The CMB, inflation and modified dispersion relation

Stefano Bianco, Nicolai V. Friedhoff, Edward Wilson-Ewing
stefanobianco3@gmail.com

Max Planck Institute for Gravitational Physics

11th July 2016

Introduction

- Several results in quantum gravity suggest dimensional reduction to 2 in the UV[1]

[1] Atick and Witten, Nucl. Phys B310 (1988), Ambjorn et al., Phys. Rev. Lett. (5 (2005); Lauscher and Reuter, JHEP 10 (2005); Phys. Rev Lett. 92 (2004), Benedetti, Phys. Rev. Lett 102 (2009)

[2] Horava, Phys. Rev. Lett 102 (2009); Visser et al., Phys. Rev D84 (2011)

[3] Amelino-Camelia et al, Nature 393 (1998); Martin and Brandemberger,Phys. Rev. D63 (2001)

Introduction

- Several results in quantum gravity suggest dimensional reduction to 2 in the UV[1]
- One way to obtain this is via a Planck-scale modified dispersion relation $E = k^2 + \sigma^4 k^6$ considering the notion of spectral dimension[2]

[1] Atick and Witten, Nucl. Phys B310 (1988), Ambjorn et al., Phys. Rev. Lett. (5 (2005); Lauscher and Reuter, JHEP 10 (2005); Phys. Rev Lett. 92 (2004), Benedetti, Phys. Rev. Lett 102 (2009)

[2] Horava, Phys. Rev. Lett 102 (2009); Visser et al., Phys. Rev D84 (2011)

[3] Amelino-Camelia et al, Nature 393 (1998); Martin and Brandemberger,Phys. Rev. D63 (2001)

Introduction

- Several results in quantum gravity suggest dimensional reduction to 2 in the UV[1]
- One way to obtain this is via a Planck-scale modified dispersion relation $E = k^2 + \sigma^4 k^6$ considering the notion of spectral dimension[2]
- Planck-scale deformed dispersion relation widely studied in quantum gravity and cosmology literature[3]

---

[1] Atick and Witten, Nucl. Phys B310 (1988), Ambjorn et al., Phys. Rev. Lett. (5 (2005); Lauscher and Reuter, JHEP 10 (2005); Phys. Rev Lett. 92 (2004), Benedetti, Phys. Rev. Lett 102 (2009)

[2] Horava, Phys. Rev. Lett 102 (2009); Visser et al., Phys. Rev D84 (2011)

[3] Amelino-Camelia et al, Nature 393 (1998); Martin and Brandemberger,Phys. Rev. D63 (2001)

Introduction

- Several results in quantum gravity suggest dimensional reduction to 2 in the UV[1]
- One way to obtain this is via a Planck-scale modified dispersion relation $E = k^2 + \sigma^4 k^6$ considering the notion of spectral dimension[2]
- Planck-scale deformed dispersion relation widely studied in quantum gravity and cosmology literature[3]
- We will study the interaction of the dispersion relation $E = k^2 + \sigma^4 k^6$ with the inflationary scenario and the effect for the description of the CMB spectrum

---

[1] Atick and Witten, Nucl. Phys B310 (1988), Ambjorn et al., Phys. Rev. Lett. (5 (2005); Lauscher and Reuter, JHEP 10 (2005); Phys. Rev Lett. 92 (2004), Benedetti, Phys. Rev. Lett 102 (2009)

[2] Horava, Phys. Rev. Lett 102 (2009); Visser et al., Phys. Rev D84 (2011)

[3] Amelino-Camelia et al, Nature 393 (1998); Martin and Brandemberger,Phys. Rev. D63 (2001)

Mukhanov-Sasaki equation with modified dispersion relation

The modified Mukhanov-Sasaki equation is

$$v_k'' + \left(k^2 + \frac{\sigma^4 k^6}{a^4}\right) v_k - \frac{z''}{z} v_k = 0 \ , \tag{1}$$

where, as in the standard case, v encodes the scalar perturbation, a is the scale factor of the background metric and $z = \frac{a\sqrt{\rho}\sqrt{1+w}}{H}$.

Mukhanov-Sasaki equation with modified dispersion relation

The modified Mukhanov-Sasaki equation is

$$v_k'' + \left(k^2 + \frac{\sigma^4 k^6}{a^4}\right) v_k - \frac{z''}{z} v_k = 0 \ , \tag{1}$$

where, as in the standard case, v encodes the scalar perturbation, a is the scale factor of the background metric and $z = \frac{a\sqrt{\rho}\sqrt{1+w}}{H}$.

As usual in cosmology, the solution to (1) has been found finding the solutions in the two regimes "inside the horizon" $\left(k^2 + \frac{\sigma^4 k^6}{a^4}\right) \gg \frac{z''}{z}$ and "outside the horizon" $\left(k^2 + \frac{\sigma^4 k^6}{a^4}\right) \ll \frac{z''}{z}$ and then matching them.

Scale invariance from modified dispersion relation

The calculation goes as in standard cosmology.

Interestingly for short wavelenght such that one can neglect k^2 compared to $\frac{\sigma^4 k^6}{a^4}$, the spectrum for scalar perturbations is

$$\Delta(k) = \frac{k^3}{2\pi^2}|\frac{v}{z}|^2 = \frac{\hbar}{4\pi^2\sigma^2} \cdot \frac{a^2}{z^2} \ , \tag{2}$$

which is scale invariant already inside the horizon[4]! In fact $v_{in} = \sqrt{\frac{\hbar}{2\omega_k}} e^{-i\int d\tau\omega_k}$, where $\omega_k = \frac{\sigma^2 k^3}{a^2}$.

---

[4] Magueijo, Class. Quant. Grav. 25 (2008)

[5] Amelino-Camelia et al, Phys. Rev. D88 (2013)

Scale invariance from modified dispersion relation

The calculation goes as in standard cosmology.

Interestingly for short wavelenght such that one can neglect k^2 compared to $\frac{\sigma^4 k^6}{a^4}$, the spectrum for scalar perturbations is

$$\Delta(k) = \frac{k^3}{2\pi^2}|\frac{v}{z}|^2 = \frac{\hbar}{4\pi^2\sigma^2} \cdot \frac{a^2}{z^2} \ , \tag{2}$$

which is scale invariant already inside the horizon[4]! In fact $v_{in} = \sqrt{\frac{\hbar}{2\omega_k}} e^{-i\int d\tau \omega_k}$, where $\omega_k = \frac{\sigma^2 k^3}{a^2}$.

Moreover [5]

- It is possible to obtain deviation from exact scale invariance using $k^{6-\delta}$ instead of k^6 or slow transient to $k^{\gamma(k)}$.
- It is possible to obtain tensor to scalar ratio r postulating different σ for scalar and tensor modes and one finds $r = (\frac{\sigma_s}{\sigma_t})^2$

[4] Magueijo, Class. Quant. Grav. 25 (2008)

[5] Amelino-Camelia et al, Phys. Rev. D88 (2013)

Necessary conditions to reproduce the CMB spectrum

For $E = k^2 + \sigma^4 k^6$ to be the key ingredient to describe the CMB it is needed that[6]

1. the short wavelenght modes should be streched to cosmological scales.

[6] Bianco, Friedhoff and Wilson-Ewing, in preparation

Necessary conditions to reproduce the CMB spectrum

For $E = k^2 + \sigma^4 k^6$ to be the key ingredient to describe the CMB it is needed that[6]

1. the short wavelenght modes should be streched to cosmological scales.
2. the short wavelenght modes must exit the horizon before k^2 becomes comparable to $\sigma^4 k^6/a^4$:

$$\frac{\sigma^4 k^6}{a^4} = \frac{z''}{z} \gg k^2 \ . \qquad (3)$$

[6] Bianco, Friedhoff and Wilson-Ewing, in preparation

Necessary conditions to reproduce the CMB spectrum

For $E = k^2 + \sigma^4 k^6$ to be the key ingredient to describe the CMB it is needed that[6]

1. the short wavelenght modes should be streched to cosmological scales.
2. the short wavelenght modes must exit the horizon before k^2 becomes comparable to $\sigma^4 k^6 / a^4$:

$$\frac{\sigma^4 k^6}{a^4} = \frac{z''}{z} \gg k^2 \ . \tag{3}$$

We find that inflationary expansion can realize these conditions.

[6] Bianco, Friedhoff and Wilson-Ewing, in preparation

Inflation and modified dispersion relation

For a constant equation of state $w < -1/3$, the scale factor behaves as

$$a(\tau) = a_o \, (-\tau)^{\frac{2}{1+3w}} \,, \tag{4}$$

where $-\infty < \tau < 0$ for an expanding universe.

The modified Mukhanov-Sasaki equation becomes

$$v_k'' + \left(k^2 + \frac{\sigma^4 k^6}{a_o^4 |\tau|^{8/(1+3w)}} \right) v_k - \frac{2(1-3w)}{(1+3w)^2 \tau^2} v_k = 0 \ . \tag{5}$$

Inflation and modified dispersion relation

Imposing

$$\frac{\sigma^4 k^6}{a^4} = \frac{z''}{z} \gg k^2 \ , \tag{6}$$

we find

$$H \gg \sqrt{\frac{2}{1-3w}}\,\sigma^{-1} \ . \tag{7}$$

The amplitude of the scalar perturbations fixes $\sigma \sim 10^4\sqrt{G\hbar}$, so that we estimate $H \gg 10^{-4}/\sqrt{G\hbar}$.

Slow-roll inflation and modified dispersion relation

The two slow-roll parameters are

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon} \ , \tag{8}$$

which satisfy the conditions $0 < \epsilon \ll 1$ and $|\eta| \ll 1$.
Near the horizon, for a short period of time both H and ϵ are nearly constant and the scale factor is given to a good approximation by

$$a(\tau) = |H\tau|^{-(1+\epsilon)} \ . \tag{9}$$

Then the modified Mukhanov-Sasaki equation takes the form

$$v_k'' + \left(k^2 + \frac{\sigma^n k^{n+2}}{a^n}\right) v_k - \frac{2 + 9\epsilon - 3\eta}{\tau^2} v_k = 0 \ , \tag{10}$$

where we allowed for a more generic dispersion relation.

Slow-roll inflation and modified dispersion relation

The spectral indeces for scalar and tensor modes are

$$n_s - 1 = -6\epsilon + 2\eta + \frac{3n\epsilon}{2+n} \tag{11}$$

and

$$n_t = -2\epsilon + \frac{3n\epsilon}{n+2} \ . \tag{12}$$

And the tensor to scalar ratio is

$$r = \frac{\Delta_t(k)}{\Delta_s(k)} = 16\,\epsilon \left(\frac{\sigma_s}{\sigma_t}\right)^{\frac{3n}{n+2}} \quad . \tag{13}$$

Remarks:

- a red tilt in the spectrum can be obtained for all different values of n

Slow-roll inflation and modified dispersion relation

The spectral indeces for scalar and tensor modes are

$$n_s - 1 = -6\epsilon + 2\eta + \frac{3n\epsilon}{2+n} \tag{11}$$

and

$$n_t = -2\epsilon + \frac{3n\epsilon}{n+2} \ . \tag{12}$$

And the tensor to scalar ratio is

$$r = \frac{\Delta_t(k)}{\Delta_s(k)} = 16\,\epsilon \left(\frac{\sigma_s}{\sigma_t}\right)^{\frac{3n}{n+2}} \quad . \tag{13}$$

Remarks:

- a red tilt in the spectrum can be obtained for all different values of n
- the amplitude of the perturbations is given by a combination of σ, ϵ and H.

Slow-roll inflation and modified dispersion relation

The spectral indeces for scalar and tensor modes are

$$n_s - 1 = -6\epsilon + 2\eta + \frac{3n\epsilon}{2+n} \tag{11}$$

and

$$n_t = -2\epsilon + \frac{3n\epsilon}{n+2}\ . \tag{12}$$

And the tensor to scalar ratio is

$$r = \frac{\Delta_t(k)}{\Delta_s(k)} = 16\,\epsilon\left(\frac{\sigma_s}{\sigma_t}\right)^{\frac{3n}{n+2}}\quad . \tag{13}$$

Remarks:

- a red tilt in the spectrum can be obtained for all different values of n
- the amplitude of the perturbations is given by a combination of σ, ϵ and H.
- possible observational test from modified consistency relation $r = \frac{-8n_t}{1-\frac{3n}{2n+4}}$ (for the case $\sigma_s = \sigma_t$).

Conclusions

- Planck-scale modified dispersion relation related to quantum-gravity dimensional reduction can provide a description for the CMB in presence of an expansionary phase that stretches the short wavelenght modes.

Conclusions

- Planck-scale modified dispersion relation related to quantum-gravity dimensional reduction can provide a description for the CMB in presence of an expansionary phase that stretches the short wavelenght modes.
- if given by inflation, this expansion phase should be characterized by $H \gg 10^4 \sqrt{G\hbar}$.

Conclusions

- Planck-scale modified dispersion relation related to quantum-gravity dimensional reduction can provide a description for the CMB in presence of an expansionary phase that stretches the short wavelenght modes.
- if given by inflation, this expansion phase should be characterized by $H \gg 10^4 \sqrt{G\hbar}$.
- if the expansionary phase is given by slow-roll inflation, one obtains new relations between observables that can provide observational tests