ELECTRODYNAMIC EFFECTS OF INFLATIONARY GRAVITONS

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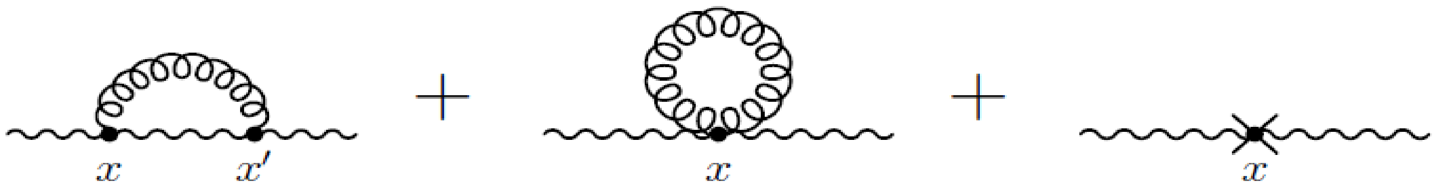
What We Did

- $\mathcal{L} = \frac{(R-2\Lambda)\sqrt{-g}}{16\pi G} - \frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\sigma\nu} \sqrt{-g}$ +c-terms

1. Compute $i[^\mu\Pi^\nu](x;x')$ from gravitons at 1 loop in dim. reg.



2. Renormalize with BPHZ counter terms (cf. LEFT)

$$\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} + \bar{C} H^2 F^{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} + \Delta C H^2 F_{ij} F_{kl} g^{ik} g^{jl} \sqrt{-g}$$

3. Quantum-correct Maxwell's equations

$$\partial_\nu\left[\sqrt{-g}\, g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)\right] + \int d^4x' [^\mu\Pi^\nu](x;x') A_\nu(x') = J^\mu(x)$$

- $J^\mu = 0$ ➔ dynamical photons
- $J^\mu \neq 0$ ➔ electromagnetic forces

Why there should be an effect

- Neither photons nor gravitons are charged
  - So how can they polarize the vacuum?
- But they DO carry momentum
  - Dynamical photons have $\vec{p} = \hbar c \vec{k}$
  - Virtual photons carry $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$
- Virtual gravitons can add or subtract to this
  - de Sitter gravitons $\left(\vec{k}, \lambda\right)$ have $N(t,k) = \left[\frac{Ha(t)}{2ck}\right]^2$
  - Gravitons with $k_{phys} \sim H \gg k_{photon} = \frac{k}{a(t)}$

What happens on a flat background

arXiv:1202.5800 by Leonard & Woodard

- $\kappa^2 \equiv 16\pi G$ $\qquad \Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - (|t - t'| - i\epsilon)^2$
- $i[^{\mu}\Pi^{\nu}](x;x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^{\mu}\partial^{\nu}) \left[\frac{\#\kappa^2}{\Delta x^{2D-2}}\right]$
- $\Delta\mathcal{L} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g}$
- $i[^{\mu}\Pi^{\nu}_{ren}](x;x') = (\eta^{\mu\nu}\partial' \cdot \partial - \partial'^{\mu}\partial^{\nu})\partial^4 \left[\frac{\kappa^2}{192\pi^2} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2}\right]$
- No change in dynamical photons
- Coulomb: $\Phi(r) = \frac{q}{4\pi r}\left[1 + \frac{2G}{3\pi r^2} + O(G^2)\right]$
  - Radkowski (1970)
  - 0th order field distorts virtual gravitons nearby & these add momentum to virtual photons carrying force

What happens on de Sitter

arXiv:1304.7265 by Leonard &Woodard ; arXiv:1308.3453 by Glavan, Miao, Prokopec & Woodard

- de Sitter breaking ➔ 2 structure functions
- $i[{}^{\mu}\Pi^{\nu}] = (\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho})\partial_\rho\partial'_\sigma F(x;x') + (\bar{\eta}^{\mu\nu}\bar{\eta}^{\rho\sigma} - \bar{\eta}^{\mu\sigma}\bar{\eta}^{\nu\rho})\partial_\rho\partial'_\sigma G(x;x')$
- Dynamical photons: E grows and B falls
  - $F^{0i}_{(1)} \to F^{0i}_{(0)} \times \frac{\kappa^2 H^2 \ln(a)}{8\pi^2}$, $F^{ij}_{(1)} \to F^{ij}_{(0)} \times \frac{\kappa^2 H^2}{8\pi^2}\frac{ik\ln(a)}{Ha}$
- Co-moving charge & magnetic dipole
- $J^\mu(\eta,\vec{x}) = q\delta^\mu_0\delta^3(\vec{x})$ ➔ $\Phi = \frac{q}{4\pi x}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3a^2H^2x^2} + \ln(aHx)\right] + \cdots\right\}$
- $J^\mu(\eta,\vec{x}) = \epsilon^{0\mu jk} m_j \partial_k \delta^3(\vec{x})$ ➔ $\vec{A} = \vec{m} \times \vec{\nabla}\left(\frac{1}{4\pi x}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3a^2H^2x^2} - \frac{2}{3}\ln(Hx)\right] + \cdots\right\}\right)$
- Static charge & magnetic dipole
- Point Charge ➔ $\widetilde{\Phi} = \frac{q}{4\pi r}\left\{1 + \frac{\kappa^2 H^2}{8\pi^2}\left[\frac{1}{3H^2r^2} + \ln(Hr)\right] + \cdots\right\}$
- Point magnetic dipole ➔ show secular growth

The Gauge Issue:

Photon & gravitons require fixing!

- EM gauge irrelevant because coupling to $F_{\mu\nu}$
- But flat $\partial^\mu h_{\mu\nu} = (\frac{b}{2})\partial_\nu h$ in flat space gives

$$i\left[{}^\mu\Pi^\nu\right](x;x') = \frac{\kappa^2}{384\pi^4}\left(\frac{2b-1}{b-2}\right)^2\left[\eta^{\mu\nu}\partial'\cdot\partial - \partial'^\mu\partial^\nu\right]\partial^2\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right]$$

  - Spin 2 part vanishes at D = 4
- The result of Bjerrum-Bohr (hep-th/0206236)
  - Effect in gauge independent S-matrix

Our Conjecture:

arXiv:1204.1784 by Miao & Woodard

- Gauge fixed GF's are not complete nonsense
  - We construct the flat space S-matrix from them!
  - Need to separate physical information from unphysical
- Gauge invariant GF's cannot solve everything
  - Every gauge fixed GF represents some invariant

$$A_0(t,x) = 0 = A_1(0,x) \quad \Rightarrow \quad A_1(t,x) = \int_0^t ds\, F_{01}(s,x)$$

- $i[^{\mu}\Pi^{\nu}]$ on de Sitter must inherit the gauge dependence of $i[^{\mu}\Pi^{\nu}]$ on flat background
- Maybe leading secular effects gauge independent
  - NB these effects aren't present in flat space
  - Cf. poles terms of gauge fixed GF's in flat space QFT
- How to check? ➔ re-compute in other gauges
  - EM gauge doesn't matter
  - General covariant GR gauge: $D^{\mu}h_{\mu\nu} = \frac{b}{2}\partial_{\nu}h_{\mu}^{\mu}$
  - arXiv:1205.4468 by Mora, Tsamis & Woodard

Conclusions

- Dynamical photons:
- Spin 2 shows secular growth & no gauge dependence
- Spin 0 doesn’t have leading order contribution
  - Consistent with our conjecture
- Intuition says the effect is real
  - Inflation really does produce huge ensemble of IR
  - Should scatter photons more the further they go
  - This would not even be doubted in flat space
- But proving it IS an important step in cosmo QFT