Dynamics of Superrotations and Supertranslations in 2+1 Dimensions

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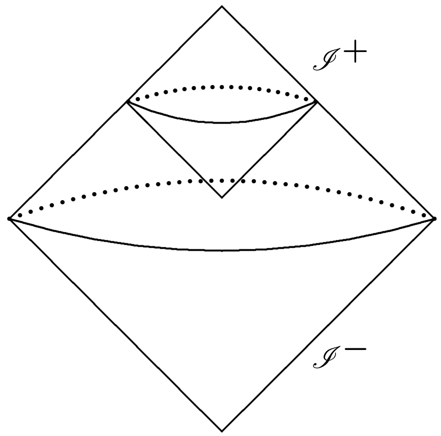
The setting



– Asymptotically flat spacetime has large symmetry group at $\mathscr{I}^\pm$

– Strominger et al.: BMS group is spontaneously broken

$\Rightarrow$ Goldstone modes? Degenerate vacuum?

– This *might* help with black hole information loss problem

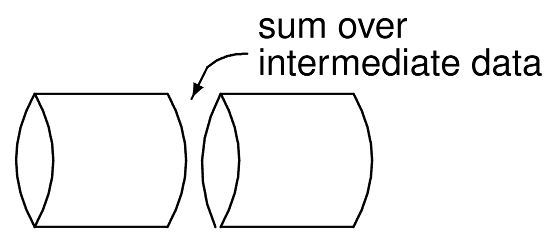
Can one extract a dynamical description of the new degrees of freedom?

The idea in a nutshell

- Action on manifold with boundary has two pieces:

$$I = I_{bulk} + I_{bdry} \qquad \text{with} \quad I_{bulk} = \int \mathscr{L}\, d^n x$$

  Boundary piece needed
  - Classically: to allow extrema
  - Quantum mechanically: to ensure proper "sewing" of path integrals

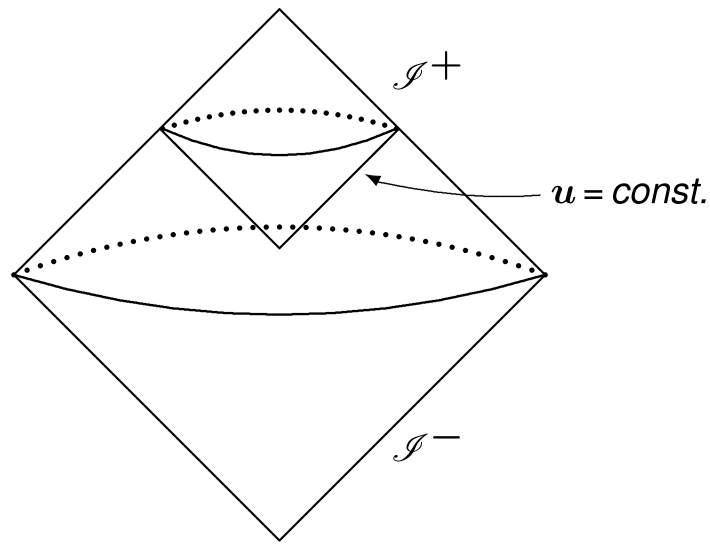


- Gauge symmetries of I_{bulk} will typically be broken by I_{bdry}
  - Formerly nonphysical degrees of freedom become dynamical at boundary
- Action for new degrees of freedom is induced from I_{bdry}
- Results for (2+1)-dimensional gravity:
  - Asymptotically AdS: Liouville action
  - Asymptotically flat: relation to chiral Liouville theory

$(2+1)$-dimensional asymptotically flat gravity

Partially gauge-fix metric: "Bondi" coordinates

$$ds^2 = -2dudr + g_{uu}\,du^2 + 2g_{u\phi}\,dud\phi + r^2e^{2\varphi}d\phi^2$$



$u = $ *const.*: outgoing null surfaces
r is an affine parameter along null geodesics $u =$ *const.*, $\phi =$ *const.*

Behavior of metric near $\mathscr{I}^{\pm}$

Symmetries: supertranslations $\sim u \to u + e^{\varphi}T(\phi)$

superrotations $\sim \phi \to \phi + Y(\phi)$

From field equations (Barnich & Troessaert)

$$g_{uu} = -2r\partial_u\varphi + e^{-2\varphi}\left[-(\partial_\phi\varphi)^2 + 2\partial_\phi^2\varphi + \Theta\right]$$
$$g_{u\phi} = e^{-\varphi}\left[\Xi + \int^u d\tilde{u}\left\{\frac{1}{2}\partial_\phi\Theta - \partial_\phi\varphi[\Theta - (\partial_\phi\varphi)^2 + 3\partial_\phi^2\varphi] + \partial_\phi^3\varphi\right\}\right]$$

with $\partial_u\Theta = \partial_u\Xi = 0$

Θ and Ξ are charges for supertranslations and superrotations

$$\mathcal{Q} \sim \frac{1}{16\pi G}\int d\phi\,(\Theta T + \Xi Y)$$

Boundary terms

$$\delta I_{grav} = \textit{bulk piece}$$
$$+ \frac{1}{16\pi G}\int_{\partial M} d^2x\,[-\partial_r(re^{\varphi}\delta g_{uu}) - 2g_{uu}\partial_r(re^{\varphi}\delta\varphi)] + \mathcal{O}\left(\tfrac{1}{r}\right)$$

Use asymptotic expression to integrate: find

$$\delta I = -\frac{1}{8\pi G}\,\delta\int_{\partial M} d^2x\, e^{\tilde{\varphi}}\tilde{g}_{uu} + \frac{1}{16\pi G}\int_{\partial M} d^2x\, e^{\tilde{\varphi}}\delta\tilde{g}_{uu}$$

where $\tilde{X}$ means the $\mathcal{O}(1)$ part of X

Fix either $\tilde{g}_{uu}$ (i.e., Θ) or $e^{\tilde{\varphi}}$ (or some combination)

(Same result from looking directly at canonically conjugate variables)

Fix $\tilde{\varphi}$:

$$I_{\textit{bdry}} = \frac{1}{16\pi G}\int_{\partial M} d^2x \, e^{\tilde{\varphi}}\tilde{g}_{uu}$$

Not quite extrinsic curvature of $r =$ *const.*:

– unit normal n^a to $r =$ *const.*

– induced metric q_{ab} on $r =$ *const.*

– null vector $\ell \propto du$, normalized so $n_a \ell^a = 1$

$$I_{\textit{bdry}} = \frac{1}{16\pi G}\int_{\partial M} d^2x \, \sqrt{q}\, \nabla_a \ell^a$$

Relation to diffeomorphisms

Start with flat base metric

$$ds^2 = -2d\bar{u}d\bar{r} + d\bar{u}^2 + \bar{r}^2 d\bar{\phi}^2$$

Diffeomorphism

$$\bar{u} = u_0 + \frac{u_1}{r} + \ldots, \qquad \bar{\phi} = \phi_0 + \frac{\phi_1}{r} + \ldots, \qquad \bar{r} = ar + b_0 + \frac{b_1}{r} + \ldots$$

Form invariance of metric $\Rightarrow$ relations among coefficients u_0, ϕ_0, b_0

Confirm form $g_{uu} = -2r\partial_u\varphi + e^{-2\varphi}\left[-(\partial_\phi\varphi)^2 + 2\partial_\phi^2\varphi + \Theta\right]$

with $\quad e^{\varphi} = \dfrac{\partial_\phi \phi_0}{\partial_u u_0}, \qquad \Theta = -(\partial_\phi\phi_0)^2 - 2\{\phi_0;\phi\}$

Schwarzian derivative $\{f;z\} = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$

An action for supertranslations and superrotations

Boundary action (for fixed φ) is then

$$I_{\mathit{bdry}} = \frac{1}{16\pi G}\int_{\partial M} d^2x\, e^{-\varphi}\left[-(\partial_\phi\varphi)^2 + 2\partial_\phi^2\varphi - (\partial_\phi\phi_0)^2 - 2\{\phi_0;\phi\}\right]$$

What can we say about this action?

- Depends on ϕ_0 (superrotation parameter)
- But $e^{\varphi} = (\partial_\phi\phi_0)/(\partial_u u_0)$ fixed,
  so equivalently depends on u_0 (supertranslation parameter)
- Schwarzian derivative in $\Theta \Leftrightarrow \Theta$ transforms as stress-energy tensor
- Many connections to Liouville theory and Virasoro group

- Vary φ: Hill's equation

$$\partial_\phi^2 \chi + \frac{6}{c} T \chi = 0 \quad \text{with } \chi = \frac{1}{\sqrt{2\pi G}} e^{-\varphi/2}, \;\; T = \frac{c}{24}(\partial_\phi \phi_0)^2 + \frac{c}{12}\{\phi_0; \phi\}$$

  – Solutions χ_1, χ_2 give orbits of Virasoro group:

  – $\delta_\varepsilon T = -\frac{c}{12}\varepsilon''' - 2\varepsilon' T - \varepsilon T' = 0$ with $\varepsilon = \chi_1^2,\ \chi_1\chi_2,\ \chi_2^2$

- Action for ϕ_0 related to Alexe'ev-Shatashvili action for coadjoint quantization

- φ fixed $\Rightarrow$ either ϕ_0 or u_0 can be treated as independent:

  – $\partial_\phi \phi_0 = e^{\sigma} \;\Rightarrow$ chiral Liouville action for σ

  – $\partial_u u_0 = e^{\tilde{\sigma}} \;\Rightarrow$ chiral Liouville action for $\tilde{\sigma}$

Clearly a strong connection to CFT (or BMS/Galilean theory?)

Generalization to 3+1 dimensions?