Radiation by an Unruh-DeWitt detector in oscillatory motion:

Identify Unruh effect in radiation

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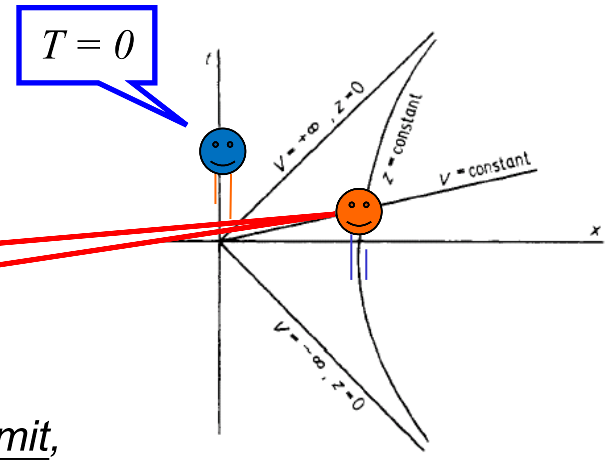
Unruh Effect

Vacuum is not vacuous [Unruh 1976]

A detector linearly, uniformly accelerated in Minkowski vacuum will experience a thermal bath at the **Unruh temperature**:

$T_U = 1\text{K}$ for
$a = 2.4 \times 10^{20}\ \text{m/s}^2$

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

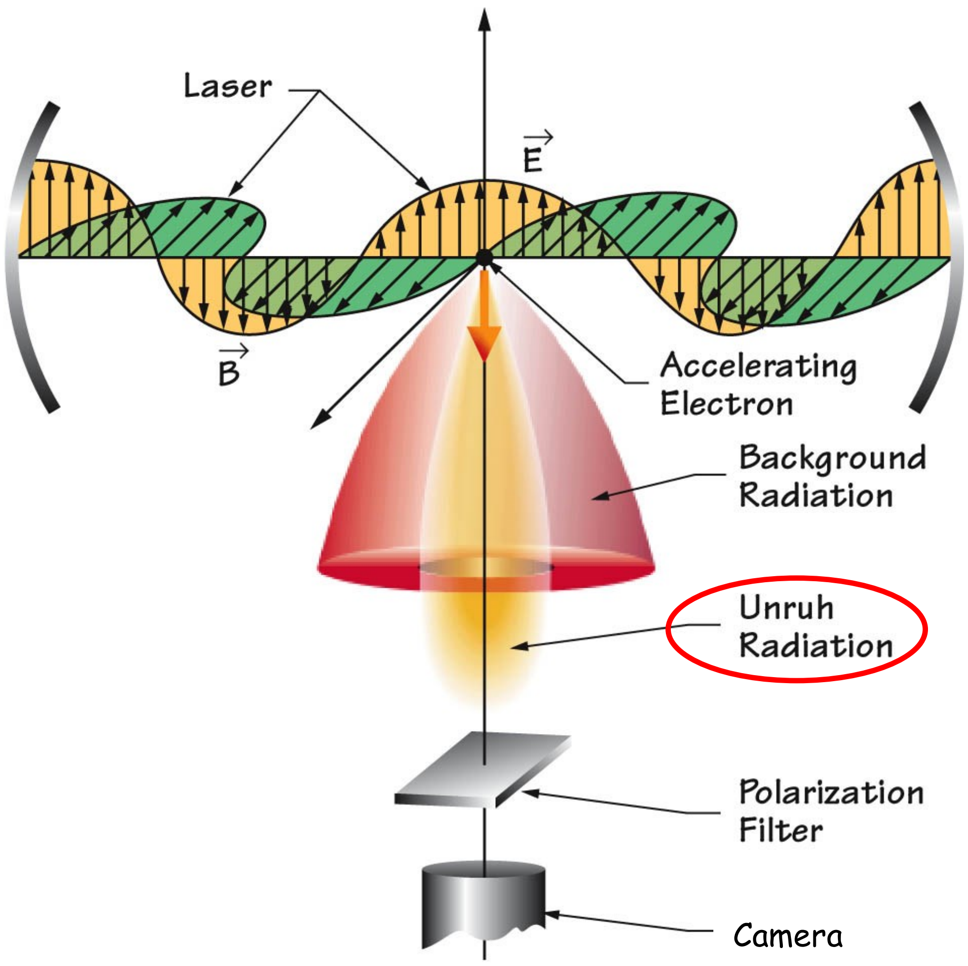


** If ☺ is initially in its ground state, <u>in weak-coupling limit</u>, the transition probability to its 1st excited state is*

$$\rho^R_{1,1}\big|_{\gamma\eta\to 0} \xrightarrow{\eta \gg a^{-1}} \frac{\lambda_0^2}{4\pi m_0}\left[\frac{\eta}{e^{2\pi\Omega_r/a}-1}\right] \qquad \eta \equiv \tau - \tau_0$$

- uniform acceleration : $a_\mu a^\mu = a^2 = \text{constant}$ (a: proper acceleration)
- Minkowski vacuum: <u>No particle</u> (field quanta) state of the field for Minkowski observer

A Conceptual Design of an Experiment for Detecting the Unruh Effect



Schematic Diagram for Detecting Unruh Radiation

[Chen, Tajima, PRL83('99)256]

A 10^{13} *W (10 TW) laser focusing on a spot of* $10^{-10}\ m^2$ *can produce*

$a \sim 3 \times 10^{24}\ m/s^2$

$T_U \sim 7 \times 10^{4}\ K$

for an electron.

[Courtesy of Pisin Chen]

Accelerated Spinless Point-Charge

- $z^\mu(\tau)$ are dynamical variables.

$$S = \int d\tau \;\; m_0 \sqrt{\dot{z}^\mu \dot{z}_\mu} \;\; + \int d^4x \sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j_\mu(x)A^\mu(x)\right]$$

$$j_\mu(x) \equiv e \int d\tau \, v_\mu(\tau)\, \delta^4(x^\mu - z^\mu(\tau))$$

too hard…

- Toy model: Unruh-DeWitt “detector" with internal HO

$$S = S_Q + S_\Phi + S_I,$$ where

$S_Q = \int d\tau \frac{m_0}{2}\left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2\right]$	*Internal: harmonic oscillator*
$S_\Phi = -\int d^4x \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi$	*Massless scalar field*
$S_I = \lambda_0 \int d\tau \int d^4x Q(\tau)\Phi(x)\underline{\delta^4\left(x^\mu - z^\mu(\tau)\right)}$	*Point-like object* [DeWitt 1979]

prescribed worldline

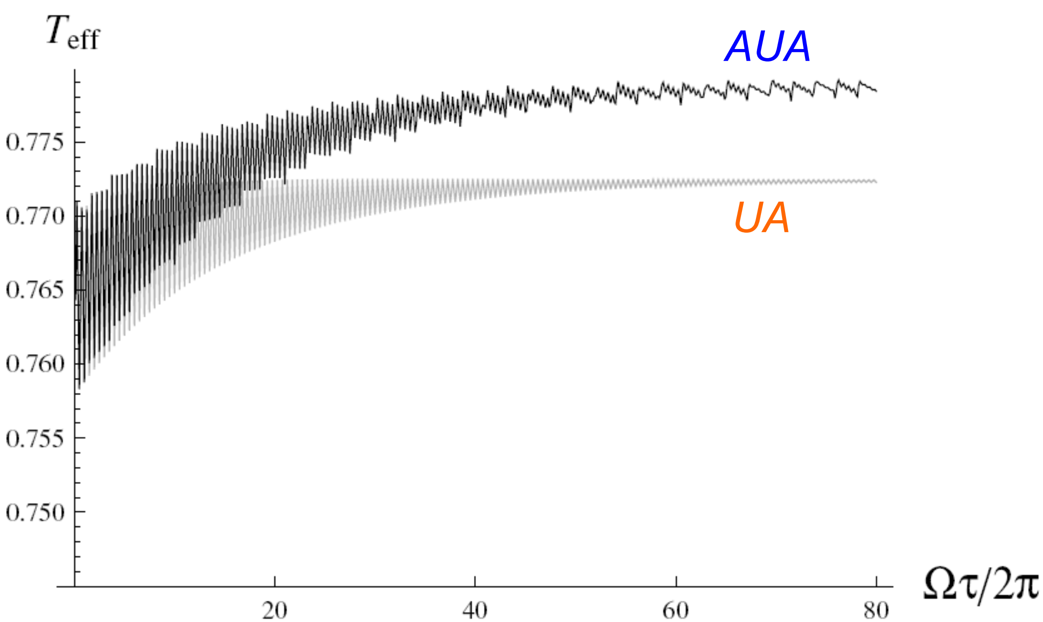
Effective Temperature

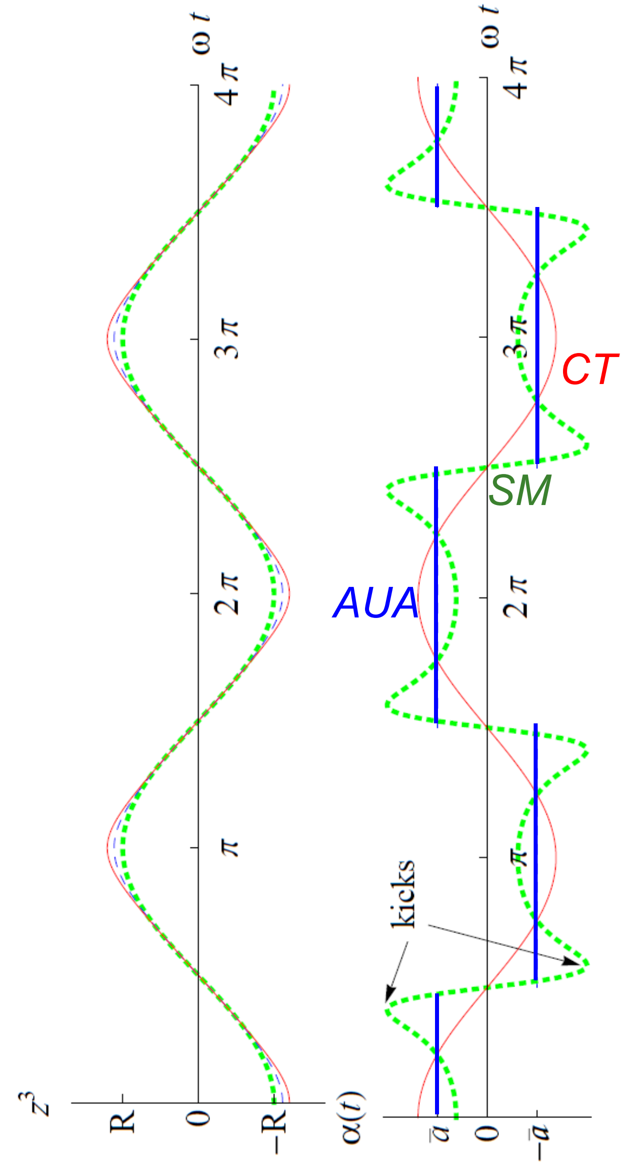
Diagonalize the reduced density matrix of the detector to obtain

[SYL, B. L. Hu, PRD76(07)064008]

$$T_{\rm eff}(\tau) = \left[\frac{k_B}{\hbar\Omega_r}\ln\left(\frac{\mathcal{U}(\tau)+\hbar/2}{\mathcal{U}(\tau)-\hbar/2}\right)\right]^{-1}$$

$$\mathcal{U}(\tau) \equiv \sqrt{\langle \hat{P}^2(\tau)\rangle\langle \hat{Q}^2(\tau)\rangle - \langle \hat{Q}(\tau),\hat{P}(\tau)\rangle^2}$$





[J. Doukas, SYL, B. L. Hu and R. B. Mann, JHEP11(2013)119]

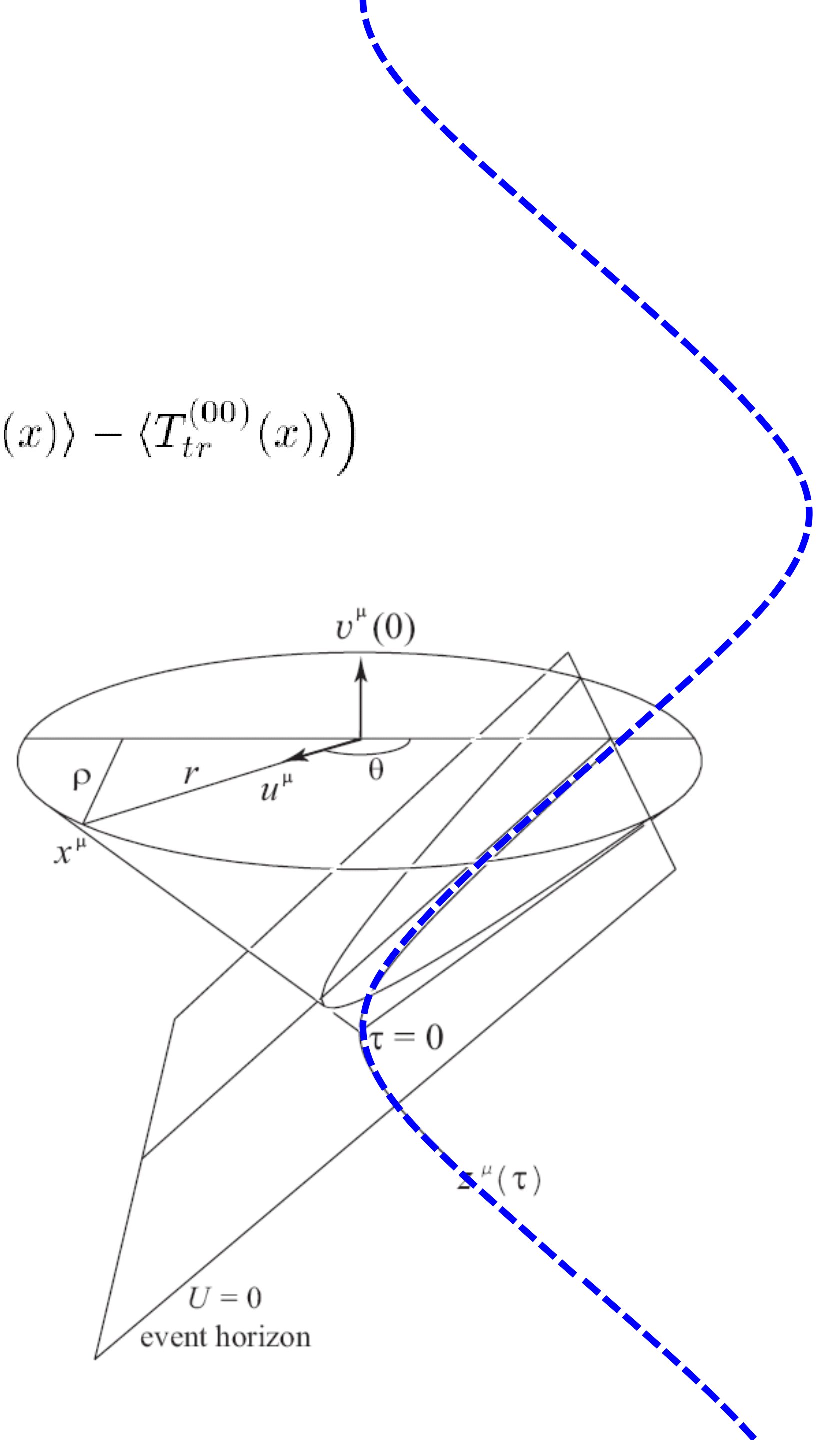
Radiation by UD Detectors

- We calculate

$$\frac{d\mathcal{P}}{d\Omega_{\rm II}}(t_0,\theta,\varphi) = -\lim_{r\to\infty} r^2\left(\langle T_{tr}(x)\rangle - \langle T_{tr}^{(00)}(x)\rangle\right)$$

where

$$\langle T_{\mu\nu}\rangle = \lim_{x'\to x}\left[\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'^\nu} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\frac{\partial}{\partial x^\rho}\frac{\partial}{\partial x'^\sigma}\right]\langle\hat\Phi(x)\hat\Phi(x')\rangle$$

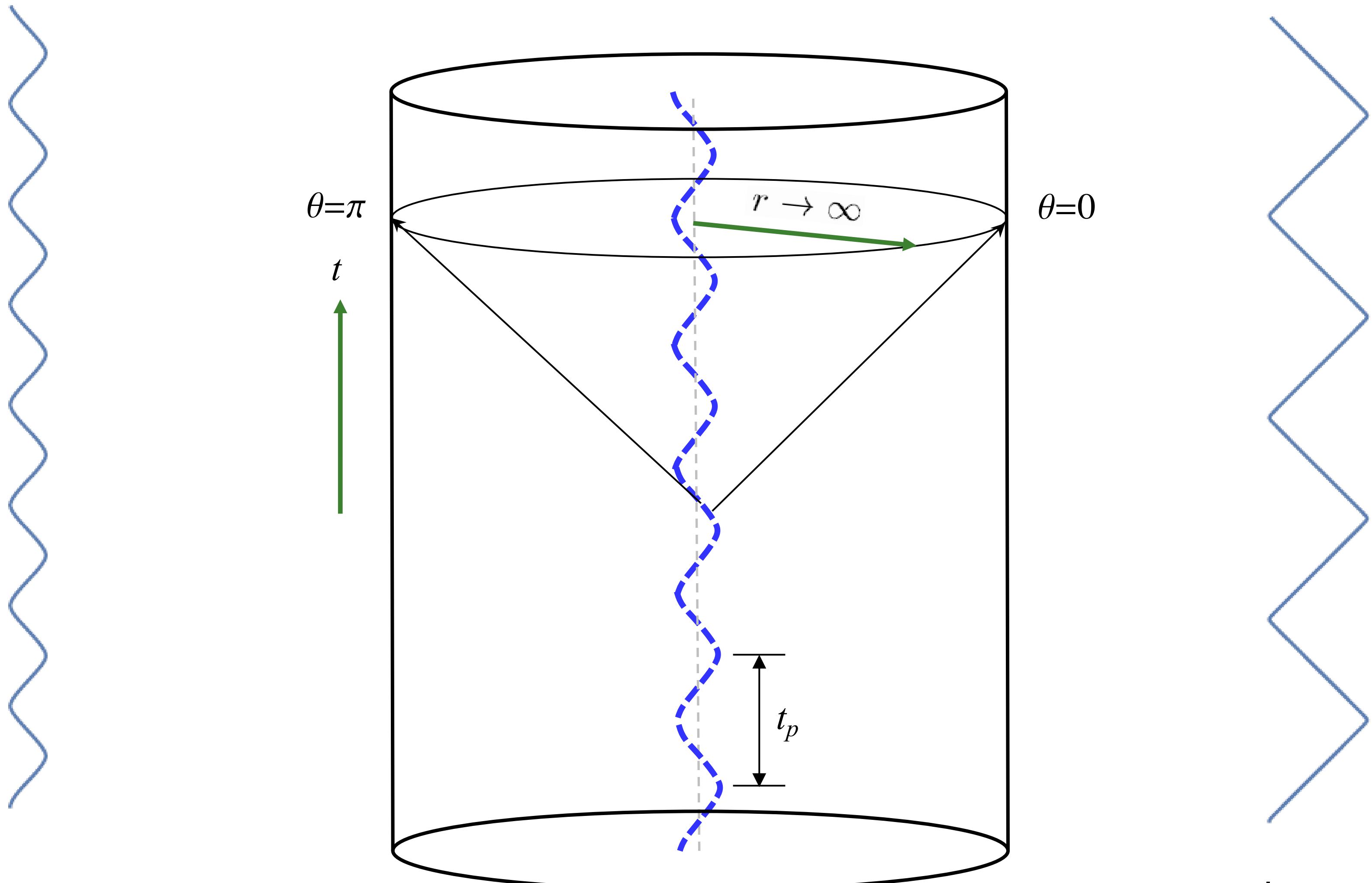


at null future infinity in the lab frame.

[SYL, B. L. Hu, PRD73(2006)124018]

Radiation by UD Detectors

- Radiated energy flux in the radiation zone (null infinity?) of the lab frame



higher ω
lower a_0

lower ω
higher a_0

$$z^{\mu}_{\rm CT}(t) = \left(t, 0, 0, -\frac{1}{\omega}\sin^{-1}\frac{2a_0\cos\omega t}{\sqrt{1+4a_0^2}} \right)$$

[Chen, Tajima '99]
($a_0 \equiv eE_0/mc\omega$)

Radiation by UD Detectors

- In (3+1)D, we calculate

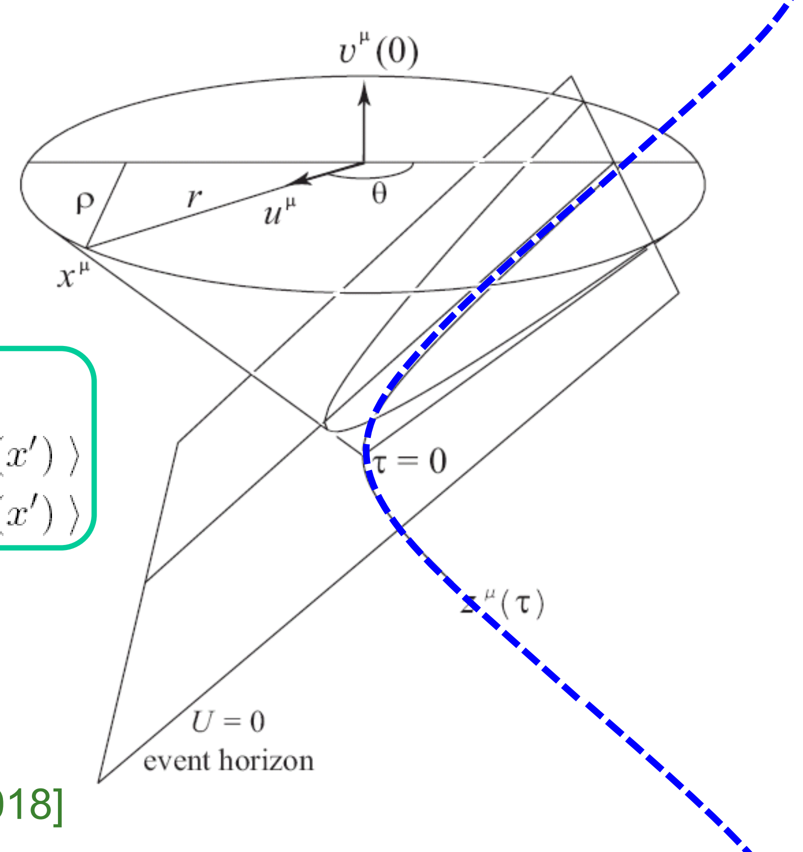
$$\frac{d\mathcal{P}}{d\Omega_{\rm II}}(t_0,\theta,\varphi) = -\lim_{r\to\infty} r^2 \left(\langle T_{tr}(x)\rangle - \langle T^{(00)}_{tr}(x)\rangle \right)$$ (normal ordered)

where $\langle T_{\mu\nu}(x)\rangle - \langle T^{(00)}_{\mu\nu}(x)\rangle$

$$= \lim_{x'\to x}\left[\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x'^\nu} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \frac{\partial}{\partial x^\rho}\frac{\partial}{\partial x'^\sigma} \right] G_{\rm ren}(x,x')$$

$$\begin{aligned} & \langle\, \Phi(x)\Phi(x')\,\rangle - \langle\, \Phi_0(x)\Phi_0(x')\,\rangle \\ \sim\ & \langle\, [\Phi_0(x)+\Phi_{\rm ret}(x)]\,[\Phi_0(x')+\Phi_{\rm ret}(x')]\,\rangle - \langle\, \Phi_0(x)\Phi_0(x')\,\rangle \\ =\ & \underbrace{\langle\, \Phi_{\rm ret}(x)\Phi_{\rm ret}(x')\,\rangle}_{G^{(11)}} + \underbrace{\langle\, \Phi_0(x)\Phi_{\rm ret}(x')\,\rangle}_{G^{(01)}} + \underbrace{\langle\, \Phi_{\rm ret}(x)\Phi_0(x')\,\rangle}_{G^{(10)}} \end{aligned}$$

$Q^2 \to \langle Q^2 \rangle$ Interfering terms



[Grove, CQG3('86)801; Raine, Sciama, Grove, PRSLondA435('91)205; SYL, Hu, PRD73(2006)124018]

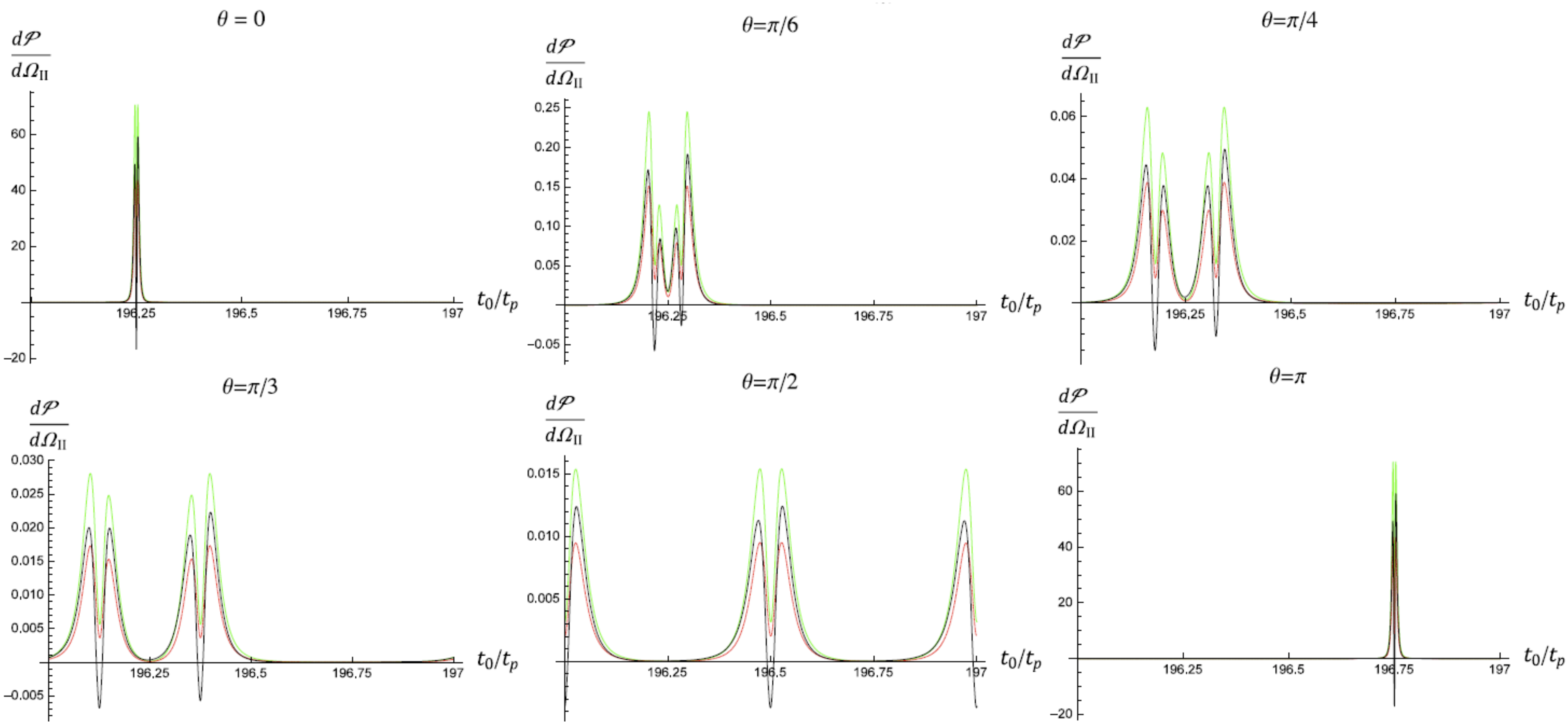
Radiation by UD Detectors

Time evolution of radiated energy flux at fixed directions

Naïve result with T_{eff}

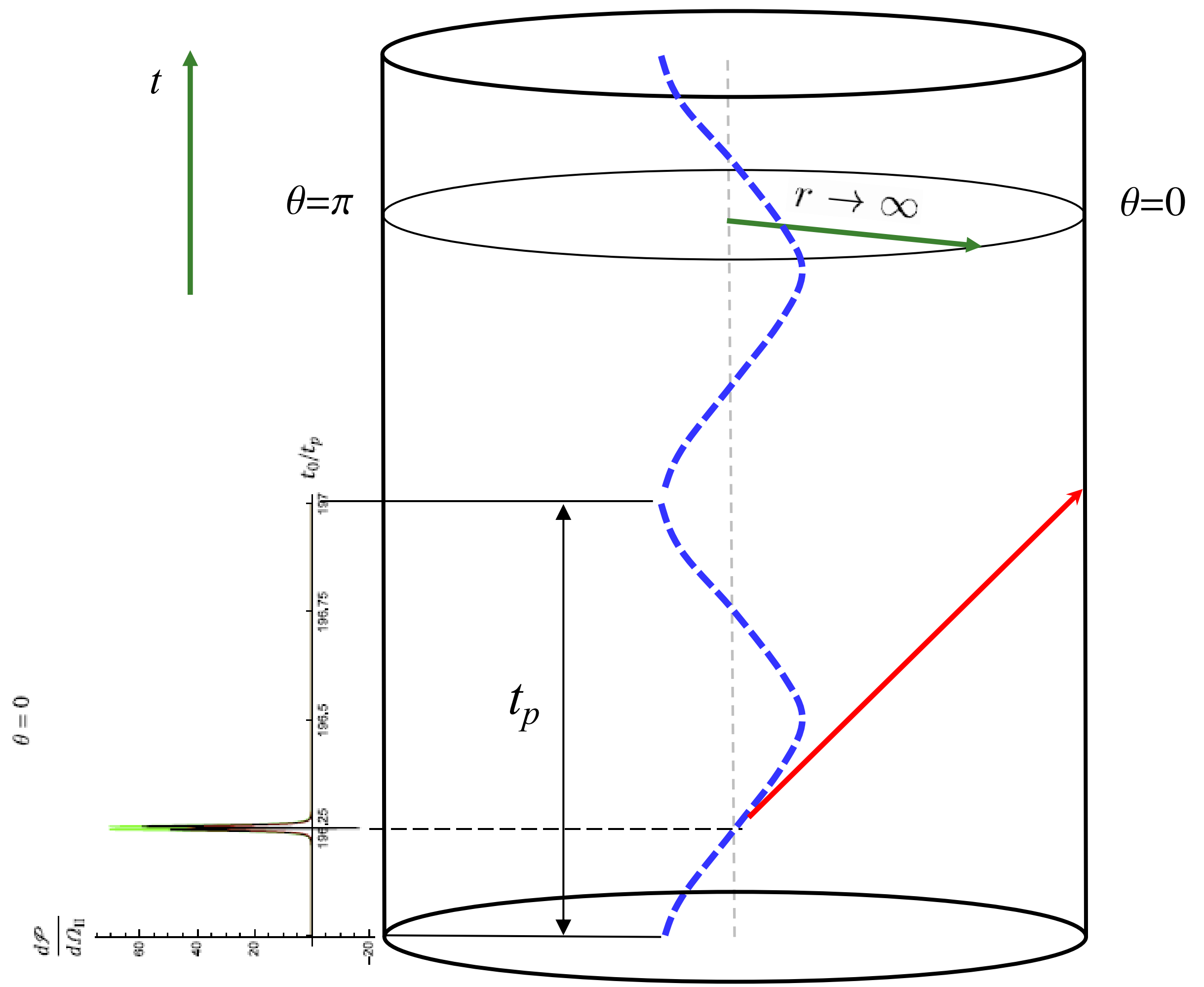
Naïve result with $T_U = 0$

Full result



Radiation by UD Detectors

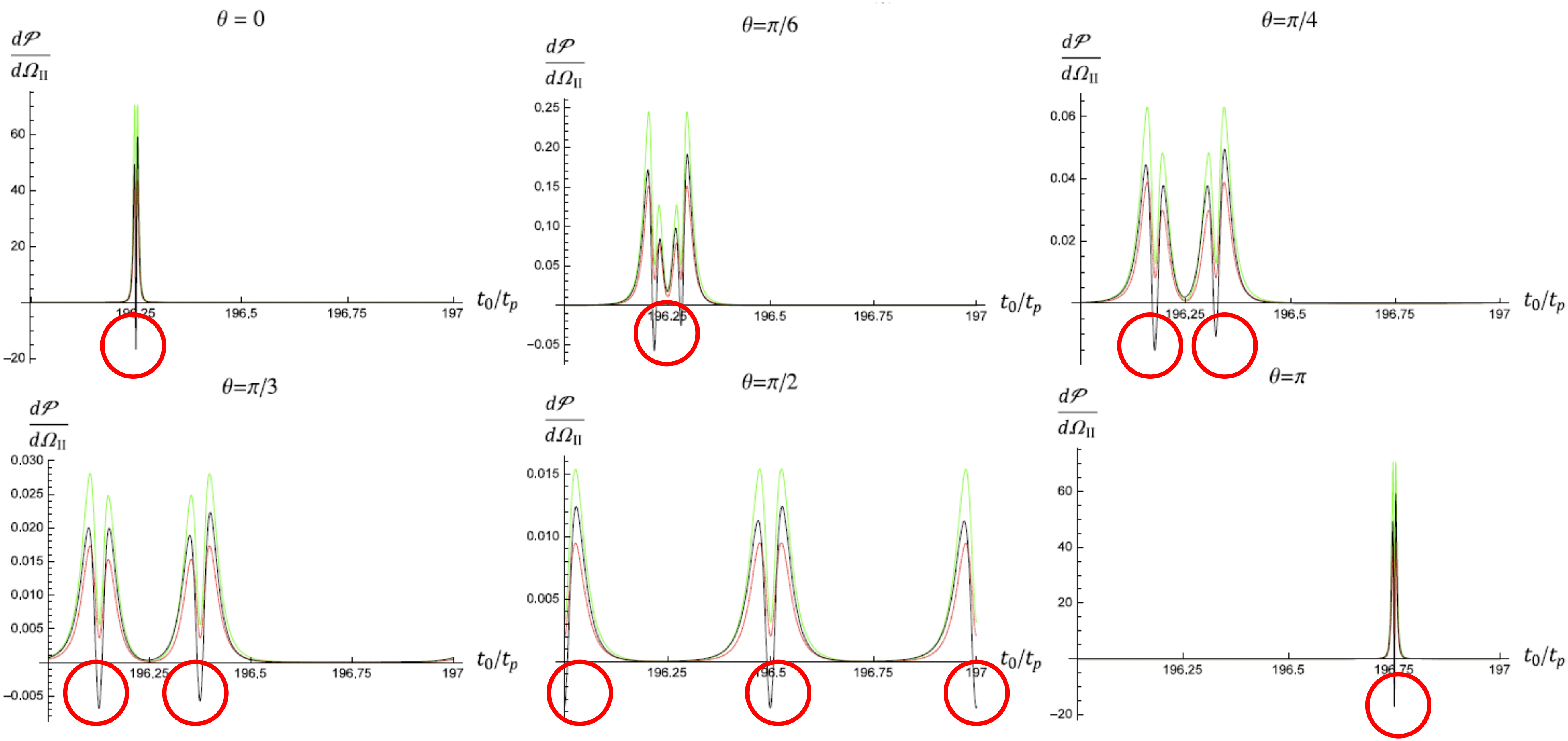
Radiated energy flux in the radiation zone of the lab frame



Radiation by UD Detectors

Time evolution of radiated energy flux at fixed direction

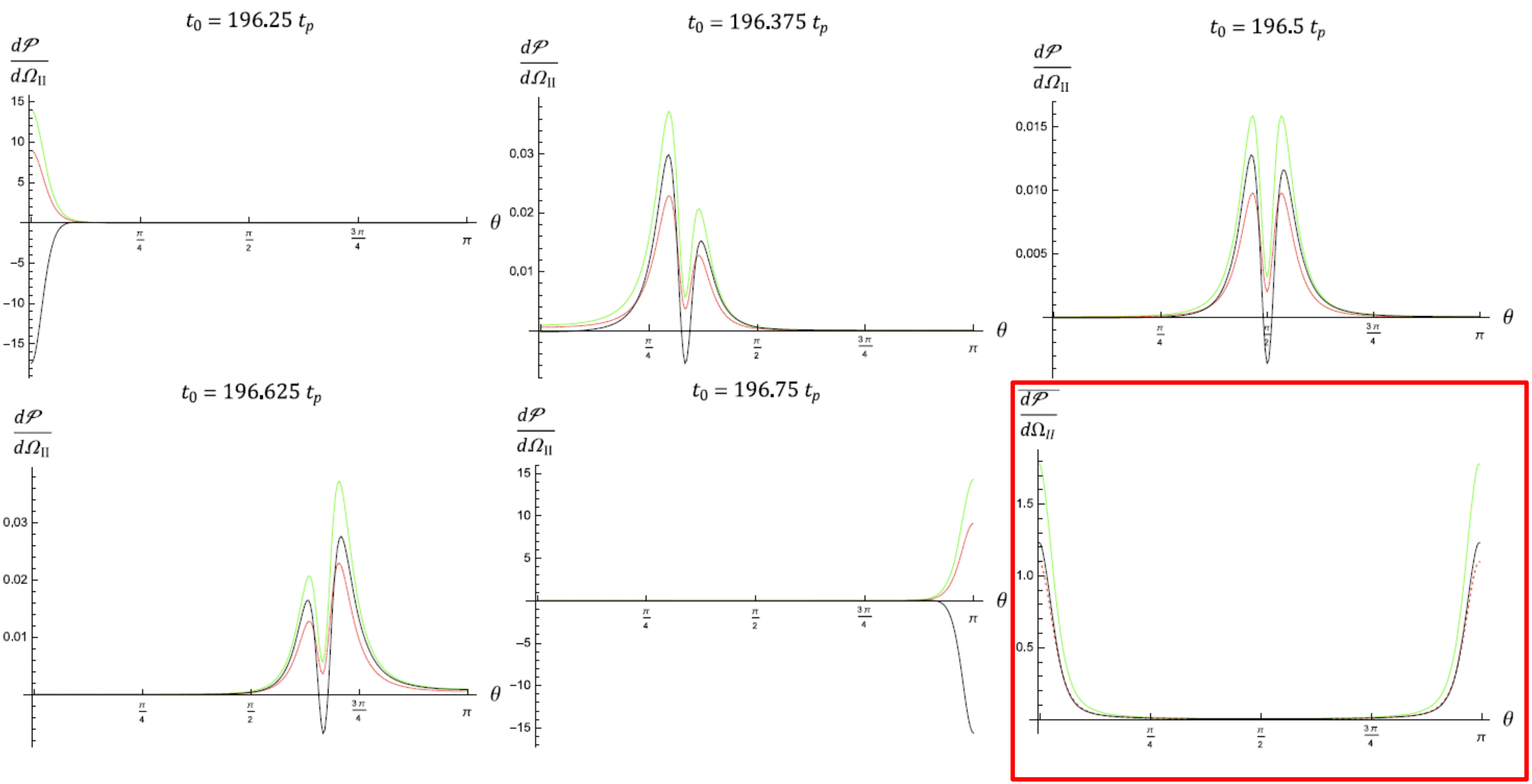
- Short-period negative energy flux



ω=3.277, a_0=2, $\bar{a}$=10

Radiation by UD Detectors

Angular distribution of radiated energy flux at fixed times



Energy flux averaged over a cycle of motion > 0

quantum inequality [Ford PRD43('91)3972]

Radiation by UD Detectors

Q1: Is the field state squeezed?

[Kuo, Ford PRD47('93)4510; Schützhold, Schaller, Habs, PRL100(08)091301]

Most likely a multi-mode squeezed state (Gaussian).

But which modes? How squeezed?

Q2: How does the Unruh temperature manifest in the Unruh radiation?

Note that in (1+1)D, a UAD has no radiation in equilibrium conditions.

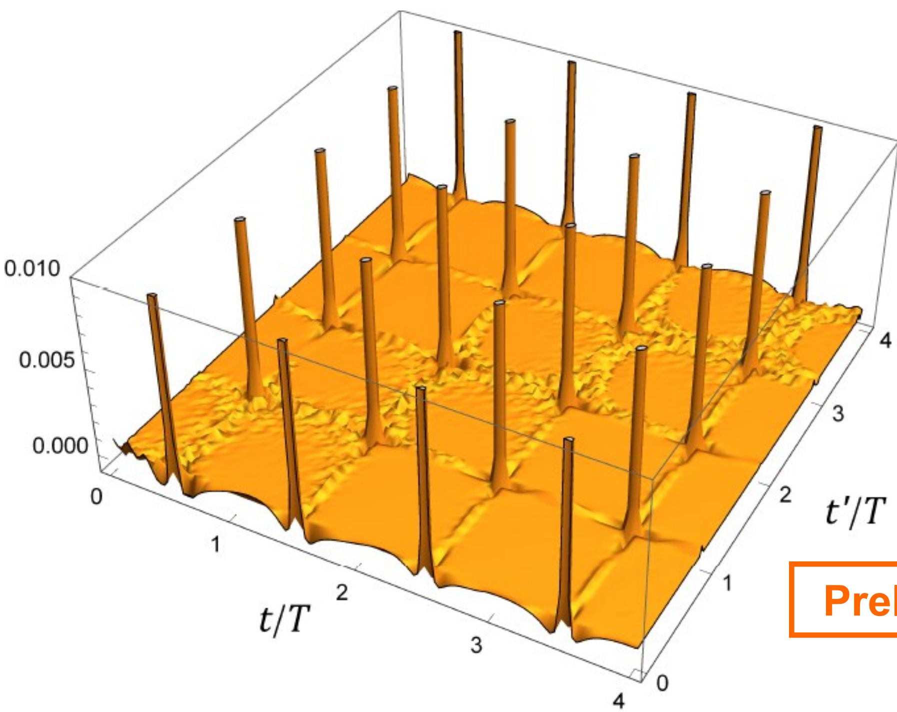
[Raine, Sciama, Grove, PRSLondA435('91)205]

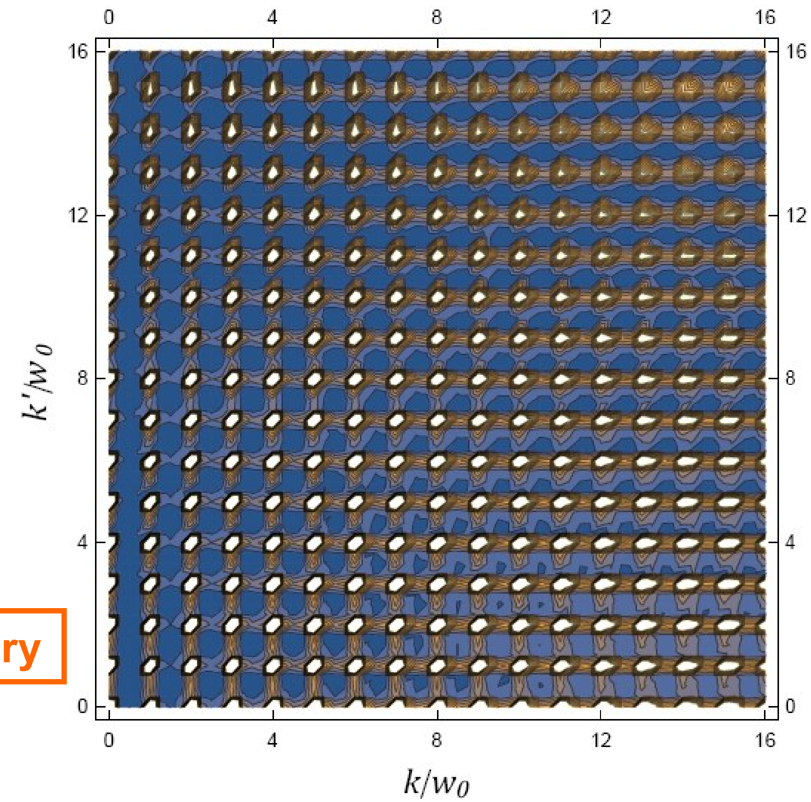
Asymptotic States (Gaussian)

- Late-time two-point correlator of the field in the radiation zone

$$\langle \hat{\Phi}^{[0]}_{\mathbf{k}}, \hat{\Phi}^{[0]}_{\mathbf{k}'} \rangle \propto \delta^3(\mathbf{k}+\mathbf{k}') \qquad \langle \hat{\Phi}^{[1]}_{x}, \hat{\Phi}^{[1]}_{x'} \rangle = \frac{\lambda_0^2}{(2\pi)^2 4\mathcal{R}\mathcal{R}'}\theta(\eta_-)\theta(\eta'_-)\langle Q(\eta_-), Q(\eta'_-)\rangle$$

$$\langle \hat{\Phi}^{[1]}_{x}, \hat{\Phi}^{[0]}_{x'} \rangle = \frac{2\hbar\gamma\theta(\eta_-(x))}{\Omega\mathcal{R}(x)}\mathrm{Re}\int_{\tau_0}^{\tau_-(x)} d\tilde{\tau} K(\tau_-(x)-\tilde{\tau})D^+(x', z(\tilde{\tau}-i\epsilon))$$





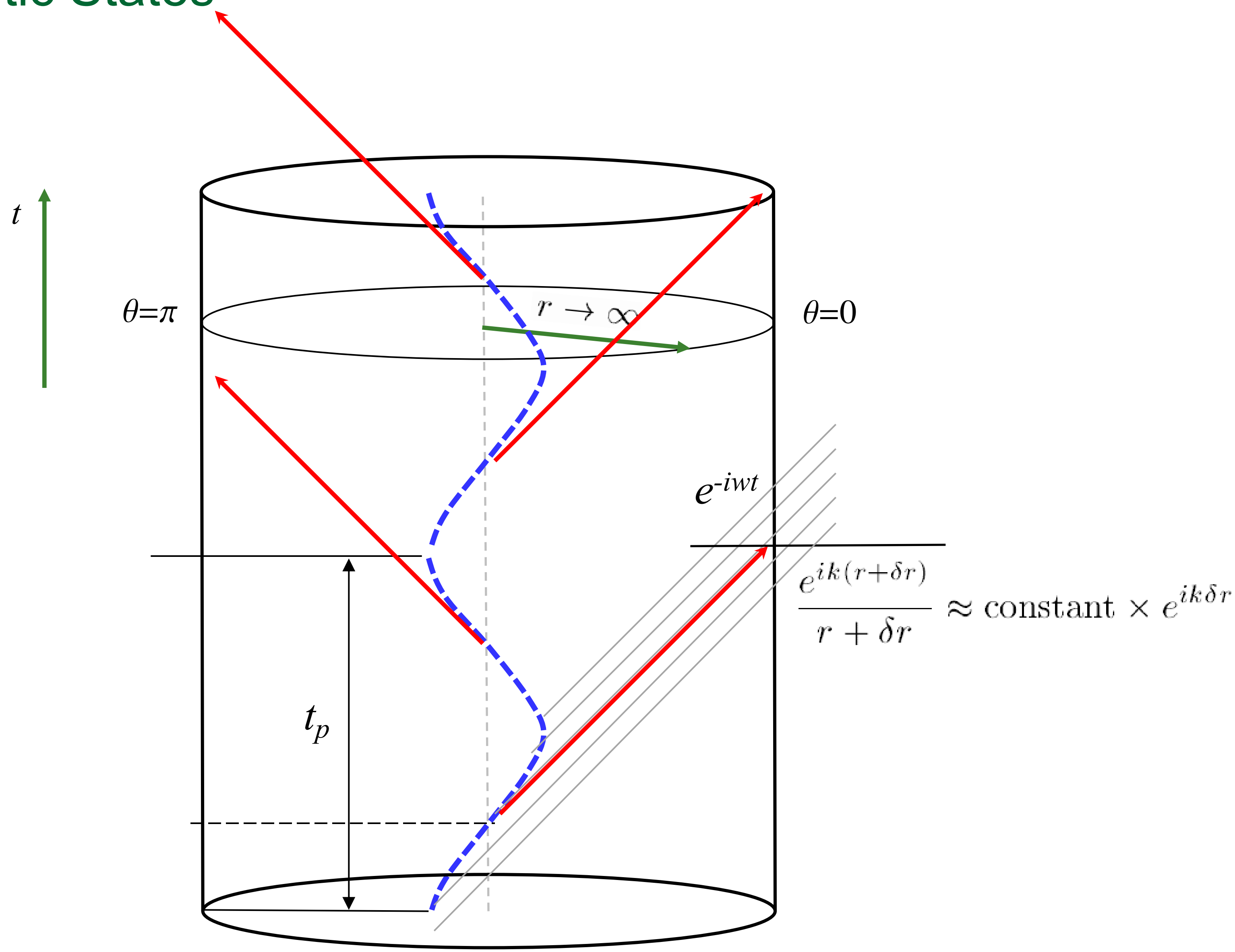
Preliminary

$$\langle \Phi_{\theta=0}(t), \Phi_{\theta=\pi}(t')\rangle - \langle \Phi^{[0]}_{\theta=0}(t), \Phi^{[0]}_{\theta=\pi}(t')\rangle$$

HHG!

$$\left| \langle \Phi_{\mathbf{k}}, \Phi_{-\mathbf{k}'}\rangle - \langle \Phi^{[0]}_{\mathbf{k}}, \Phi^{[0]}_{-\mathbf{k}'}\rangle \right|$$

Asymptotic States

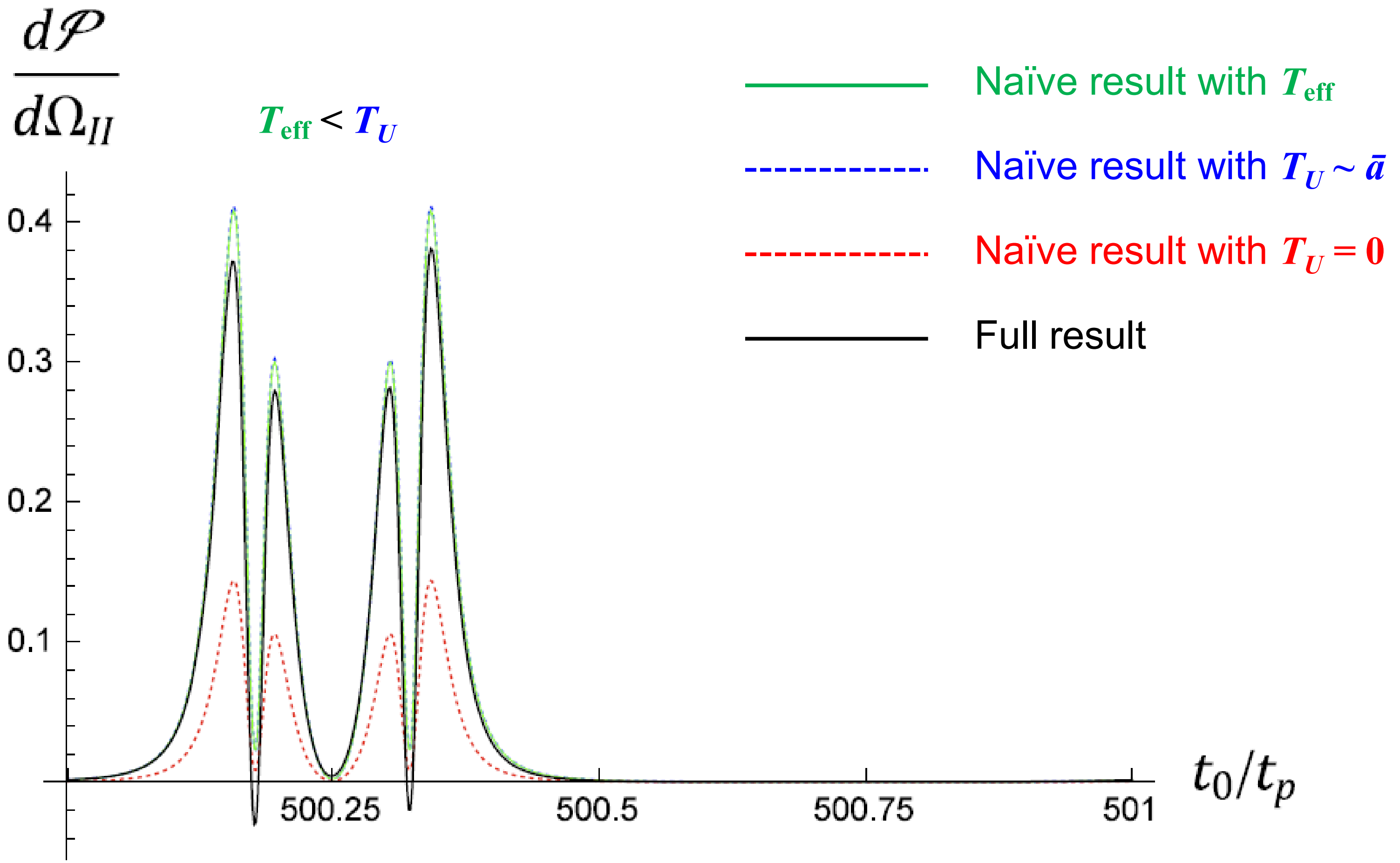


Radiation by UD Detectors

At a <u>higher</u> averaged proper acceleration:

$$\bar{a} \equiv \frac{\int_{\mathcal{P}} a(\tau) d\tau}{\int_{\mathcal{P}} d\tau}$$

$$\gamma=0.01, \theta=\pi/4, \omega=3.277*2, a_0=2, \bar{a}=20$$



The **full result** is closer to the naïve result (at finite temperature).

Deviation of T_{eff} from the Unruh temperature T_U happens to be unimportant.

Summary

- An effective temperature ‘experienced’ by an Unruh-DeWitt detector in oscillatory motion in the Minkowski vacuum is defined.
- The radiated energy flux emitted by a UD detector in oscillatory motion can be negative in short periods, while the flux averaged over a cycle is always positive. This indicates that the field state is squeezed.
- High-harmonic generation (HHG) is evident in the asymptotic state of the field in the relativistic regime. Quanta with different frequencies are correlated.
- Deviation of the effective temperature from the Unruh temperature is unimportant in the radiation in our parameter regime.
- With high $\bar{a}$ and short t_p (i.e. in the highly non-equilibrium regime), the Unruh-like effect can be pronounced in the Unruh radiation even though the interference tends to reduce it.

Thank you!

[Doukas, SYL, Hu, Mann, JHEP11(2013)119; SYL, arXiv: 1601.07006]