Black Holes @ Large D

Things we've learned so far

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Work in 2013-2016 with:

D Grumiller

K Izumi

R Luna

T Shiromizu

R Suzuki

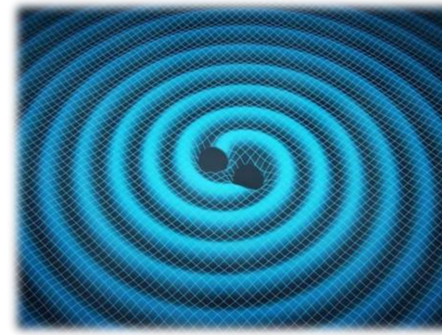
K Tanabe

T Tanaka

Parallel line of work: Bhattacharya, Minwalla et al

The Too Perfect Theory

$$R_{\mu\nu} = 0$$



No scale

No parameter

Fiendish complexity

D-diml General Relativity

Well-defined for all D

Many problems can be formulated keeping D arbitrary

→ D = continuous parameter

→ expand in 1/D

Large-D in General Relativity

$D^2 \sim$ # local degrees of freedom at a point

akin to Large N SU(N) gauge theory

$D \sim$ # connections between nearby points

= directions out of a point

akin to Mean Field Theory limit in Stat Mech

Thing we've learned:

What's useful is

$D^2 \sim$ # local degrees of freedom at a point

akin to Large N SU(N) gauge theory

$D \sim$ # connections between nearby points

= directions out of a point

Exploit large gradients of gravitational potential $\frac{1}{r^{D-3}}$

BH in D dimensions

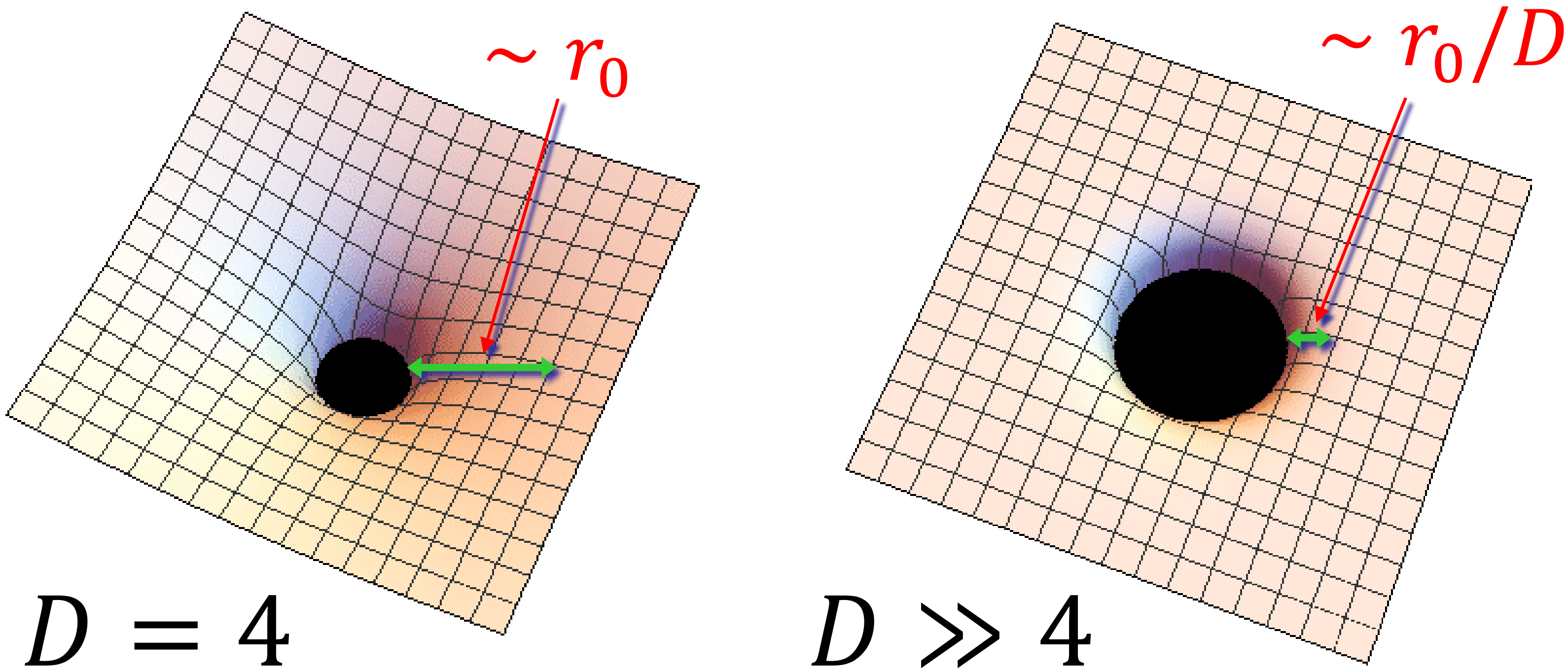
$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3} \qquad \nabla\Phi|_{r_0} \sim \frac{D}{r_0} \gg \frac{1}{r_0}$$

Thing we've learned:

<u>Large D introduces new, parametrically separated scales</u>

$$r_0 \gg \frac{r_0}{D}$$

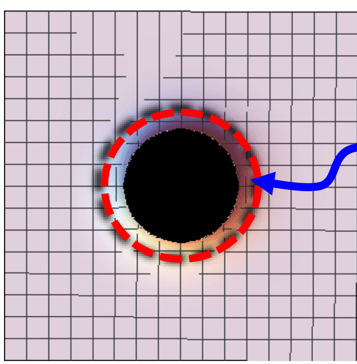


$\sim r_0$
$\sim r_0/D$
$D = 4$
$D \gg 4$

Thing we've learned:

∃ well-defined, universal near-horizon geometry

Take $D \to \infty$ keeping finite $\left(\frac{r}{r_0}\right)^{D-3}$



$r - r_0 \sim \frac{r_0}{D}$

Small fluctuations of black hole horizon

Quasinormal modes

Thing we've learned:

Most QN modes have high frequencies

$$\omega \sim D/r_0$$

featureless oscillations of a hole in space

<u>A few long-lived QN modes localized</u>

<u>in near-horizon region</u>

$$\omega \sim 1/r_0$$

Decoupled from far-zone

They capture interesting horizon dynamics

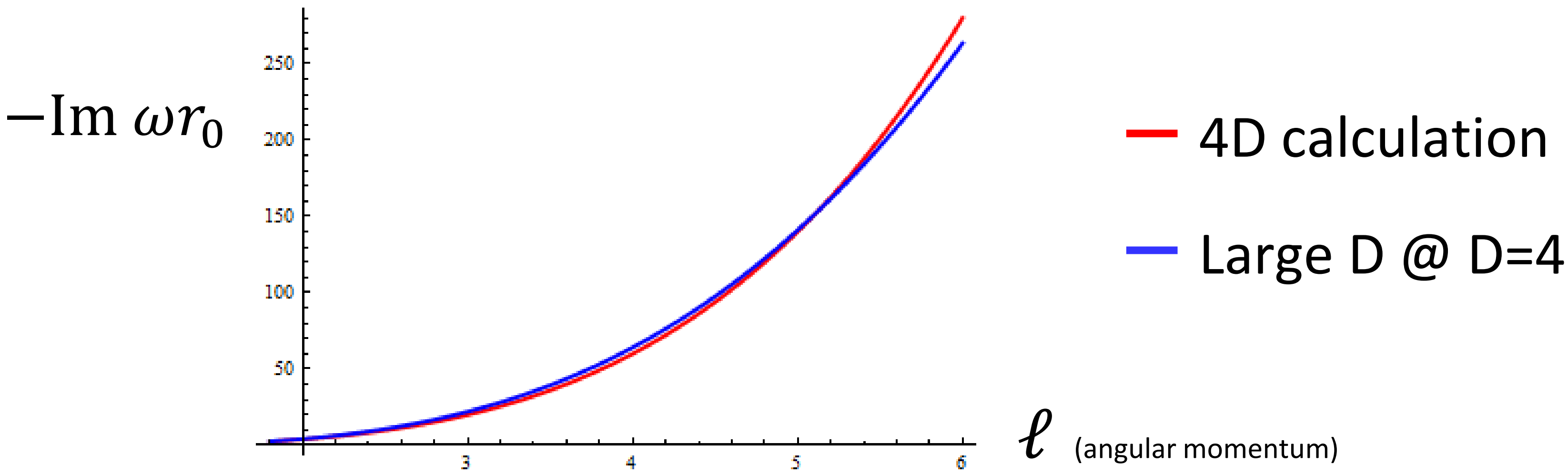
Thing we've learned:

Large D can be a very good approximation

for moderate, even small D

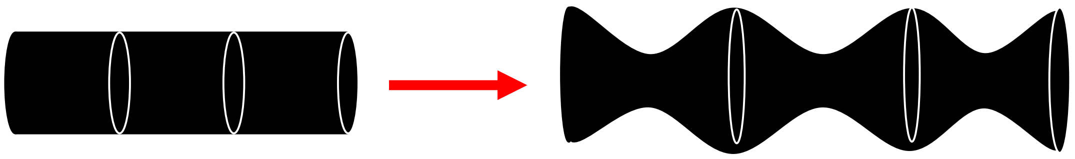
Quasinormal frequency of Schw bh in $D = 4$

(vector-type)



6% accuracy in $D = 4$

Gregory-Laflamme threshold mode of black string in $D = n + 4$

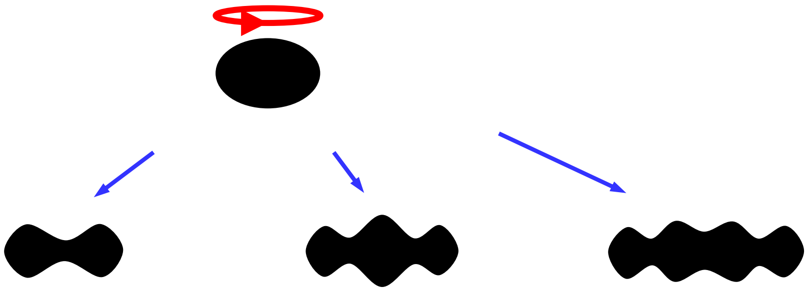


$$k_{GL}|_{\color{red}{\boldsymbol{n=2}}} = 1.238 \quad \text{large-D analytical}$$

$$1.269 \quad \text{numerical}$$

2.4% accuracy

Ultraspinning bifurcations of Myers-Perry black holes



Numerical: $\frac{a}{r_+} = 1.77, \quad 2.27, \quad 2.72 \ldots$ (D=8)

Dias et al

Large D: $\frac{a}{r_+} = \sqrt{3}, \quad \sqrt{5}, \quad \sqrt{7}, \ldots$

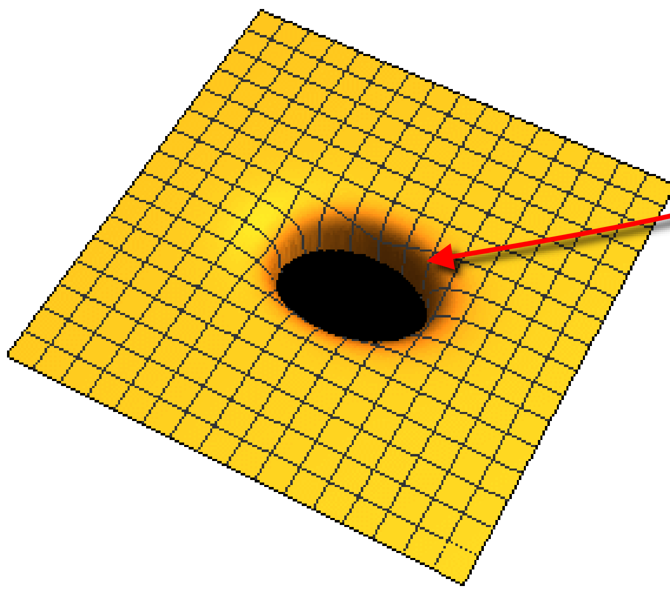
Suzuki+Tanabe

Error $\lesssim$ 2.7%

Non-linear fluctuations of black hole horizon

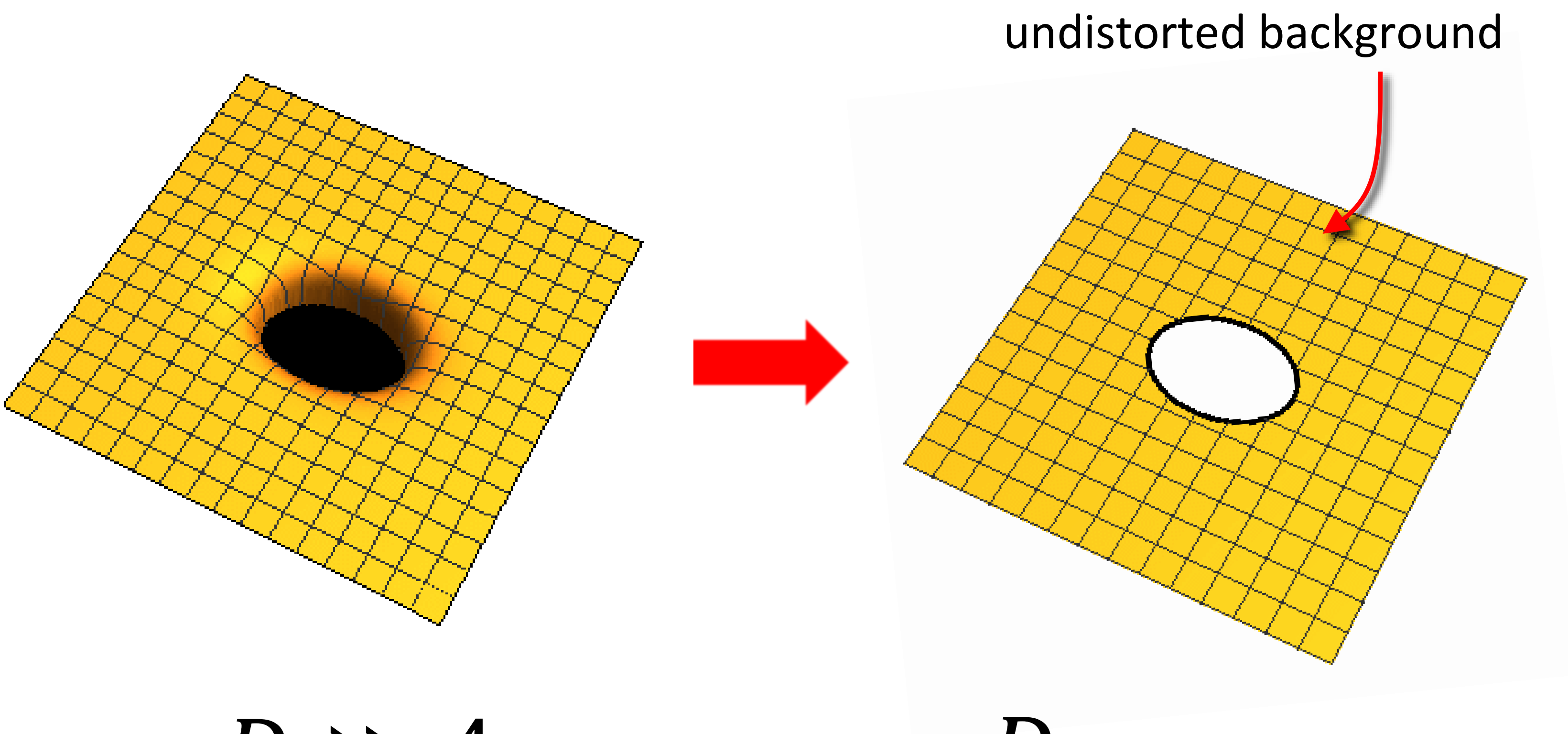
Effective Theory of black holes

All the black hole physics is concentrated here



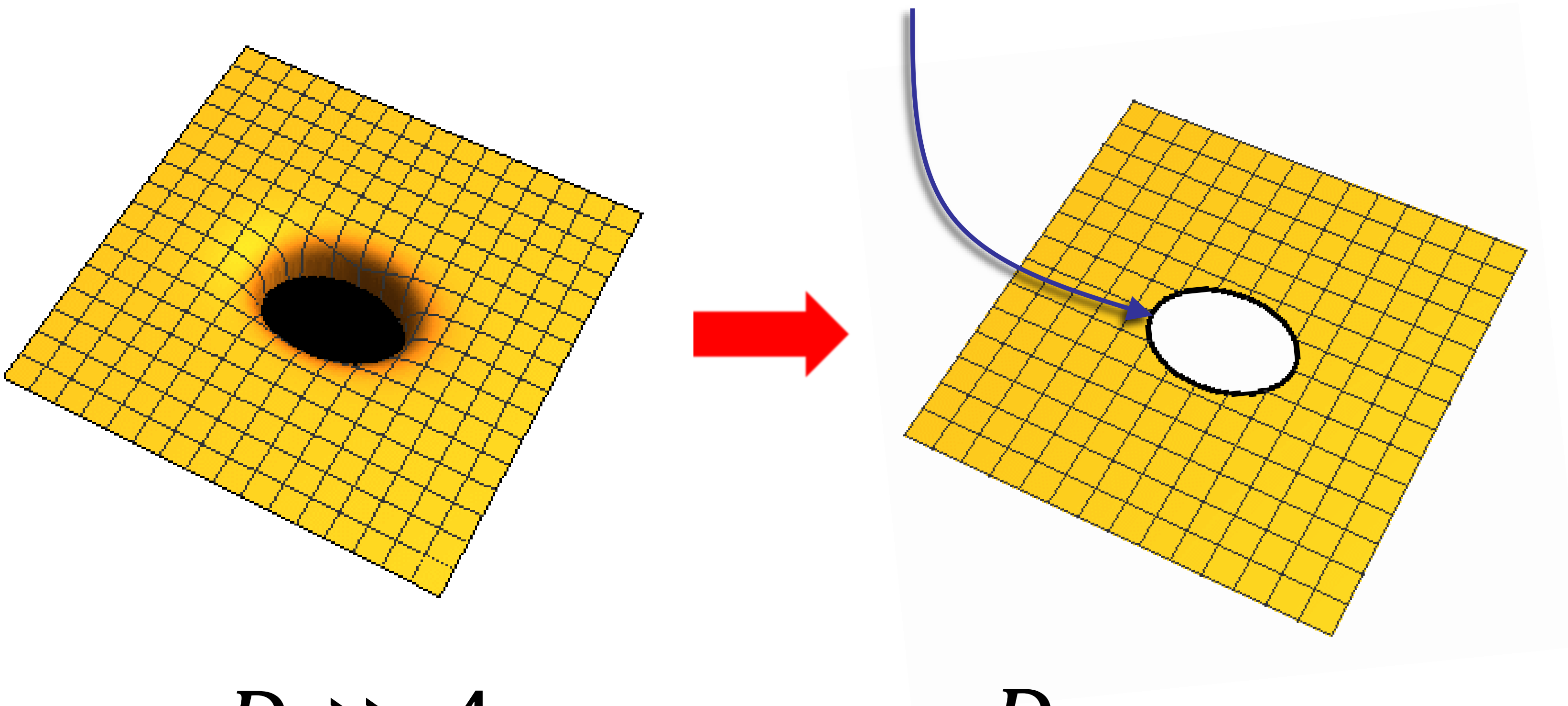
$$D \gg 4$$

Replace bh → Surface (‘membrane’)



$D \gg 4$ $\quad$ $D \to \infty$

What's the dynamics of this membrane?



$D \gg 4$ $\qquad$ $D \to \infty$

Solve Einstein equations in near-horizon

→ *Effective membrane theory*

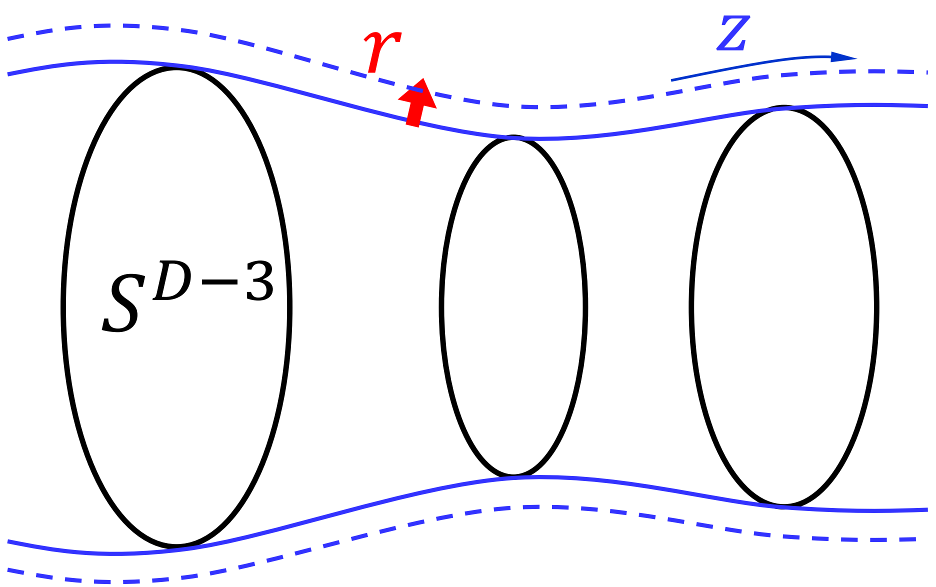
Non-linear effective theory of lightest

quasinormal modes

Gradient hierarchy

$\perp$ Horizon: $\partial_r \sim D$

$\parallel$ Horizon: $\partial_z \sim 1$ (or $\sim \sqrt{D}$)



Stationary solution

Soap-bubble equation (redshifted)

$$\boxed{K = 2\gamma\kappa}$$

K = trace **extrinsic curvature** of membrane

γ = **redshift** factor on membrane

κ = **surface gravity**

Static soap bubble in Minkowski (AdS) =

Schwarzschild (AdS) BH

Rotating soap bubble =

Myers-Perry rotating BH

Time-dependent effective theory of black strings and branes

Effective equations

effective fields for fluctuating horizon

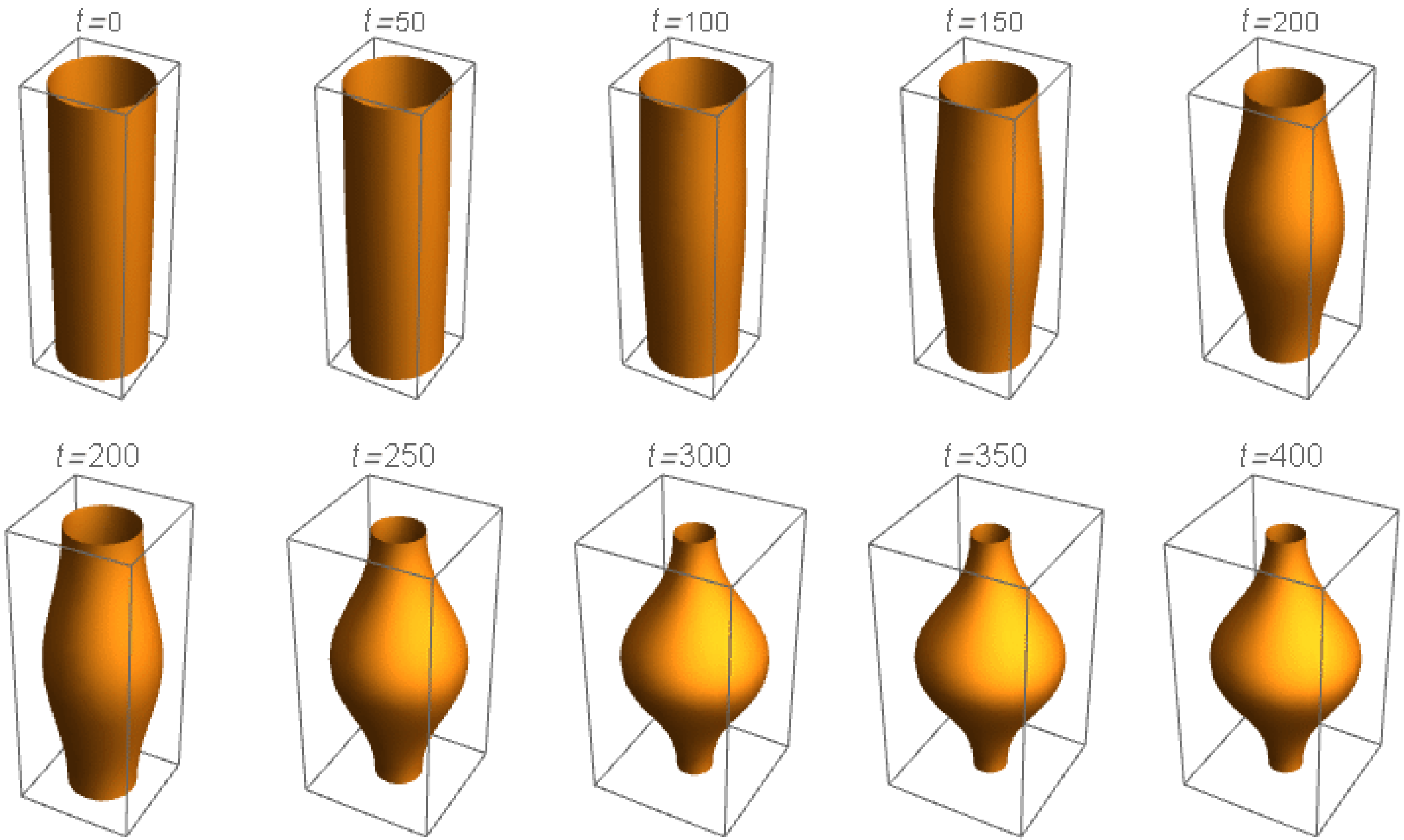
$\rho(t, z^i)$, $v_i(t, z^j)$

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

$$\partial_t(\rho v_i) + \partial^j \left(\pm\rho\, \delta_{ij} + \rho v_i v_j - 2\,\rho\, \partial_{(i} v_{j)} - \rho\, \partial^2_{ij} \ln\rho\right) = 0$$

Hydrodynamic-like, but truncate exactly

Can study phenomena at finite wavelengths



Endpoint: **stable non-uniform** black string

Horowitz+Maeda

Extended to:

charged black holes/branes

compact horizons

cosmo constant

see Kentaro’s talk

The main thing we've learned so far

Large D is very efficient for describing and solving **horizon deformations and fluctuations**