Growing tensor perturbations on superhorizon scales in Generalized Galilean Genesis

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in preparation.

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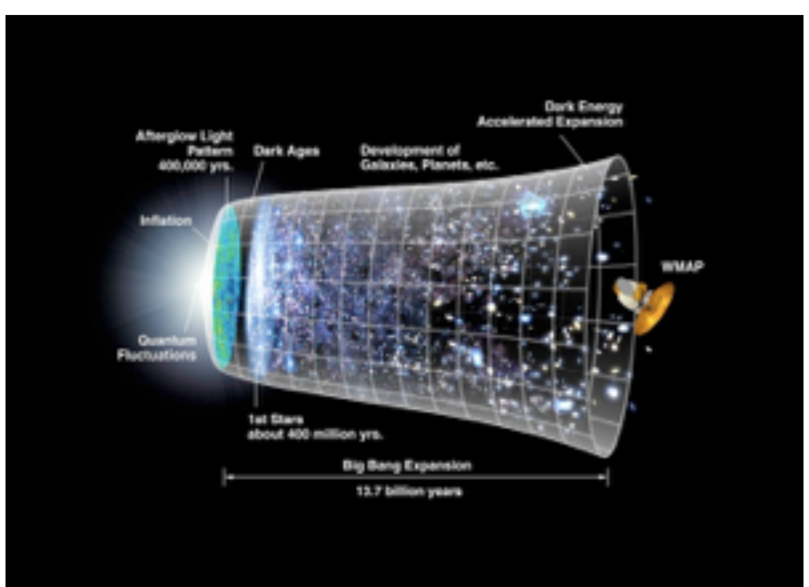
Introduction

Inflation

Alternative scenarios

…

Galilean Genesis



Inflation is a very successful scenario.

There are many kinds of models which explain the early universe.

↓

We want to compare genesis to other inflation models and discuss observational implications

Q : If nearly scale invariant primordial GWs detected → Inflation ?

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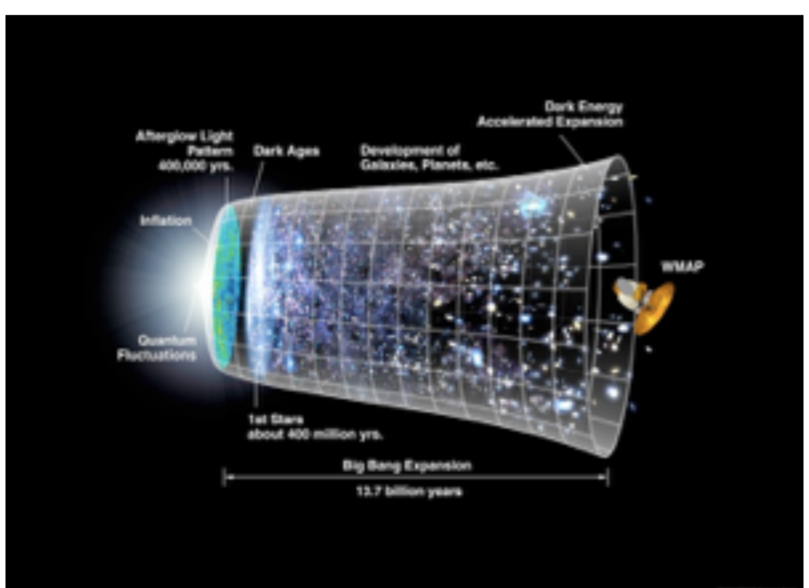

$$H_{inf} \simeq \text{const.}$$

$$\mathcal{P}_{GW} \to \text{flat}$$

(nearly scale invariant)

bouncing, genesis, …

(NEC violation)

$$H_{inf} \to \text{grow}$$

$$\mathcal{P}_{GW} \to \text{Blue}$$

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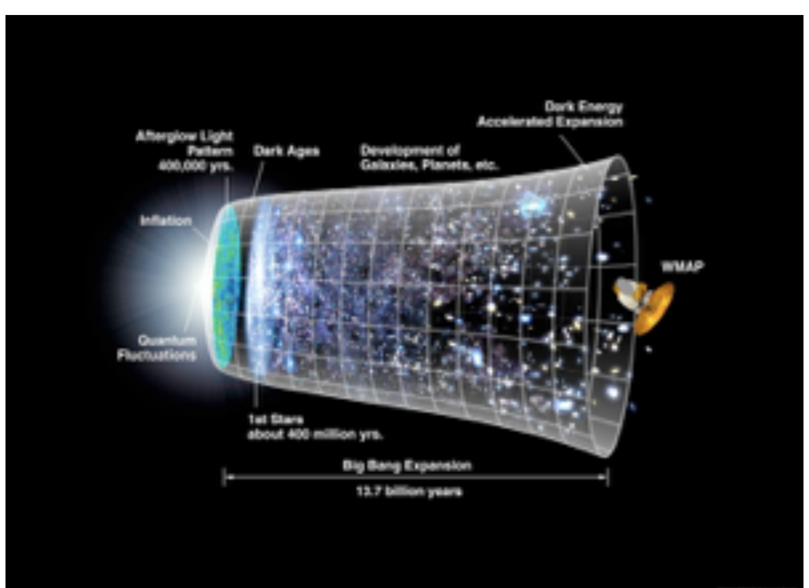
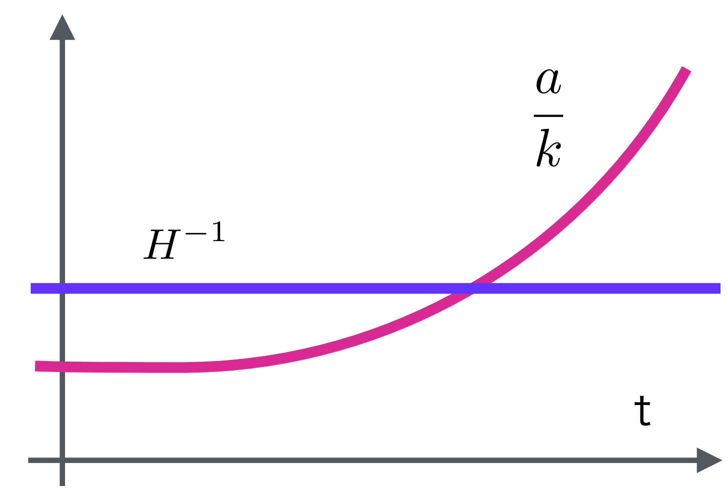
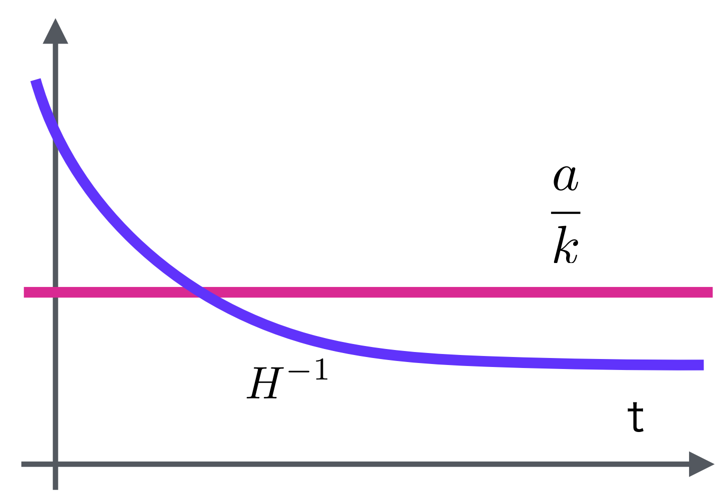
…

Galilean Genesis







$$(t \to -\infty)$$

started from Minkowski space-time

Outline

Original model

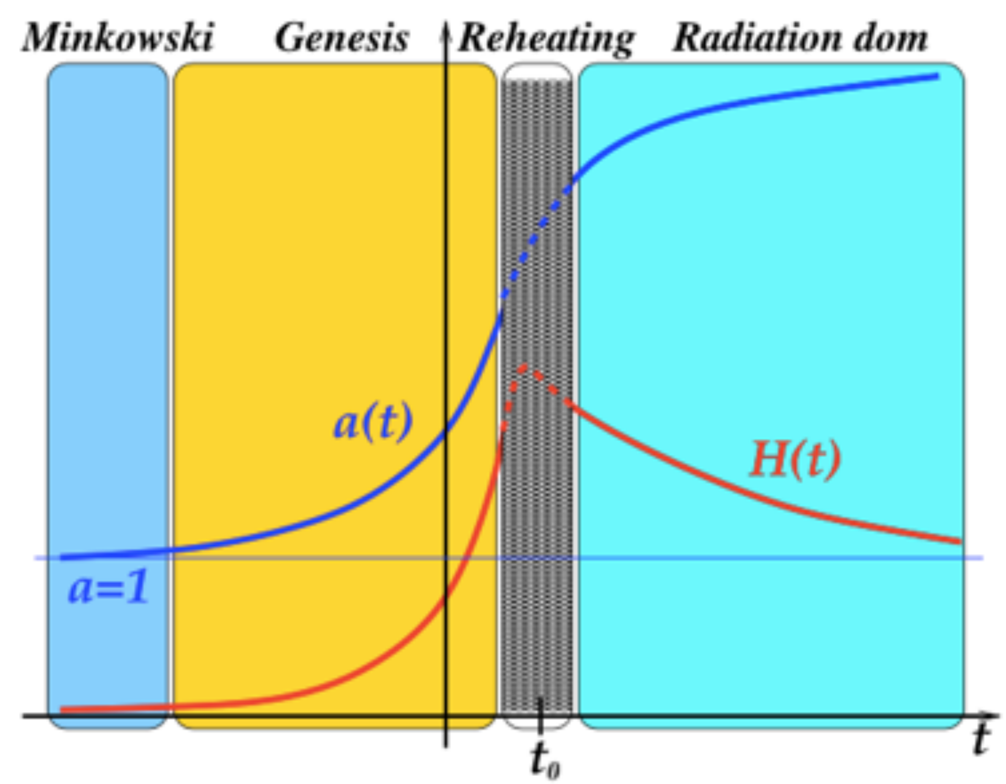
Generalized model

Extended model

Horndeski theory

Galilean Genesis

[P. Creminelli, A. Nicolis and E. Trincherini, (2010)]



- Alternative to inflation model
- Null Energy Condition is violated with stable.
- Previous work
  - action $$\mathcal{S} = \int \mathrm{d}x^4 \sqrt{-g}\left[f^2 e^{2\phi}(\partial\phi)^2 + \frac{f^3}{\Lambda^3}(\partial\phi)^2\Box\phi + \frac{f^3}{2\Lambda^3}(\partial\phi)^4\right]$$
- Solutions

$t \to -\infty$

$$e^{\lambda\phi} \propto (-t)^{-1}$$

$$H(t) \simeq -\frac{f^2}{3M_{Pl}^2}\frac{1}{H_0^2 t^3}$$

$$a(t) \simeq 1$$

$t \to t_0$ (numerical analysis)

$$a(t) = \exp\left[\frac{8f^2}{3H_0^2 M_{Pl}^2}\frac{1}{(t_0 - t)^2}\right]$$

$$H(t) \simeq \frac{16 f^2}{3M_{Pl}^2}\frac{1}{H_0^2 (t_0 - t)^3}$$

Outline

Original model

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Horndeski theory

Horndeski theory

- the most general scalar-tensor theory which has up to 2nd derivative

- Field eqs. have no 3rd and higher derivative terms

- Generalized Galilean Genesis is subclass of this theory.

$$S_{\mathrm{Hor}} = \int \mathrm{d}^4x\sqrt{-g}\Big\{ G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

[G. W. Horndeski (1974)]

[C. Deffayet, Xian Gao, D. A. Steer, and G. Zahariade (2011)]

[T. Kobayashi, M. Yamaguchi and J. Yokoyama (2011)]

Outline

Original model

Generalized model

Extended model

Horndeski theory

$$S_{\rm Hor} = \int \mathrm{d}^4x\sqrt{-g}\Big\{ G_2(\phi, X) - G_3(\phi,$$

$$+G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G$$

$$-\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu$$

Generalized Galilean Genesis

- include the various models of Genesis
- Introduce a parameter α, arbitrary functions $g_i(Y)$

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi} g_3(Y),$$
$$G_4 = \frac{M_{\rm Pl}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y), \quad G_5 = e^{-2\lambda\phi} g_5(Y). \quad Y := e^{-2\lambda\phi} X$$

- Solution $(-\infty < t < 0)$

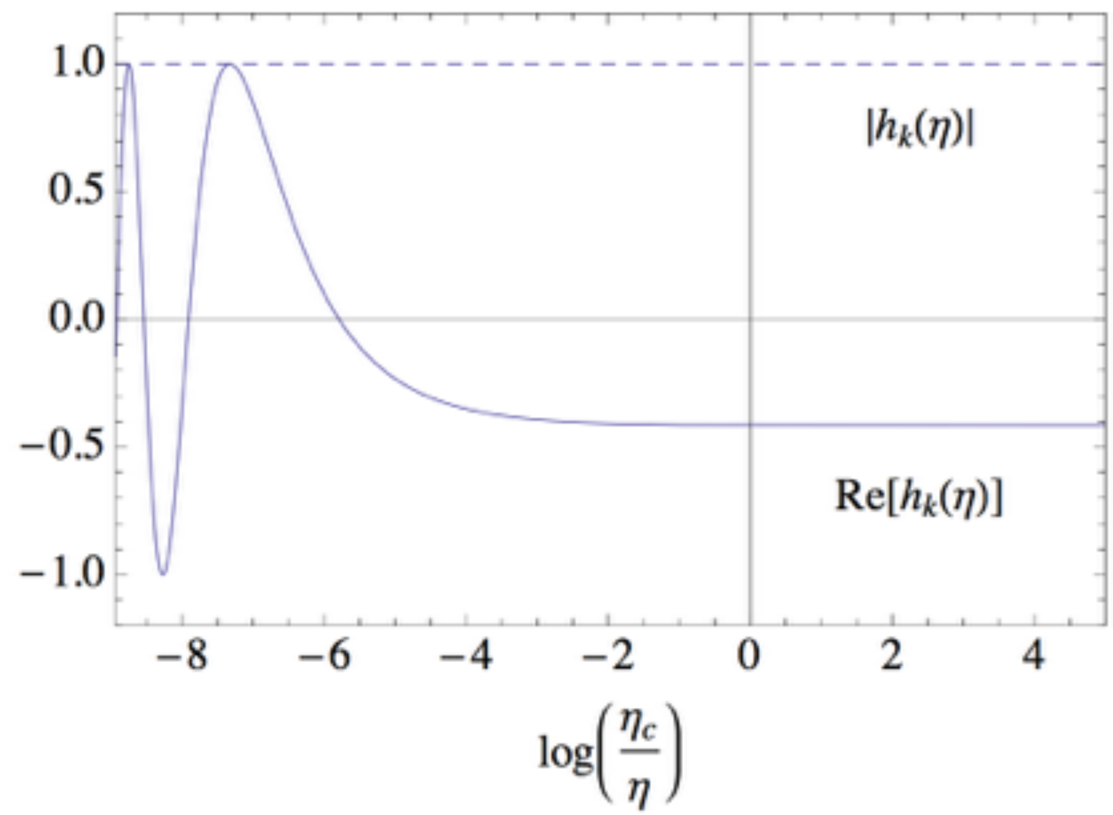
$$e^{\lambda\phi} \propto (-t)^{-1}, \; H(t) \propto \frac{1}{(-t)^{2\alpha+1}}, \; a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}}\right]$$

[S. Nishi, T. Kobayashi, [arXiv:1501.02553 [hep-th]]]

Generalized Galilean Genesis

- tensor perturbations
  - action $$S_h^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, \boxed{a^3\mathcal{G}(Y_0)}\left[\dot{h}_{ij}^2 - \boxed{\frac{c_t^2}{a^2}}(\nabla h_{ij})^2\right]$$
  - propagation speed $$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \text{const.}$$
  - power spectrum $$\mathcal{P}_T \propto k^2$$

horizon cross ↓



GWs do not grow

Generalized Galilean Genesis

- scalar perturbation
  - action

$$\mathcal{G}_S \propto (-t)^{2\alpha}$$

$$S_\zeta^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \boxed{\mathcal{G}_S} \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\nabla\zeta)^2\right]$$

  - propagation speed $\qquad c_S^2 = \dfrac{\mathcal{F}_S}{\mathcal{G}_S} = \mathrm{const.}$

  - power spectrum

$\mathcal{P}_S \propto k^{2+2\alpha} \quad (0 < \alpha < 1/2)$ (ζ : decaying mode + const.)

$\mathcal{P}_S \propto k^{4-2\alpha} \quad (\alpha > 1/2)$ (ζ : growing mode + const.)

Outline

Original model

↓

Generalized model

↓

Extended model

Horndeski theory

scalar power spectrum

→ flat, red , blue

tensor power spectrum

→ blue

Extended Generalized Galilean Genesis

- action (in Horndeski theory) (subclass → Y. Cai, Y. Piao, (2016))

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y) + e^{-2\beta\lambda\phi} a_2(Y) + e^{-2(\alpha+2\beta)} b_2(Y)$$
$$G_3 = -e^{2\alpha\lambda\phi} g_3(Y) + e^{-2\beta\lambda\phi} a_3(Y) + e^{-2(\alpha+2\beta)} b_3(Y)$$
$$G_4 = e^{-2\beta\lambda\phi} a_4(Y) + e^{-2(\alpha+2\beta)\lambda\phi} b_4(Y)$$
$$G_5 = e^{-2(\alpha+2\beta+1)\lambda\phi} b_5(Y) \qquad (\alpha+\beta>0)$$

$$a_2(Y) = 8\lambda^2 Y (Y\partial_Y + \beta)^2 A(Y)$$
$$a_3(Y) = 2\lambda(2Y\partial_Y + 1)(Y\partial_Y + \beta) A(Y)$$
$$a_4(Y) = Y\partial_Y A(Y)$$

$$b_2(Y) = 16\lambda^3 Y^3 (Y\partial_Y + \alpha + \beta + 1)^3 B(Y)$$
$$b_3(Y) = 4\lambda^2 (2Y\partial_Y + 3)(Y\partial_Y + \alpha + \beta + 1)^2 B(Y)$$
$$b_4(Y) = 2\lambda Y (Y\partial_Y + 1)(Y\partial_Y + \alpha + \beta + 1) B(Y)$$
$$b_5(Y) = -(2Y\partial_Y + 1)(Y\partial_Y + 1) B(Y)$$

$A(Y), B(Y)$: arbitrary functions

Extended Generalized Galilean Genesis

- Generalized

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y) \qquad G_4 = \frac{M_{pl}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y)$$

$$G_3 = e^{2\alpha\lambda\phi} g_3(Y) \qquad G_5 = e^{-2\lambda\phi} g_5(Y)$$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \simeq \frac{h_0}{(-t)^{2\alpha+1}}, \quad a(t) \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}}\right]$$

- Extended

$(\alpha + \beta > 0)$

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y) + e^{-2\beta\lambda\phi} a_2(Y) + e^{-2(\alpha+2\beta)} b_2(Y)$$

$$G_3 = -e^{2\alpha\lambda\phi} g_3(Y) + e^{-2\beta\lambda\phi} a_3(Y) + e^{-2(\alpha+2\beta)} b_3(Y)$$

$$G_4 = e^{-2\beta\lambda\phi} a_4(Y) + e^{-2(\alpha+2\beta)\lambda\phi} b_4(Y)$$

$$G_5 = e^{-2(\alpha+2\beta+1)\lambda\phi} b_5(Y)$$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \simeq \frac{h_0}{(-t)^{2\alpha+2\beta+1}}, \quad a(t) \simeq a_G \left[1 + \frac{1}{2(\alpha+\beta)} \frac{h_0}{(-t)^{2(\alpha+\beta)}}\right]$$

Extended Generalized Galilean Genesis

- scalar perturbation (Generalized model)
  - propagation speed
    $$c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \text{const.}$$
  - power spectrum
    $$\mathcal{P}_S \propto k^{2+2\alpha} \quad (0 < \alpha < 1/2) \qquad \text{or} \qquad \mathcal{P}_S \propto k^{4-2\alpha} \quad (\alpha > 1/2)$$

- scalar perturbation (Extended model)

$$\mathcal{G}_S := \frac{\boxed{\Sigma}\mathcal{G}_T^2}{\Theta^2} + 3\mathcal{G}_T$$

$\Sigma \neq 0$	$\Sigma = 0$
$c_S^2 = \text{const.}$	$c_S^2 \propto (-t)^{2(\alpha+\beta)}$
$\nu = 1/2 - \alpha - 2\beta$	$\nu = \dfrac{1/2 - \beta}{\alpha + \beta + 1}$

  - propagation speed
  - power spectrum

$$\mathcal{P}_S \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$$H_\nu^{(1)}$$

Extended Generalized Galilean Genesis

- tensor perturbation (Generalized model)
  - propagation speed $$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \text{const.}$$
  - power spectrum $$\mathcal{P}_T \propto k^2$$

- tensor perturbation (Extended model)

	$b_5 = 0$	$b_5 \neq 0$
propagation speed	$c_T^2 = \text{const.}$	$c_T^2 \propto (-t)^{2(\alpha+\beta)}$
power spectrum	$\nu = 1/2 - \beta$	$\nu = \dfrac{1/2-\beta}{\alpha+\beta+1}$

$$\mathcal{P}_T \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$$H_\nu^{(1)}$$

Extended Generalized Galilean Genesis

The solution on superhorizon

$$h = \int^t \frac{\mathrm{d}t'}{a^3 \mathcal{G}_T} + \mathrm{const.}$$

$$(-\infty < t < 0)$$

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} \qquad \mathcal{G}_T \propto (-t)^{2\beta}$$

-> grow on superhorizon scales

lized model)

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \mathrm{const.}$$

$$\mathcal{P}_T \propto k^2$$

ed model)

$b_5 = 0$

$= 1/2 - \beta$

$b_5 \neq 0$

$$c_T^2 \propto (-t)^{2(\alpha+\beta)}$$

$$\nu = \frac{1/2 - \beta}{\alpha + \beta + 1}$$

$H_\nu^{(1)}$

Extended Generalized Galilean Genesis

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  - propagation speed $$c_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \text{const.}$$
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- scalar perturbation (Extended model)

$$\mathcal{G}_S := \frac{\boxed{\Sigma}\mathcal{G}_T^2}{\Theta^2} + 3\mathcal{G}_T$$

  - propagation speed
  - power spectrum

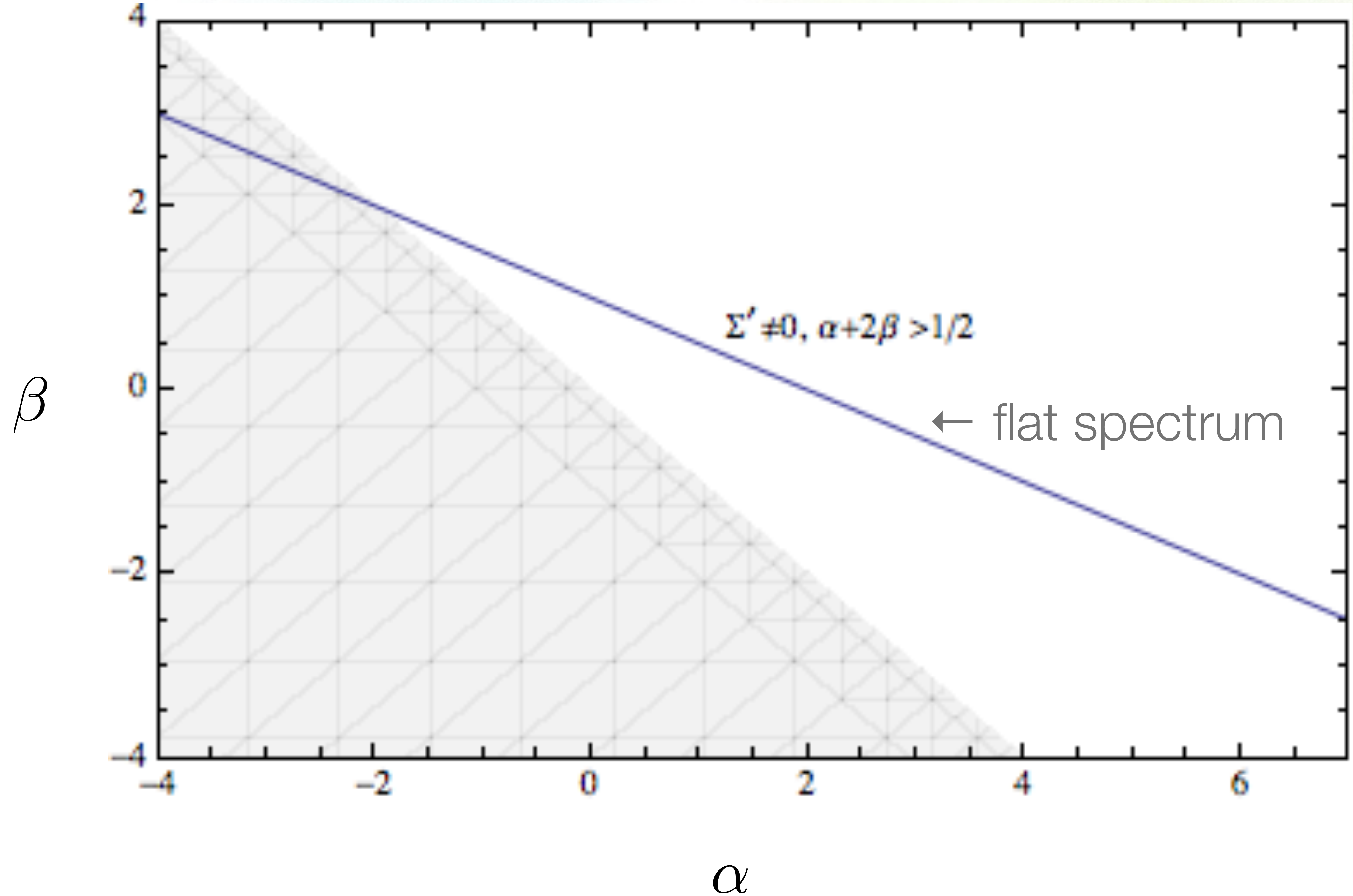
$$\mathcal{P}_S \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$\Sigma \neq 0$	$\Sigma = 0$
$c_S^2 = \text{const.}$	$c_S^2 \propto (-t)^{2(\alpha+\beta)}$
$\nu = 1/2 - \alpha - 2\beta$	$\nu = \dfrac{1/2 - \beta}{\alpha + \beta + 1}$

$H_\nu^{(1)}$

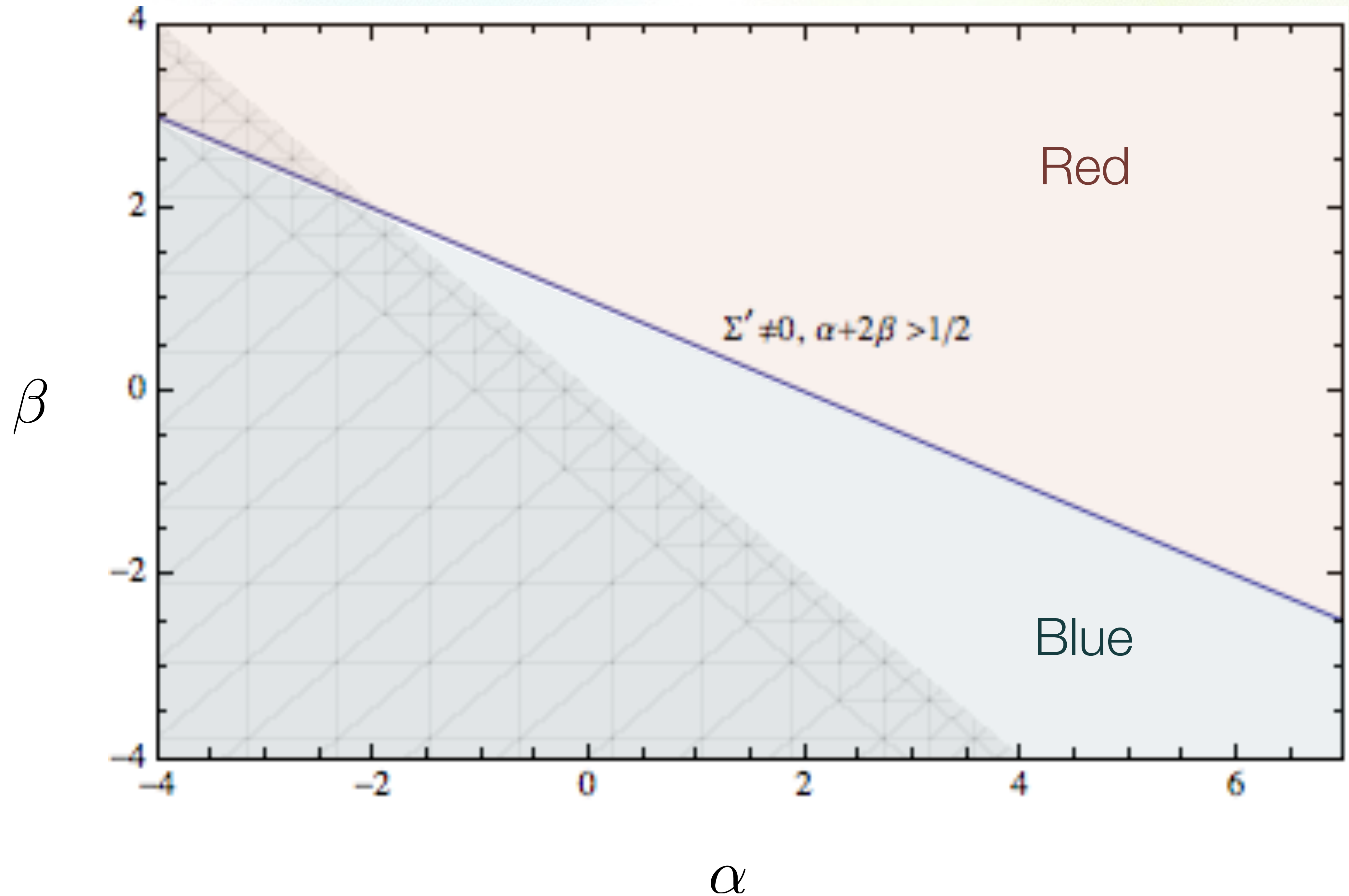
Extended Generalized Galilean Genesis

- flat spectrum on the line (scalar perturbations)



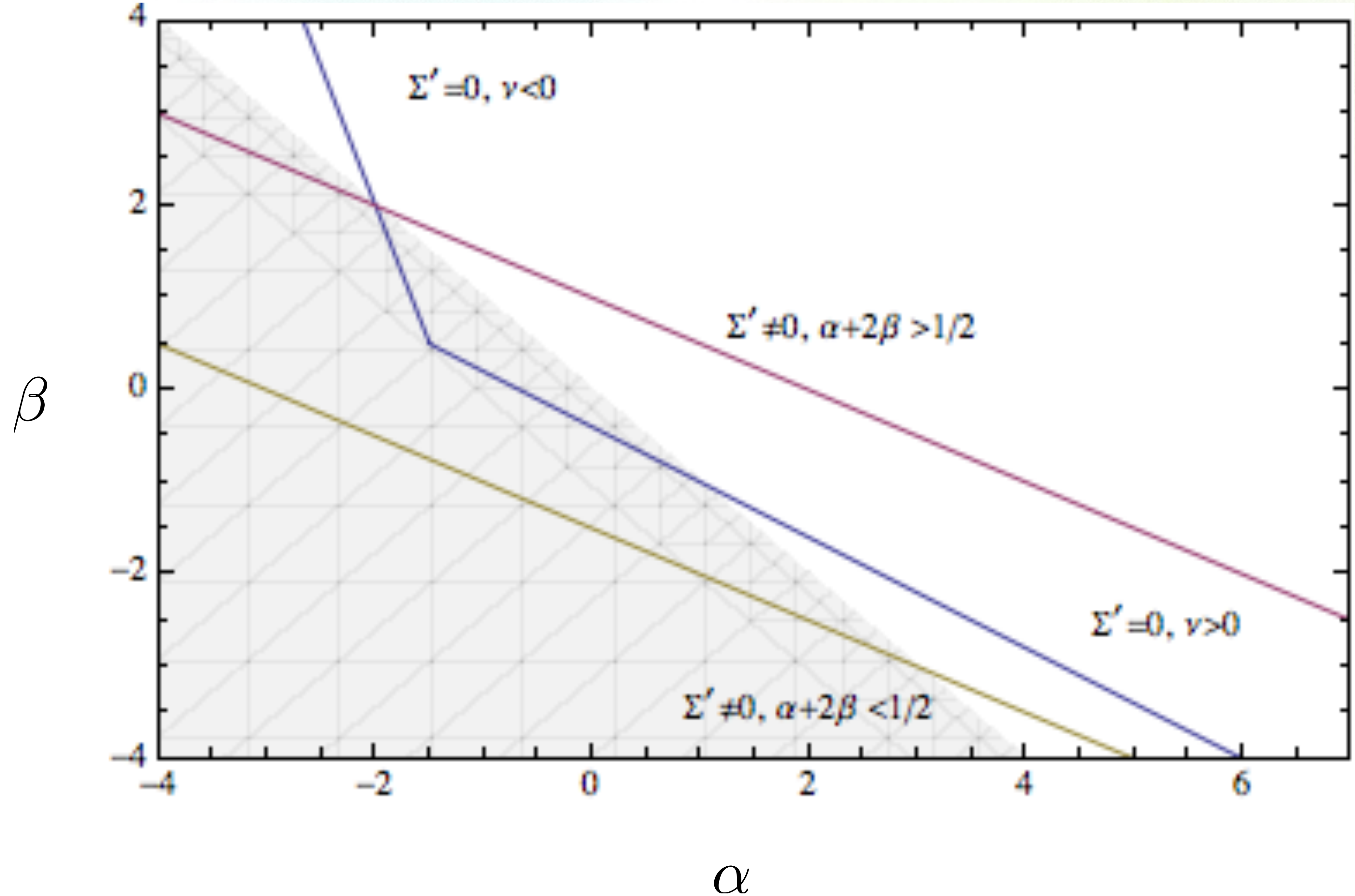
Extended Generalized Galilean Genesis

- flat spectrum on the line (scalar perturbations)



Extended Generalized Galilean Genesis

- flat spectrum on the lines (Scalar perturbations)



Extended Generalized Galilean Genesis

- tensor perturbation (Generalized model)
  - propagation speed $$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \text{const.}$$
  - power spectrum $$\mathcal{P}_T \propto k^2$$

- tensor perturbation (Extended model)

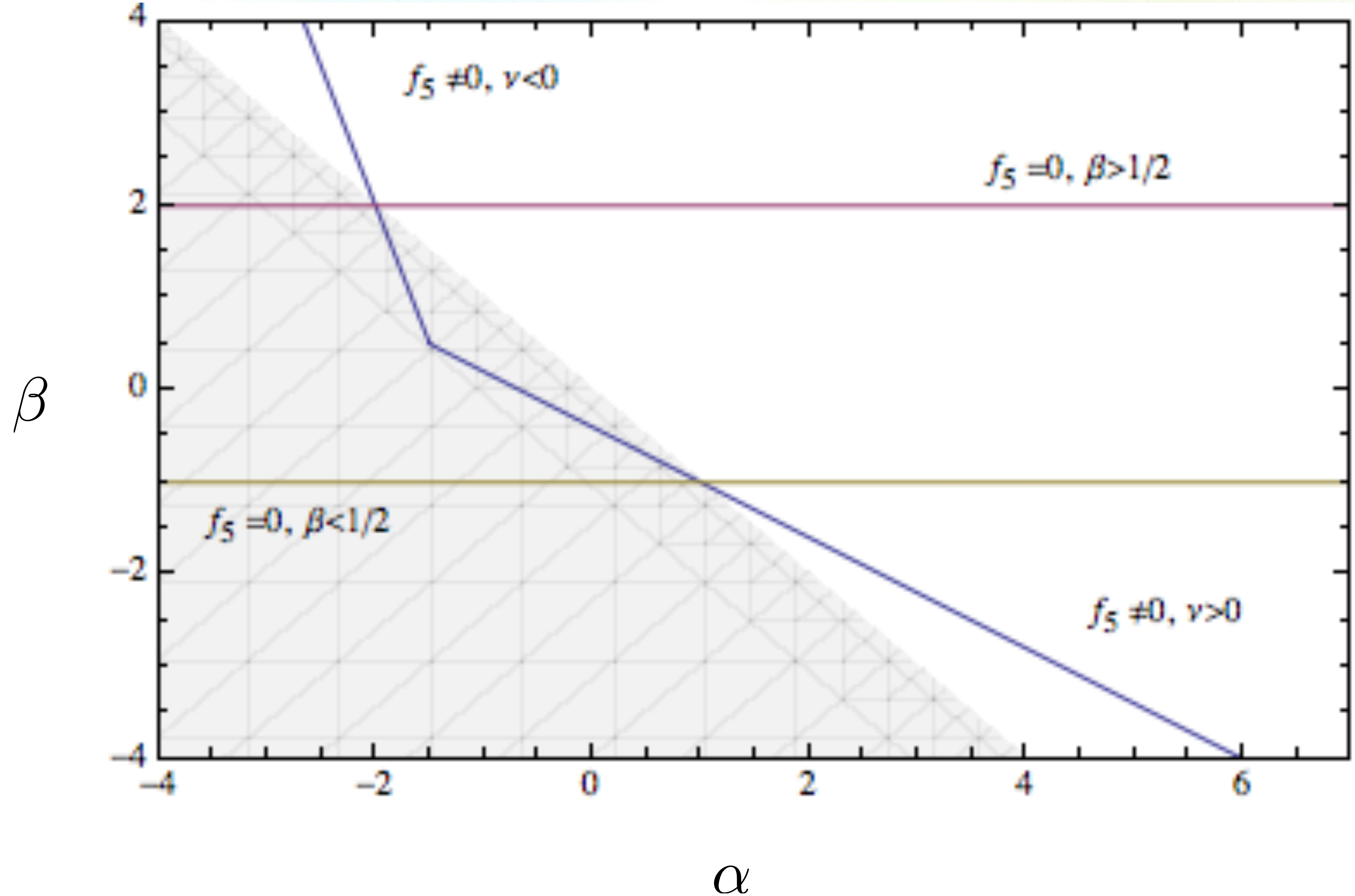
	$b_5 = 0$	$b_5 \neq 0$
propagation speed	$c_T^2 = \text{const.}$	$c_T^2 \propto (-t)^{2(\alpha+\beta)}$
power spectrum	$\nu = 1/2 - \beta$	$\nu = \dfrac{1/2-\beta}{\alpha+\beta+1}$

$$\mathcal{P}_T \propto \begin{cases} k^{3-2\nu} & (\nu \geq 0) \\ k^{3+2\nu} & (\nu < 0) \end{cases}$$

$$H_\nu^{(1)}$$

Extended Generalized Galilean Genesis

- flat spectrum on the lines (Tensor perturbations)



Conclusions

- Galilean genesis as an alternative scenario to inflation
- We extended generalized galilean genesis model so that the tensor perturbations grow on superhorizon scales.
- Q : If nearly scale invariant primordial GWs detected

  → Inflation ?

  A : This work → No

Genesis scenario gives…

nearly scale-invariant GWs → possible

red / blue GWs → also possible

background equations

- Friedmann equation

$$e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) \simeq 0$$

$$\hat{\rho}(Y) \quad := \quad 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right)$$

- Evolution equation

$$4e^{-2\beta\lambda\phi}\mathcal{G}_f(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0) \simeq 0$$

$$\hat{p}(Y) \quad := \quad g_2 - 4\alpha\lambda Y g_3 + 8(2\alpha+1)\lambda^2 Y(\alpha g_4 - Yg_4')$$

background equations

- Friedmann equation

$$\underbrace{e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0)}_{\propto(-t)^{-2(\alpha+1)}} - \underbrace{\boxed{\frac{3\mathcal{G}_T K}{a^2}}}_{\propto(-t)^{2\beta}} \simeq 0$$

$\alpha + \beta > 0$

→ Flatness problem is solved

$$\hat{\rho}(Y) \quad := \quad 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right)$$

- Evolution equation

$$\underbrace{4e^{-2\beta\lambda\phi}\mathcal{G}_f(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0)}_{\propto(-t)^{-2(\alpha+1)}} + \underbrace{\boxed{\frac{\mathcal{F}_T K}{a^2}}}_{\propto(-t)^{2\beta} \text{ or } (-t)^{2(\alpha+2\beta)}} \simeq 0$$

$$\hat{p}(Y) \quad := \quad g_2 - 4\alpha\lambda Y g_3 + 8(2\alpha+1)\lambda^2 Y(\alpha g_4 - Yg_4')$$

Growing tensor perturbations

[Y. Cai, Y. Piao, (2016)]

- Action $S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{-g}(\mathcal{L}_1 + \mathcal{L}_2) + S_{\mathrm{matter}}$

$$\mathcal{L}_1 = -e^{4\phi/\mathcal{M}}X + \frac{1}{\mathcal{M}}X^3 - \alpha\mathcal{M}^4 e^{6\phi/\mathcal{M}}$$

$$\mathcal{L}_2 = \frac{M_p^2}{2}\left(\frac{\mathcal{M}^8}{X^2} + 1\right)R + \frac{M_p^2\mathcal{M}^8}{X^3}\left[-(\Box\phi)^2 + \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right]$$

- solutions $a \simeq a_G\left(1 + \frac{1}{\mathcal{M}^6M_p^2}\frac{1}{(t_* - t)^8}\right)$ $H \sim \frac{1}{\mathcal{M}^6M_p^2}(t_* - t)^{-9}$

- perturbations

	scalar	tensor
sound speed	$c_S^2 \propto (-t)^8 \text{ or } (-t)^{12}$	$c_T^2 \propto 1/5$
power spectrum	$\mathcal{P}_S \propto k^{12/5}$	$\boxed{\mathcal{P}_T \propto k^0}$

Growing tensor perturbations

[T,Kobayashi, M, Yamaguchi, and J, Yokoyama (2010)]

- Horndeski theory

$$c_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right]$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

  - G_4 and G_5 determine the sound speed

$$S_{\rm Hor} = \int \mathrm{d}^4x\sqrt{-g}\Big\{G_2(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi,X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$X := -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

ADM -> covariant form

- EOM

$$M_2^4(Na_2)'f_2 + 3M_3^3 Na_3' f_3 H + 6M_4^2 N^2 (N^{-1}a_4)' f_4 H^2 + 6M_5 N^3 (N^{-2}a_5)' f_5 H^3 = 0$$

$$M_2^4 a_2 f_2 - 6M_4^2 a_4 f_4 H^2 - 12M_5 a_5 f_5 H^3 - \frac{1}{N}\frac{\mathrm{d}}{\mathrm{d}t}(M_3^3 a_3 f_3 + 4M_4^2 a_4 f_4 H + 6M_5 a_5 f_5 H^2) = 0$$

we have…

$$H(t) \simeq \frac{h_0}{(-t)^{2\alpha+2\beta+1}}$$

$$(Na_2)' = 0$$

$$Na_2 \boxed{f_2} \sim (a_4 \boxed{f_4} H)^{\cdot}$$

- Perturbations

$$\mathcal{G}_T = -2M_4^2 \boxed{f_4} a_4 - 6M_5 \boxed{f_5} a_5 H$$

$$\mathcal{F}_T = 2M_4^2 \boxed{f_4} b_4 + \frac{1}{N} M_5 \boxed{f_5'} b_5$$

$$\mathcal{G}_T \simeq 2(1+2\beta) f_4 Y_0^{-\beta} e^{-2\beta\lambda\phi} + 2(1+\alpha+2\beta) f_5 Y_0^{-(1+\alpha+2\beta)} e^{-2(1+\alpha+2\beta)\lambda\phi} H\dot{\phi}$$

$$\mathcal{F}_T \simeq 2f_4 Y_0^{-\beta} e^{-2\beta\lambda\phi} + 2(1+\alpha+2\beta) f_5 Y^{-(1+\alpha+2\beta)} e^{-2(1+\alpha+2\beta)\lambda\phi}\ddot{\phi}$$

in the same way, f3 is evaluated from G_S

Galilean Genesis

solutions

$$(-\infty < t < 0)$$

- Friedmann eq.

$$\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi}\hat{\rho}(Y_0) \simeq 0$$

$$\hat{\rho}(Y) := 2Yg_2' - g_2 - 4\lambda Y\left(\alpha g_3 - Yg_3'\right)$$

- $\hat{\rho} = 0 \qquad Y_0 = e^{-2\lambda\phi}X = const.$

higher order of t^-1

$$\boxed{e^{\lambda\phi} \propto (-t)^{-1}\,, \quad H(t) \propto \frac{1}{(-t)^{2\alpha+1}}, \quad a \simeq a_G\left[1 + \frac{1}{2\alpha}\frac{h_0}{(-t)^{2\alpha}}\right]} + \cdots$$

- Evolution eq.

$$\mathcal{P} \simeq 2\mathcal{G}(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi}\hat{p}(Y_0) \simeq 0$$

NEC violated

$$\hat{p} < 0$$