Fully relativistic Zeldovich approximation & multiple structures with Szekeres models.

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This talk is based on results obtained in the following two articles:

1. arXiv:1508.03127 [pdf, other]
Multiple non-spherical structures from the extrema of Szekeres scalars
Roberto A. Sussman, Ismael Delgado Gaspar
Comments: 27 pages, 9 figures. Typos corrected and references added
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO);

Phys. Rev. D **92**, 083533 (2015)

2. arXiv:1507.02306 [pdf, other]
Coarse-grained description of cosmic structure from Szekeres models
Roberto A. Sussman, I. Delgado Gaspar, Juan Carlos Hidalgo
Comments: V3: Discussion of the expansion eigenvalues and of the Zeldovich approximation added. Figures modified accordingly. References updated. Version accepted for publication in JCAP
Subjects: **General Relativity and Quantum Cosmology (gr-qc)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)

JCAP 03 (2016) 012

Spherical LTB models allow for the description of the collapse of a single structure, but cosmic structure is NOT spherically symmetric

Can we hope to provide a "decent" (at least coarse grained) description of multiple collapsing structures with an exact solution of Einstein's equations?

YES, with Szekeres models !!

Szekeres metric in spherical coordinates.

$$ds^2 = -dt^2 + a^2 \left\{ \left[\frac{(\Gamma - W)^2}{1 - K_0 r^2} + W_1 \right] dr^2 + \frac{2W_2}{1+\cos^2\theta}\, dr\, d\theta \right.$$
$$\left. + \frac{2W_3}{1+\cos^2\theta}\, dr\, d\phi + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\}$$

$$\boxed{a = a(t,r), \quad \Gamma = \Gamma(t,r), \quad K_0 = K_0(r), \quad W,\, W_1,\, W_2,\, W_3 \quad \text{depend on } (r,\theta,\phi)}$$

Szekeres Dipole

$$\boxed{\mathbf{W} = -\left[X(r)\, \sin\theta\cos\phi + Y(r)\, \sin\theta\sin\phi + Z(r)\, \cos\theta \right]}$$

Dipole free parameters

$$[X(r),\, Y(r),\, Z(r)]$$

Control Dipole Orientation & Magnitude

Axial Symmetry

$$X = Y = 0, \quad Z \neq 0$$

Dipole along Z direction

Spherical Symmetry

$$X = Y = Z = 0$$

Zero dipole (only monopole)

What do we mean by a "structure" ?

Over-density = density maximum

Density void = density minimum

Transition = density saddle

Necessary & sufficient conditions for the 3-d spatial extrema of the density

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial \rho}{\partial \phi} = \frac{\partial \rho}{\partial r} = 0 \quad \text{at an arbitrary fixed } t$$

Nature & classification (**maxima, minima, saddle points**) depends on the signs of the Hessian matrix (2nd derivatives)

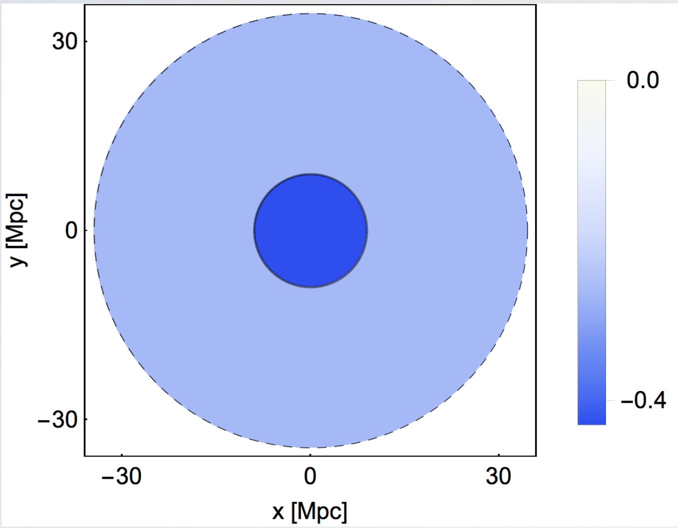
IMPORTANT RESULT

The extrema of the density (and all other covariant scalars) are located along the curves given by the direction of the Szekeres dipole

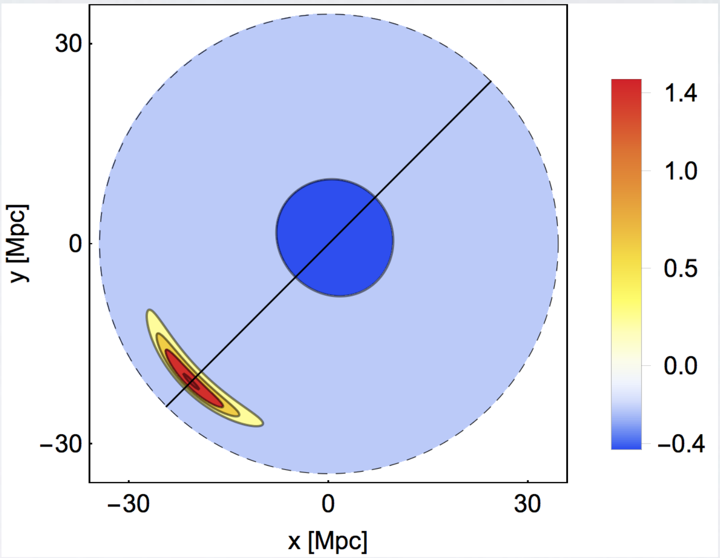
The over-densities & voids can be placed in arbitrary locations

Szekeres models allow for the description of multiple structures (density multipoles) by superposing a monopole with many dipoles

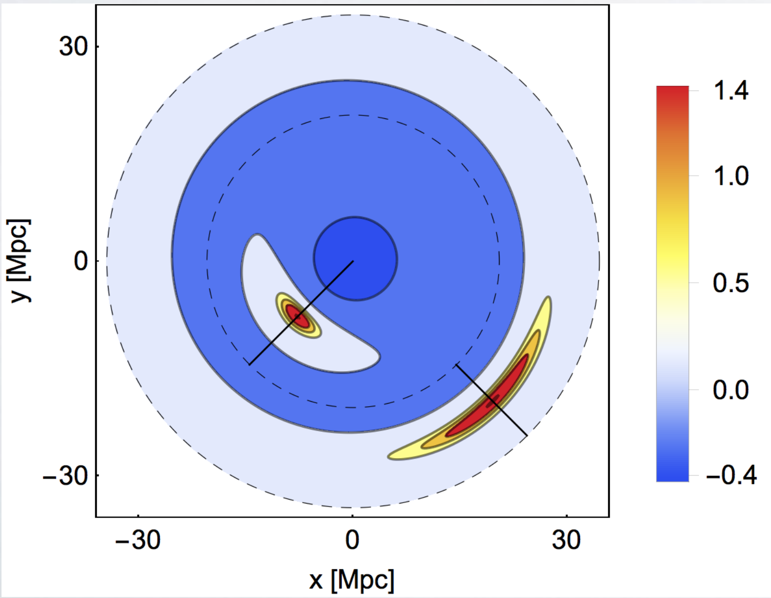
Spherical Symmetry (zero dipole) = one structure



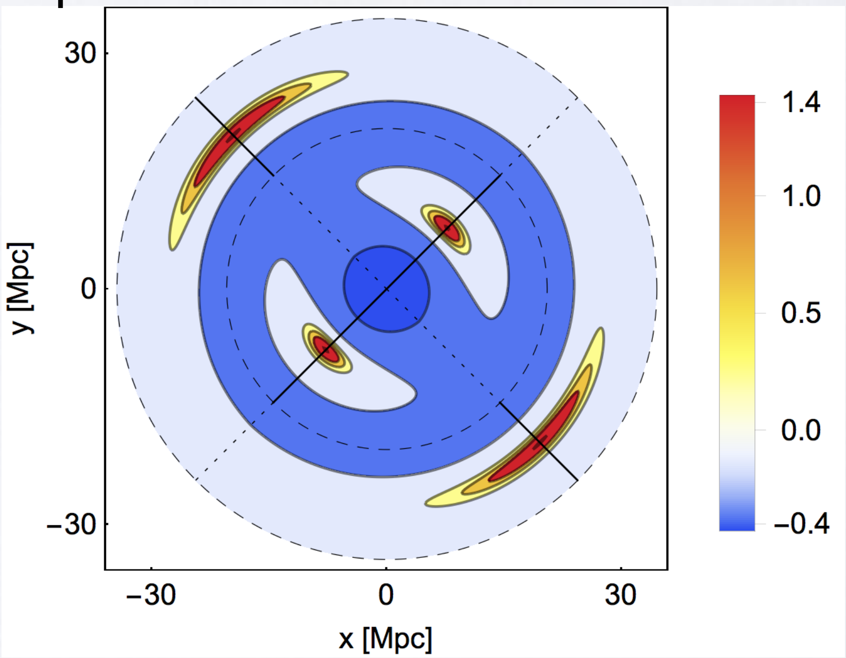
2 structures = superposition of a monopole and a single dipole (Axial-like Symmetry)



3 structures = superposition of a monopole PLUS 2 dipoles in different radial shells



5 structures = superposition of a monopole PLUS 4 radial & angular dipoles



The Newtonian Zeldovich approximation

Zeldovich proposed a first order correction in the relation between Eulerian x^i and Lagrangian y^i coordinates

$$y^i = \bar{a}(t)\left[x^i + \Psi^i(t, x^j)\right] \qquad \text{where} \qquad \Psi^i(t_0, x^j) = 0$$

The density takes the approximate from (valid up to the mild non-linear regime)

$$\rho = \frac{\rho_0}{\det(y^i_{,j})} = \frac{\rho_0}{\bar{a}^3\left[1-\xi^{(1)}\right]\left[1-\xi^{(2)}\right]\left[1-\xi^{(3)}\right]}$$

where $-\xi^{(A)}$ $(A = 1, 2, 3)$ are the eigenvalues of the "deformation" tensor $\xi^i_j = \Psi^i_{,j}$

and collapse occurs as $\det(y^i_{,j}) \to 0$

Assuming that $0 \leq \xi^{(3)} \leq \xi^{(2)} \leq \xi^{(1)}$, leads to the following collapse morphologies

Pancake collapse: $\xi^{(1)} \to 1 \quad \text{while} \quad \xi^{(2)} < \xi^{(3)} < 1,$ — One direction collapses, two directions expand

Filamentary collapse: $\xi^{(1)}, \xi^{(2)} \to 1 \quad \text{while} \quad \xi^{(3)} < 1,$ — Two direction collapse, one directions expands

Spherical or isotropic collapse: $\xi^{(1)}, \xi^{(2)}, \xi^{(3)} \to 1,$ — Three directions collapse

The Szekeres collapse: an exact relativistic analogue of the Zeldovich approximation

Time dependence of Szekeres models is given by the two functions $a(t,r),\ \Gamma(t,r)$

$$ds^2 = -dt^2 + a^2 \left\{ \left[\frac{(\Gamma - W)^2}{1 - K_0 r^2} + W_1 \right] dr^2 + \frac{2W_2}{1+\cos^2\theta}\, dr\, d\theta + \frac{2W_3}{1+\cos^2\theta}\, dr\, d\phi + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\}$$

Szekeres collapse is governed by the expansion tensor whose trace is the Hubble scalar

$$H^a_b = \nabla_b u^a = H\, h^a_b + \sigma^a_b, \qquad 3H = \nabla_c u^c = \mathrm{tr}\, H^a_b,$$

and its 3 eigenvalues are $$\mathrm{H}^{(1)} = \frac{\dot a}{a} + \frac{\dot\Gamma}{\Gamma - \mathrm{W}}, \qquad \mathrm{H}^{(2)} = \mathrm{H}^{(3)} = \frac{\dot a}{a}$$

Scale factors $\ell^{(A)}$ **such that**
$\dot\ell^{(A)}/\ell^{(A)} = \mathbf{H}^{(A)}$ **&** $\ell^{(A)}_0 = 1$

$$\ell^{(1)} = \frac{a(\Gamma - W)}{1 - W}, \qquad \ell^{(2)} = \ell^{(3)} = a,$$

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the Szekeres density takes the form

$$\rho = \frac{\rho_0\, \mathcal{J}_0}{\mathcal{J}} = \frac{\rho_0}{\ell^{(1)}\, \ell^{(2)}\, \ell^{(3)}}, \qquad \mathcal{J} = \sqrt{\det(g_{ij})}$$

The Szekeres density takes the same form as in the Zeldovich approximation

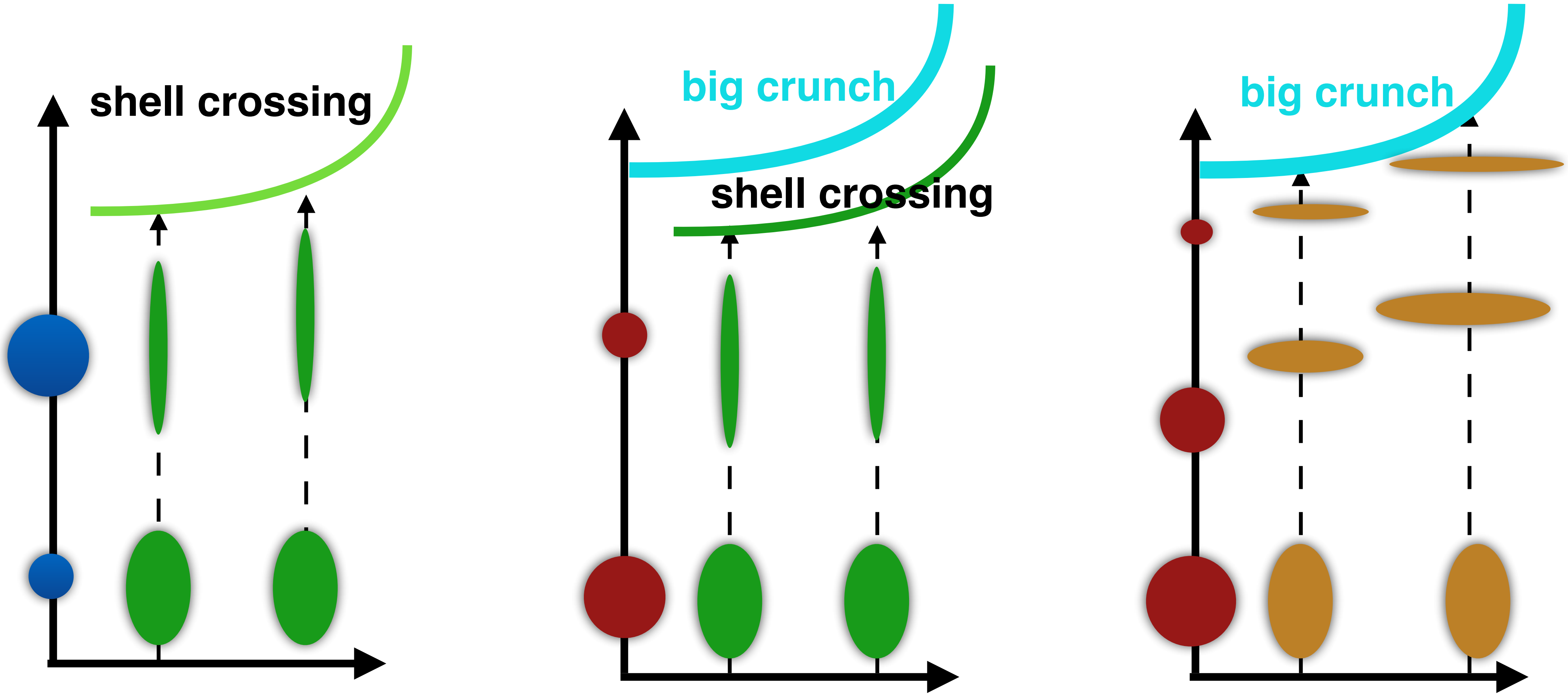
$$\rho = \frac{\rho_0\, \mathcal{J}_0}{\mathcal{J}} = \frac{\rho_0}{\ell^{(1)}\, \ell^{(2)}\, \ell^{(3)}}, \qquad \mathcal{J} = \sqrt{\det(g_{ij})}$$

However, it is an EXACT and FULLY RELATIVISTIC form !!

The following collapse morphologies are possible:

Spheroidal collapse	$\mathrm{H}^{(1)},\, \mathrm{H}^{(2)} = \mathrm{H}^{(3)} \to -\infty,$ $\ell^{(1)},\, \ell^{(2)} = \ell^{(3)} \to 0,$	**Contraction in three directions**
Pancake collapse in expanding background	$\mathrm{H}^{(1)} \to -\infty, \quad \mathrm{H}^{(2)} = \mathrm{H}^{(3)} > 0$ $\ell^{(2)} = \ell^{(3)} > 0, \;\; \ell^{(1)} \to 0$	**Contraction in one direction expansion in two directions**
Pancake collapse in collapsing background	$\mathrm{H}^{(1)} \to -\infty,$ $\mathrm{H}^{(2)} = \mathrm{H}^{(3)} < 0 \quad \text{finite},$ $\ell^{(2)} = \ell^{(3)} > 0, \;\; \ell^{(1)} \to 0$	**Fast contraction in one direction slow contraction in two directions**
Filamentary collapse in a collapsing background	$\mathrm{H}^{(2)} = \mathrm{H}^{(3)} \to -\infty,$ $\mathrm{H}^{(1)} < 0 \quad \text{finite},$ $\ell^{(2)} = \ell^{(3)} \to 0, \quad \ell^{(1)} > 0,$	**Fast contraction in two direction slow contraction in one directions**

Multi-structure collapse scenarios



Spheroidal expansion
CENTRAL VOID

PANCAKE STRUCTURES in an expanding background

Spheroidal collapse
CENTRAL OVERDENSITY

PANCAKE STRUCTURES in a collapsing background

Spheroidal collapse
CENTRAL OVERDENSITY

FILAMENTARY STRUCTURES in a collapsing background

Conclusions & Future Research

- We have show how effective coarse grained modelling of cosmic structure can be achieved with Szekeres models. The maxima and minima of the density and the Hubble scalar can be located in assorted positions and orientations in the spherical coordinate system.

- Explore non-perturbative & relativistic effects in the testing of cosmological observations that is currently undertaken with Newtonian gravity and/or linear perturbations

- Fully relativistic & exact multi-structure collapse model. Useful to test numerical codes and relativistic corrections of Newtonian N-body simulations

That's all folks !