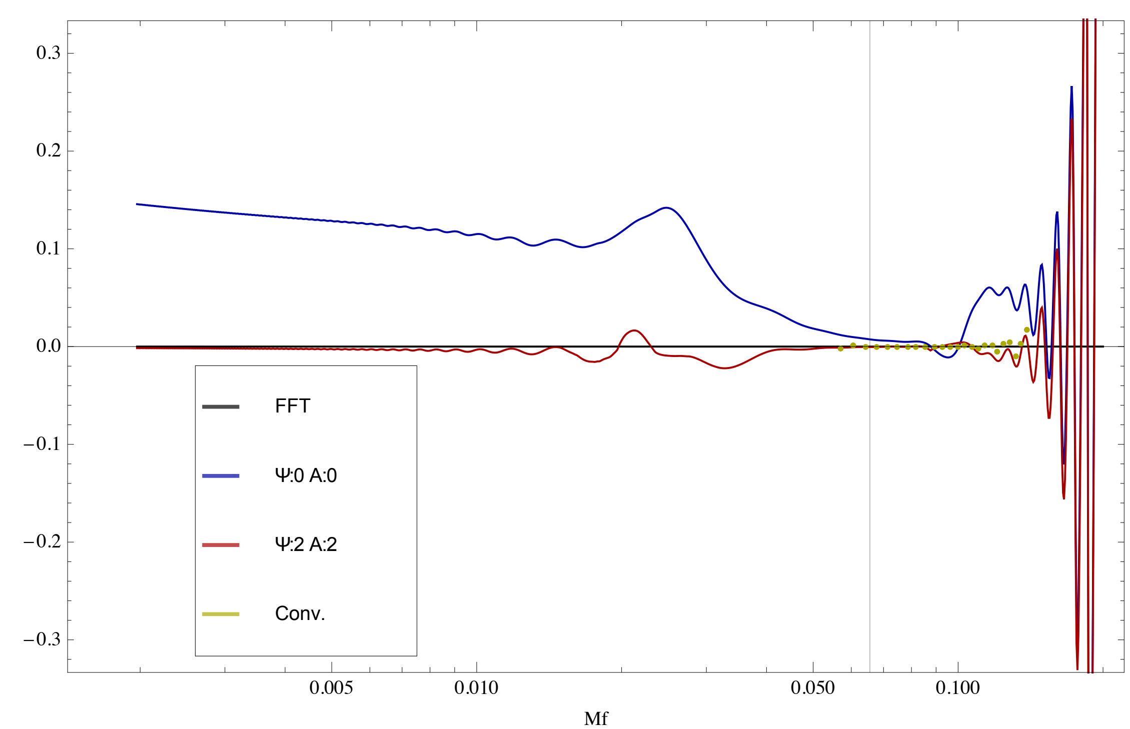
$h_{22}^{\mathrm{P}} \rightarrow h_{22}^{\mathrm{I}}$





$h_{22}^{\mathrm{P}} \rightarrow h_{21}^{\mathrm{I}}$



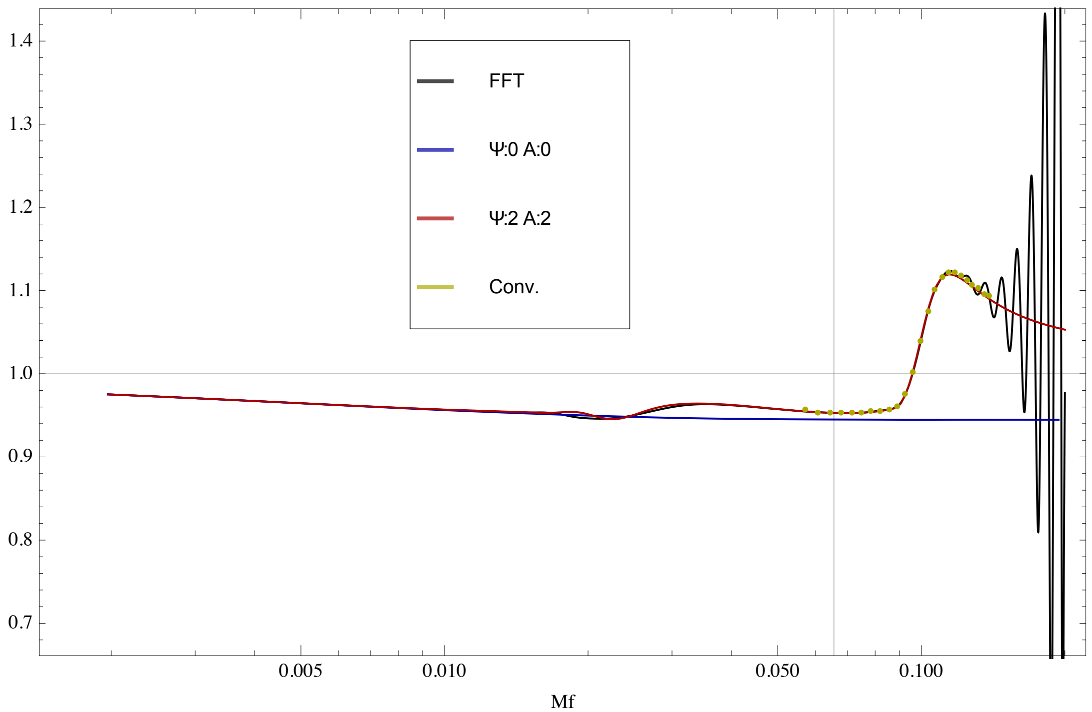


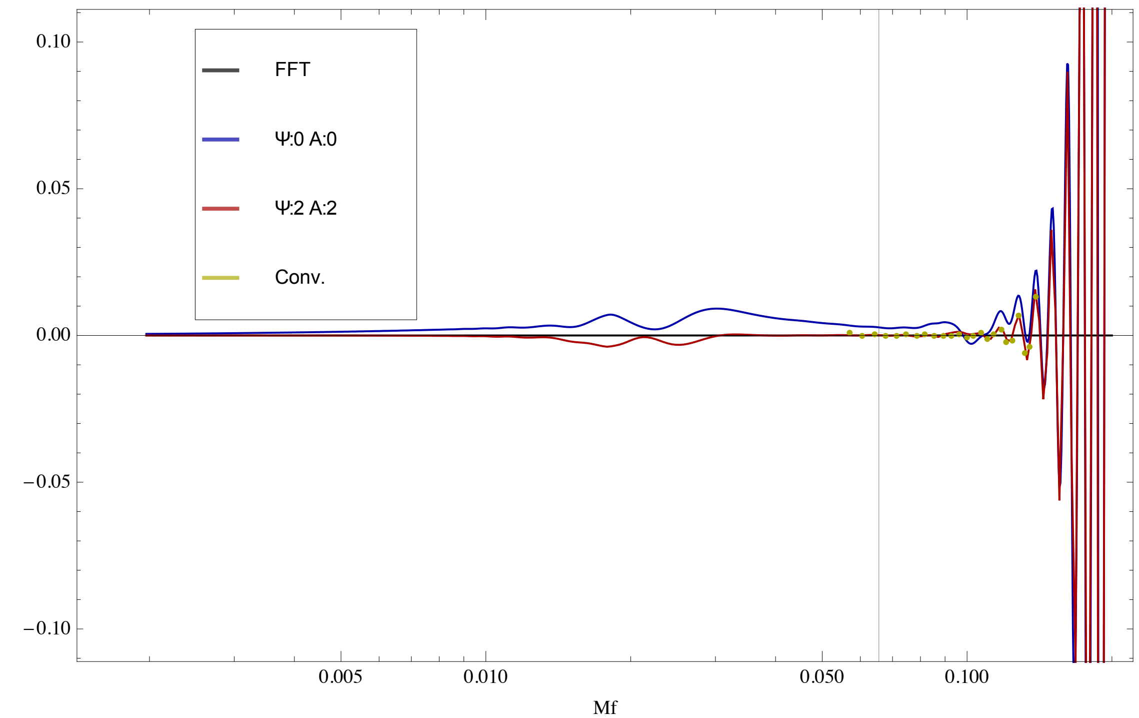
Precession: errors - case II

Amplitude relative to h22P

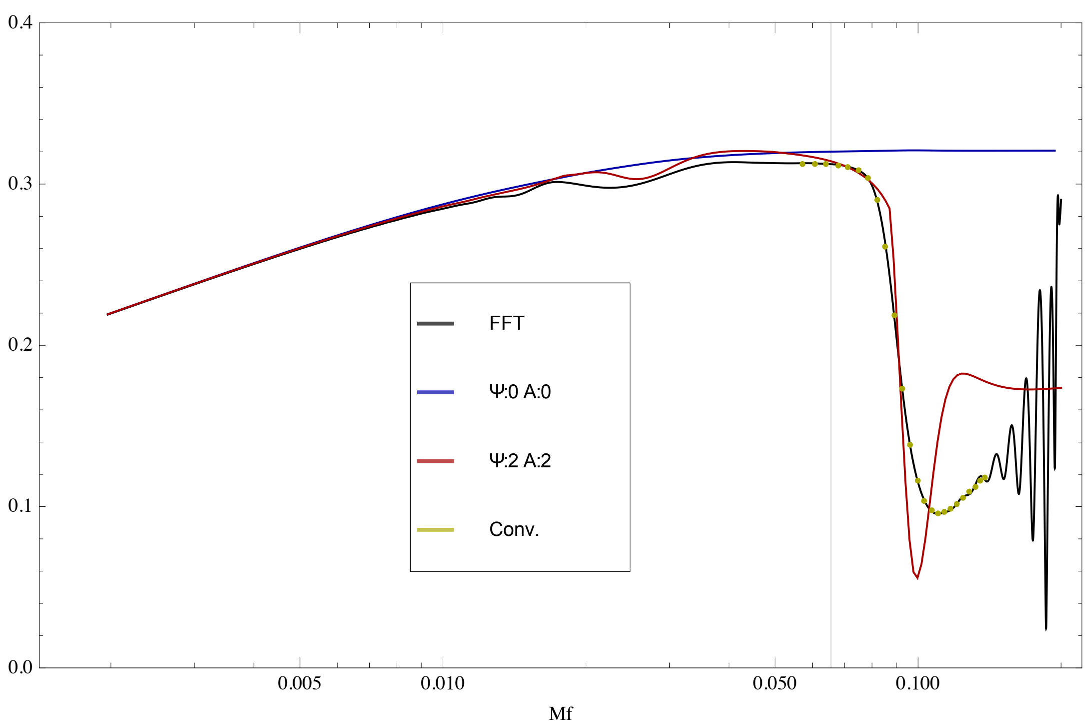
Phase difference

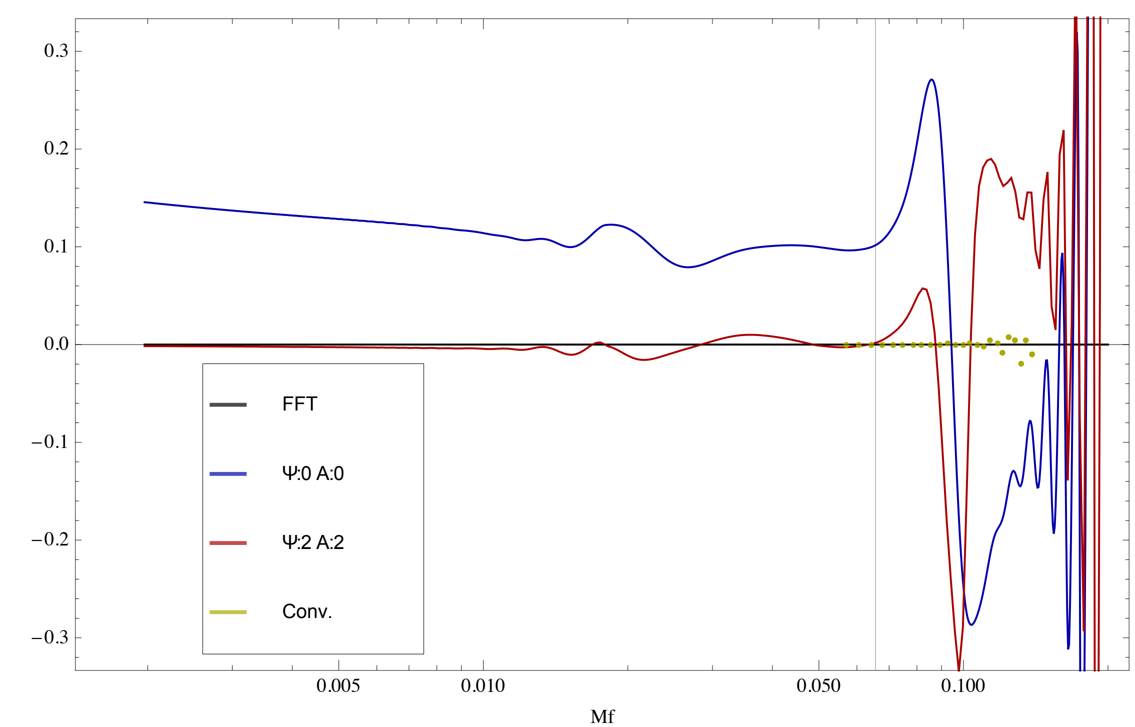
$h_{22}^{\mathrm{P}} \to h_{22}^{\mathrm{I}}$





$h_{22}^{\mathrm{P}} \to h_{21}^{\mathrm{I}}$

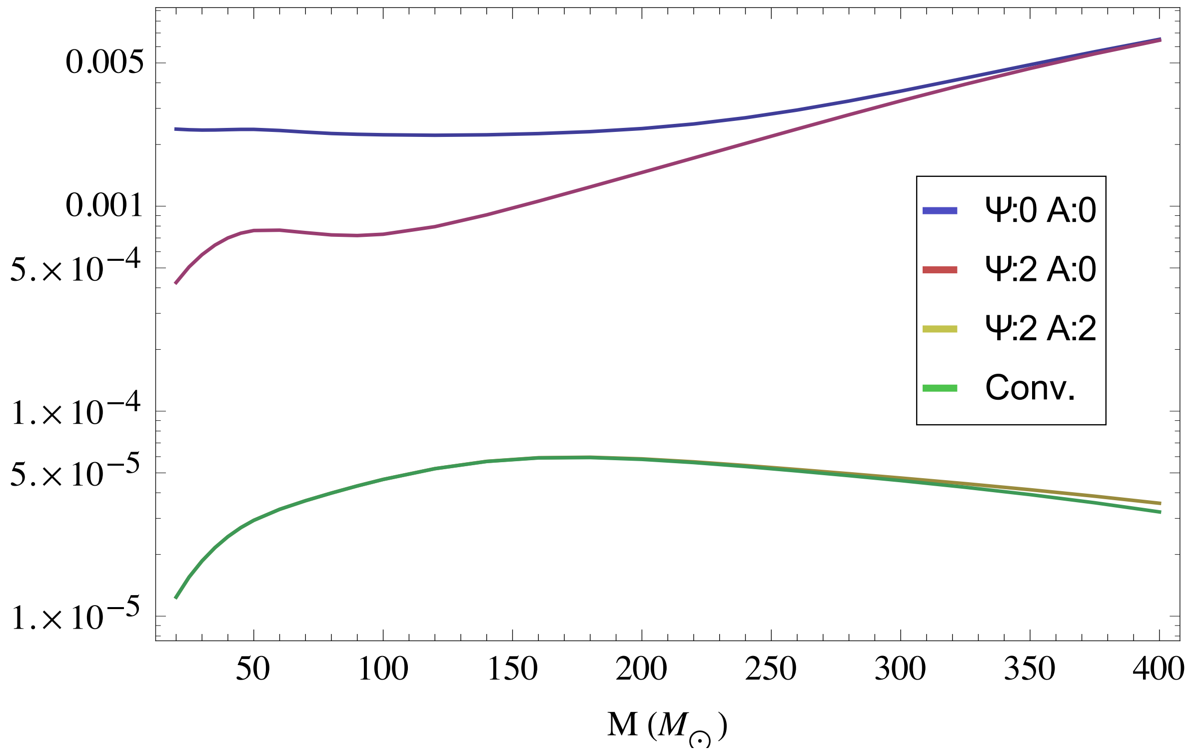




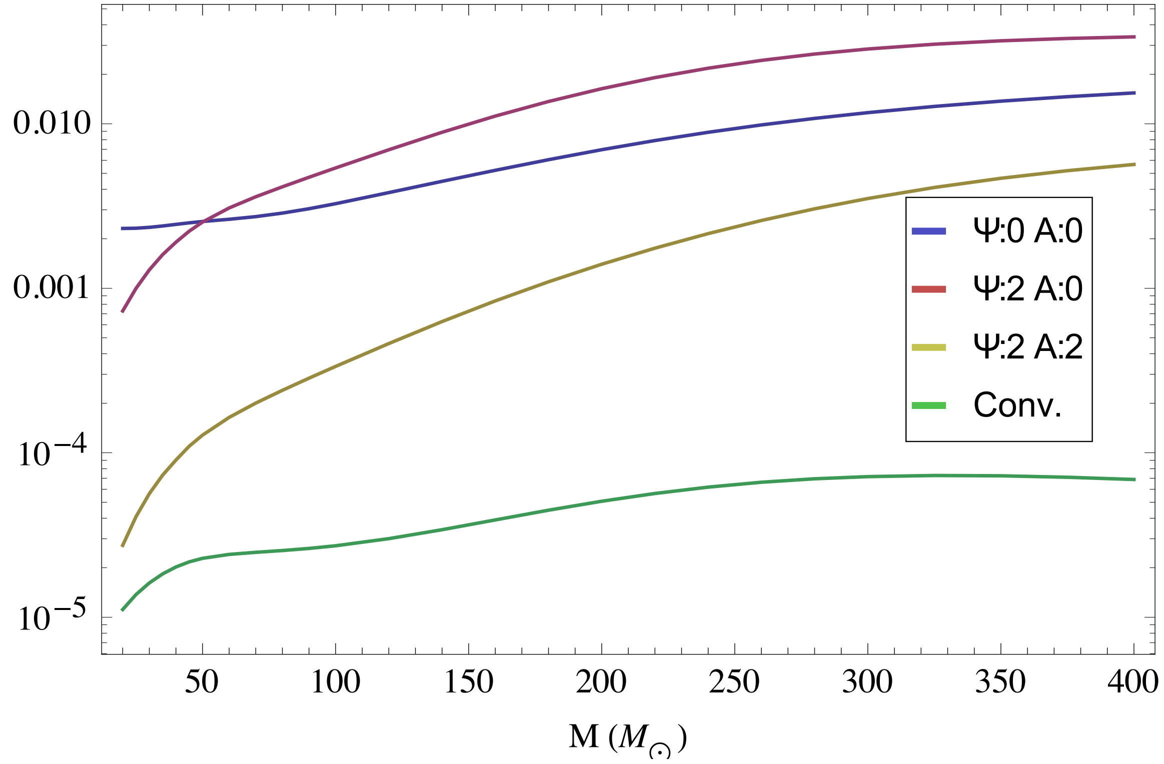
Precession: mismatches

[Preliminary]
[Broader exploration of parameter space needed]

Case I



Case II



$$\iota = \pi/2$$

Summary

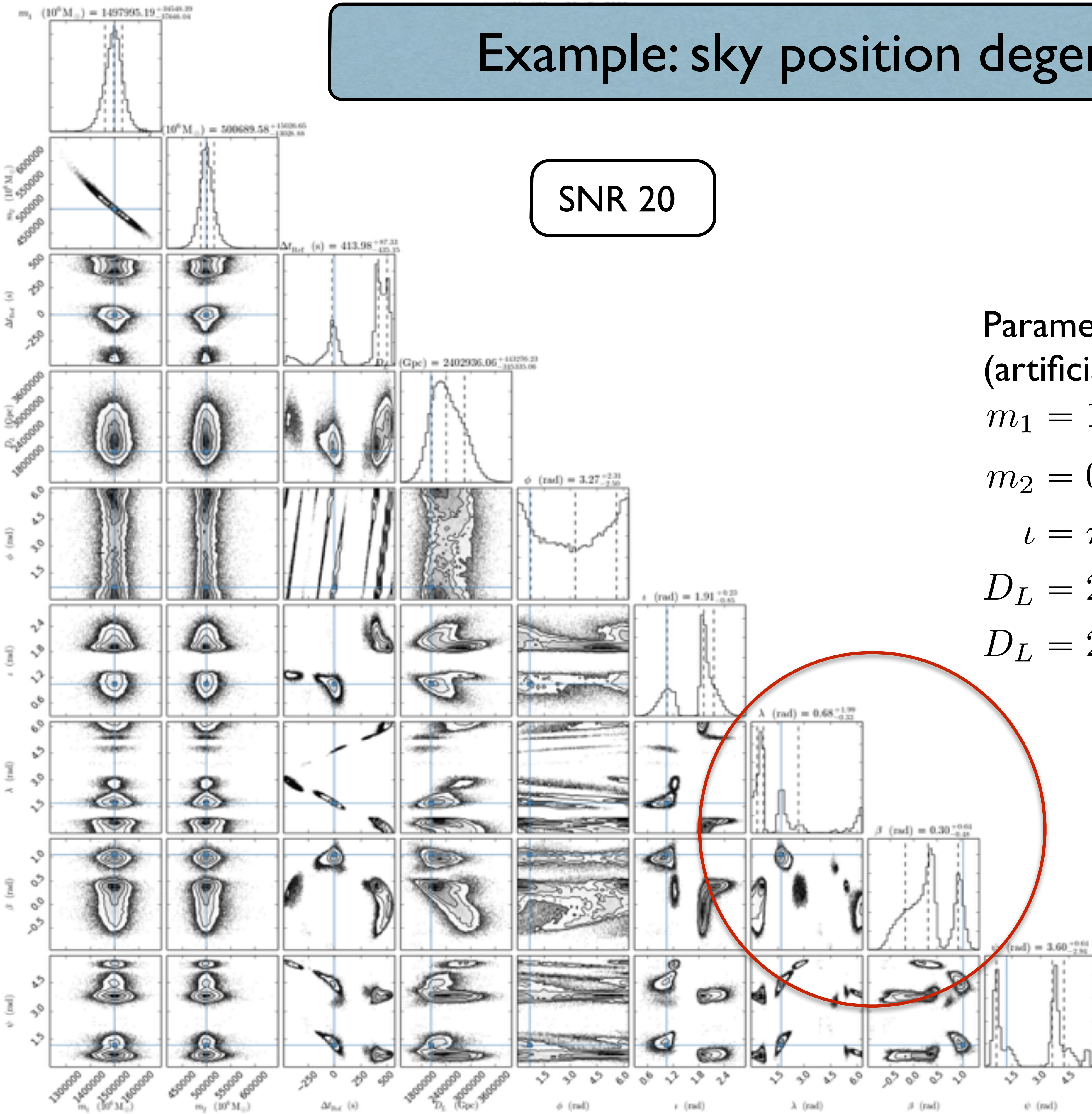
(e)LISA prospective parameter estimation

- Fourier-domain processing through the response of the instrument using t(f) correspondence
- Higher-order corrections available
- Applicable to all FD IMR waveform models: aligned spins (SEOBNRv2, PhenomD), precession (PhenomP), compact FD amplitude/phase representation
- Implementation using accelerated no-noise overlaps: few ms/likelihood
- Ongoing and future exploration: impact of MR, HM, spins and of instrument design

Modeling waveforms from precessing binaries

- Formally recover and extend previous results on FD frame rotation
- Inclusion of FD amplitude corrections, direct convolution approach for post-merger
- 'Mild' frame rotation: captured by amplitude corrections
- 'Fast' frame rotation: requires direct convolution
- New corrections are local to MR, and mostly in amplitude — small mismatches !
- More systematic exploration of parameter space is needed

Example: sky position degeneracies



SNR 20

Parameters:
(artificially distant)

$$m_1 = 1.5 \times 10^6 M_\odot$$

$$m_2 = 0.5 \times 10^6 M_\odot$$

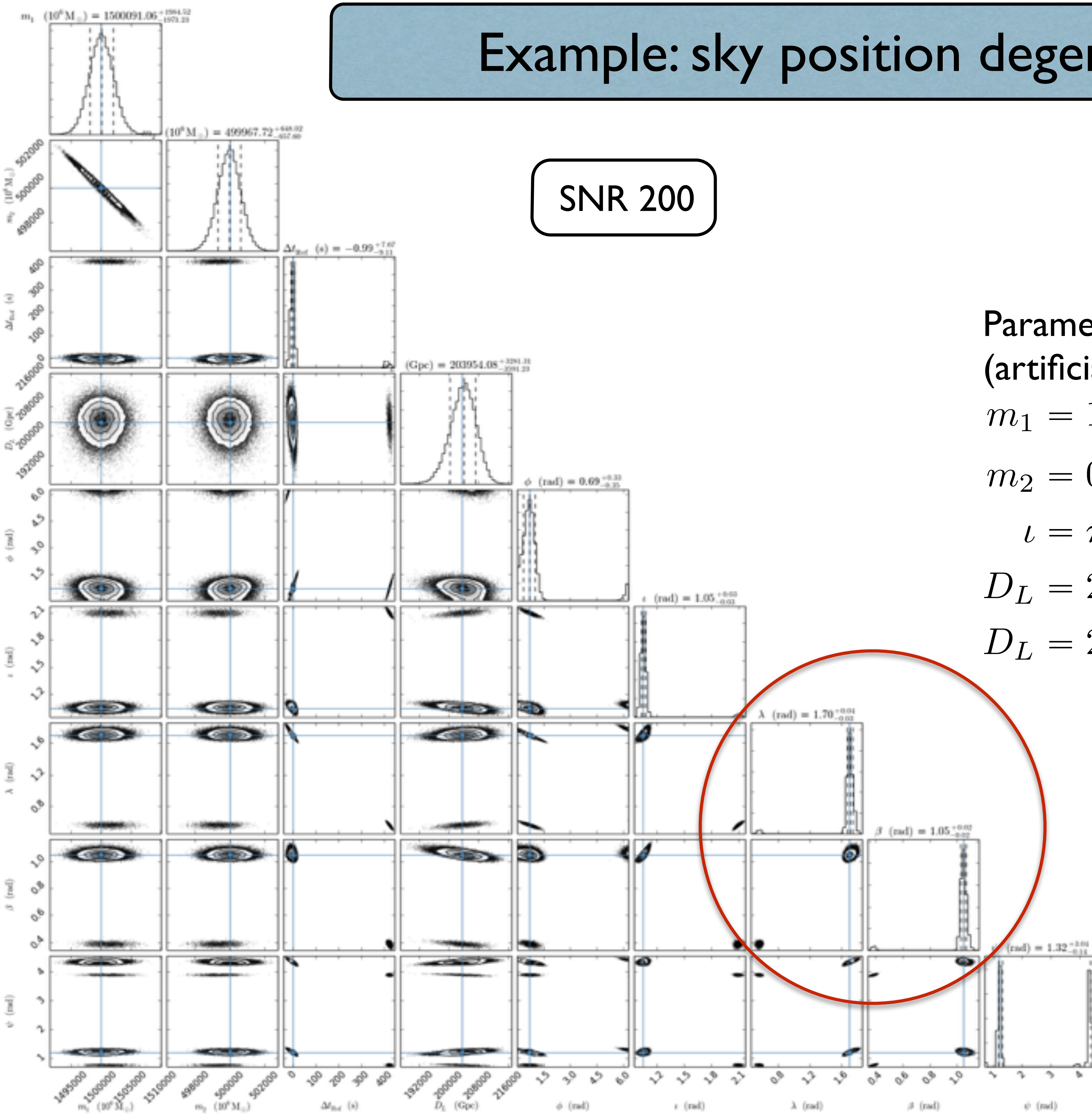
$$\iota = \pi/3$$

$$D_L = 2036\mathrm{Gpc}\ (\mathrm{SNR}\ 20)$$

$$D_L = 203.6\mathrm{Gpc}\ (\mathrm{SNR}\ 200)$$

Example: sky position degeneracies

SNR 200



Parameters:
(artificially distant)

$$m_1 = 1.5 \times 10^6 M_\odot$$

$$m_2 = 0.5 \times 10^6 M_\odot$$

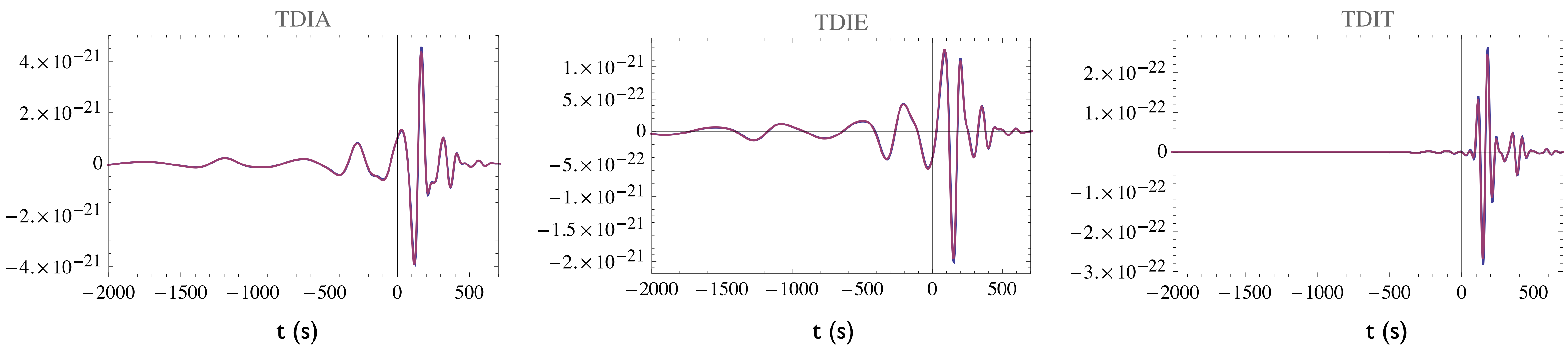
$$\iota = \pi/3$$

$$D_L = 2036\text{Gpc (SNR 20)}$$

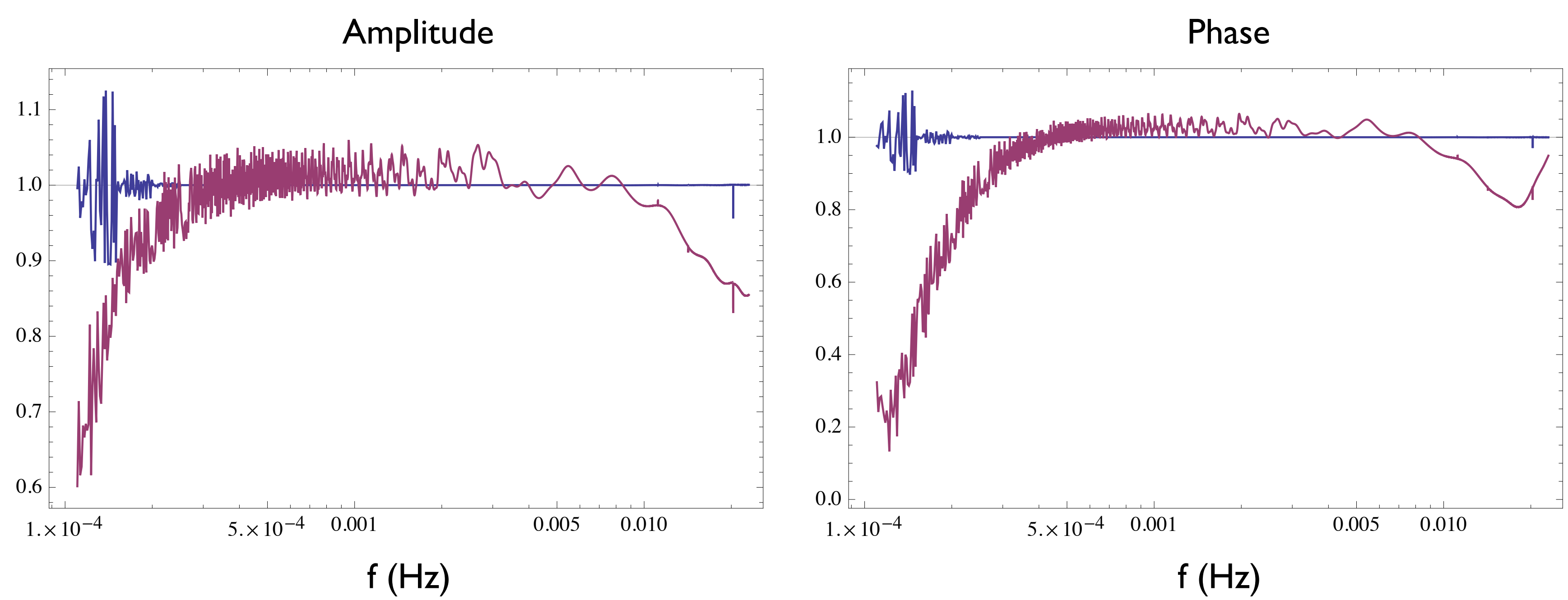
$$D_L = 203.6\text{Gpc (SNR 200)}$$

Example: sky position degeneracies

TD comparison to 2nd peak

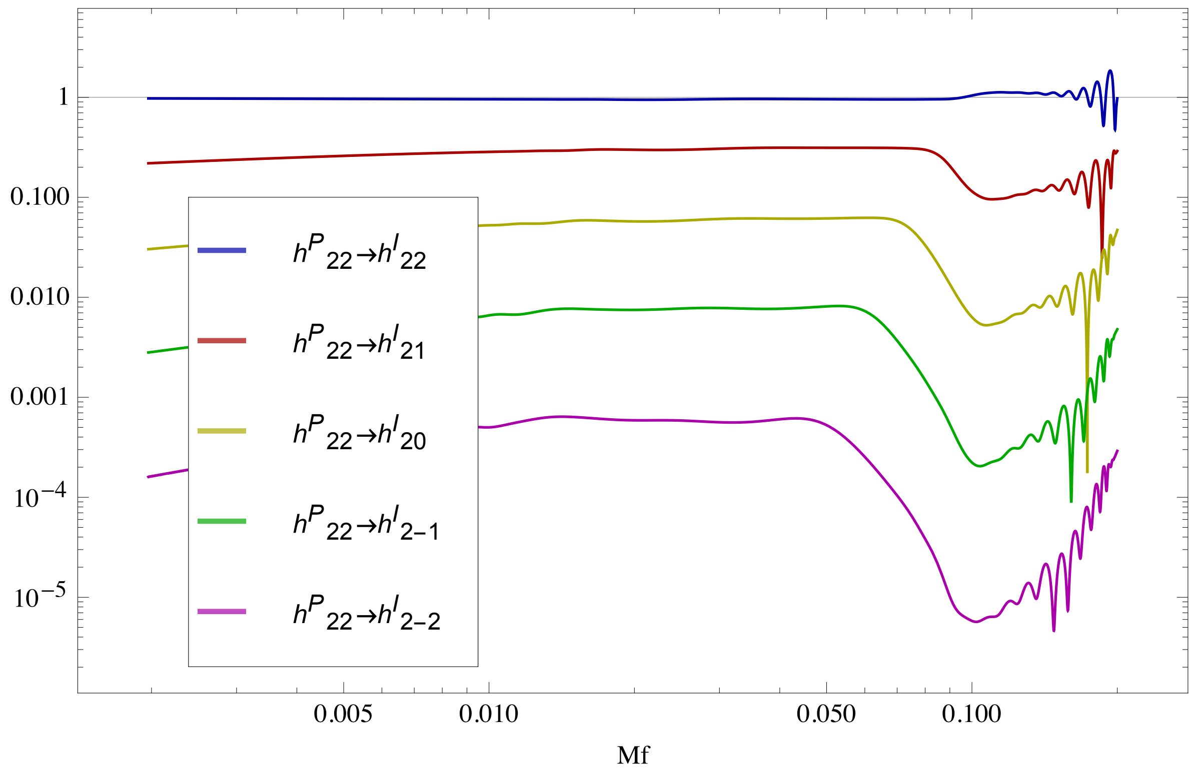


FD comparison to 2nd peak

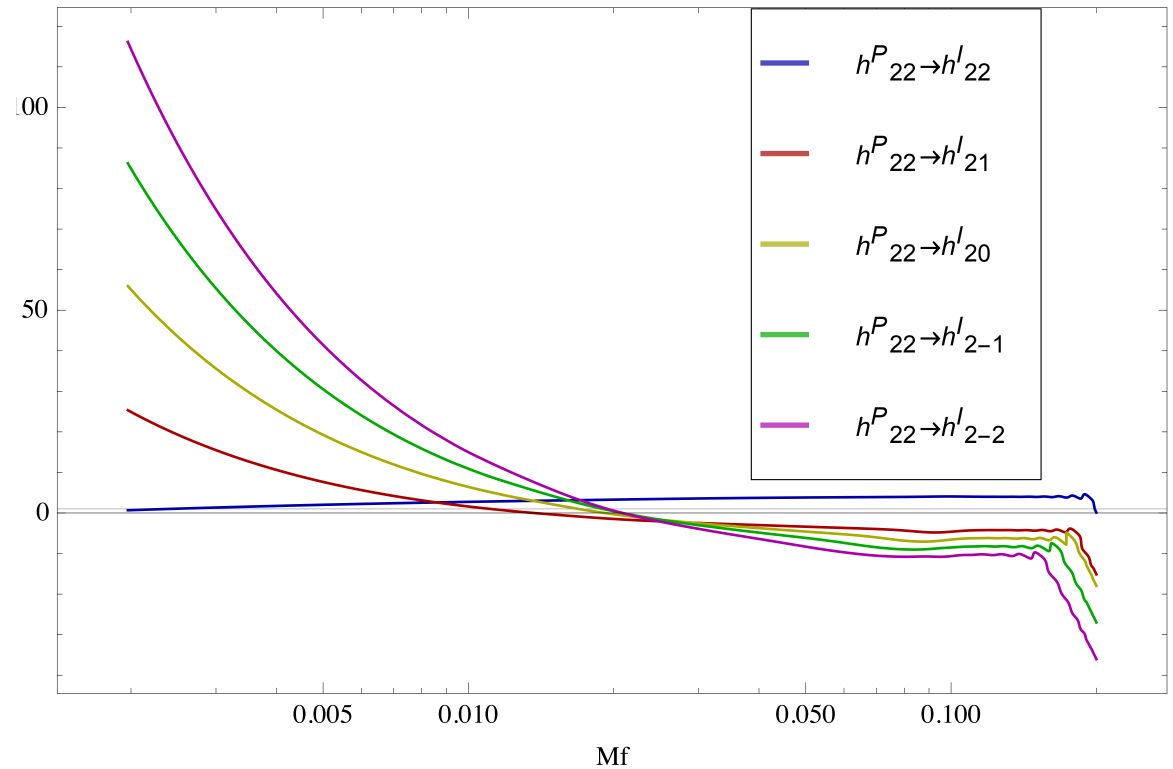


FD transfer functions for different modes

Normalized amplitude

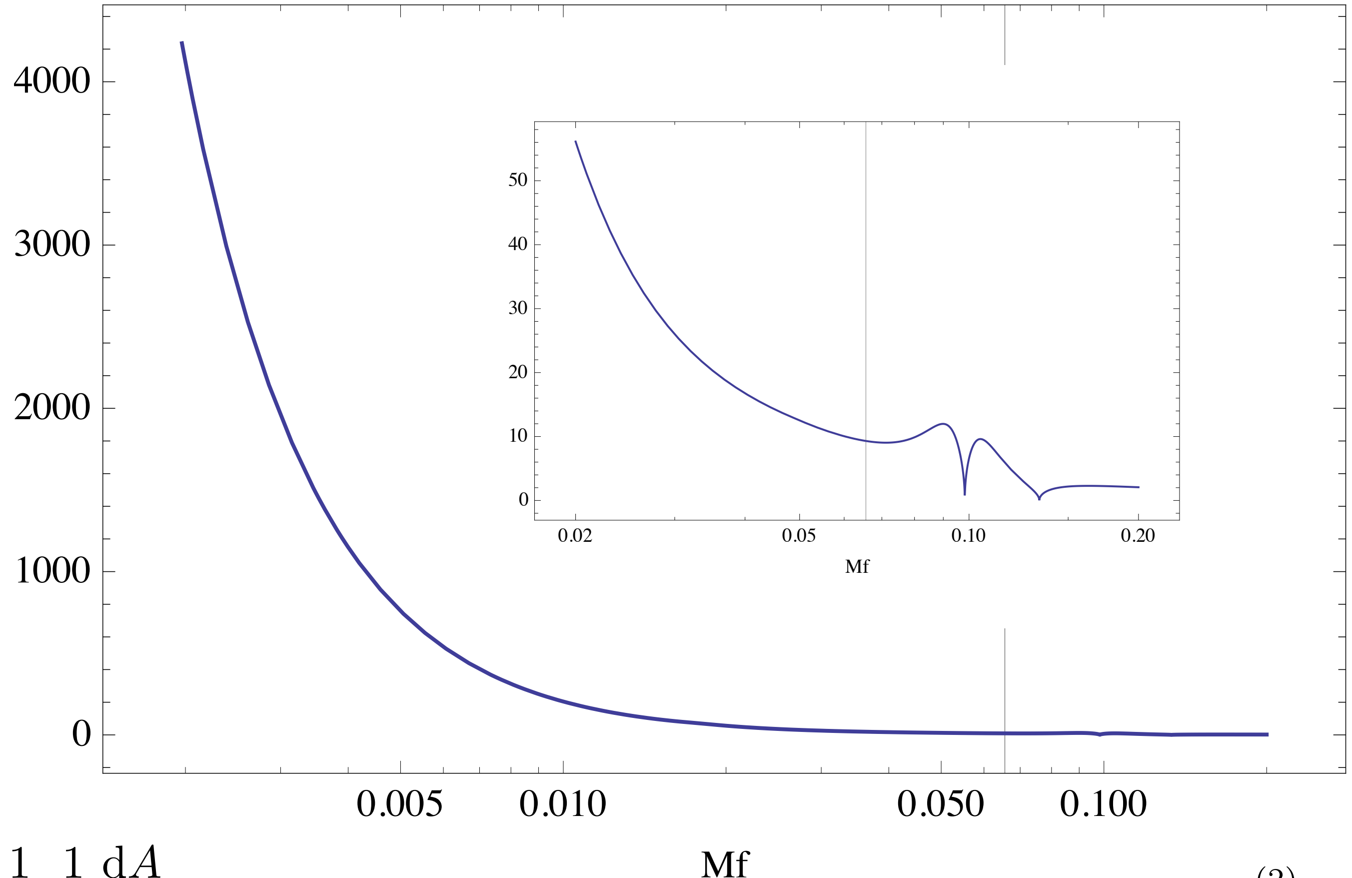


Phase

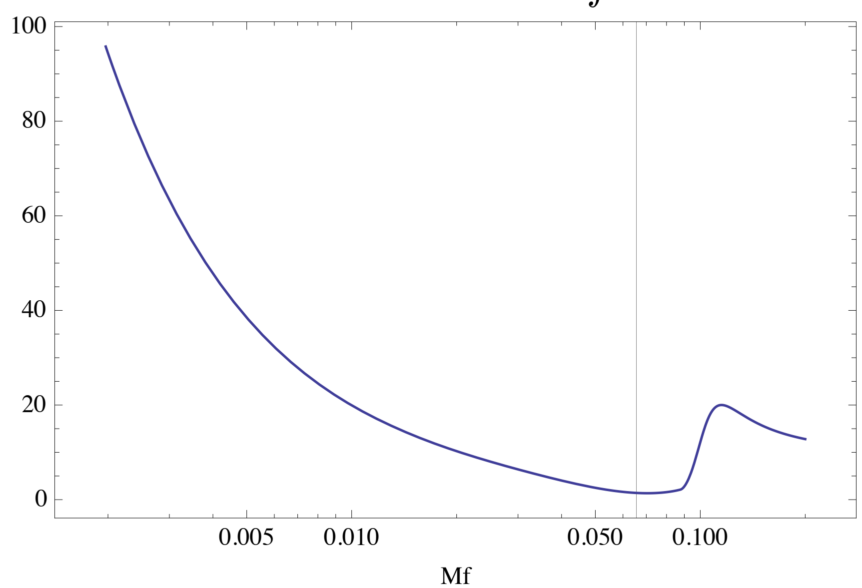


FD timescales

$$T_f = \sqrt{\frac{1}{4\pi^2}\left|\frac{\mathrm{d}^2\Psi}{\mathrm{d}f^2}\right|}$$



$$T_A^{(1)} = \frac{1}{2\pi}\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}f}$$



$$T_A^{(2)} = \sqrt{\frac{1}{4\pi^2}\frac{1}{A}\frac{\mathrm{d}^2A}{\mathrm{d}f^2}}$$