21st International Conference on General Relativity and Gravitation
Columbia University, New York

Accelerating CBC parameter estimation with multi-band frequency domain waveforms

Serena Vinciguerra

Supervisors: Ilya Mandel, John Veitch

GraWIToN
GW Initial Training Network

MARIE CURIE ACTIONS

UNIVERSITY OF BIRMINGHAM

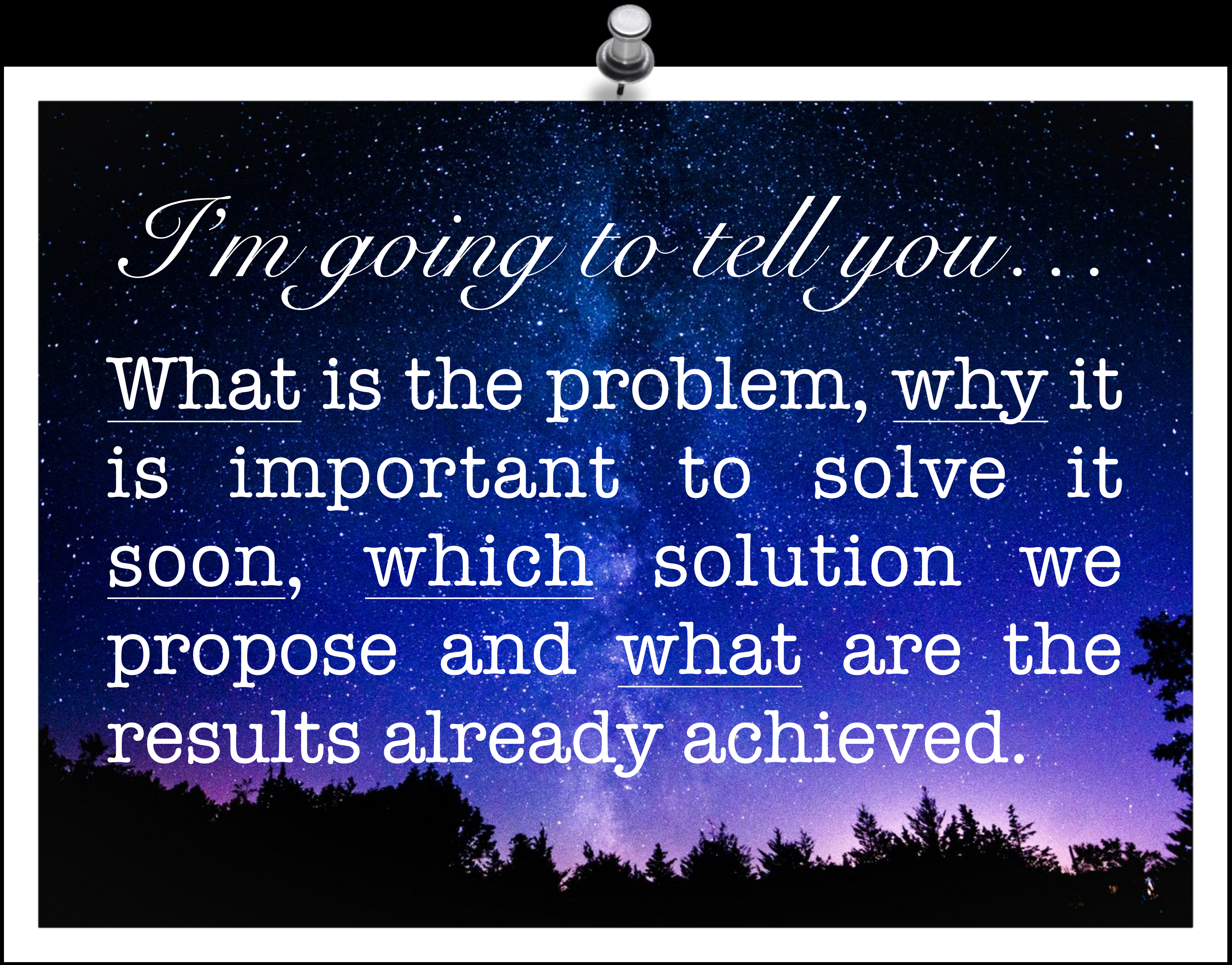
What?

why?

when?

which?

what?



What?

Why?

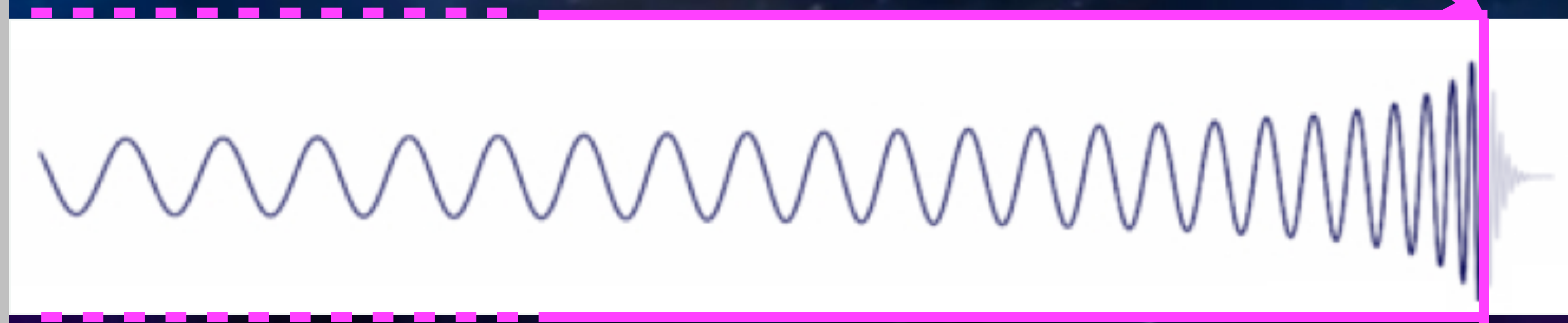
When?

Which?

What?

COMPACT BINARY COALESCENCES

- Neutron Star-Neutron Star [NS-NS];
- Neutron Star-Black Hole [NS-BH];
- Black Hole-Black Hole [BH-BH]



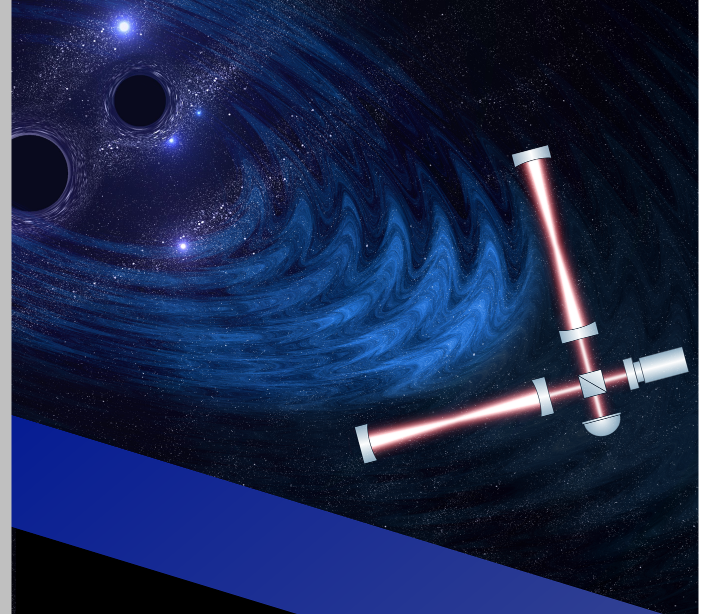
What?

Why?

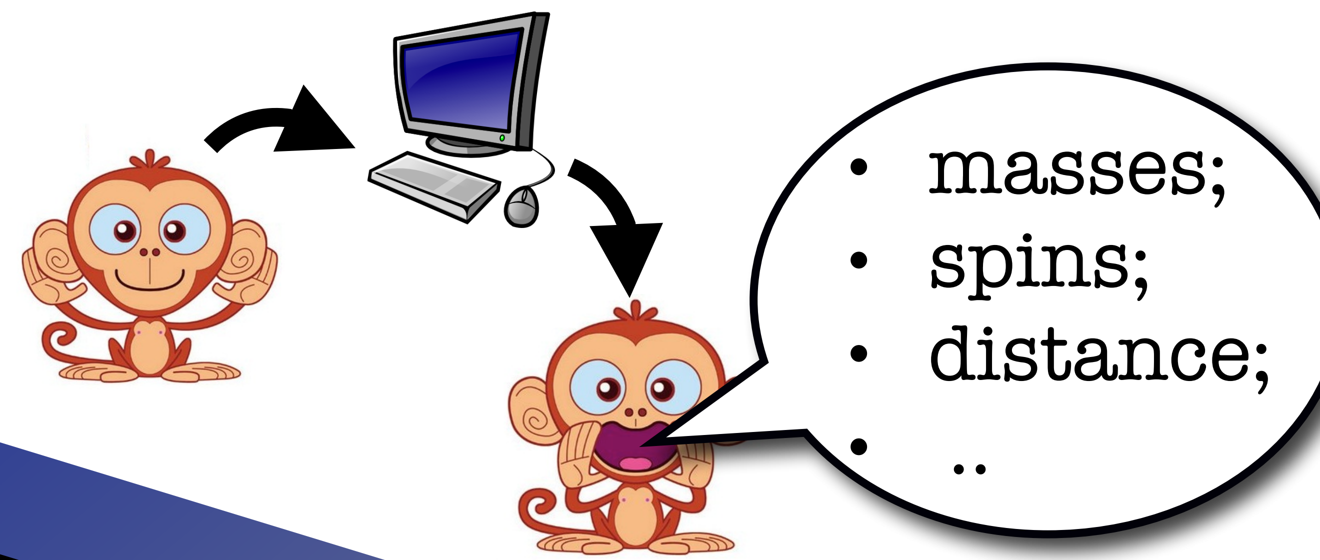
When?

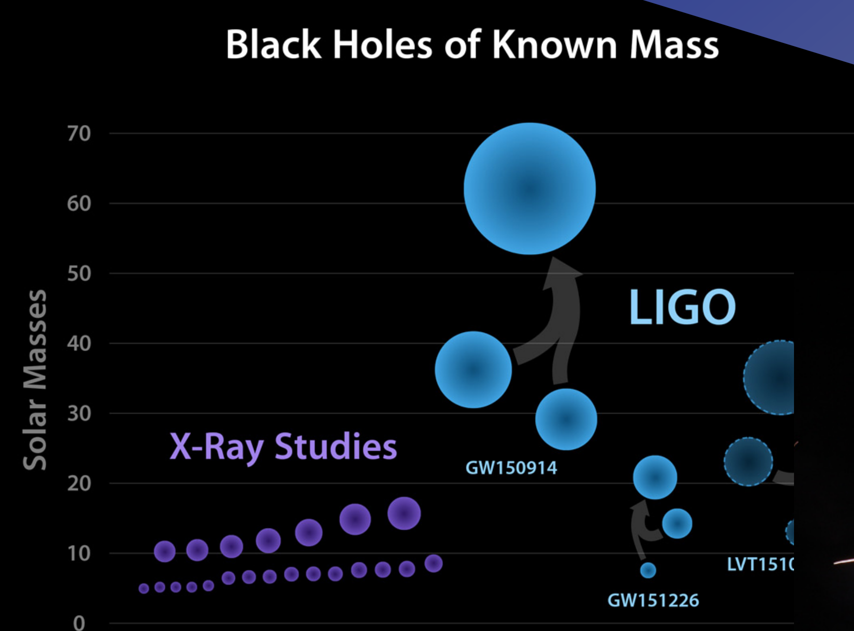
Which?

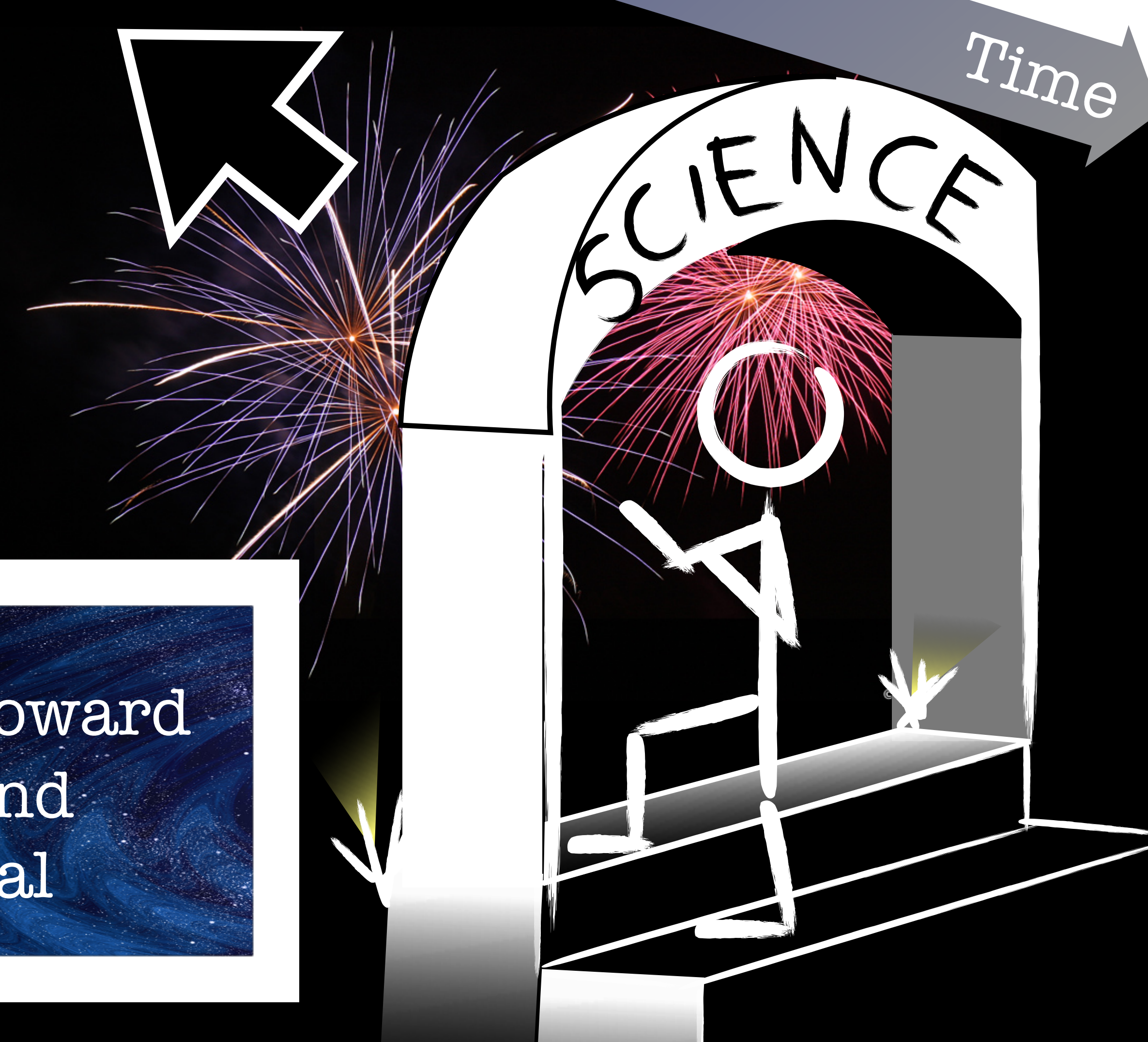
What?



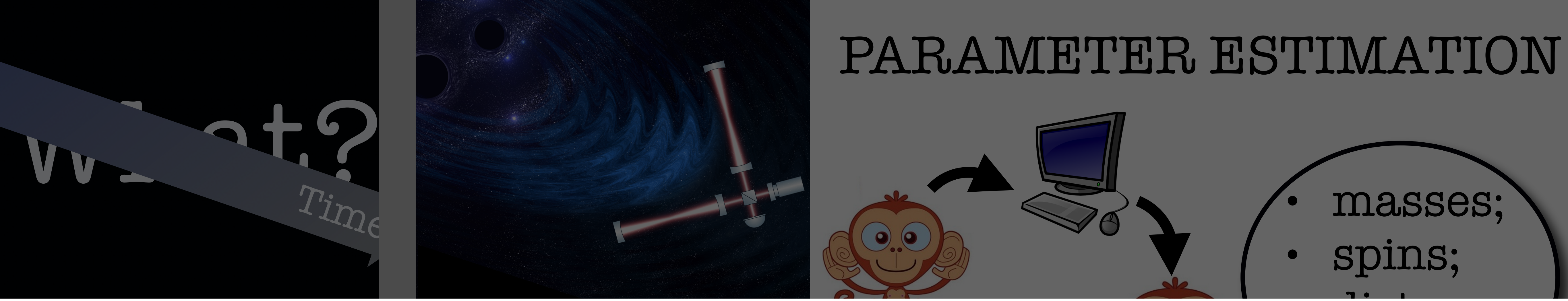
PARAMETER ESTIMATION







PE is our **first step** toward GW astronomy and science in general



PE IS CRUCIAL

What?

Why?

When?

Which?

What?

PE is based on the Bayes' theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

What?

Why?

When?

Which?

What?

PE is based on the Bayes' Theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

What?

Why?

When?

Which?

What?

PE is based on the Bayes' theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

Priors

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)\,p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

What?

Why?

When?

Which?

What?

PE is based on the Bayes' Theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

Priors

Likelihood

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)\, p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

What?

Why?

When?

Which?

What?

PE is based on the Bayes' Theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

Priors

Likelihood

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)\, p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

Evidence

What?

Why?

When?

Which?

What?

PE is based on the Bayes' theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{n}$$

parameters

$\mathbf{h}$: signal

$\mathbf{n}$: noise

Priors

Likelihood

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)\,p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

Evidence

Gaussian noise

- zero mean
- known variance

$$p(\mathbf{d}|\boldsymbol{\theta}, H) \propto \exp\left[-\delta f \sum_{i=0}^{N} \frac{2|\tilde{d}(f_i) - \tilde{h}(\boldsymbol{\theta}, f_i)|^2}{S_n(f_i)}\right]$$

What?

Why?

When?

Which?

What?

PE is based on the Bayes' Theorem

$\mathbf{d}$: data

H : model

$$\mathbf{d} = \mathbf{h}(\underset{\substack{\uparrow \\ \text{parameters}}}{\boldsymbol{\theta}}) + \mathbf{n}$$

$\mathbf{h}$: signal

$\mathbf{n}$: noise

Priors

Likelihood

$$p(\boldsymbol{\theta}|\mathbf{d}, H) = \frac{p(\boldsymbol{\theta}|H)\,p(\mathbf{d}|\boldsymbol{\theta}, H)}{p(\mathbf{d}|H)}$$

Posterior

Evidence

Gaussian noise

- zero mean
- known variance

$$p(\mathbf{d}|\boldsymbol{\theta}, H) \propto \exp\left[-\delta f \sum_{i=0}^{N} \frac{2|\tilde{d}(f_i) - \tilde{h}(\boldsymbol{\theta}, f_i)|^2}{S_n(f_i)}\right]$$

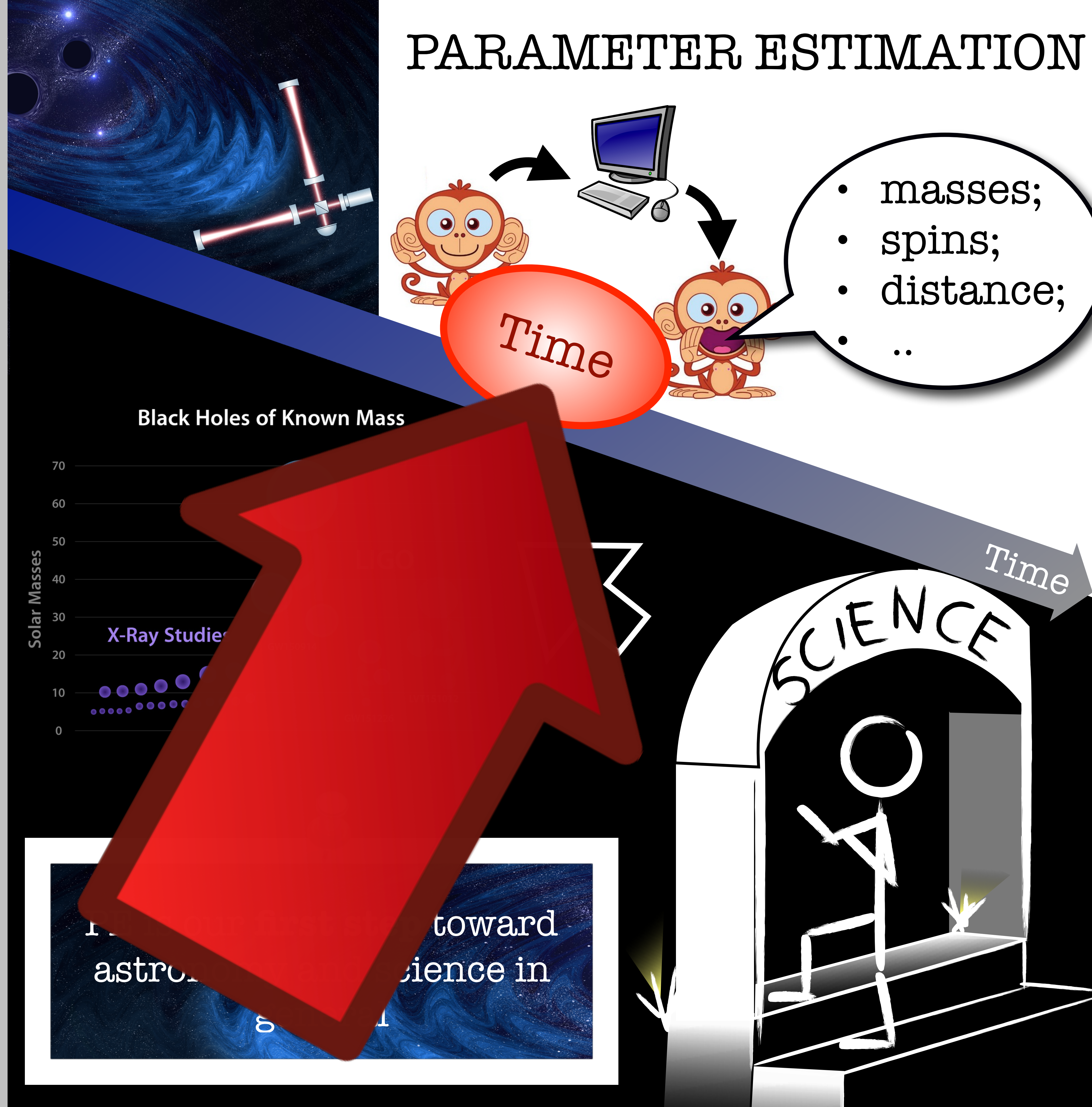
What?

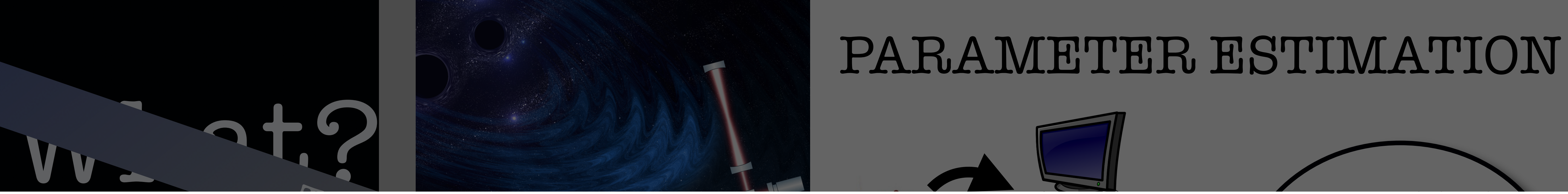
Why?

When?

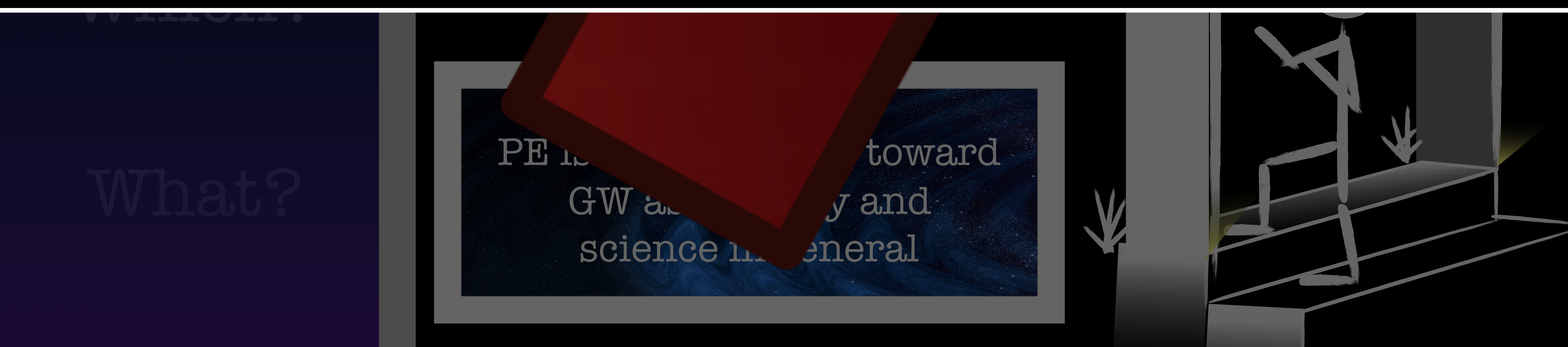
Which?

What?





PE IS EXPENSIVE



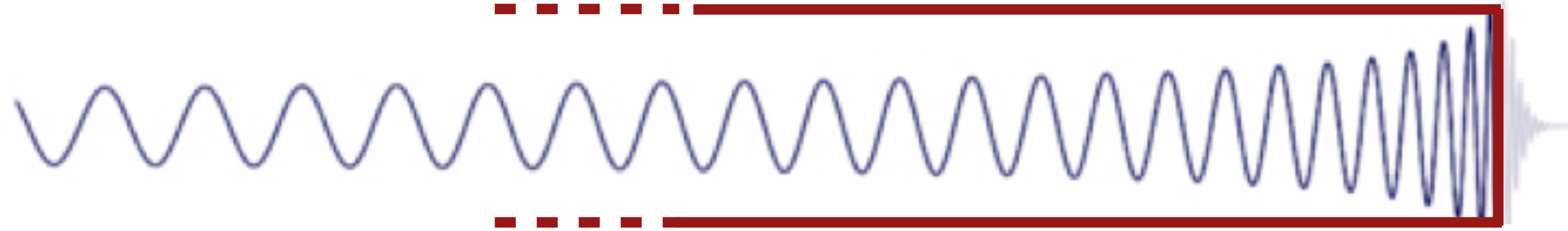
What?

Why?

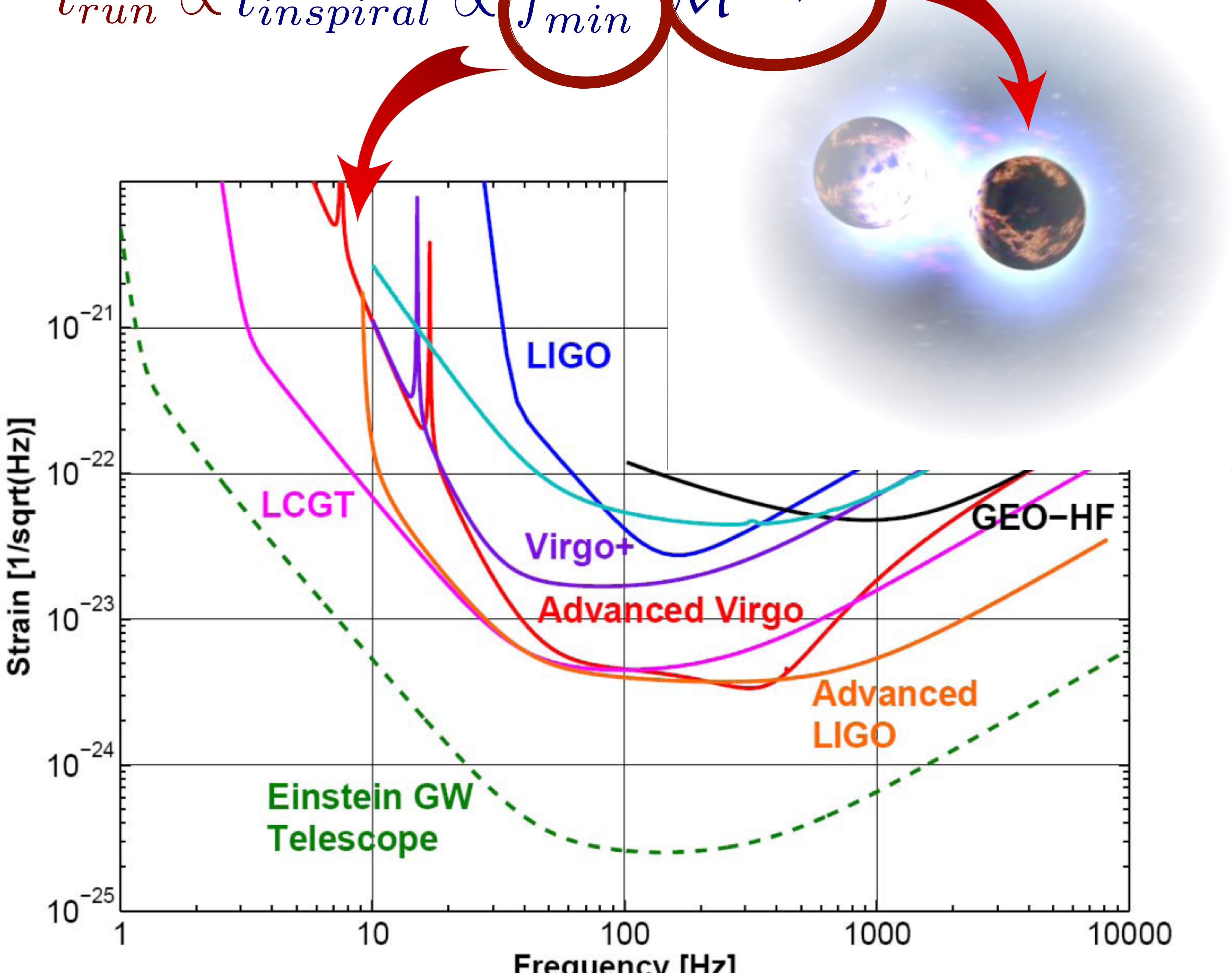
When?

Which?

What?



$$t_{run} \propto t_{inspiral} \propto f_{min}^{-8/3} \mathcal{M}^{-5/3}$$



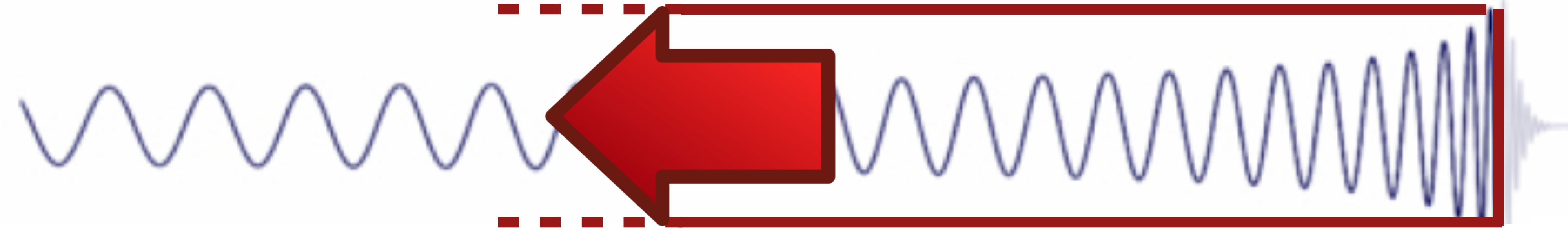
What?

Why?

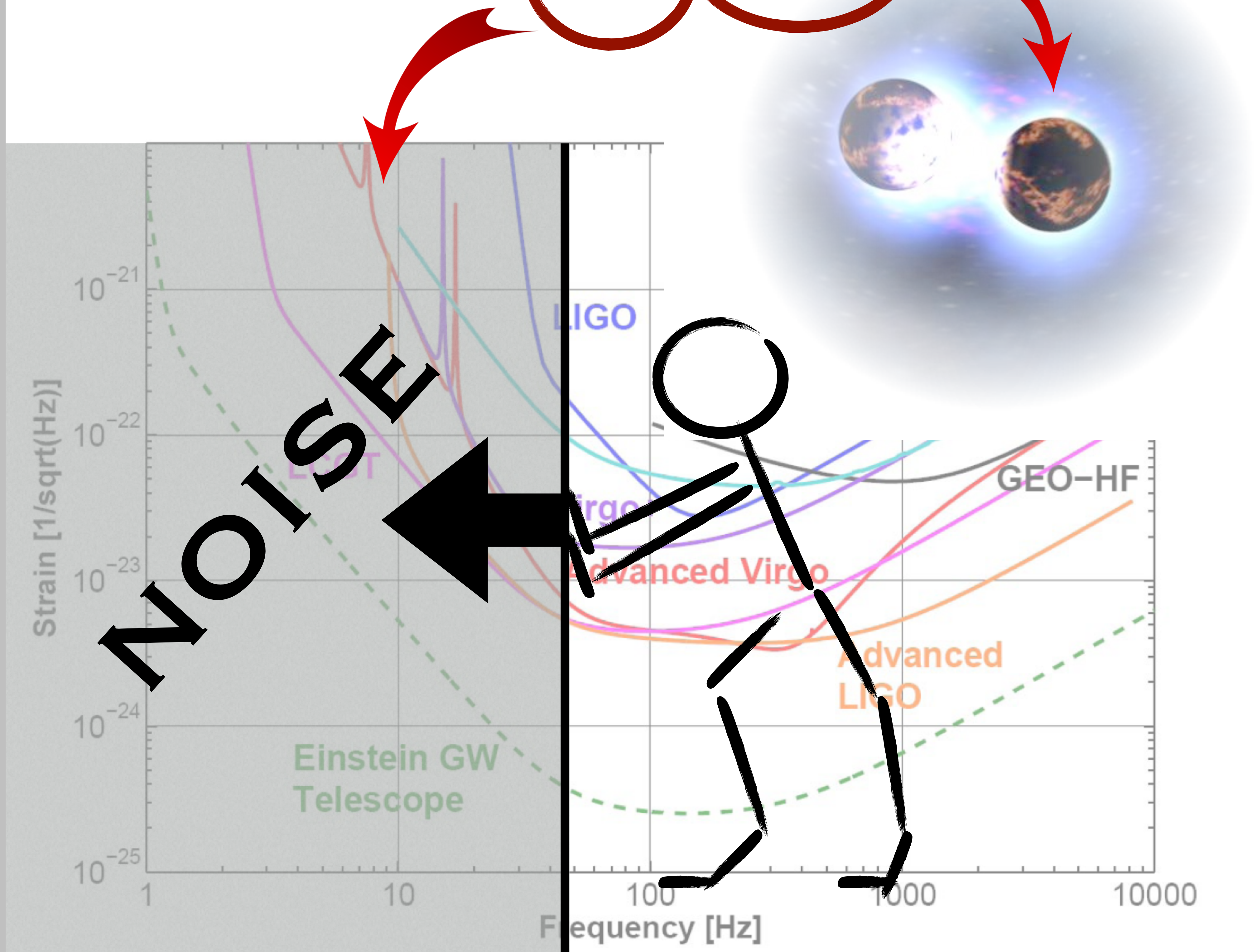
When?

Which?

What?



$$t_{run} \propto t_{inspiral} \propto f_{min}^{-8/3} \mathcal{M}^{-5/3}$$



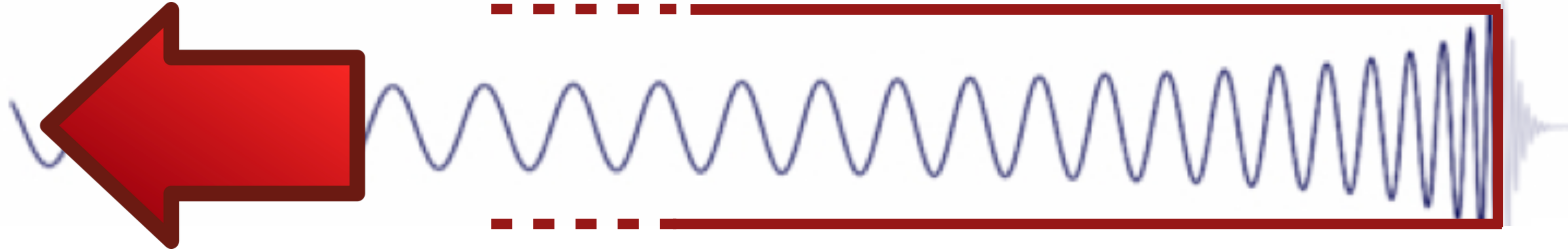
What?

Why?

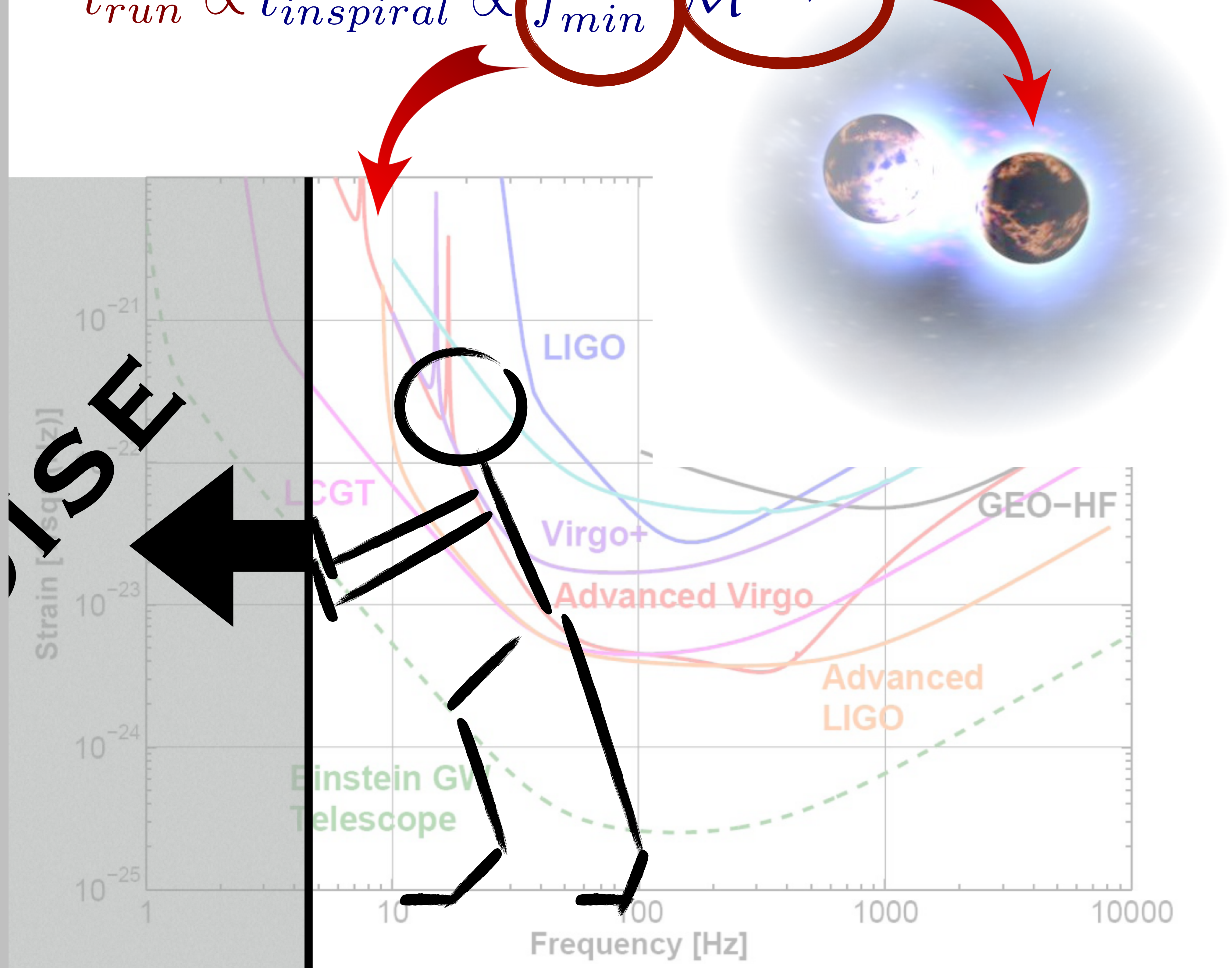
When?

Which?

What?



$$t_{run} \propto t_{inspiral} \propto f_{min}^{-8/3} \mathcal{M}^{-5/3}$$



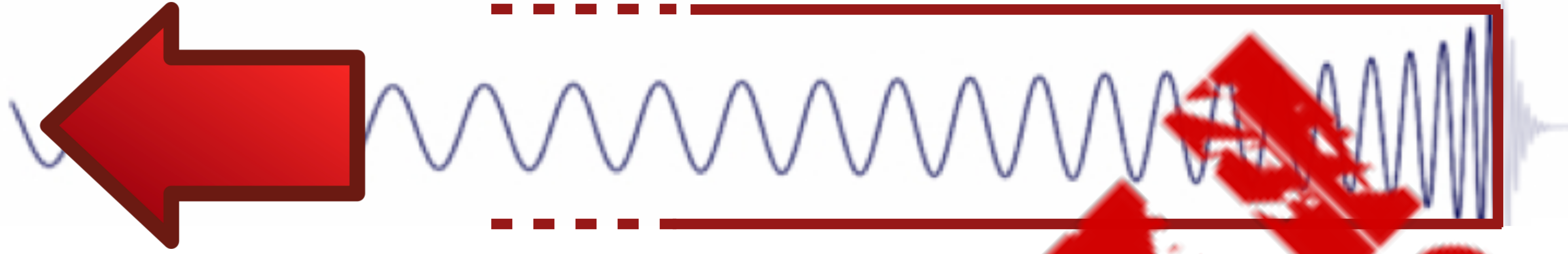
What?

Why?

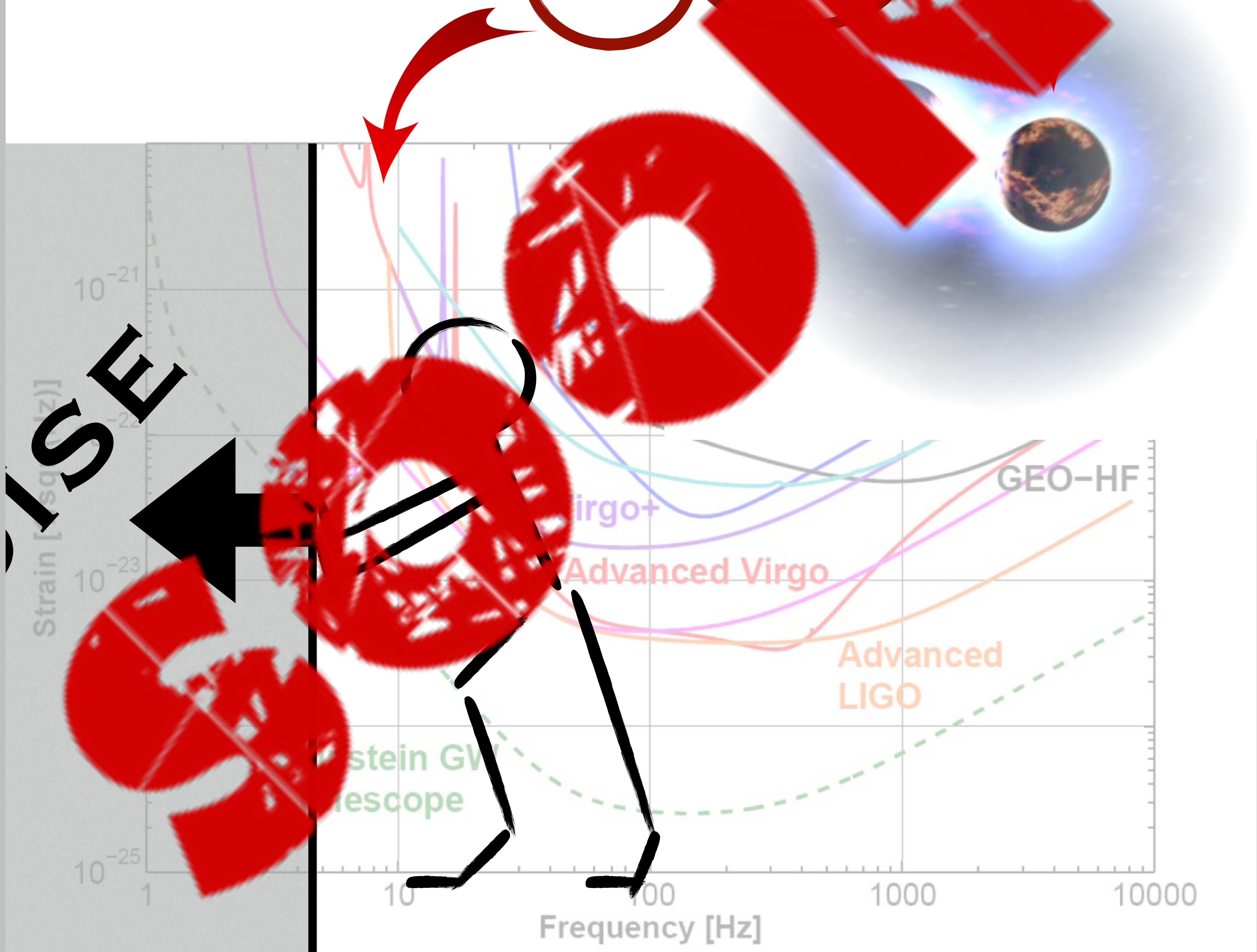
When?

Which?

What?



$$t_{run} \propto t_{inspiral} \propto f_{min}^{-8/3} \mathcal{M}^{-5/3}$$



What?

Why?

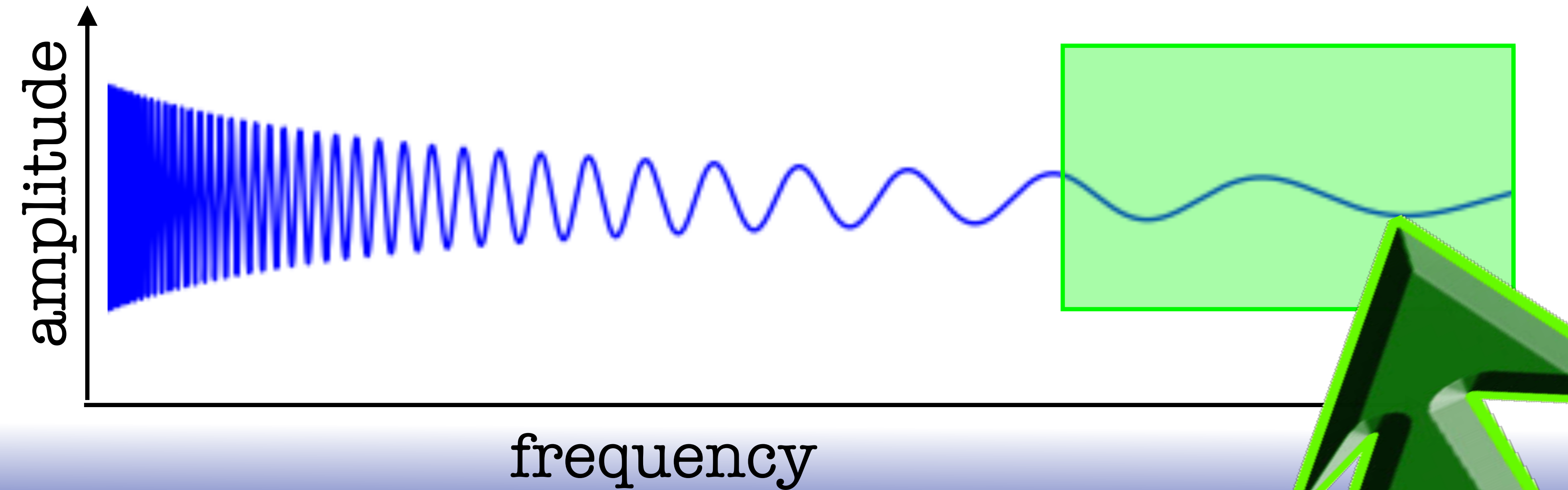
When?

Which?

What?

FREQUENCY DOMAIN WAVEFORM

THE CHIRPING FEATURE IN FREQUENCY DOMAIN



The latest parts of the waveform don't need the same sampling of the first ones

SAMPLING THEORY

$$\delta f \leq T[s]^{-1}$$

~ONE POINT EVERY CYCLE

What?

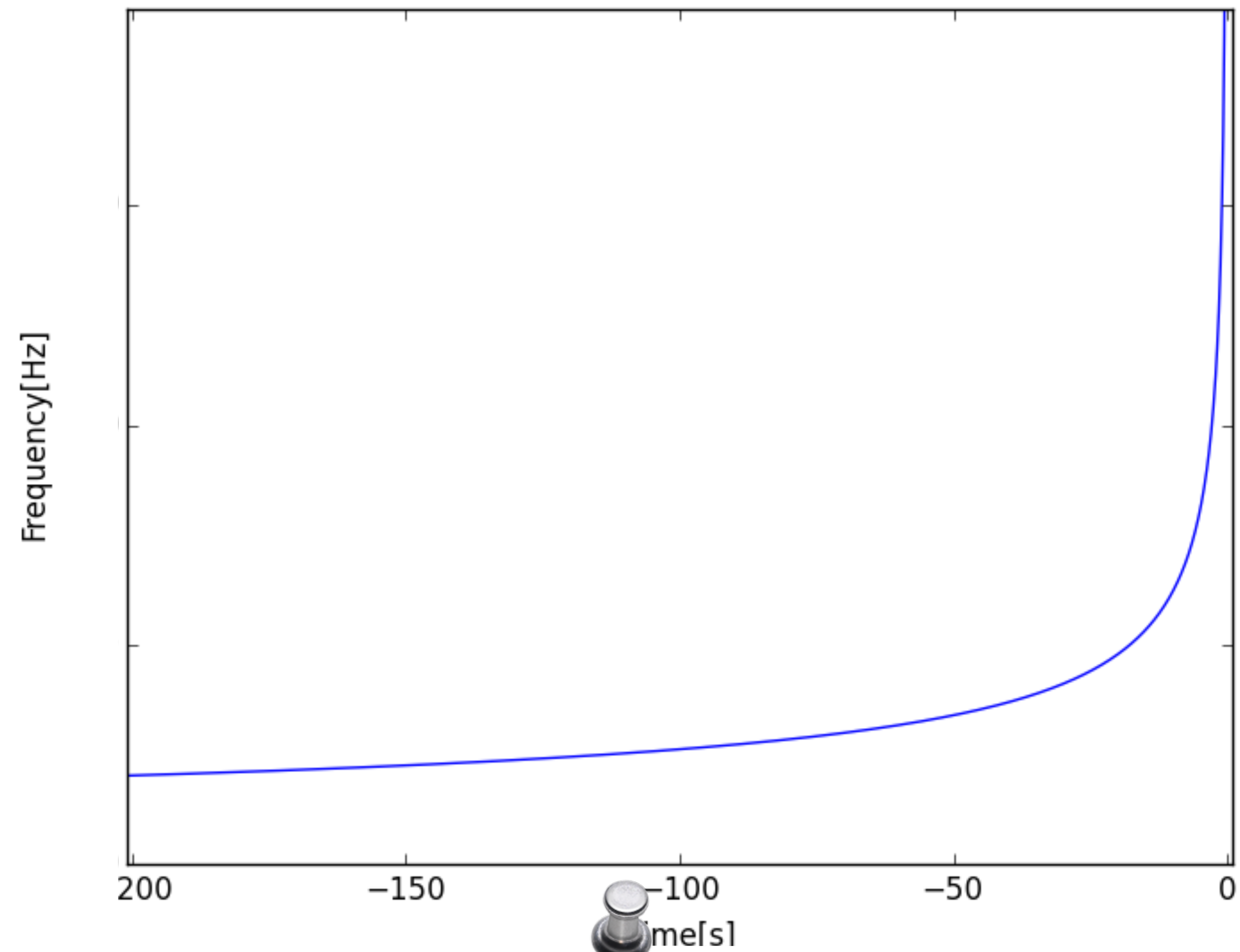
Why?

When?

Which?

What?

FREQUENCY DOMAIN WAVEFORM



SAMPLING THEORY

$$\delta f(t) \leq (T(t))^{-1}$$

WE CAN ADAPT THE SAMPLING FREQUENCY

What?

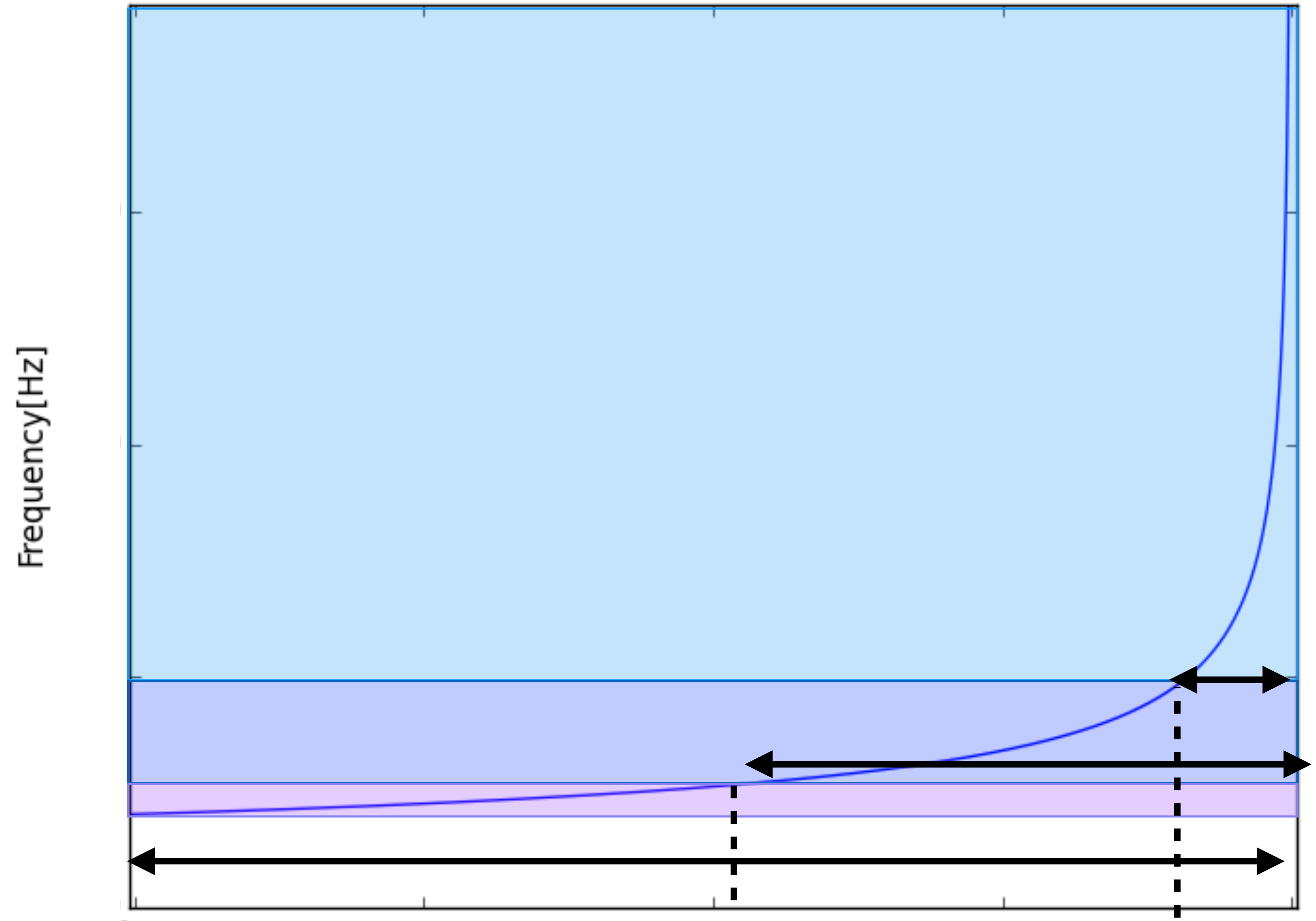
Why?

When?

Which?

What?

FREQUENCY DOMAIN WAVEFORM



SAMPLING THEORY

$$\delta f(t) \leq (T(t))^{-1}$$

We can adapt the sampling frequency

What?

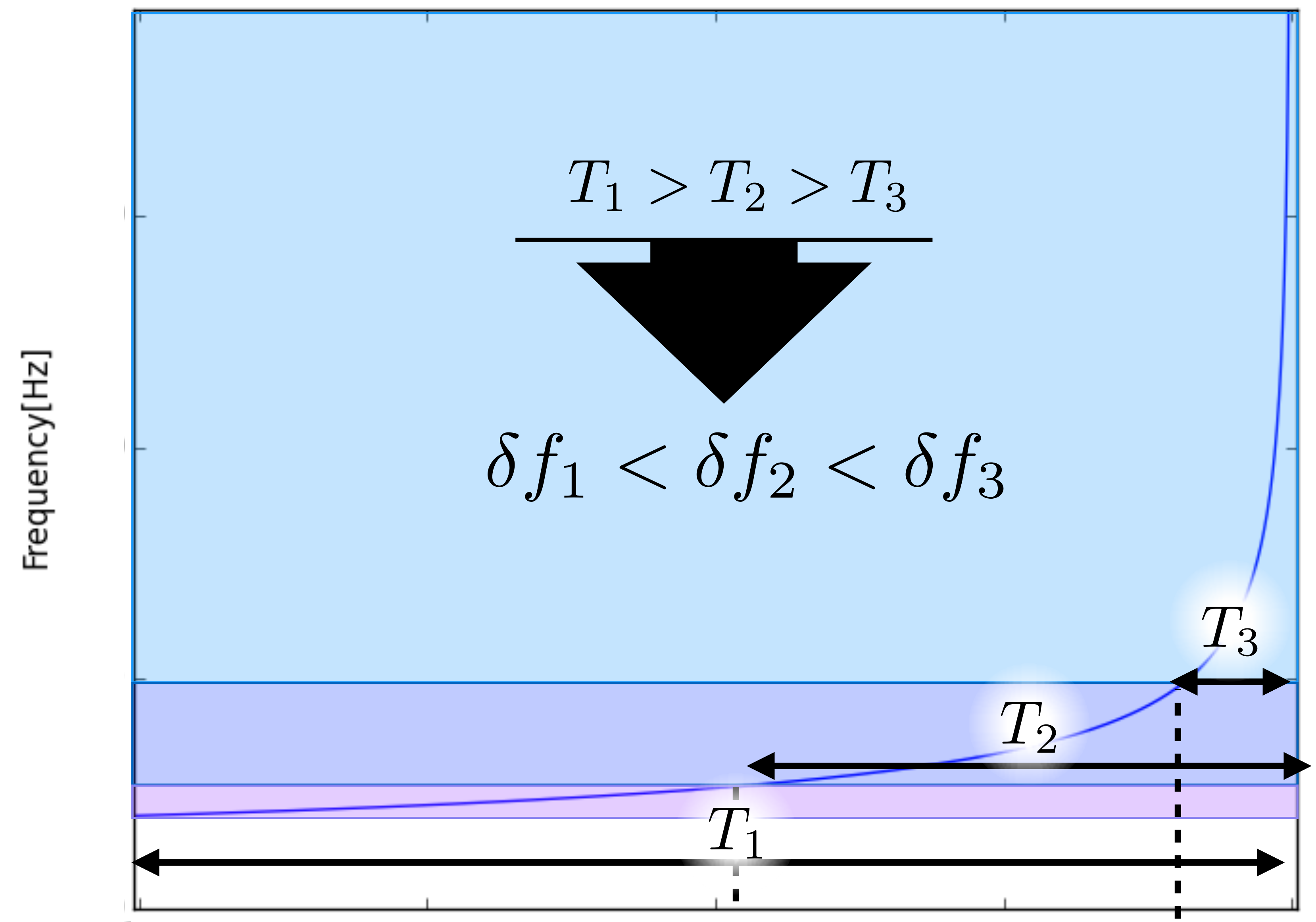
Why?

When?

Which?

What?

FREQUENCY DOMAIN WAVEFORM



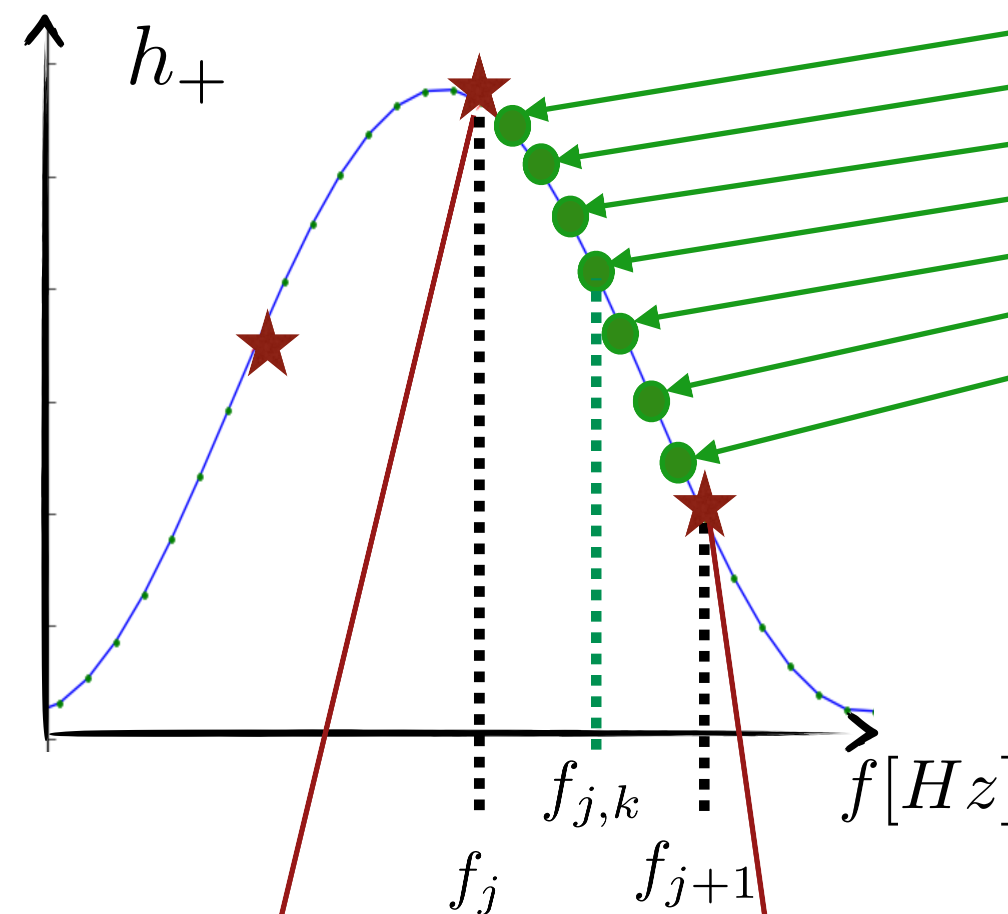
SAMPLING THEORY

$$\delta f(t) \leq (T(t))^{-1}$$

We can adapt the sampling frequency

PHASE INTERPOLATION

$$\tilde{h}(\boldsymbol{\theta}, f) \sim A(\boldsymbol{\theta}, f)e^{i\psi(\boldsymbol{\theta}, f)}$$



$$\tilde{h}_{jk} \approx \chi_{jk}$$

$$\chi_{jk} \equiv \tilde{h}(f_j)e^{i\Delta\psi_j k}$$

OUT

$$\psi(f) = \int_{f_j}^{f} \frac{d\psi(\hat{f})}{d\hat{f}} \, d\hat{f}$$

$$\psi(f_{jk}) \cong \frac{\psi(f_{j+1}) - \psi(f_j)}{f_{j+1} - f_j} k\delta f_0$$

IN

$$\tilde{h}(f_j) \quad , \quad \tilde{h}(f_{j+1})$$

What?

Why?

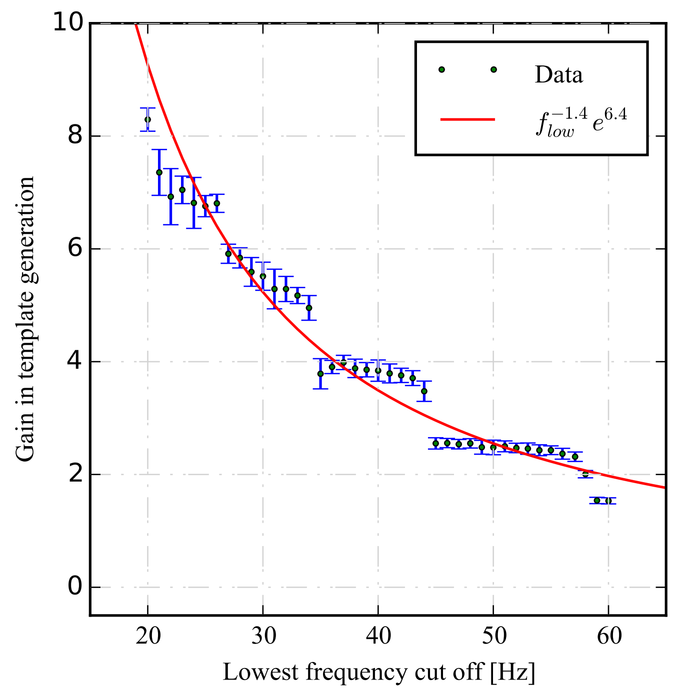
When?

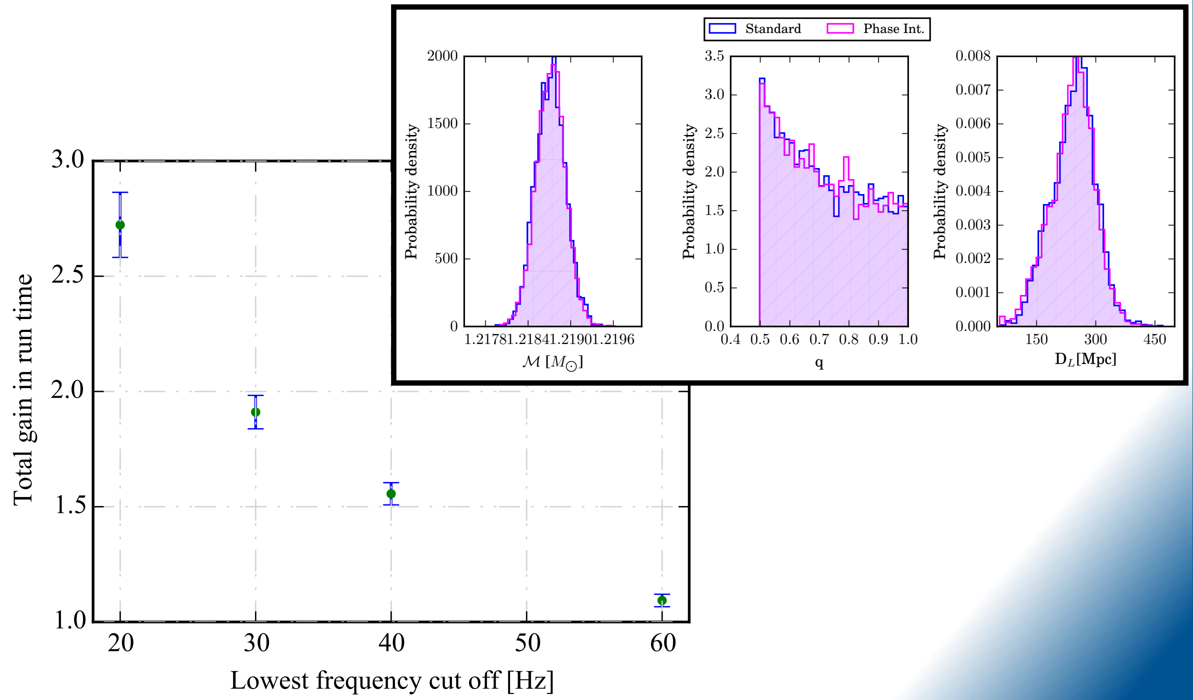
Which?

What?

Results & Model Dependence

For one of the simplest waveform models : **TaylorF2**





What?

Why?

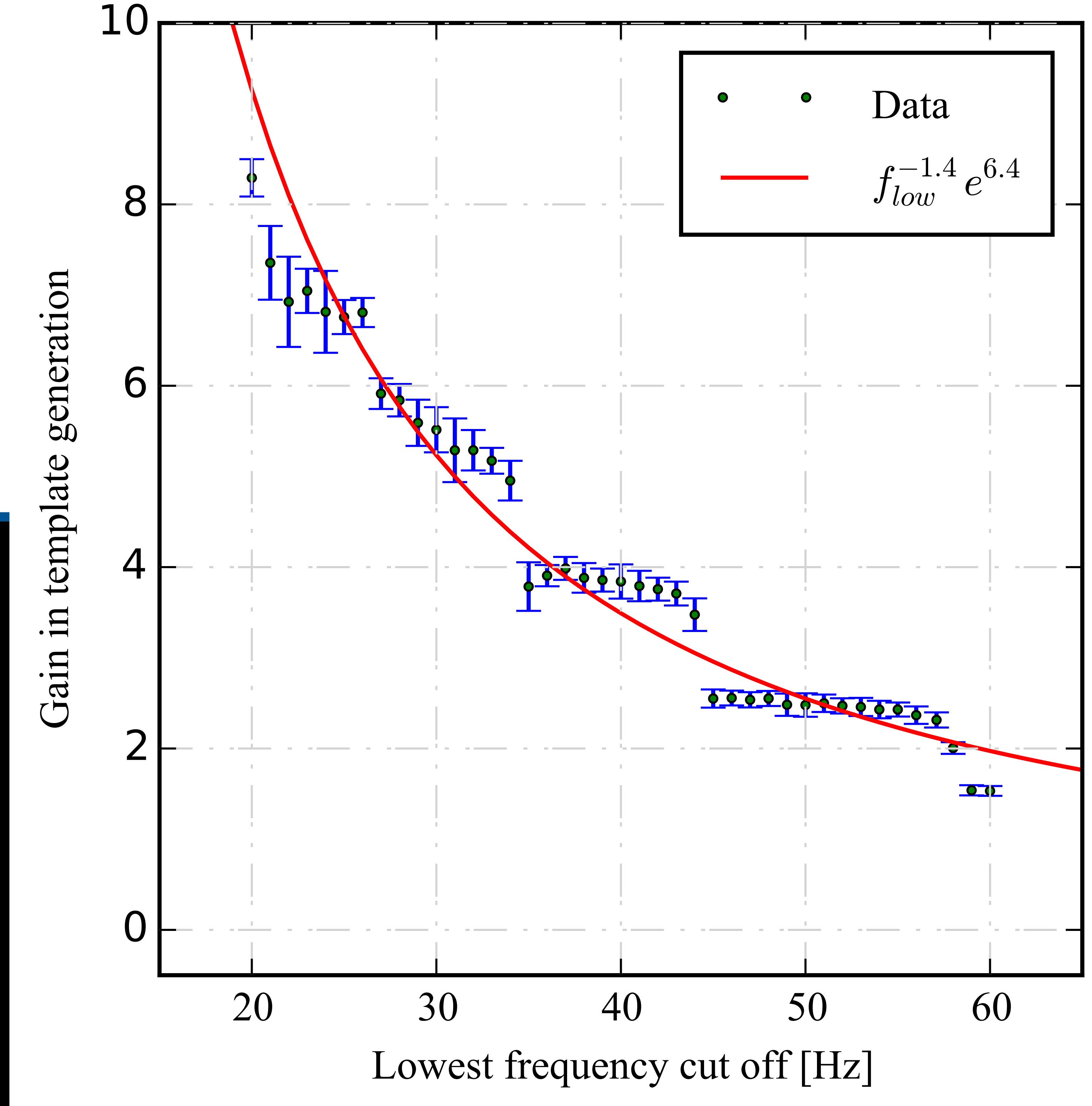
When?

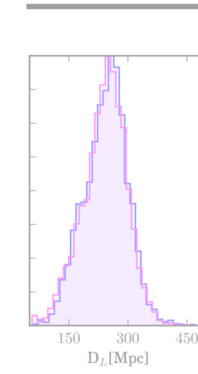
Which?

What?

RESULTS & MODEL DEPENDENCE

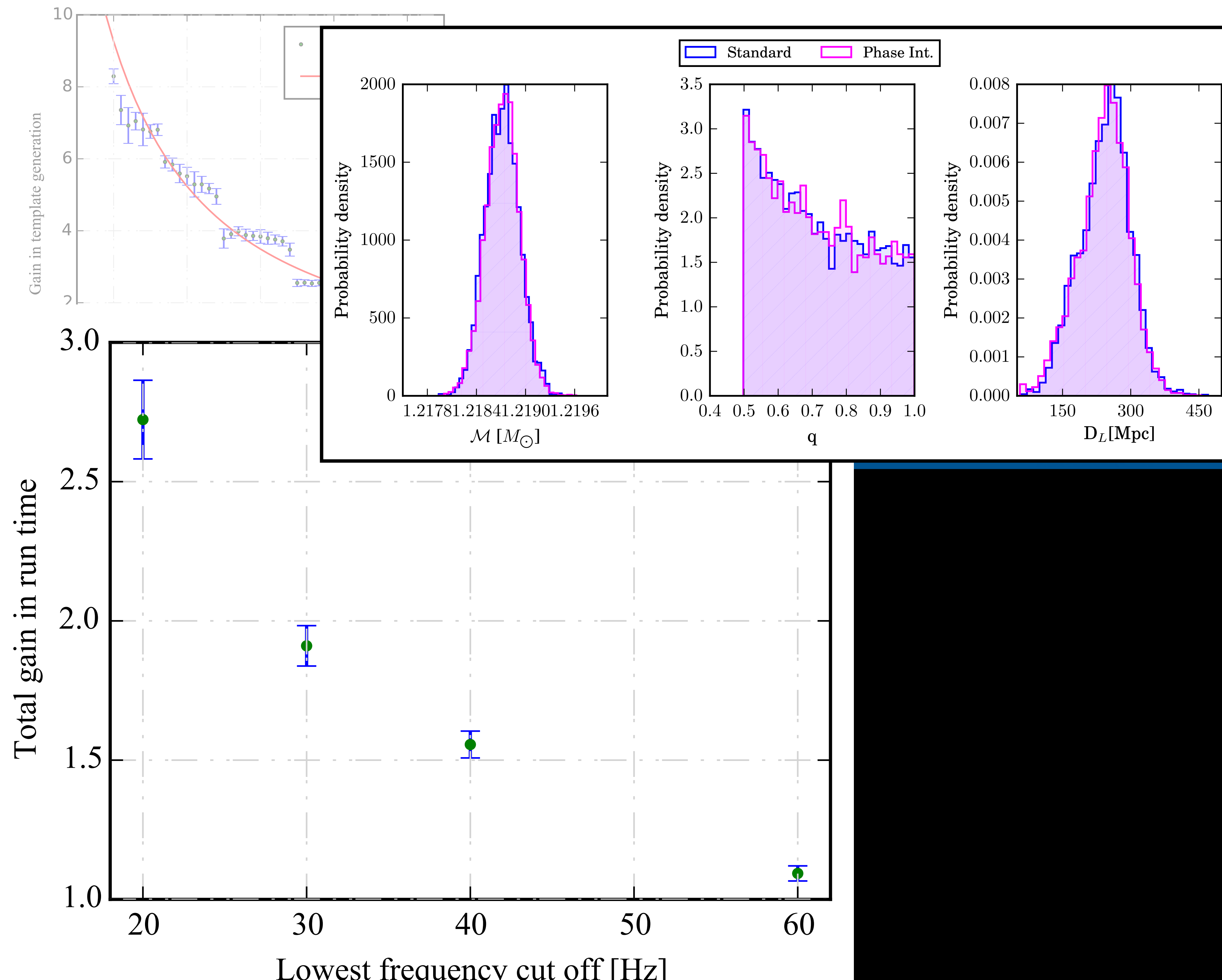
For one of the simplest waveform models : TaylorF2





RESULTS & MODEL DEPENDENCE

For one of the simplest waveform models : TAYLORF2



What?

Why?

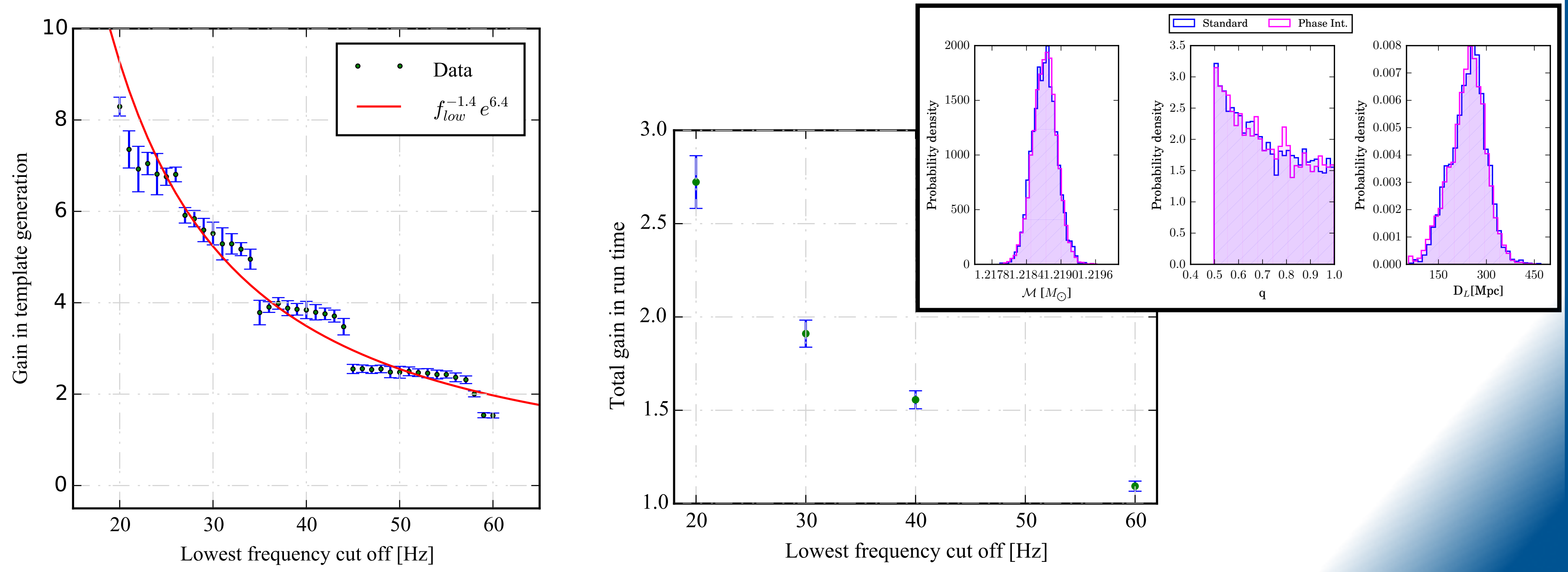
When?

Which?

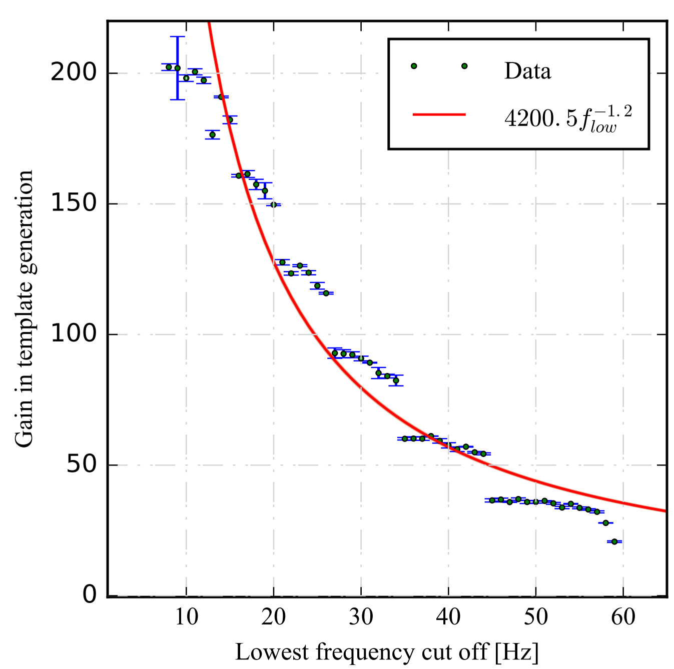
What?

RESULTS & MODEL DEPENDENCE

For one of the simplest waveform models : TAYLORF2



For a more sophisticated waveform model : IMRPHENOMPV2



What?

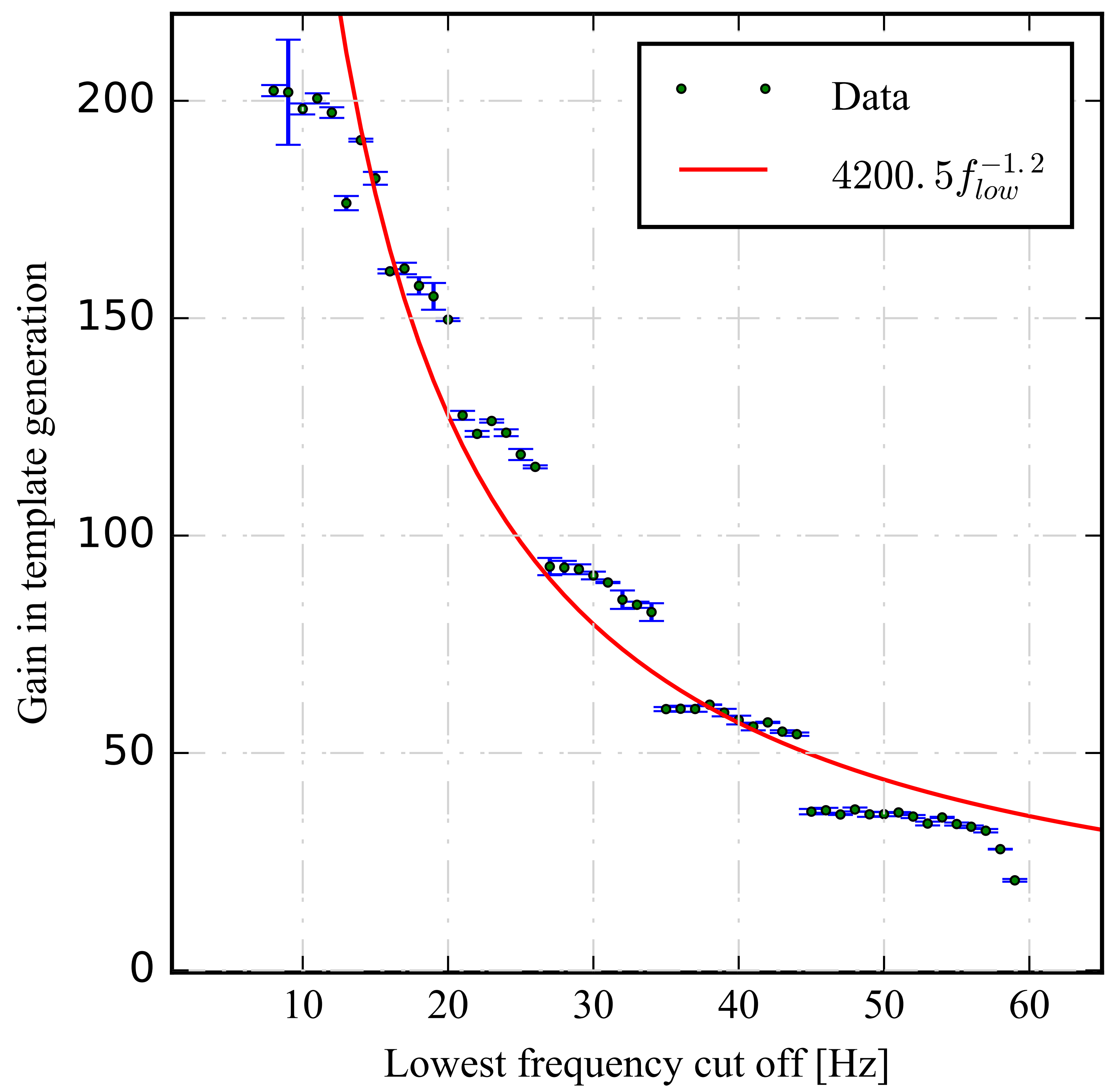
Why?

When?

Which?

What?

RESULTS & MODEL DEPENDENCE



What?

Why?

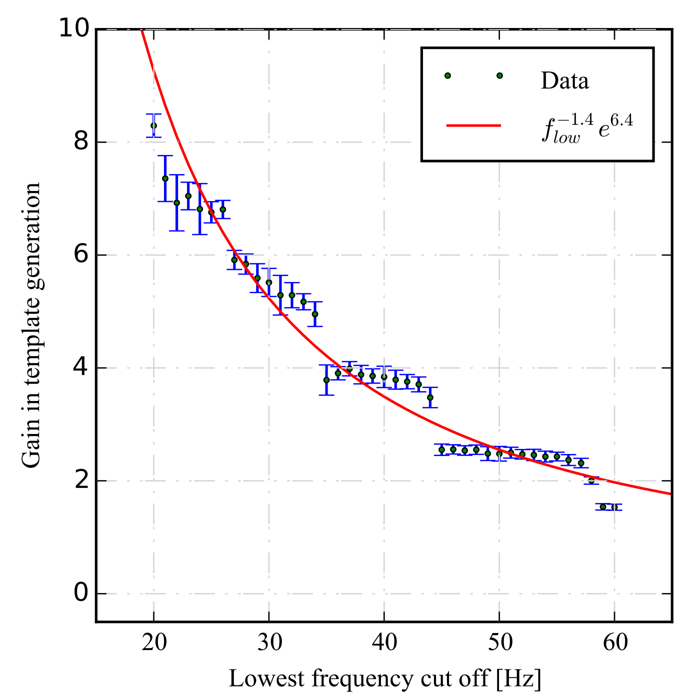
When?

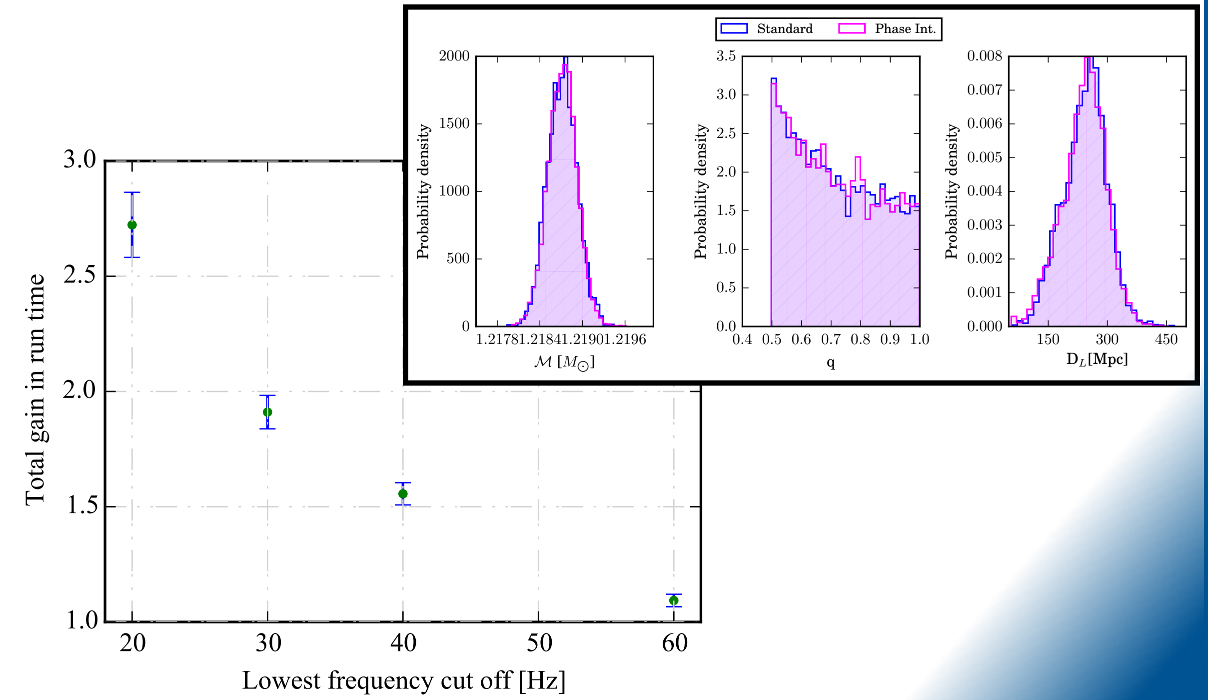
Which?

What?

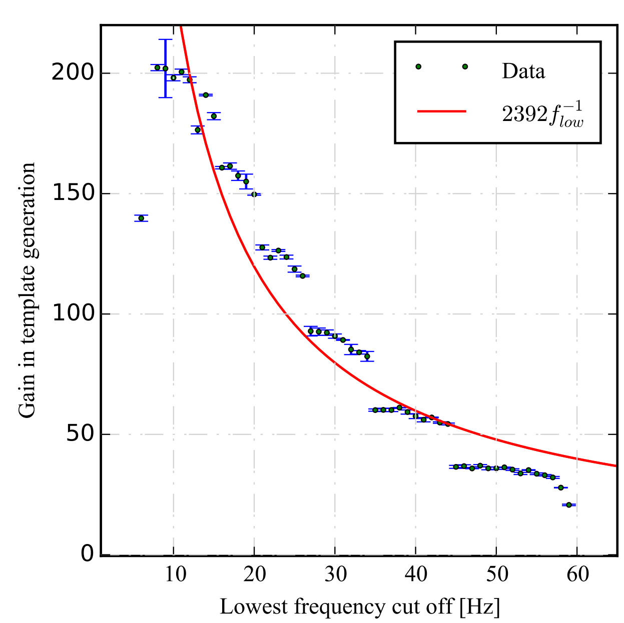
RESULTS & MODEL DEPENDENCE

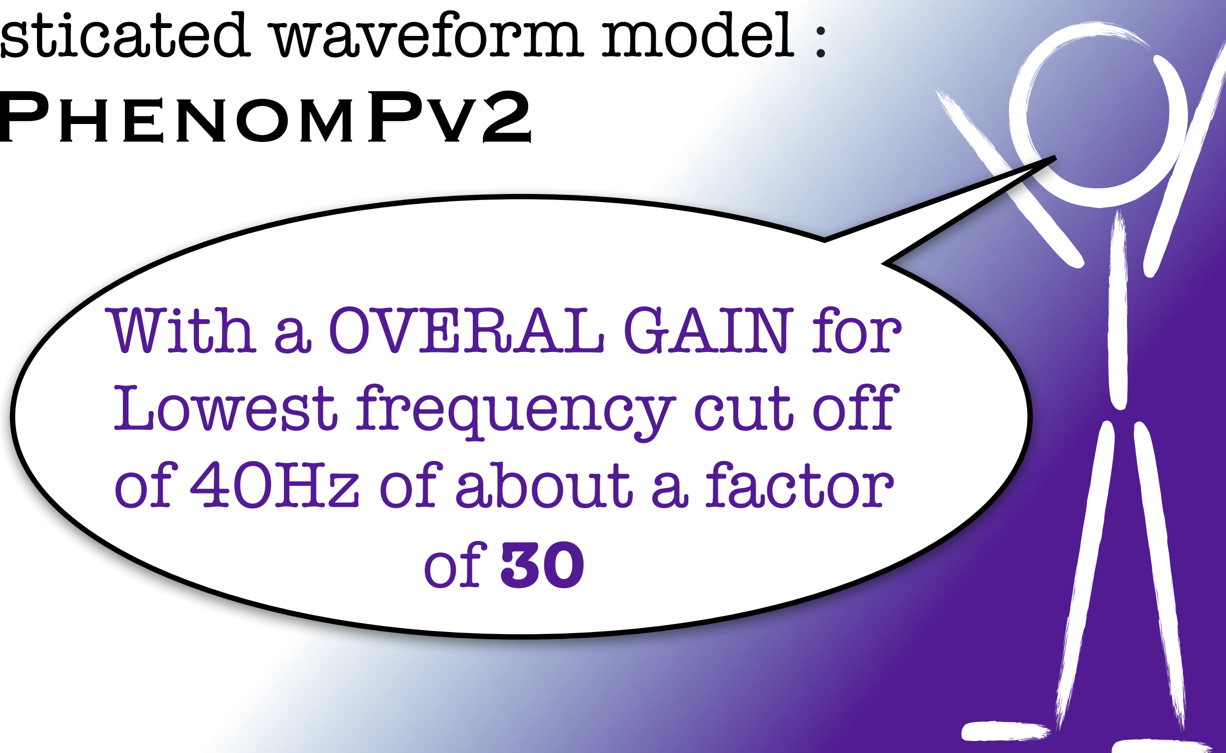
For one of the simplest waveform models : TAYLORF2





For a more sophisticated waveform model :
IMRPHENOMPV2





SUMMARY

GW astronomy is here ➡ Need to process data faster

New instruments ➡ better low-frequency sensitivity
better low-frequency sensitivity ➡ more and longer data

multi-banding + phase-interpolation
➡ wide range of applications

multi-banding + phase-interpolation
➡ good results already

WORK IN PROGRESS:

- can we improve the results? Probably yes
  on going further investigations

Thank you for your attention

EXTRA-SLIDES

What?

Why?

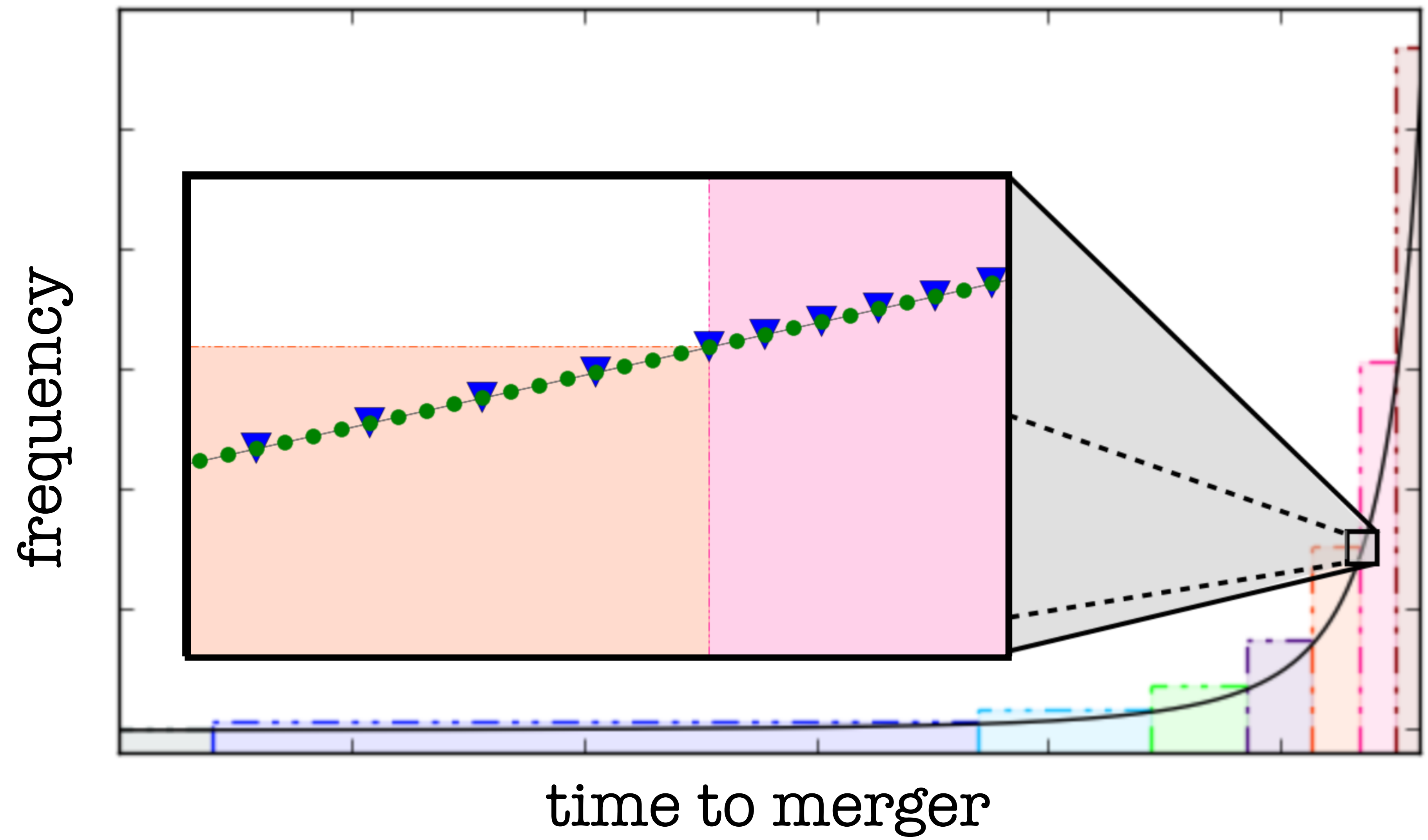
When?

Which?

What?

FREQUENCY DOMAIN WAVEFORM

EXAMPLE OF PRACTICAL RESULTS



In our case

DEFINITION OF FREQUENCY STEP IN BAND “l”

$$\delta f_l = 2^{n_l} \delta f_0$$

DEFINITION OF THE FREQUENCY SET

$$\boxed{\delta f(f) \leq \frac{1}{|t(f)| + t_{pad}}}$$

$$\delta f_l = 2^{n_l} \delta f_0 \quad \Rightarrow \quad G = \frac{N_0}{N_T} = \frac{\sum_{l=0}^{NBands} N_{l,0}}{\sum_{l=0}^{NBands} N_{l,0}/2^{n_l}}$$

$$N_{l,0} = (f_{max.l} - f_{min,l})\, \delta f_0^{-1}$$

Step1 : Finding the maximum n_l allowed by the frequency range taken into account.

$$\delta f_{max} = \frac{1}{t_{pad}}$$

$$n_{max} = -log_2 \left(\delta f_0 \cdot t_{pad}\right)$$

Step 2 : Finding the minimum necessary resolution.

$$\delta f_{min} = \frac{1}{|T_{tot}| + t_{pad}}$$

$$n_{min} = -log_2 \left[\delta f_0 \left(|T_{tot}| + t_{pad}\right)\right]$$

DEFINITION OF THE FREQUENCY SET

Step 3 : Finding the frequency limits of each band. It is convenient to start from the highest frequencies where $n_l = n_{max}$.

Step 4 : Define the frequencies at the extrema of all the new bins (now spaced by δf_l) contained inside every frequency band. From f_{max} we define the new frequency set by $f_i = f_{max} - i\delta f_{max} = f_{max} - i2^{n_{max}}\delta f_0$ until $f_i \geq f_{lim_{max}}$. When this last condition is no more satisfied a new band begins. Generalising the process, the frequencies inside the l band are fixed starting from the last frequency defined in the $l-1$ band and then keeping subtracting $\delta f_l = 2^{n_l}\delta f_0$ until $f_i \geq f_{lim_\alpha}$ is satisfied.

STATIONARY PHASE APPROXIMATION

An alternative description can be performed in the frequency domain. [REF Cutler and Flanagan] In this context the waveform is usually computed by adopting and developing the *Stationary Phase Approximation* (SPA). This consists in the approximation to the Fourier transforms of the two GW polarisations. Given a function of time $h(t) = \mathcal{A}(t)\cos\phi(t)$, its Fourier transform :

$$\tilde{h}(f') = \int_{-\infty}^{\infty} h(t) e^{-i2\pi f' t} dt \tag{2.61}$$

can be estimated by the formula for positive frequencies $f' \geq 0$:

$$\tilde{h}(f') \approx \frac{1}{2}\mathcal{A}(t)\sqrt{\frac{dt}{df'}} e^{i(2\pi f' t(f') - \phi(f') - \pi/4)} \tag{2.62}$$

. where the time t is defined at $d\phi/dt = 2\pi f'$. This approximation holds whenever the variation of the angular velocity is much slower than itself, i.e. $\frac{d^2\phi}{dt^2} \ll \left(\frac{d\phi}{dt}\right)$ and the time dependence of the function $h(t)$ is led by the phase term, meaning that

$$\frac{dh}{dt}(t) = \frac{d\mathcal{A}}{dt} cos\phi(t) - \mathcal{A}(t)\sin\phi(t)\frac{d\phi}{dt} \approx \mathcal{A}(t)\sin\phi(t)\frac{d\phi}{dt} \tag{2.63}$$

This last condition can be formulated by the requirement $\frac{dln(\mathcal{A})}{dt} \ll \frac{d\phi}{dt}$.
At the Newtonian approximation, the amplitude $\mathcal{A}$ is given by the expression (2.20), the frequency evolution is defined by equation (2.28) and the time as function of the frequency and the other physical parameters is given by (??).
The time and phase as function of the be expressed as function of the gravitational frequency:

$$\begin{aligned} t(f) &= t_c - 5(8\pi f)^{-8/3}(G\mathcal{M})^{-5/3}c^5 \\ \phi(f) &= \phi_c - 2\left[8\pi f G\mathcal{M}\right]^{-5/3} c^5 \end{aligned} \tag{2.64}$$

These relations lead to the following estimation of $\tilde{h}(f)$

$$\tilde{h}(f) = \frac{Q(angles)}{d_L}\frac{(G\mathcal{M})^{5/6}}{c^{3/2}} f^{-7/6} e^{i\psi(f)} = A(f)e^{i\psi(f)} \tag{2.65}$$

where $Q(angles)$ is a function of the angle-parameters, $A(f)$ is the amplitude and

$$\psi(f) = 2\pi f t_c - \phi_c + \frac{3}{4}c^5(8\pi G\mathcal{M} f)^{-5/3} - \frac{\pi}{4} \tag{2.66}$$

The frequency domain amplitude derived under the SPA can be intuitively understood by considering that the oscillatory terms contribute almost as ~ 1 when in phase and almost 0 otherwise. How long is the time at which this condition is satisfied is determined by the term $d^2\phi/dt^2 \approx df'/dt$. A rough approximation can thus be given by requiring $d^2\phi/dt^2\Delta t^2 \sim 1$, which can be used to determine the effective time duration in the Fourier transform definition: $\Delta t \sim \sqrt{\frac{dt}{df}}$.
More accurate waveforms in frequency domain can be reached by developing the phase $\psi(f)$ at higher orders of v or x.