

Holography: From Discretum to Continuum

Seth Kurankyi Asante
Perimeter Institute
sasante@pitp.ca

at

GR21 Conference
Columbia University

July 14, 2016

Outline

- 1 Motivation
- 2 3D Continuum Computations
- 3 3D Discrete Computations
 - Boundary field theory
- 4 Extension to 4D

Motivation

- Study quantum gravity at semi-classical level for generalised boundaries
- Consider coarse graining and how diffeomorphism symmetry is restored
- Study Hamilton Jacobi functional for GR for general boundaries

Continuum 1-Loop Computations

[Brown & Henneaux '86, Giombi, Maloney & Yin '08, Barnich, González, Oblak, Maloney '15]

- One loop partition function of AdS reproduces character of double-Virasoro algebra
- One loop partition function is exact
- Classical central extension for asymptotic symmetries at null infinity(3D) gives BMS group
- One loop partition function computed in flat space
- vacuum character of BMS reproduced

3D thermal
spinning flat
space

$$ds^2 = dt^2 + dr^2 + r^2 d\phi^2$$

$$(r, t, \phi) \sim (r, t + \beta, \phi + \gamma)$$

periodic time

angular twist

$$\blacksquare Z(\beta, \gamma) \sim e^{-\hbar^{-1}S^{(0)} + S^{(1)} + \mathcal{O}(\hbar)} = e^{\frac{\beta}{8G}} \prod_{k=2}^{\infty} \frac{1}{|1 - q^k|^2}, \quad q = e^{i\gamma}$$

Discrete Computations

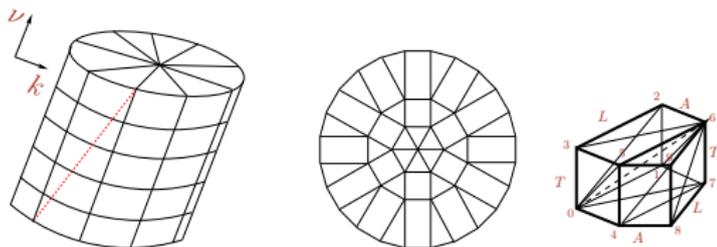
[B Dittrich , V Bonzom '15]

- Use Regge action (semi classical limit of spin foams)

$$-8\pi G S_R[l_e] = \sum_{e \in \mathcal{T}^0} l_e \epsilon_e + \sum_{e \in \partial \mathcal{T}} l_e \psi_e, \quad \epsilon_e = 2\pi - \sum_{\sigma \supset e} \theta_e^\sigma, \quad \psi_e = \pi - \sum_{\sigma \supset e} \theta_e^\sigma$$

- Path Integral $Z_R = \int \mathcal{D}\mu(l) \exp(-\frac{1}{\hbar} S_R)$

- make use of topological nature of the theory



- Vary $l_e = L_e + \delta_e$: $S_R \sim \sum_e \delta_e H_{ee'} \delta_{e'} = \sum_{k, \nu} \hat{\delta}(k, \nu) \tilde{M}(k, \nu) \hat{\delta}(k, \nu)$

- One loop is determined by determinant of Hessian $H_{ee'} = \frac{1}{2} \frac{\partial^2 S_R}{\partial l_e \partial l_{e'}} \Big|_{l_e = L_e}$
(restricted to the bulk)

Results

- $H_{ee'}$ has gauge modes which only affects $k = 0, \pm 1$ modes in angular direction. (No further regularization) Corresponds to displacement of bulk vertices in time direction ($\tilde{M}(k = 0, \nu)$) and on spatial hypersurface ($\tilde{M}(\pm 1, \nu)$)
- One loop correction is:

$$\frac{1}{\sqrt{\det M_{red}}} \approx \left[\prod_{\nu} \frac{(ALT)^{\frac{3}{2}}}{L^2 T} (1 + \mathcal{O}(x)) \right] \prod_{k=2}^{(N_A-1)/2} \frac{1}{2x} \frac{1}{|1 - q^k|}; \quad q = e^{i\gamma}$$

absorbed into measure term

Remarks

- Need continuum limit in angular direction to get product over all modes
- identifies contributing degrees of freedom (\hat{r} variables)
- Product over angular fourier modes
- $k = 2$ is due to diffeomorphism invariance

Effective boundary action

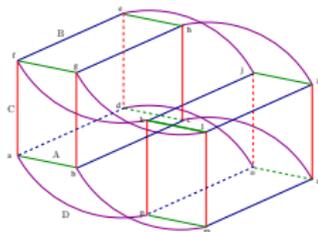
- Effective boundary action encoded in matrix $M_h^{bdry}(k, \nu)$
- Take continuum limit $A = \varepsilon \tilde{A}$, $T = \varepsilon \tilde{T}$, $\varepsilon \ll 1$
- Evaluate HJ functional on infinitesimal deformations
- Compare to continuum action evaluated on perturbations around flat metric induced by infinitesimal diffeomorphisms $\nabla_{[\mu} \xi_{\nu]}$
- Look for dual scalar field coupled to boundary metric; integrating out gives $M_h^{bdry}(k, \nu)$
- Discretizing Liouville action and integrating out using linear field equation lead to equivalent boundary action up to quadratic order

Extending to 4D ?

- Regge calculus convenient in computing HJ functional and 1-loop correction for finite boundaries
- Shown example of ‘thermal flat 3D space’ :flat space-CFT correspondence
- Regge calculus allowed to identify dual boundary and a finite boundary
- Made heavy use of bulk triangulation invariance in 3D
- Would it be possible to use Regge Calculus to study semi-classical partition function in 4D?
 - General boundary proposal: studying HJ functional for gravity
 - Coarse graining associated via amplitudes of generalised boundaries
 - Understand how diffeomorphism symmetry is restored
 - Determine PI measure for 4D

4D Regge Calculus

- Consider linearised Regge action for a regular lattice with flat background



$$8\pi G S_R \sim \frac{1}{2} \sum_{\sigma} \sum_{t \supset \sigma} \left. \frac{\partial A_t}{\partial l_e} \frac{\partial \theta_t^{\sigma}}{\partial l_{e'}} \right|_{flat} \delta_e \delta_{e'}$$

- Integrate out one direction (hope to consider all other directions)
- Provide canonical decomposition of lattice action $\mathcal{M} = T^3 \times \mathcal{I}$
- Find measure invariant under integrating out gauge degrees of freedom and compare to proposals from canonical and covariant context

Outlook

- Find effective action $\tilde{S}(\delta_t^{\Sigma_t}, \delta_{t+1}^{\Sigma_{t+1}})$ as Hamilton Jacobi functional
- Study coarse graining techniques
- To determine how or where diffeomorphism symmetry is restored
- Continuum limit in the bulk

Thank You