# Supernova core collapse in scalar-tensor theory with massive fields

**Roxana Roșca**

DAMTP, University of Cambridge

Work in progress in collaboration with:
U. Sperhake, D. Gerosa and C. Ott

# Overview

- Introduction
- Formalism
- Constraints
- Static case
- Dynamic case

# Introduction

- General Relativity has passed all tests successfully (even in the strong regime), so why bother?
- Many physical concepts left unexplained by GR:
  - Dark energy
  - Dark matter
  - Quantum Field Theory etc.
- Scalar-tensor theories show interesting behaviour: e.g. spontaneous scalarization (see *Damour and Esposito-Farèse 1993*).
- Adding the potential to these theories allows a larger range for the parameters. (see *F. M. Ramazanoğlu and F. Pretorius 2016*).

# Formalism 1

Action for the scalar tensor theory with a potential in the Einstein frame*:*

$$S = \frac{1}{16\pi G}\int dx^4\sqrt{-\bar{g}}\left[\bar{R} - 2\bar{g}^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - 4W(\varphi)\right] + S_m\left[\psi_m, \frac{\bar{g}_{\mu\nu}}{F(\varphi)}\right]$$

$$g_{\alpha\beta} = \frac{1}{F}\bar{g}_{\alpha\beta}$$

Energy-momentum tensor:

$$\bar{T}^{\alpha\beta} \equiv \frac{2}{\sqrt{-\bar{g}}}\frac{\delta S_m}{\delta\bar{g}_{\alpha\beta}} \equiv \frac{1}{F^3}T^{\alpha\beta}$$

Coupling function:

$$F = e^{-2\alpha_0\varphi - \beta_0\varphi^2}$$

# Formalism 2

Tensor equations in the Jordan frame are:

$$G_{\alpha\beta} = \frac{8\pi}{F}\left(T^{F}_{\alpha\beta} + T^{\phi}_{\alpha\beta} + T_{\alpha\beta}\right),$$

$$T^{F}_{\alpha\beta} = \frac{1}{8\pi}\left(\nabla_\alpha \nabla_\beta F - g_{\alpha\beta}\nabla^\mu \nabla_\mu F\right),$$

$$T^{\phi}_{\alpha\beta} = \partial_\alpha \phi \partial_\beta \phi - g_{\alpha\beta}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi + V(\phi)\right],$$

$$\nabla^\mu \nabla_\mu \phi = -\frac{1}{16\pi} F_{,\phi} R + V_{,\phi}.$$

Where the potential is represented by:

$$V(\phi) = \frac{F^2}{4\pi} W(\varphi)$$

and

$$\frac{\partial \varphi}{\partial \phi} = \sqrt{\frac{3}{4}\frac{{F_{,\phi}}^2}{F^2} + \frac{4\pi}{F}}$$

**Radial gauge, polar sclicing in the Einstein frame:**

$$\boldsymbol{ds^2 = -\bar{N}^2 dt^2 + \bar{A}^2 dr^2 + r^2\, d\Omega^2}$$
$$\boldsymbol{= -\alpha^2 dt^2 + X^2 dr^2 + \frac{r^2}{F} d\Omega^2}$$

# Constraints

Massless case:

$$|\alpha_0| \lesssim 10^{-3} \text{ and } \beta_0 \gtrsim -4.5$$

Massive case:

$$3 \lesssim -\beta_0 \lesssim 10^3$$

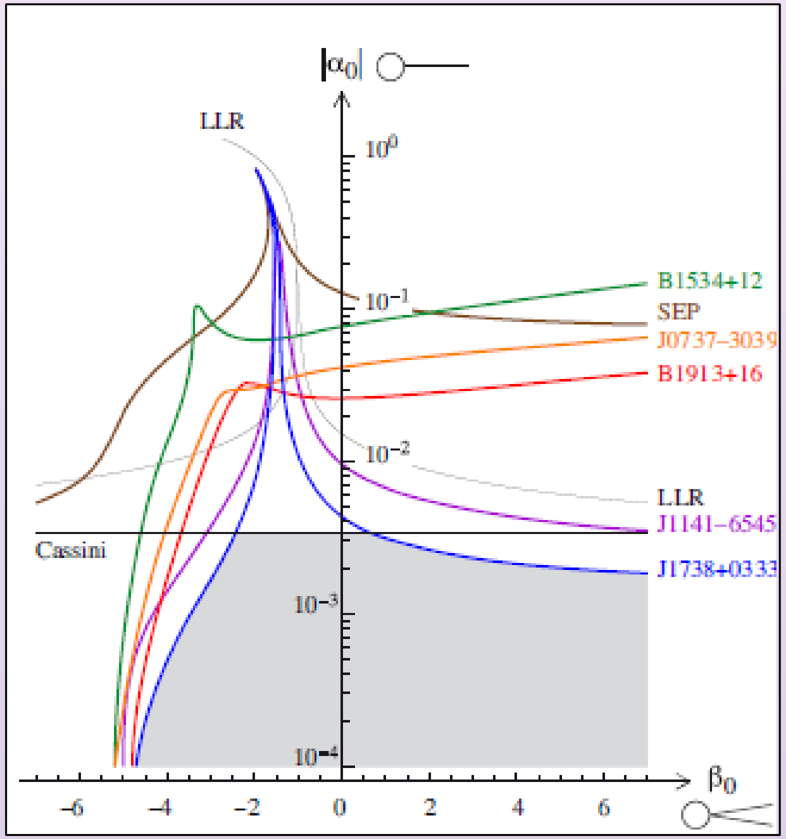
For a potential

$$W = \frac{m_\varphi^2 \varphi^2}{2}$$

In order to get spontaneous scalarization, the mass parameter should take the following values values:

$$10^{-16} eV \lesssim m_\varphi \lesssim 10^{-9} eV$$

(see *F. M. Ramazanoğlu and F. Pretorius 2016*)




Solar system and binary pulsar 1-σ constraints for $\alpha_0$ and $\beta_0$
(*from P. C. C. Freire et al. 2012*).

# Static case

$$\partial_r X = \frac{4\pi r X^3}{F}(\rho h - P) + \frac{r X^3 \eta^2}{2} - \frac{X^3 F}{2r} + \frac{X}{2r} - \frac{F_{,\varphi} X^2 \eta}{2F} + X^3 F W r,$$

$$\partial_r \eta = -\frac{3\eta}{2r} - \frac{2\pi X F_{,\varphi}}{F^2}(\rho h - 4P) - \frac{X^2 \eta F}{2r} - \frac{4\pi r X^2 \eta \pi P}{F} - \frac{X^2 \eta^3 r}{2} + \frac{F_{,\varphi} X \eta^2}{2F} + X^2 \eta F W r + X F W F_{,\varphi}$$

$$\frac{\partial_r \alpha}{\alpha} = \frac{F X^2 - 1}{2r} + \frac{4\pi r X^2}{F} + \frac{r X^2 \eta^2}{2} - W r X^2 F,$$

$$\partial_r \varphi = X\eta,$$

$$\partial_r P = -\rho h F X^2 \left( \frac{1}{2r}\left(1 - \frac{1}{F X^2}\right) + 4\pi r \frac{P}{F^2} + \frac{r}{2F}\eta^2 - rW \right) + \rho h \frac{F_{,\varphi}}{2F} X \eta.$$

A relaxation code is used to solve these .

# Dynamic case

**Metric equations:**

$$\frac{\partial_r \alpha}{\alpha} = FX^2\left[\frac{r}{2}\left(1-\frac{1}{FX^2}\right)+4\pi r\left(\bar{S}^r v+\frac{P}{F^2}\right)+\frac{r}{2F}(\eta^2+\psi^2)\right]-\frac{F_{,\varphi}}{2F}\eta X - WrX^2F$$

$$\frac{\partial_r X}{X} = 4\pi r F X^2(\bar{\tau}+\bar{D})+\frac{rX^2}{2}(\eta^2+\psi^2)-\frac{1}{2r}(FX^2-1)-\frac{F_{,\varphi}}{2F}\eta X + WrX^2F$$

$$\frac{\partial_t X}{X} = rX\alpha(\eta\psi - 4\pi F\bar{S}^r)+\frac{F_{,\varphi}}{F}$$

**Wave equation:**

$$\partial_t\partial_t\varphi = \frac{\alpha^2}{X^2}\left[\partial_r\partial_r\varphi+\frac{2}{r}\partial_r\varphi+\left(\frac{\partial_r\alpha}{\alpha}-\frac{\partial_r X}{X}\right)\partial_r\varphi\right]$$
$$+\left(\frac{\partial_t\alpha}{\alpha}-\frac{\partial_t X}{X}\right)\partial_t\varphi+2\pi\alpha^2\left(\bar{\tau}-\bar{S}^r v+\bar{D}-3\frac{P}{F^2}\right)F_{,\varphi}$$
$$-\alpha^2 \mathrm{F} W_{,\varphi}$$

**Matter equations:**

$$\partial_t\bar{D}+\frac{1}{\sqrt{F}r^2}\partial_r\left(r^2\frac{\alpha}{X}\sqrt{F}f_{\bar{D}}\right)=s_{\bar{D}}$$

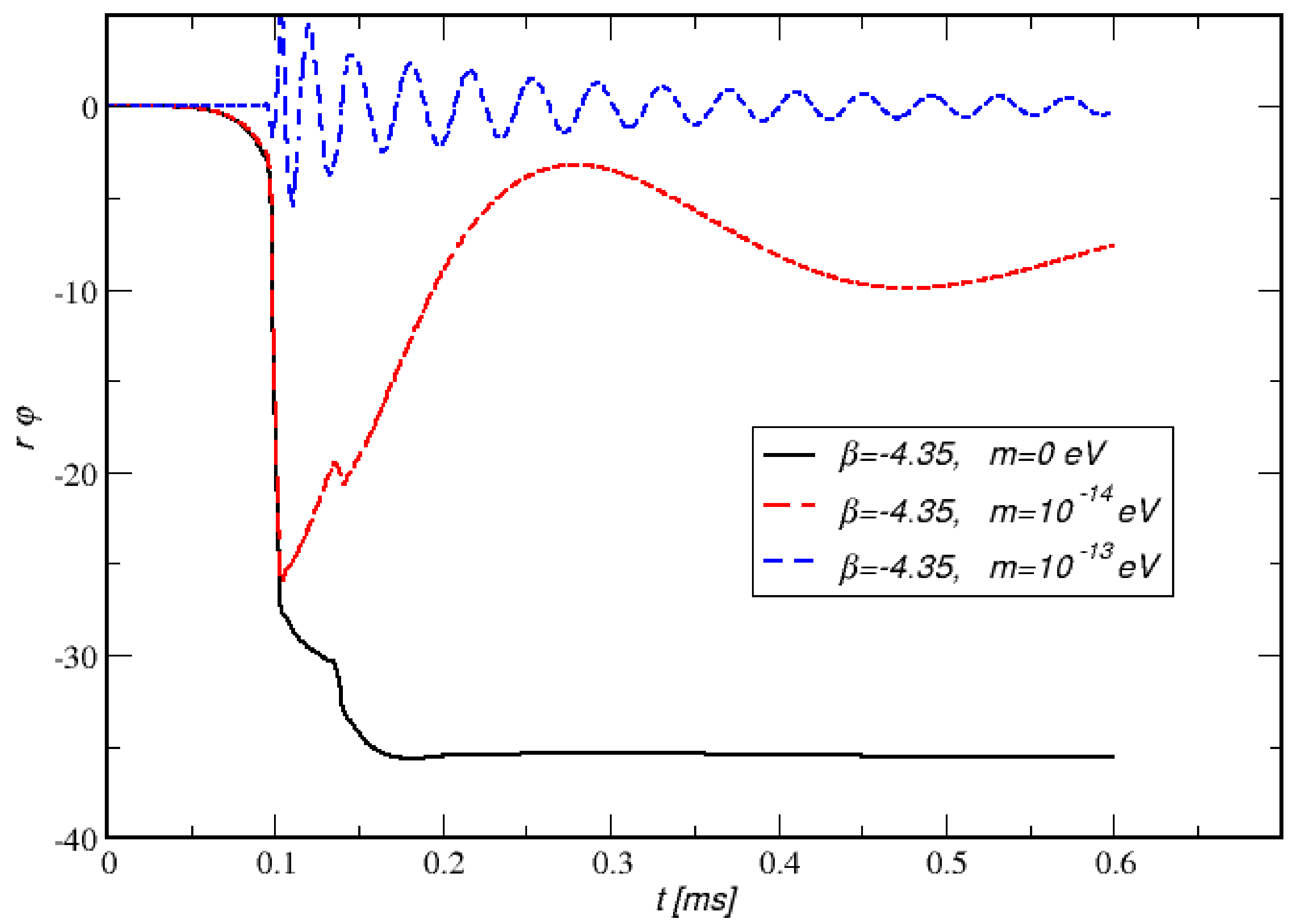
$$\partial_t\bar{S}^r+\frac{1}{r^2}\partial_r\left(r^2\frac{\alpha}{X}f_{\bar{S}^r}\right)=s_{\bar{S}^r}$$

$$\partial_t\bar{\tau}+\frac{1}{r^2}\partial_r\left(r^2\frac{\alpha}{X}f_{\bar{\tau}}\right)=s_{\bar{\tau}}$$

$$\partial_r\varphi = X\eta$$
$$\partial_t\varphi = \alpha\psi$$

$$\boldsymbol{s_{\bar{D}} \rightarrow s_{\bar{D}}}$$
$$\boldsymbol{s_{\bar{S}^r} \rightarrow s_{\bar{S}^r} - W\left[r\alpha XF(\bar{S}^r v - \bar{\tau} - \bar{D}) + r\alpha X\frac{P}{F}\right]}$$
$$\boldsymbol{s_{\bar{\tau}} \rightarrow s_{\bar{\tau}}}$$

For full evolution equations: please see *D. Gerosa et al. 2016* (arXiv:1602.06952).



0
-10
-20
-30
-40
r φ
0
0.1
0.2
0.3
0.4
0.5
0.6
t [ms]
β=-4.35, m=0 eV
β=-4.35, m=10^-14 eV
β=-4.35, m=10^-13 eV

# Thank you for your attention!!!