Conformal loop quantum gravity: first steps

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Idea:

The Brans-Dicke theory with ω=-3/2 is conformally invariant and in vacuum reproduces ordinary general relativity when one chooses a certain family of gauges. Solutions to the field equations of the theory can be mapped via a conformal transformation to solutions of general relativity.

We argue that the conformal theory is the fundamental theory that one should quantize and that it is amenable to a loop quantum gravity quantization.

There exist several potential advantages in having a conformally invariant theory at the time of quantization.

Why conformal invariance?

-Our investigations on quantum field theory on quantum space-times in spherical symmetry shows that, the discreteness of the quantum space-time naturally regulates the quantum field theory.
Quantities that in quantum field theory in curved space-time become infinite, on a quantum space-time become finite, but large, and depend on the details of the Planck-scale microphysics, especially the separation of the vertices.
-Dependence on the micro scale physics requires a (finite) renormalization to make the macro scale physics of the quantum field independent of the micro scale degrees of freedom.The problem is that the renormalization depends on the details of the background quantum state, particularly the spacing.

One should need to add state dependent counterterms to the action. This is not what is usually encountered in renormalization and appears quite artificial.

In a conformally invariant theory, state dependent counterterms are not required.

Continuum limit

It has been observed that the existence of a Planck scale imposes restrictions at the process of going to the continuum limit in that if one adds additional points to the spin network the continuum approximation of volumes and areas does not Improve. There is no spatial structure at physical scales smaller than Planck scale

This makes it difficult to take the various limits involved in the definition of the Hamiltonian constraint in a non-trivial way.

There have been some extensions of the kinematical setup proposed to deal with this, in particular with the issue of the continuum limit, but none is still widely accepted (see for instance (Dittrich et al., Zapata, Thiemann et al., Freidel, and many others)

In a conformal theory of gravity conformal spin networks can be be indefinitely refined.

The Theory:

The theory we wish to consider is the conformally invariant ω=-3/2 case of the Brans-Dicke theory. The action in metric variables is given by:

$$S = \int d^4x\sqrt{-g}\left[\frac{\phi^2 R}{2} + 3g^{ab}\partial_a\phi\partial_b\phi\right] = \int d^4x\sqrt{-g}3\left[\frac{\phi^2 R}{6} + g^{ab}\partial_a\phi\partial_b\phi\right]$$

In the latter form it is clear that the dilaton is conformally invariant.
Recall that under conformal transformations:

$$g_{ab} \to \overline{g_{ab}} = \exp(\Theta(x))g_{ab},$$

$$\phi \to \bar{\phi} = \exp(\frac{-\Theta(x)}{2})\phi.$$

In the gauge $\phi = \kappa^{-1/2}$ with $\kappa = 8\pi G$ the theory is identical to Einstein gravity.

One can introduce a conformal invariant “metric” given by $g^{(c)}_{ab} := \phi^2 g_{ab}$

The Brans-Dicke action takes the manifestly invariant form:

$$S = \frac{1}{2}\int d^4x\sqrt{-g^{(c)}}R^{(c)}$$

Notice that the conformal metric has different dimensionality than the usual metric: “intervals” are dimensionless.
For the loop quantization we consider the Holst version of the action

$$S^{(c)}_H = \int \frac{1}{2} e^{(c)} e^{a(c)}_I e^{b(c)}_J \left(\Omega^{IJ(c)}_{ab} + \frac{1}{\gamma} {}^*\Omega^{IJ(c)}_{ab}\right),$$

With γ the Immirzi parameter and $\Omega^{IJ(c)}_{ab}$ the curvature of the SL(2,C) connection $\omega^{IJ(c)}_a$.

One can write this action in terms of the geometrical triad and connection by substituting in the action:

$$
\begin{aligned}
e_a^{I(c)} &= \phi e_a^I, \\
e_I^{a(c)} &= \frac{e_I^a}{\phi}, \\
\omega_a^{IJ(c)} &= \omega_a^{IJ} - 2\phi^{-1}\partial_b \phi e^{b[I} e_a^{J]}
\end{aligned}
$$

And introducing the standard Ashtekar Barbero canonical variables we get

$$\{K_a^i(x), E_j^b\} = \kappa\, \delta_a^b \delta_j^i \delta^3(x,y), \quad \{\phi(x), \pi(y)\} = \delta^3(x,y).$$

Besides the Hamiltonian, diffeomorphisms and Gauss constraints the theory has an additional conformal constraint

$$H = \frac{\phi^2}{2}\epsilon_i^{lm} E_l^a E_m^b \left[F_{ab}^i - \left(\gamma^2 + (\kappa\phi^2)^{-2}\right) K_a^j K_b^k \epsilon_{jk}^i\right] - E^{ai} E_i^b \partial_a \phi \partial_b \phi + 2E^{ai} E_i^b \left(\nabla_a \partial_b \phi\right) \phi$$

$$\frac{2}{\kappa} K_a^i E_i^a + \pi\phi =: \mathcal{S} \qquad \mathcal{C}_a = \frac{1}{\gamma} F_{ab}^i E_i^b + \pi \partial_a \phi,$$

$$\mathcal{G}^i = \partial_a E_i^a + \epsilon_{ijk} A_a^j E^{ak}$$ with $$A_a^i = \gamma K_a^i + \Gamma_a^i,$$

One can introduce canonical variables that are conformally invariant:

$$E_i^{a(c)} = \phi^2 E_i^a \quad K_a^{i(c)} = \frac{1}{8\pi G\phi^2} K_a^i \quad A_a^{i(c)} = \Gamma_a^{i(c)} + \gamma K_a^{i(c)}$$

$$\left\{A_a^{(c)i}(x), E_j^{b(c)}(y)\right\} = \delta_a^b \delta_j^i \delta^3(x,y)$$

In terms of the conformally invariant connection one can introduce conformally invariant parallel propagators
along a path η,

$$U\left(A^{(c)}, \eta\right) = P \exp \int A_a^{(c)} dy^a$$

And construct in the usual fashion spin network states

$$\psi_S(A^{(c)}) = \otimes_l R^{(j_l)}(U(A^{(c)}, \eta_l)) \otimes_n i_n$$

One can solve the diffeomorphism constraint via group averaging and obtain conformal spin knots, $|S^{(c)}>$, orthogonality and other properties are similar to the usual case and one can introduce conformal operators that characterized the conformal properties of the space and are similar to the standard area and volume operators.

Coupling the Standard Model

A common worry about conformal theories of gravity is that matter is not conformal in nature, so coupling realistic matter to conformal gravity theories is problematic. We would like to argue that this can be overcome via a variation of the Higgs mechanism.

One starts by coupling the massless Standard Model to conformal gravity by using conformal invariant fields,

$$\mathcal{L}_T = \mathcal{L}_{\mathrm{GR}}\left(g^{(c)}\right) + \mathcal{L}_M\left(g^{(c)}, \frac{\Psi^M}{\phi^d}\right),$$

Where Ψ^M are the matter fields and d is a suitable power to make the matter action conformally invariant. In the matter fields we include a Higgs field. Let us concentrate on its portion of the action.

Recall that the Higgs field Lagrangian has the following form

$$\mathcal{L}_H = \left[-\frac{1}{2}\left(\partial_a H^{\dagger\alpha} + g A_a^i \tau_\beta^{i\alpha} H^{\dagger\beta}\right)\left(\partial^a H^\alpha + g A^{ak} \tau_{k\gamma}^{\alpha} H^\gamma\right) - \frac{\lambda}{4} V(\mu, H)\right]\sqrt{-g},$$

And the potential is a “Mexican hat” with a negative mass $-\mu^2$

$$V(\mu, H) = \frac{\lambda}{4}\left(H^{\dagger\alpha}H^{\alpha}\right)^2 - \mu^2 H^{\dagger\alpha}H^{\alpha} + \text{const.}$$

Bars, Steinhardt and Turok arXiv:1307.8106 (2013); arXiv:1307.1848 (2014) have proposed a conformal extension of the previous action,

$$V(H,\phi) = \left[-\frac{\lambda}{4}\left(H^{\dagger\alpha}H^{\alpha} - \alpha^2\phi^2\right)^2 + \frac{\lambda'}{4}\phi^4\right]\frac{\sqrt{-g^{(c)}}}{\phi^4}$$

λ, λ', α, are dimensionless coupling constants. This is not conformally invariant when coupled to general relativity so a term $H^{\dagger\alpha}H^{\alpha}R/6$ needs to be added to the action in that case. As we shall see such a term is not necesarilly required.

The matter action therefore would read,

$$S^{(c)}_{\text{Matter}} = \int d^4x\sqrt{-g}\left[g^{ab}D_aH^\dagger D_bH - \frac{1}{6}H^\dagger HR + V(H,\phi) + \mathcal{L}_{SM}\right]$$

Where D_a is covariant derivative with the connection associated with the weak interactions. $\mathcal{L}_{SM}$ is the massless Lagrangian of the Standard Model, which is conformally invariant. Inspired in the form of the loop invariants under all the kinematikal constraints we introduce conformal invariant variables

$$\Psi^{(c)} = \Psi/\phi^{3/2} \;\; H^{\alpha(c)} = H^\alpha/\phi \qquad g^{(c)}_{ab} = \phi^2 g_{ab}$$

And the resulting Matter Lagrangian takes a simpler form

$$S_{\text{Higgs}} = \int d^4x\sqrt{-g}g^{ab(c)}D_aH^{(c)\dagger}D_bH^{(c)} - \frac{\lambda}{4}\left(H^{(c)\dagger}H^{(c)} - \alpha^2\right)^2 + \frac{\lambda'}{4} + \mathcal{L}_{SM}\left(g^{(c)},\Psi^{(c)},A_a\right).$$

With the Standard Model portion of the action as we discussed before.

The kinematics of loop invariants may be easily extended to the matter case by including open spin networks.

If one considers a gauge fixing $\phi(x)=\phi_0$=constant, one can write the dimensionful parameters in terms of ϕ_0,

$$
\begin{aligned}
G &= \frac{1}{8\pi\phi_0^2}, \\
\frac{\Lambda}{16\pi G} &= \frac{\lambda'\phi_0^4}{4},
\end{aligned}
$$

If one now carries out the Higgs mechanism for the resulting theory one gets the mass and expectation value of the Higgs,

$$
\begin{aligned}
\langle H^{(c)\dagger} H^{(c)} \rangle &= \alpha^2 \phi_0^2, \\
m^2_{\mathrm{Higgs}} &= 2g^2\alpha^2\phi_0^2,
\end{aligned}
$$

With g the Yang-Mills coupling constant for the weak interactions. In turn this endows with mass all the particles in the Standard Model in the usual fashion.

All masses and expectation values get fixed in terms of the Planck scale and dimensionless parameters, no matter what gauge fixing chosen, implying that the physics is gauge invariant. Ratios of fundamental masses are independent of the gauge (units) chosen.

Advantages:

-It is possible to consider conformal invariant extensions of the Standard Model coupled to gravity.

-In quantum field theory in quantum space times calculations, the resulting (finite) renormalized results do not depend on the chosen quantum state, in particular on the spacing of the vertices of the spin network. The dependence on the Planck scale structure of space time may be absorbed with a redefinition of the dilaton field, i.e with a different gauge fixing.

-New perspectives are opened on the issue of taking continuum limits in LQG.

-At the quantum level all the physical constants become running coupling constants. Choosing a fixed Planck scale G, relations of particle masses to the Planck mass could change at the quantum level due to running of the dimensionless coupling constants. At low energies we live in a world where particle masses have a fixed relation to the Planck mass, at high energies and high curvatures quantum gravity corrections will change them. In that context the best way to describe things are conformally invariant.

t hooft has suggested that in a conformal invariant field theory one should demand the β functions all vanish and all coupling constants are fixed by this condition.

The terms quadratic in the curvatures that appear in renormalization do not involve G_B so they cannot be absorbed by changing the conformal gauge. The corrections that appear in QFT in CST are all finite. Logarithmically divergent terms in the continuum now just provide small corrections that can be viewed as stemming from quantum gravity effects and are unambiguously determined. As a consequence *there is not trace anomaly for the stress energy tensor.*

Summary

- A conformal extension of general relativity can be quantized via loop quantization techniques.
- It is equivalent to general relativity by fixing a family of gauges.
- It can be coupled to the usual Standard Model through a modified Higgs mechanism in which the gauge fixing endows the Higgs boson with mass.
- It opens new possibilities in QFT in CST calculations and in continuum limits of LQG.