Gravitational radiation from compact binaries in scalar-tensor gravity

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Based on:
Mirshekari and Will, PRD 87, 084070 (2013)
Lang PRD 89, 084014 (2014)
Lang PRD 91, 084027 (2015)

Scalar-tensor theories of gravity

- Scalar-tensor (ST) theories are popular alternatives to general relativity (GR).
  - Simple modification
  - Well-motivated:
    - Possible low-energy limit of string theory
    - $f(R)$ theories designed to explain cosmic acceleration can be recast as ST
- Constrained by solar system and binary pulsar tests, but not in strong-field, dynamical regime (e.g., coalescing compact binaries made of neutron stars and/or black holes)
  - Can be tested by gravitational-wave (GW) detectors!
- Goal: Calculate highly accurate waveforms for inspiral of nonspinning compact binaries in scalar-tensor theories

The starting point: ST field equations

$$G_{\mu\nu} = \frac{8\pi}{\phi}T_{\mu\nu} + \frac{\omega(\phi)}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\lambda}\phi^{,\lambda}\right) + \frac{1}{\phi}\left(\phi_{;\mu\nu} - g_{\mu\nu}\Box_g\phi\right)$$

$$\Box_g\phi = \frac{1}{3+2\omega(\phi)}\left(8\pi T - 16\pi\phi\frac{\partial T}{\partial\phi} - \frac{d\omega}{d\phi}\phi_{,\lambda}\phi^{,\lambda}\right)$$

- Assumptions:
  - No potential/mass for the scalar field.
  - Coupling $\omega(\phi)$ is not limited to a constant (i.e., not Brans-Dicke).
- To solve: Redefine variables.

$$\varphi \equiv \frac{\phi}{\phi_0} \qquad \tilde{h}^{\mu\nu} \equiv \eta^{\mu\nu} - \sqrt{-\tilde{g}}\tilde{g}^{\mu\nu} \qquad \tilde{g}_{\mu\nu} \equiv \varphi g_{\mu\nu}$$

  - Gauge condition: $\tilde{h}^{\mu\nu}{}_{,\nu} = 0$

The starting point: ST field equations

- The "relaxed" field equations:

$$\Box_\eta \tilde{h}^{\mu\nu} = -16\pi\tau^{\mu\nu} \qquad \tau^{\mu\nu} \equiv (-g)\frac{\varphi}{\phi_0}T^{\mu\nu} + \frac{1}{16\pi}(\Lambda^{\mu\nu} + \Lambda_s^{\mu\nu})$$

$$\Lambda^{\mu\nu} \equiv 16\pi(-\tilde{g})\tilde{t}^{\mu\nu}_{LL} + \tilde{h}^{\mu\alpha}{}_{,\beta}\tilde{h}^{\nu\beta}{}_{,\alpha} - \tilde{h}^{\alpha\beta}\tilde{h}^{\mu\nu}{}_{,\alpha\beta}$$

$$\Lambda_s^{\mu\nu} \equiv \frac{(3+2\omega)}{\varphi^2}\varphi_{,\alpha}\varphi_{,\beta}\left(\tilde{\mathfrak{g}}^{\mu\alpha}\tilde{\mathfrak{g}}^{\nu\beta} - \frac{1}{2}\tilde{\mathfrak{g}}^{\mu\nu}\tilde{\mathfrak{g}}^{\alpha\beta}\right)$$

$$\Box_\eta \varphi = -8\pi\tau_s \qquad \tau_s \equiv -\frac{1}{3+2\omega}\sqrt{-g}\frac{\varphi}{\phi_0}\left(T - 2\varphi\frac{\partial T}{\partial\varphi}\right) - \frac{1}{8\pi}\tilde{h}^{\alpha\beta}\varphi_{,\alpha\beta} + \frac{1}{16\pi}\frac{d}{d\varphi}\left[\ln\left(\frac{3+2\omega}{\varphi^2}\right)\right]\varphi_{,\alpha}\varphi_{,\beta}\tilde{\mathfrak{g}}^{\alpha\beta}$$

- What about the source $T_{\mu\nu}$? We can consider the compact bodies (NS, BH) to be point masses, with each mass a function of ϕ. Key quantity: sensitivity of a body's mass to variations in the scalar field.
  - For weak fields, proportional to gravitational binding energy per unit mass.
  - For black holes, s_A = 0.5 and all derivatives are zero.

Fundamental parameters

Parameter	Definition
Scalar-tensor parameters	
G	$\phi_0^{-1}(4+2\omega_0)/(3+2\omega_0)$
ζ	$1/(4+2\omega_0)$
λ_1	$(d\omega/d\varphi)_0\zeta^2/(1-\zeta)$
λ_2	$(d^2\omega/d\varphi^2)_0\zeta^3/(1-\zeta)$
Sensitivities	
s_A	$[d\ln M_A(\phi)/d\ln\phi]_0$
s'_A	$[d^2\ln M_A(\phi)/d\ln\phi^2]_0$
s''_A	$[d^3\ln M_A(\phi)/d\ln\phi^3]_0$

Solving the field equations

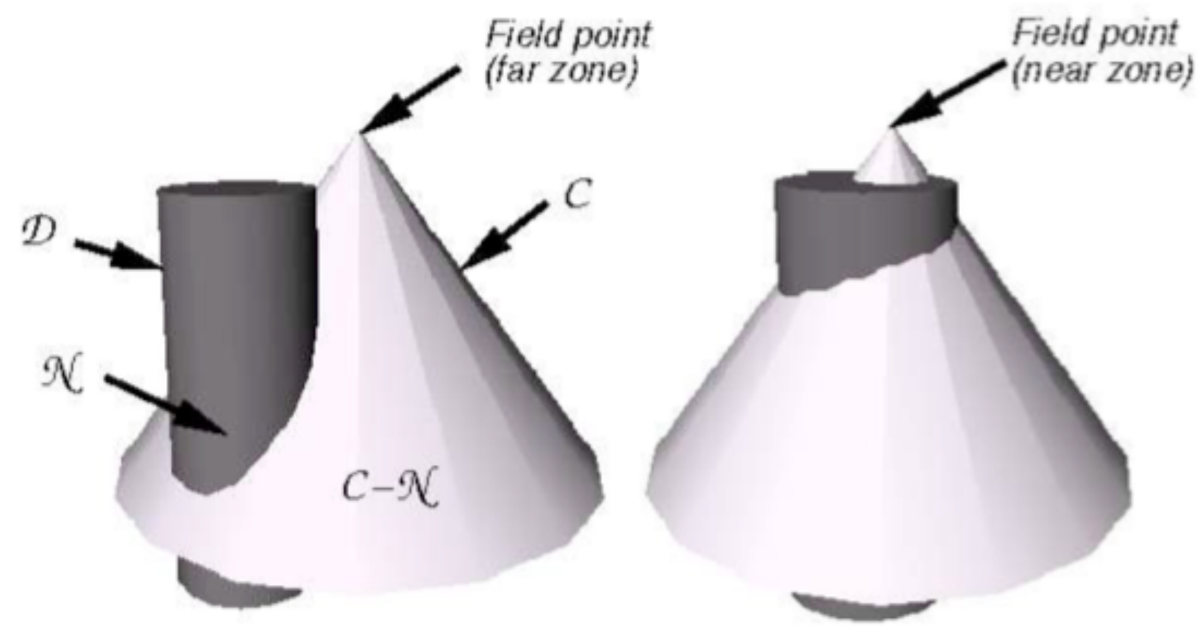
- The "relaxed" field equations:

$$\Box_\eta \tilde{h}^{\mu\nu} = -16\pi\tau^{\mu\nu} \qquad \Box_\eta \varphi = -8\pi\tau_s$$

- Flat-spacetime wave equations can be solved formally using a retarded Green's function:

$$\tilde{h}^{\mu\nu}(t,\mathbf{x}) = 4\int \frac{\tau^{\mu\nu}(t',\mathbf{x}')\delta(t'-t+|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|} d^4x'$$

$$\varphi(t,\mathbf{x}) = 2\int \frac{\tau_s(t',\mathbf{x}')\delta(t'-t+|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|} d^4x'$$



- Practical solution method differs for near zone and radiation zone

Pieces of the calculation I

- Equations of motion to 2.5PN order (Mirshekari and Will 2013)
    - Evaluate integrals in near zone.
    - New terms include dipole radiation reaction at 1.5PN order.

- Tensor gravitational waves to 2PN order (Lang 2014)
    - Evaluate $\tilde{h}^{\mu\nu}$ in radiation zone.
    - Key piece: “Epstein-Wagoner moments” $I^{ij}_{\rm EW} \equiv \int_{\mathcal{M}} \tau^{00} x^{ij} d^3x + I^{ij}_{\rm EW\ (surf)}$
        - Quadrupole: Generates 0PN+ GWs
        - Octupole: Generates 0.5PN+ GWs...

Tensor waves: 0PN, 0.5PN, 1PN

$$\tilde{h}^{ij} = \frac{2G(1-\zeta)\mu}{R}[\tilde{Q}^{ij} + P^{1/2}Q^{ij} + PQ^{ij} + P^{3/2}Q^{ij}_{\mathcal{N}} + P^{3/2}Q^{ij}_{\mathcal{C}-\mathcal{N}} + P^{2}Q^{ij}_{\mathcal{N}} + P^{2}Q^{ij}_{\mathcal{C}-\mathcal{N}} + O(\epsilon^{5/2})]_{TT}$$

$$\tilde{Q}^{ij} = 2\left(v^{ij} - \frac{G\alpha m}{r}\hat{n}^{ij}\right)$$

$$P^{1/2}Q^{ij} = \frac{\delta m}{m}\left\{3(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})\frac{G\alpha m}{r}[2\hat{n}^{(i}v^{j)} - \dot{r}\hat{n}^{ij}] + (\hat{\mathbf{N}}\cdot\mathbf{v})\left[\frac{G\alpha m}{r}\hat{n}^{ij} - 2v^{ij}\right]\right\}$$

$$\begin{aligned} PQ^{ij} = \frac{1}{3}(1-3\eta)\bigg\{&(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^2\frac{G\alpha m}{r}\left[\left(3v^2 - 15\dot{r}^2 + 7\frac{G\alpha m}{r}\right)\hat{n}^{ij} + 30\dot{r}\hat{n}^{(i}v^{j)} - 14v^{ij}\right] \\ &+(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{N}}\cdot\mathbf{v})\frac{G\alpha m}{r}[12\dot{r}\hat{n}^{ij} - 32\hat{n}^{(i}v^{j)}] + (\hat{\mathbf{N}}\cdot\mathbf{v})^2\left[6v^{ij} - 2\frac{G\alpha m}{r}\hat{n}^{ij}\right]\bigg\} \\ &+\frac{1}{3}\bigg\{\left[3(1-3\eta)v^2 - 2(2-3\eta)\frac{G\alpha m}{r}\right]v^{ij} + 4\frac{G\alpha m}{r}\dot{r}(5+3\eta+3\bar{\gamma})\hat{n}^{(i}v^{j)} \\ &+\frac{G\alpha m}{r}\left[3(1-3\eta)\dot{r}^2 - (10+3\eta+6\bar{\gamma})v^2 + \left(29+12\bar{\gamma}+12\bar{\beta}_+ - 12\frac{\delta m}{m}\bar{\beta}_-\right)\frac{G\alpha m}{r}\right]\hat{n}^{ij}\bigg\} \end{aligned}$$

Tensor waves: 1.5PN (near zone)

$$
\begin{aligned}
P^{3/2}Q^{ij}_{\mathcal{N}} = {}& \frac{\delta m}{m}(1-2\eta)\left\{(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^3\frac{G\alpha m}{r}\left[\frac{5}{4}\left(3v^2-7\dot{r}^2+6\frac{G\alpha m}{r}\right)\dot{r}\hat{n}^{ij}-\frac{17}{2}\dot{r}v^{ij}\right.\right.\\
&-\left.\frac{1}{6}\left(21v^2-105\dot{r}^2+44\frac{G\alpha m}{r}\right)\hat{n}^{(i}v^{j)}\right]+\frac{1}{4}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{N}}\cdot\mathbf{v})\frac{G\alpha m}{r}\left[58v^{ij}\right.\\
&+\left.\left(45\dot{r}^2-9v^2-28\frac{G\alpha m}{r}\right)\hat{n}^{ij}-108\dot{r}\hat{n}^{(i}v^{j)}\right]+\frac{3}{2}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{N}}\cdot\mathbf{v})^2\frac{G\alpha m}{r}[10\hat{n}^{(i}v^{j)}-3\dot{r}\hat{n}^{ij}]\\
&\left.+\frac{1}{2}(\hat{\mathbf{N}}\cdot\mathbf{v})^3\left[\frac{G\alpha m}{r}\hat{n}^{ij}-4v^{ij}\right]\right\}\\
&+\frac{1}{12}\frac{\delta m}{m}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})\frac{G\alpha m}{r}\left\{2\hat{n}^{(i}v^{j)}\left[\dot{r}^2(63+54\eta+36\bar{\gamma})-\frac{G\alpha m}{r}\left(128-36\eta+48\bar{\gamma}+72\bar{\beta}_+\right.\right.\right.\\
&\left.\left.-72\frac{\delta m}{m}\bar{\beta}_-\right)+v^2(33-18\eta+24\bar{\gamma})\right]+\hat{n}^{ij}\dot{r}\left[\dot{r}^2(15-90\eta)-v^2(63-54\eta+36\bar{\gamma})\right.\\
&\left.\left.+\frac{G\alpha m}{r}\left(242-24\eta+96\bar{\gamma}+96\bar{\beta}_+-96\frac{\delta m}{m}\bar{\beta}_-\right)\right]-\dot{r}v^{ij}(186+24\eta+96\bar{\gamma})\right\}\\
&+\frac{\delta m}{m}(\hat{\mathbf{N}}\cdot\mathbf{v})\left\{\frac{1}{2}v^{ij}\left[\frac{G\alpha m}{r}(3-8\eta)-2v^2(1-5\eta)\right]-\hat{n}^{(i}v^{j)}\frac{G\alpha m}{r}\dot{r}(7+4\eta+4\bar{\gamma})\right.\\
&\left.-\hat{n}^{ij}\frac{G\alpha m}{r}\left[\frac{3}{4}(1-2\eta)\dot{r}^2+\frac{1}{3}\left(26-3\eta+12\bar{\gamma}+6\bar{\beta}_+-6\frac{\delta m}{m}\bar{\beta}_-\right)\frac{G\alpha m}{r}-\frac{1}{4}(7-2\eta+4\bar{\gamma})v^2\right]\right\}\\
&+\frac{16}{3}\eta\left(\frac{G\alpha m}{r}\right)^2\zeta\mathcal{S}_-^2\left(\dot{r}\hat{n}^{ij}-\frac{1}{3}\hat{n}^{(i}v^{j)}\right)
\end{aligned}
$$

Tensor waves: 1.5PN (radiation zone)

$$P^{3/2}Q^{ij}_{\mathcal{C}-\mathcal{N}} = 4m\int_0^\infty \left\{\frac{G\alpha m}{r^3}\left[\left(3v^2+\frac{G\alpha m}{r}-15\dot{r}^2\right)\hat{n}^{ij}+18\dot{r}\hat{n}^{(i}v^{j)}-4v^{ij}\right]\right\}_{\tau-s}$$
$$\times\left\{G(1-\zeta)\left[\ln\left(\frac{s}{2R+s}\right)+\frac{11}{12}\right]-\frac{1}{12}G\alpha\zeta\left(\mathcal{S}_+ + \frac{\delta m}{m}\mathcal{S}_-\right)\left(\mathcal{S}_+ - \frac{\delta m}{m}\mathcal{S}_-\right)\right\}ds$$
$$+8G\alpha\mu\zeta\mathcal{S}_-^2\int_0^\infty\left\{\frac{G\alpha m}{r^3}\left[\left(-\frac{1}{6}v^2+\frac{1}{9}\frac{G\alpha m}{r}+\frac{5}{6}\dot{r}^2\right)\hat{n}^{ij}-\dot{r}\hat{n}^{(i}v^{j)}+\frac{2}{9}v^{ij}\right]\right\}_{\tau-s}ds$$

Tensor waves: 2PN (near zone)

$$
\begin{aligned}
P^2Q^{ij}_{\mathcal{N}} = {} & \frac{1}{60}(1-5\eta+5\eta^2)\Bigg\{24(\hat{\mathbf{N}}\cdot\mathbf{v})^4\left[5v^{ij}-\frac{G\alpha m}{r}\hat{n}^{ij}\right] \\
& +\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^4\Bigg[2\left(175\frac{G\alpha m}{r}-465\dot{r}^2+93v^2\right)v^{ij}+30\dot{r}\left(63\dot{r}^2-50\frac{G\alpha m}{r}-27v^2\right)\hat{n}^{(i}v^{j)} \\
& +\left(1155\frac{G\alpha m}{r}\dot{r}^2-172\left(\frac{G\alpha m}{r}\right)^2-945\dot{r}^4-159\frac{G\alpha m}{r}v^2+630\dot{r}^2v^2-45v^4\right)\hat{n}^{ij}\Bigg] \\
& +24\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^3(\hat{\mathbf{N}}\cdot\mathbf{v})\Bigg[87\dot{r}v^{ij}+5\dot{r}\left(14\dot{r}^2-15\frac{G\alpha m}{r}-6v^2\right)\hat{n}^{ij} \\
& +16\left(5\frac{G\alpha m}{r}-10\dot{r}^2+2v^2\right)\hat{n}^{(i}v^{j)}\Bigg]+288\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{N}}\cdot\mathbf{v})^3[\dot{r}\hat{n}^{ij}-4\hat{n}^{(i}v^{j)}] \\
& +24\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{N}}\cdot\mathbf{v})^2\left[\left(35\frac{G\alpha m}{r}-45\dot{r}^2+9v^2\right)\hat{n}^{ij}-76v^{ij}+126\dot{r}\hat{n}^{(i}v^{j)}\right]\Bigg\} \\
& +\frac{1}{15}(\hat{\mathbf{N}}\cdot\mathbf{v})^2\Bigg\{\Bigg[5\left(25-78\eta+12\eta^2+4(1-3\eta)\left(3\bar{\gamma}+\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r} \\
& -(18-65\eta+45\eta^2+10(1-3\eta)\bar{\gamma})v^2+9(1-5\eta+5\eta^2)\dot{r}^2\Bigg]\frac{G\alpha m}{r}\hat{n}^{ij} \\
& +3\left[5(1-9\eta+21\eta^2)v^2-2(4-25\eta+45\eta^2)\frac{G\alpha m}{r}\right]v^{ij} \\
& +18\left[6-15\eta-10\eta^2+\frac{10}{3}(1-3\eta)\bar{\gamma}\right]\frac{G\alpha m}{r}\dot{r}\hat{n}^{(i}v^{j)}\Bigg\} \\
& +\frac{1}{15}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})(\hat{\mathbf{N}}\cdot\mathbf{v})\frac{G\alpha m}{r}\Bigg\{\Bigg[3(36-145\eta+150\eta^2+20(1-3\eta)\bar{\gamma})v^2 \\
& -5\left(127-392\eta+36\eta^2+56(1-3\eta)\bar{\gamma}+32(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r} \\
& -15(2-15\eta+30\eta^2)\dot{r}^2\Bigg]\dot{r}\hat{n}^{ij}+6[98-295\eta-30\eta^2+50(1-3\eta)\bar{\gamma}]\dot{r}v^{ij} \\
& +2\Bigg[5\left(66-221\eta+96\eta^2+26(1-3\eta)\bar{\gamma}+32(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r} \\
& -9(18-45\eta-40\eta^2+10(1-3\eta)\bar{\gamma})\dot{r}^2-(66-265\eta+360\eta^2+50(1-3\eta)\bar{\gamma})v^2\Bigg]\hat{n}^{(i}v^{j)}\Bigg\}
\end{aligned}
$$

Tensor waves: 2PN (near zone)

$$
\begin{aligned}
&+\frac{1}{60}(\hat{\mathrm{N}}\cdot\hat{\mathrm{n}})^2\frac{G\alpha m}{r}\Bigg\{\Bigg[3(33-130\eta+150\eta^2+20(1-3\eta)\bar{\gamma})v^4+105(1-10\eta+30\eta^2)\dot{r}^4\\
&\quad+15\left(181-572\eta+84\eta^2+72(1-3\eta)\bar{\gamma}+64(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r}\dot{r}^2\\
&\quad-\left(131-770\eta+930\eta^2-80(1-3\eta)\bar{\gamma}+160(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r}v^2\\
&\quad-60(9-40\eta+60\eta^2+5(1-3\eta)\bar{\gamma})v^2\dot{r}^2\\
&\quad-8\left(131-390\eta+30\eta^2+60(1-3\eta)\bar{\gamma}+65(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\left(\frac{G\alpha m}{r}\right)^2\Bigg]\hat{n}^{ij}\\
&\quad+4\Bigg[(12+5\eta-315\eta^2-10(1-3\eta)\bar{\gamma})v^2-9(39-115\eta-35\eta^2+20(1-3\eta)\bar{\gamma})\dot{r}^2\\
&\quad+5\left(29-104\eta+84\eta^2+8(1-3\eta)\bar{\gamma}+28(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r}\Bigg]v^{ij}\\
&\quad+4\Bigg[15(18-40\eta-75\eta^2+10(1-3\eta)\bar{\gamma})\dot{r}^2\\
&\quad-5\left(197-640\eta+180\eta^2+76(1-3\eta)\bar{\gamma}+80(1-3\eta)\left(\bar{\beta}_+-\frac{\delta m}{m}\bar{\beta}_-\right)\right)\frac{G\alpha m}{r}\\
&\quad+3(21-130\eta+375\eta^2+20(1-3\eta)\bar{\gamma})v^2\Bigg]\dot{r}\hat{n}^{(i}v^{j)}\Bigg\}\\
&+\frac{1}{15}\eta\left(\frac{G\alpha m}{r}\right)^2\zeta\mathcal{S}_-\Bigg\{(\hat{\mathrm{N}}\cdot\hat{\mathrm{n}})\Bigg[192\left(\mathcal{S}_+-\frac{\delta m}{m}\mathcal{S}_-\right)\dot{r}\hat{n}^{(i}v^{j)}\\
&\quad+\left(-120\left(\mathcal{S}_+-\frac{\delta m}{m}\mathcal{S}_-\right)\dot{r}^2+24\left(\mathcal{S}_+-\frac{\delta m}{m}\mathcal{S}_-\right)v^2+4\left(11\mathcal{S}_++19\frac{\delta m}{m}\mathcal{S}_-\right)\frac{G\alpha m}{r}\right)\hat{n}^{ij}\\
&\quad-4\left(17\mathcal{S}_+-7\frac{\delta m}{m}\mathcal{S}_-\right)v^{ij}\Bigg]+(\hat{\mathrm{N}}\cdot\mathrm{v})\left[-16\left(8\mathcal{S}_++7\frac{\delta m}{m}\mathcal{S}_-\right)\hat{n}^{(i}v^{j)}+12\left(7\mathcal{S}_++3\frac{\delta m}{m}\mathcal{S}_-\right)\dot{r}\hat{n}^{ij}\right]\Bigg\}
\end{aligned}
$$

Tensor waves: 2PN (near zone)

$$
\begin{aligned}
&+\frac{1}{60}\Bigg\{\Bigg[\Bigg(467+780\eta-120\eta^2+120(2+3\eta)\bar\gamma+10\bar\gamma^2+40\left(\bar\delta_++\frac{\delta m}{m}\bar\delta_-\right)\\
&\quad -40(1-3\eta)\left(\bar\beta_+-\frac{\delta m}{m}\bar\beta_-\right)\Bigg)\frac{G\alpha m}{r}v^2-15\Bigg(61-96\eta+48\eta^2+\frac{8}{3}(7-12\eta)\bar\gamma-\frac{4}{3}\bar\gamma^2\\
&\quad -\frac{16}{3}\left(\bar\delta_++\frac{\delta m}{m}\bar\delta_-\right)+\frac{32}{3}(2-3\eta)\left(\bar\beta_+-\frac{\delta m}{m}\bar\beta_-\right)\Bigg)\frac{G\alpha m}{r}\dot r^2-(144-265\eta-135\eta^2+20(4-9\eta)\bar\gamma)v^4\\
&\quad +6(24-95\eta+75\eta^2+10(1-3\eta)\bar\gamma)v^2\dot r^2-2\Bigg(642+545\eta+20(29+6\eta)\bar\gamma+15(9-2\eta)\bar\gamma^2\\
&\quad +80(8+6\eta+3\bar\gamma)\bar\beta_+-80(8+3\bar\gamma)\frac{\delta m}{m}\bar\beta_-+60(1-2\eta)(\bar\delta_+-2\bar\chi_+)+60\frac{\delta m}{m}(\bar\delta_-+2\bar\chi_+)\\
&\quad -1440\eta\frac{\bar\beta_1\bar\beta_2}{\bar\gamma}\Bigg)\left(\frac{G\alpha m}{r}\right)^2-45(1-5\eta+5\eta^2)\dot r^4\Bigg]\frac{G\alpha m}{r}\hat n^{ij}\\
&\quad +\Bigg[4(69+10\eta-135\eta^2+10(4-3\eta)\bar\gamma)\frac{G\alpha m}{r}v^2-12(3+60\eta+25\eta^2+40\eta\bar\gamma)\frac{G\alpha m}{r}\dot r^2\\
&\quad +45(1-7\eta+13\eta^2)v^4-10\Bigg(56+165\eta-12\eta^2+4(7+24\eta)\bar\gamma+\bar\gamma^2\\
&\quad +4\left(\bar\delta_++\frac{\delta m}{m}\bar\delta_-\right)-4(1+3\eta)\bar\beta_++4(1-3\eta)\frac{\delta m}{m}\bar\beta_-\Bigg)\left(\frac{G\alpha m}{r}\right)^2\Bigg]v^{ij}\\
&\quad +4\Bigg[2(36+5\eta-75\eta^2+5(4-3\eta)\bar\gamma)v^2-6(7-15\eta-15\eta^2+5(1-3\eta)\bar\gamma)\dot r^2\\
&\quad +5\Bigg(35+45\eta+36\eta^2+8(1+6\eta)\bar\gamma-\bar\gamma^2+16(1-3\eta)\bar\beta_+-8(2-3\eta)\frac{\delta m}{m}\bar\beta_-\\
&\quad -4\left(\bar\delta_++\frac{\delta m}{m}\bar\delta_-\right)\Bigg)\frac{G\alpha m}{r}\Bigg]\frac{G\alpha m}{r}\dot r\hat n^{(i}v^{j)}\Bigg\}
\end{aligned}
$$

Tensor waves: 2PN (radiation zone)

$$
\begin{aligned}
P^2 Q^{ij}_{\mathcal{C}-\mathcal{N}} = 2m \int_0^\infty \Bigg\{ &\frac{G\alpha m}{r^3}\Bigg[15\left(3v^2 + 2\frac{G\alpha m}{r} - 7\dot{r}^2\right)\dot{r}\hat{n}^{ij}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) \\
&- \left(13v^2 + \frac{22}{3}\frac{G\alpha m}{r} - 65\dot{r}^2\right)\left(\hat{n}^{ij}(\hat{\mathbf{N}}\cdot\mathbf{v}) + 2\hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})\right) - 40\dot{r}\left(v^{ij}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) + 2\hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\mathbf{v})\right) \\
&+20v^{ij}(\hat{\mathbf{N}}\cdot\mathbf{v})\Bigg]\Bigg\}_{\tau-s} \\
&\times\left\{G(1-\zeta)\frac{\delta m}{m}\ln\left[\left(\frac{s}{2R+s}\right) + \frac{97}{60}\right] - \frac{1}{20}G\alpha\zeta\left(\mathcal{S}_+ + \frac{\delta m}{m}\mathcal{S}_-\right)\left(\frac{\delta m}{m}\mathcal{S}_+ - (1-2\eta)\mathcal{S}_-\right)\right\} ds \\
&+ 8G(1-\zeta)\delta m\int_0^\infty\Bigg\{\frac{G\alpha m}{r^3}\Bigg[\left(v^2 - \frac{2}{3}\frac{G\alpha m}{r} - 5\dot{r}^2\right)\left(\hat{n}^{ij}(\hat{\mathbf{N}}\cdot\mathbf{v}) - \hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})\right) \\
&-2\dot{r}\left(v^{ij}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) - \hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\mathbf{v})\right)\Bigg]\Bigg\}_{\tau-s}\left[\ln\left(\frac{s}{2R+s}\right) + \frac{7}{6}\right] ds \\
&+\frac{1}{15}G\alpha\mu\zeta\mathcal{S}_-\left(\mathcal{S}_+ - \frac{\delta m}{m}\mathcal{S}_-\right)\int_0^\infty\Bigg\{\frac{G\alpha m}{r^3}\Bigg[\left(225v^2 + 18\frac{G\alpha m}{r} - 525\dot{r}^2\right)\dot{r}\hat{n}^{ij}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) \\
&+\left(-9v^2 - 6\frac{G\alpha m}{r} + 45\dot{r}^2\right)\hat{n}^{ij}(\hat{\mathbf{N}}\cdot\mathbf{v}) + \left(-162v^2 + 44\frac{G\alpha m}{r} + 810\dot{r}^2\right)\hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) \\
&-144\dot{r}\hat{n}^{(i}v^{j)}(\hat{\mathbf{N}}\cdot\mathbf{v}) - 276\dot{r}v^{ij}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}}) + 56v^{ij}(\hat{\mathbf{N}}\cdot\mathbf{v})\Bigg]\Bigg\}_{\tau-s} ds
\end{aligned}
$$

Parameters in these equations

- Only a limited number of combinations of the fundamental parameters:

Parameter	Definition
Equation of motion parameters	
Newtonian	
α	$1-\zeta+\zeta(1-2s_1)(1-2s_2)$
post-Newtonian	
$\bar{\gamma}$	$-2\alpha^{-1}\zeta(1-2s_1)(1-2s_2)$
$\bar{\beta}_1$	$\alpha^{-2}\zeta(1-2s_2)^2\left(\lambda_1(1-2s_1)+2\zeta s_1'\right)$
$\bar{\beta}_2$	$\alpha^{-2}\zeta(1-2s_1)^2\left(\lambda_1(1-2s_2)+2\zeta s_2'\right)$
2nd post-Newtonian	
$\bar{\delta}_1$	$\alpha^{-2}\zeta(1-\zeta)(1-2s_1)^2$
$\bar{\delta}_2$	$\alpha^{-2}\zeta(1-\zeta)(1-2s_2)^2$
$\bar{\chi}_1$	$\alpha^{-3}\zeta(1-2s_2)^3\left[(\lambda_2-4\lambda_1^2+\zeta\lambda_1)(1-2s_1)-6\zeta\lambda_1 s_1'+2\zeta^2 s_1''\right]$
$\bar{\chi}_2$	$\alpha^{-3}\zeta(1-2s_1)^3\left[(\lambda_2-4\lambda_1^2+\zeta\lambda_1)(1-2s_2)-6\zeta\lambda_1 s_2'+2\zeta^2 s_2''\right]$

$$\mathcal{S}_+ \equiv \alpha^{-1/2}(1-s_1-s_2)$$
$$\mathcal{S}_- \equiv \alpha^{-1/2}(s_2-s_1)$$

Pieces of the calculation II

- Scalar gravitational waves to 1.5PN order (Lang 2015)
  - Evaluate $\varphi \equiv 1 + \Psi$ in radiation zone.
  - Why only 1.5PN order?
  - Key piece: "scalar multipole moments" $\mathcal{I}_s^{k_1 \cdots k_q}(t) \equiv \int_{\mathcal{M}} \tau_s(t, \mathbf{x}') x'^{k_1} \cdots x'^{k_q} d^3x'$
    - Monopole: Generates (-1PN)+ scalar field (0PN+ waves)
    - Dipole: Generates (-0.5PN)+ scalar waves
    - Quadrupole: Generates 0PN+ scalar waves…

$$\Psi = \frac{2G\zeta\mu\alpha^{1/2}}{R}[\Psi_{-0.5} + \Psi_0 + \Psi_{0.5} + \Psi_{1,\mathcal{N}} + \Psi_{1,\mathcal{C}-\mathcal{N}} + \Psi_{1.5,\mathcal{N}} + \Psi_{1.5,\mathcal{C}-\mathcal{N}}]$$

Scalar waves: -0.5PN, 0PN, 0.5PN

$$\Psi_{-0.5} = 2\mathcal{S}_-(\hat{\mathbf{N}}\cdot\mathbf{v}) \qquad \mathcal{S}_- \equiv \alpha^{-1/2}(s_2 - s_1)$$

$$\Psi_0 = \left(\mathcal{S}_+ - \frac{\delta m}{m}\mathcal{S}_-\right)\left[-\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^2 + (\hat{\mathbf{N}}\cdot\mathbf{v})^2 - \frac{1}{2}v^2\right] + \frac{G\alpha m}{r}\left[-2\mathcal{S}_+ + \frac{8}{\bar\gamma}(\mathcal{S}_+\bar\beta_+ + \mathcal{S}_-\bar\beta_-)\right]$$

$$\begin{aligned}\Psi_{0.5} =& \left[-\frac{\delta m}{m}\mathcal{S}_+ + (1-2\eta)\mathcal{S}_-\right]\left[\frac{3}{2}\frac{G\alpha m}{r}\dot r(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^3 - \frac{7}{2}\frac{G\alpha m}{r}(\hat{\mathbf{N}}\cdot\mathbf{v})(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})^2 + (\hat{\mathbf{N}}\cdot\mathbf{v})^3\right] \\ &+ \frac{G\alpha m}{r}\dot r(\hat{\mathbf{N}}\cdot\hat{\mathbf{n}})\left[-\frac{5}{2}\frac{\delta m}{m}\mathcal{S}_+ + \frac{3}{2}\mathcal{S}_- + \frac{4}{\bar\gamma}\frac{\delta m}{m}(\mathcal{S}_+\bar\beta_+ + \mathcal{S}_-\bar\beta_-) - \frac{4}{\bar\gamma}(\mathcal{S}_-\bar\beta_+ + \mathcal{S}_+\bar\beta_-)\right] \\ &+ (\mathbf{N}\cdot\mathbf{v})\left\{\left(\frac{\delta m}{m}\mathcal{S}_+ - \eta\mathcal{S}_-\right)v^2\right. \\ &\left.\quad + \frac{G\alpha m}{r}\left[\frac{1}{2}\frac{\delta m}{m}\mathcal{S}_+ + \left(-\frac{3}{2} + 2\eta\right)\mathcal{S}_- - \frac{4}{\bar\gamma}\frac{\delta m}{m}(\mathcal{S}_+\bar\beta_+ + \mathcal{S}_-\bar\beta_-) + \frac{4}{\bar\gamma}(\mathcal{S}_-\bar\beta_+ + \mathcal{S}_+\bar\beta_-)\right]\right\}\end{aligned}$$

Pieces of the calculation III

- Total energy flux to 1PN order
  - Why only 1PN order???

$$\frac{dE_T}{dt} = \frac{R^2}{32\pi}\phi_0 \oint \dot{\tilde{h}}^{ij}_{\mathrm{TT}} \dot{\tilde{h}}^{ij}_{\mathrm{TT}} d^2\Omega \qquad \frac{dE_S}{dt} = \frac{R^2}{32\pi}\phi_0 (4\omega_0 + 6) \oint \dot{\Psi}^2 d^2\Omega$$

  - Flux at Nth PN order requires (N+1/2)th order scalar waves and (N+1)th order equations of motion

$$\frac{dE}{dt} = \dot{E}_{-1} + \dot{E}_0 + \dot{E}_{0.5,\mathcal{C}} + \dot{E}_{0.5,\mathcal{C}-\mathcal{N}} + \dot{E}_1$$

$$\dot{E}_{-1} = \frac{4}{3}\frac{\mu\eta}{r}\left(\frac{G\alpha m}{r}\right)^3 \zeta \mathcal{S}_-^2$$

Energy flux: 0PN

$$\dot{E}_0 = \frac{8}{15}\frac{\mu\eta}{r}\left(\frac{G\alpha m}{r}\right)^3 \Bigg\{ v^2 \Bigg[12 + 5\bar{\gamma} - 5(3 + \bar{\gamma} + 2\bar{\beta}_+)\zeta\mathcal{S}_-^2 + \frac{10}{\bar{\gamma}}\zeta\mathcal{S}_-(\mathcal{S}_-\bar{\beta}_+ + \mathcal{S}_+\bar{\beta}_-)$$

$$+ 10\frac{\delta m}{m}\zeta\mathcal{S}_-^2\bar{\beta}_- - \frac{10}{\bar{\gamma}}\frac{\delta m}{m}\zeta\mathcal{S}_-(\mathcal{S}_+\bar{\beta}_+ + \mathcal{S}_-\bar{\beta}_-) \Bigg]$$

$$+ \dot{r}^2 \Bigg[-11 - \frac{45}{4}\bar{\gamma} + 40\bar{\beta}_+ + 5(17 + \eta + 6\bar{\gamma} + 8\bar{\beta}_+)\zeta\mathcal{S}_-^2 - \frac{90}{\bar{\gamma}}\zeta\mathcal{S}_-(\mathcal{S}_-\bar{\beta}_+ + \mathcal{S}_+\bar{\beta}_-)$$

$$- 40\frac{\delta m}{m}\zeta\mathcal{S}_-^2\bar{\beta}_- + \frac{30}{\bar{\gamma}}\frac{\delta m}{m}\zeta\mathcal{S}_-(\mathcal{S}_+\bar{\beta}_+ + \mathcal{S}_-\bar{\beta}_-) + \frac{120}{\bar{\gamma}^2}\zeta(\mathcal{S}_+\bar{\beta}_+ + \mathcal{S}_-\bar{\beta}_-)^2 \Bigg] \Bigg\}$$

Special cases

- For two BHs, identical to GR except for rescaling of masses by $(4 + 2\omega_0)/(3 + 2\omega_0)$
  - But this rescaling is unmeasurable!
  - Hawking: Single BHs have no scalar hair.
    - Now seen for binaries (2.5PN motion, 2PN tensor waves, 1.5PN scalar waves)
    - Conjecture: True to all PN orders (with no scalar potential, no matter, ϕ_0 constant). Supported by EMRI studies (Yunes et al.), NR results (Healy et al.).

- For BH-NS (mixed) system, things also simplify.
  - Most deviations depend only upon a single parameter.

Sennett, Marsat, & Buonanno (arXiv:1607.01420)

- Calculated ready-to-use (tensor) waveforms for quasicircular orbits
  - Time and frequency domain

- Two regimes: quadrupole overpowers dipole when $1 \lesssim \left(\frac{24}{5\zeta\mathcal{S}_-^2}\right)(G\alpha M\pi f)^{2/3}$
  - BNS/NSBH: $100\mu\mathrm{Hz}$, NS-IMBH: $5\mu\mathrm{Hz}$
  - Quadrupole-driven: systems studied by GW observatories
  - Dipole-driven: low frequency, large separation (binary pulsars) OR large sensitivities (spontaneously scalarized)
  - Dynamical scalarization: both!

- For $100 - 1.4M_\odot$ system over $f \in (0.065\mathrm{Hz}, 1\mathrm{Hz})$ $\begin{cases} \text{Newtonian GR term: } 7.7\times 10^6 \text{ cycles} \\ \text{Leading-order ST correction: -600 cycles} \\ \text{Next corrections: 2, 3, 0.1 cycles} \end{cases}$

Conclusion

- Large-scale effort to determine analytic waveforms for nonspinning compact binary inspiral in a class of scalar-tensor theories of gravity

- Next steps:
    - Anna Heffernan and Cliff Will: Extend scalar waves and energy flux to 2PN order
        - 3PN equations of motion by Blanchet et al.
    - Scalarization (see talk by Sennett)
    - Parameter estimation studies
    - Test with real data?
    - Other theories: multiple scalars, nonzero potential, beyond ST

Parameter	Definition
Scalar-tensor parameters	
G	$\phi_0^{-1}(4+2\omega_0)/(3+2\omega_0)$
ζ	$1/(4+2\omega_0)$
λ_1	$(d\omega/d\varphi)_0\zeta^2/(1-\zeta)$
λ_2	$(d^2\omega/d\varphi^2)_0\zeta^3/(1-\zeta)$
Sensitivities	
s_A	$[d\ln M_A(\phi)/d\ln\phi]_0$
s'_A	$[d^2\ln M_A(\phi)/d\ln\phi^2]_0$
s''_A	$[d^3\ln M_A(\phi)/d\ln\phi^3]_0$

$$G = \frac{2}{(1+\gamma_{\rm PPN})\phi_0},$$

$$\zeta = \frac{1-\gamma_{\rm PPN}}{2},$$

$$\bar{\lambda}_1 = \frac{2\sqrt{2}(\beta_{\rm PPN}-1)\phi_0}{\sqrt{1+\gamma_{\rm PPN}}},$$

$$\bar{\lambda}_2 = \frac{(\epsilon(\gamma_{\rm PPN}-1)+24(\beta_{\rm PPN}-1)^2)\phi_0^2}{1+\gamma_{\rm PPN}},$$

Constraints from Sennett et al.

Scalar charge:

$$\alpha_A = \frac{1-2s_A}{\sqrt{3+2\omega_0}}$$

TABLE II. Constraints on the weak-field parameters in Eqs. (7)–(10) set by solar-system and binary-pulsar observations. As discussed in the text, we set $\phi_0 = 1$ for simplicity.

Parameter	Constraint	Reference
$\gamma_{\rm PPN}-1$	2.3×10^{-5}	[38]
$\beta_{\rm PPN}-1$	7.8×10^{-5}	[15, 39]
ϵ	7×10^{-2}	[17]
$G-1$	1.2×10^{-5}	
ζ	1.2×10^{-5}	
$\bar{\lambda}_1$	1.6×10^{-4}	
$\bar{\lambda}_2$	8.8×10^{-7}	

Cassini

binary pulsars

Mercury perihelion
+ helioseismology

$$|\alpha_A| \lesssim 6\times10^{-3}$$

(PSR J0348+0432)

$$|\alpha_A| \lesssim 10^{-2}$$

(Global)

$$\alpha_A \sim 1$$

(Scalarization?)