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The Λ_2 Limit of Massive Gravity

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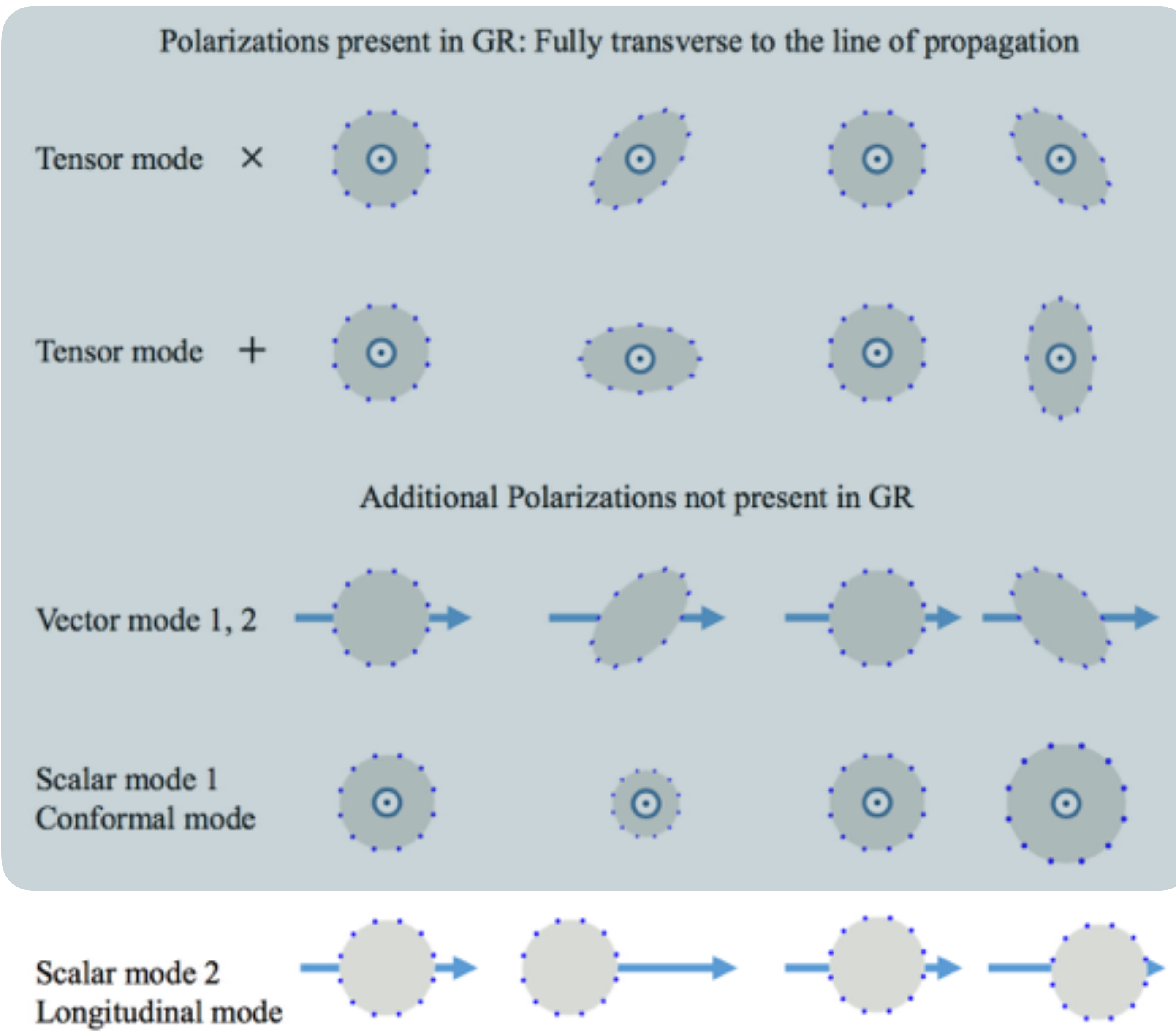
de Rham, Tolley & SYZ, arXiv:1512.06838
de Rham, Tolley & SYZ, arXiv:1602.03721

Massive Gravity: Boulware-Deser Ghost Problem



Boulware & Deser, 1972

dRGT Massive Gravity

de Rham, Gabadzdze & Tolley, 2010

$$\mathcal{L} = M_P^2\sqrt{-g}\left(\frac{R}{2} + \boxed{m^2\left(\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_{\nu]} + \alpha_3\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_\nu\mathcal{K}^\rho_{\rho]} + \alpha_4\mathcal{K}^\mu_{[\mu}\mathcal{K}^\nu_\nu\mathcal{K}^\rho_\rho\mathcal{K}^\sigma_{\sigma]}\right)} + \mathcal{L}_m\right)$$

where $\quad \mathcal{K}^\mu_\nu = \delta^\mu_\nu - \mathcal{X}^\mu_\nu \qquad \mathcal{X}^\mu_\nu = \sqrt{g^{\mu\rho}\eta_{\rho\nu}}$

BD ghost projected out by 2 second-class constraints

de Rham, Gabadzdze & Tolley, 2010
Hassan & Rosen, 2011

The unique graviton potential to eliminate BD ghost!

dRGT Massive Gravity: Peculiarities

van Dam, Veltman, 1970
Zakharov, 1970
Iwasaki, 1970

- vDVZ discontinuity around $\eta_{\mu\nu}$:

scattering amplitude between $T^{\mu\nu}_{(1)}$ and $T^{\mu\nu}_{(2)}$

$$\mathcal{A} \propto \frac{M_P^{-2}}{(k^2 - m^2 + i\epsilon)} \left(2T^{\mu\nu}_{(1)} T_{(2)\mu\nu} - \boxed{\frac{2}{3}} T_{(1)} T_{(2)} \right)$$

GR value = 1

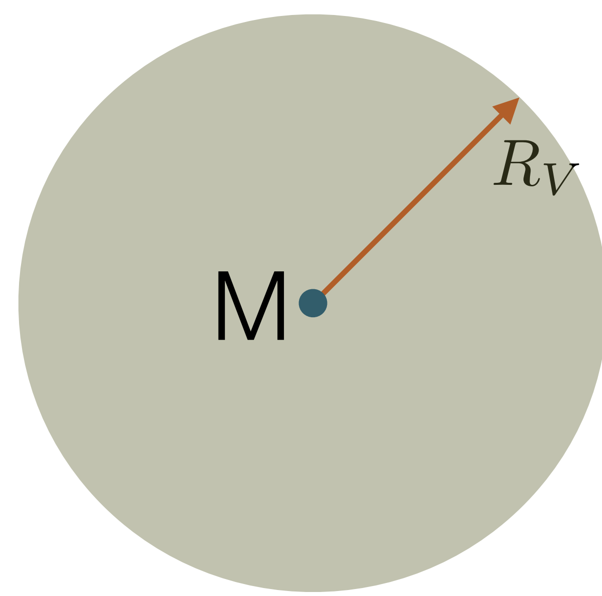
- Strong coupling scale around $\eta_{\mu\nu}$:

$$\Lambda_3 = (M_P m^2)^{\frac{1}{3}}$$

Vainshtein Mechanism

Non-linearities to restore the GR limit

Vainshtein, 1972



$$R_V = \left(\frac{M}{M_P^2 m^2} \right)^{\frac{1}{3}}$$

non-perturbativity

Dvali et al, 2010
Keltner & Tolley, 2015

Non-trivial Vacua

Non-trivial vacua in dRGT: de Rham, Tolley & SYZ, arXiv:1602.03721

$$\bar{g}_{\mu\nu} = \partial_\mu \bar{\phi}^\alpha \partial_\nu \bar{\phi}^\beta \eta_{\alpha\beta} + \mathcal{O}(m^2)$$

Properties of non-trivial vacua:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

1. Free of vDVZ discontinuity
2. Strong coupling scale raised to $\Lambda_{2*} \gg \Lambda_3$

Trivial Vacuum: Lack of a Kinetic Term

Stueckelberg formulation

$$\eta_{\mu\nu} \;\to\; \partial_\mu \phi^A \partial_\nu \phi^B \eta_{AB}$$

Around the Λ_3 vacuum: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \phi^\alpha = x^\alpha + A^\alpha + \partial^\alpha \pi$

$$\mathcal{L}_{\rm dRGT} \;\sim -M_P^2 h\mathcal{E}h - m^2 M_P^2 \partial A \partial A + m^2 M_P^2 h \partial\partial\pi + hT + \cdots$$

$$\xrightarrow{h = \tilde{h} + m^2 \pi}$$

$$\mathcal{L}_{\rm dRGT} \;\sim -M_P^2 \tilde{h}\mathcal{E}\tilde{h} - m^2 M_P^2 \partial A \partial A - m^4 M_P^2 \partial\pi\partial\pi + \tilde{h}T + m^2 \pi T + \cdots$$

$$\xrightarrow{\hat{h} \sim M_P \tilde{h}, \quad \hat{A} \sim m M_P A, \quad \hat{\pi} \sim m^2 M_P \pi}$$

$$\mathcal{L}_{\rm dRGT} \sim -\hat{h}\mathcal{E}\hat{h} - \partial\hat{A}\partial\hat{A} - \partial\hat{\pi}\partial\hat{\pi} + \frac{1}{M_P}\hat{h}T + \frac{1}{M_P}\hat{\pi}T + \cdots$$

Non-trivial Vacua: Re-gain a Kinetic Term

Around non-trivial vacua:

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(m^2) + h_{\mu\nu} \qquad \phi^\alpha = \bar{\phi}^\alpha + A^\alpha + \partial^\alpha \pi$$

- Hamiltonian analysis
- Perturbations on general backgrounds
- Perturbations on exact solutions

$$\mathcal{L} \sim -f_h(\bar{\phi})(\partial h)^2 - f_A(\bar{\phi})(\partial A)^2 - \boxed{f_\pi(\bar{\phi})(\partial \pi)^2} + \cdots$$

de Rham, Tolley & SYZ, arXiv:1602.03721

Non-trivial Vacua: Stabilities

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(m^2) + h_{\mu\nu} \qquad \phi^\alpha = \bar{\phi}^\alpha + A^\alpha + \partial^\alpha \pi$$

There exist non-trivial vacua that are

- Free of ghost instabilities
- Free of gradient instabilities

de Rham, Tolley & SYZ, arXiv:1602.03721

Λ_2 Decoupling Limit

$$\mathcal{L}_{\rm dRGT} = \sqrt{-g}\left(\frac{M_P^2}{2}R + m^2 M_P^2 \sum_{n=0}^{D} \beta_n \mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_n}_{\mu_n]} + \mathcal{L}_{\rm m}\right)$$

A double scaling limit:

$$M_{\rm Pl}\to\infty, \qquad m\to 0, \qquad \Lambda_2=\sqrt{M_{\rm Pl}m}\to \text{fixed}$$

$$S_{\rm dRGT}\to\int \mathrm{d}^4x\left(-\frac{1}{4}\hat{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma}+\frac{\hat{h}_{\mu\nu}}{2M_P}T^{\mu\nu}+\Lambda_2^4\mathcal{L}_{\rm NSM}\right)$$

$$\mathcal{L}_{\rm NSM}=\sum_{n=1}^{D}\beta_n X^{\mu_1}_{[\mu_1}X^{\mu_2}_{\mu_2}\cdots X^{\mu_n}_{\mu_n]} \qquad\qquad X^\mu_\nu=\sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

de Rham, Tolley & SYZ, arXiv:1602.03721

Free of vDVZ Discontinuity

$$S_{\rm dRGT} \to \int {\rm d}^4x \left(\boxed{-\frac{1}{4}\hat{h}^{\mu\nu}\mathcal{E}^{\rho\sigma}_{\mu\nu}\hat{h}_{\rho\sigma} + \frac{\hat{h}_{\mu\nu}}{2M_P}T^{\mu\nu}} + \Lambda_2^4 \mathcal{L}_{\rm NSM} \right)$$

No vDVZ discontinuity around Λ_2 vacua!

↓

Easily pass weak field GR tests!

Λ_2 vacua have Vainshtein mechanism already built-in!

de Rham, Tolley & SYZ, arXiv:1602.03721

Raised Strong Coupling Scale

$$\mathcal{L}_{\mathrm{dRGT}} = \sqrt{-g}\left(\frac{M_P^2}{2}R + m^2 M_P^2 \sum_{n=0}^{D} \beta_n \mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_n}_{\mu_n]} + \mathcal{L}_{\mathrm{m}}\right)$$

Explicit scale Λ_2

$$\bar{\phi}^\alpha = x^\rho\left[H^\mu_\rho + M^\alpha{}_{\rho\sigma}\left(\frac{x^\sigma}{L}\right) + \mathcal{O}\left(\left(\frac{x}{L}\right)^2\right)\right]$$

Implicit scale L^{-1}

Strong coupling scale:

$$\Lambda_{2*} \gg \Lambda_3$$

de Rham, Tolley & SYZ, arXiv:1602.03721

By-product: Non-Compact NLSM

Conventional wisdom of NLSM:

Internal symmetry group should be compact.

dRGT NLSM:

$$\mathcal{L}_{\rm NSM} = \sum_{n=1}^{D} \beta_n X^{\mu_1}_{[\mu_1} X^{\mu_2}_{\mu_2} \cdots X^{\mu_n}_{\mu_n]} \qquad X^\mu_\nu = \sqrt{\eta^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

Unique consistent non-compact NLSM

de Rham, Tolley & SYZ, arXiv:1512.06838

dRGT as Generalization of Nambu-Goto

Nambu-Goto action:

$$\mathcal{L}_{\rm NG} = \sqrt{-g}\,\mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_D}_{\mu_D]} \qquad\qquad \mathcal{X}^{\mu}_{\nu} = \sqrt{g^{\mu\rho}\partial_\rho\phi^\alpha\partial_\nu\phi^\beta\eta_{\alpha\beta}}$$

dRGT massive gravity:

$$\mathcal{L}_{\rm dRGT} = \sqrt{-g}\left(\frac{M_P^2}{2}R + m^2 M_P^2\sum_{n=0}^{D}\beta_n\mathcal{X}^{\mu_1}_{[\mu_1}\mathcal{X}^{\mu_2}_{\mu_2}\cdots\mathcal{X}^{\mu_n}_{\mu_n]} + \mathcal{L}_{\rm m}\right)$$

de Rham, Tolley & SYZ, arXiv:1512.06838

Summary

- dRGT has a Λ_2 decoupling limit.
- Non-trivial Λ_2 vacua in dRGT:
    - Free of vDVZ discontinuity
    - Strong coupling scale raised to quasi-Λ_2
- By-product: Unique consistent non-compact NLSM

Thank you!

Compact requirement for NSM

Typical nonlinear sigma model

$$S = \int \mathrm{d}^D x \left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi^A\partial_\nu\phi^B f_{AB}(\phi) - V(\phi) \right)$$

The symmetry group should be compact!

$f_{AB}(\phi)$ **:** **Riemannian (+,+,+,...)**

Auxiliary gauge trick

- **p-brane action:**

$$S_{\text{Polyakov}} = -\frac{T_p}{2}\int \mathrm{d}^{p+1}\xi\sqrt{-\gamma}\left(\gamma^{\mu\nu}\partial_\mu X^A(\xi)\partial_\nu X^B(\xi)G_{AB}(X) - p + 1\right)$$

Ghost eliminated by diff invariance

- **Cremmer–Julia NSM**

SU(1,1)/U(1): $$S_{\text{CJ}} = \int \mathrm{d}^D x\left(|\partial_\mu\phi_1 - A_\mu\phi_1|^2 - |\partial_\mu\phi_0 - A_\mu\phi_0|^2\right)$$

$$|\phi_0|^2 - |\phi_1|^2 = 1$$

Ghost eliminated by gauge invariance

Generally: G/H non-compact group/gauge subgroup

All known non-compact NSMs use 'auxiliary gauge trick'.