Nonlocal Metric Realizations of MOND

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Dark Matter versus Modified Gravity

- GR great for solar system
- But not for galaxies

  Theory: $v^2 = \frac{GM}{r}$

  Observation: $v^2 = \sqrt{a_0 GM}$

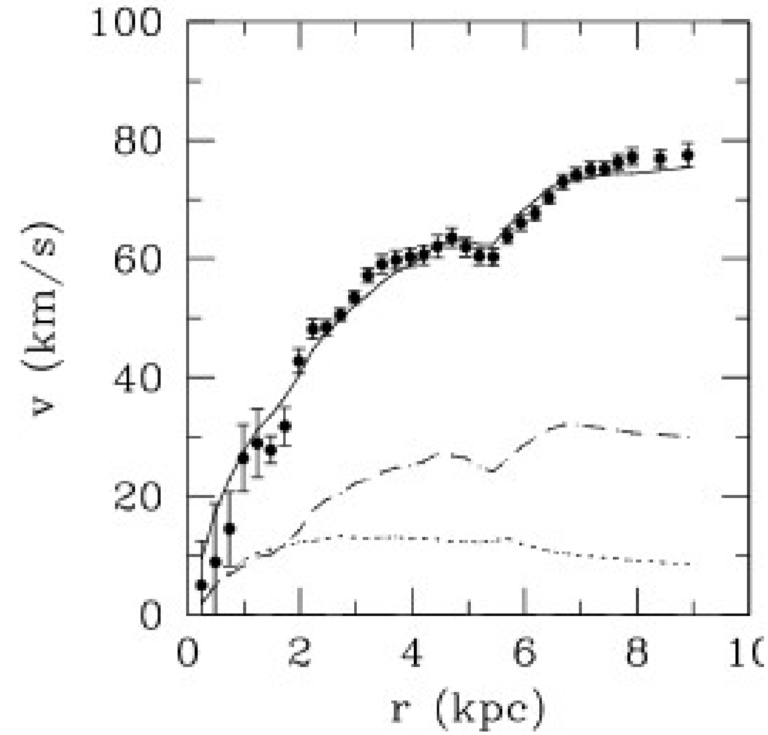
  $a_0 \sim 10^{-10} \frac{m}{s^2}$

- Maybe missing mass

  But still no direct detection!

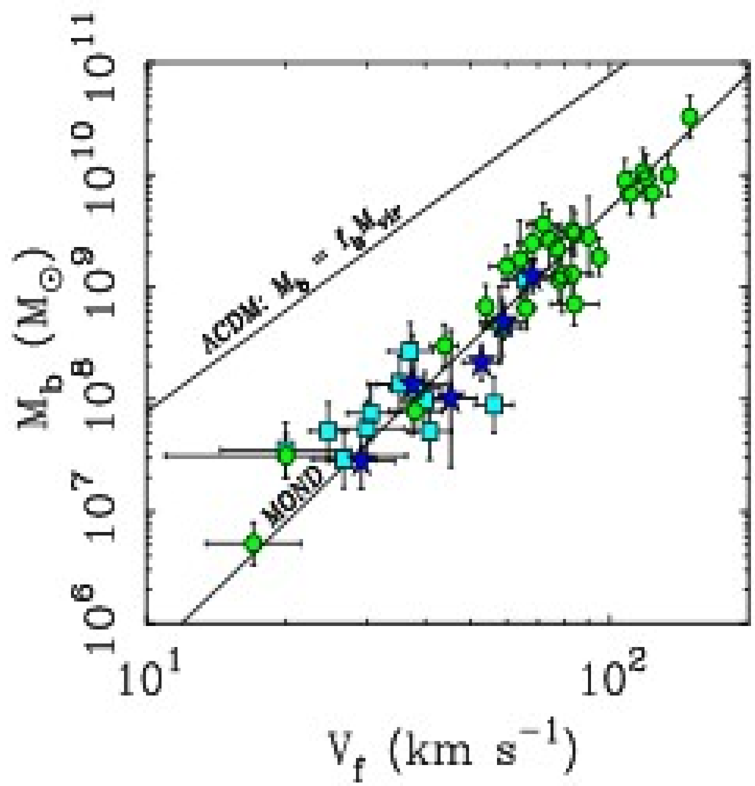
- Or modified gravity

  MOND (Milgrom 1983)



Observational Evidence for MOND in rotationally supported systems

- Baryonic Tully-Fisher Relation:
  - Asymptotic $v^4 = a_0 GM$
- Milgrom's Law:
  - Always need DM for $g(r) < a_0$
- Freeman's Law:
  - Surface density $\Sigma < \frac{a_0}{G}$
- Sancisi's Law:
  - Bumps trace baryons
- BOTTOM LINE:
  - This works for galaxies
  - Needs a relativistic extension



$$ds^2 = -[1 + b(r)]c^2dt^2 + [1 + a(r)]dr^2 + r^2 d\Omega^2$$

- Enclosed Mass ➔ $M(r) = \frac{4\pi}{c^2}\int_0^r dr' r'^2 \rho(r')$
  - Tully-Fisher ➔ $rb'(r) = \frac{2}{c^2}\sqrt{a_0 GM(r)}$
  - Lensing ➔ $a(r) = rb'(r)$
- GR ➔ $a(r) = rb'(r) = \frac{2GM(r)}{c^2 r}$
  - $\frac{\delta S}{\delta b} = \frac{c^4}{16\pi G}(ra)' - ½r^2\rho = 0$
  - $\frac{\delta S}{\delta a} = \frac{c^4}{16\pi G}[-rb' + a] = 0$
- MOND ➔ $a(r) = rb'(r) = \frac{2}{c^2}\sqrt{a_0 GM(r)}$
  - $\frac{\delta S}{\delta b} = \frac{c^4}{16\pi G}\left[(ra)' - (ra)' + \frac{c^2}{2a_0}\partial_r (rb')^2\right] - ½r^2\rho = 0$
  - $\frac{\delta S}{\delta a} = \frac{c^4}{16\pi G}[-rb' + a] = 0$

Building an Invariant Action

- $\mathcal{L} = \frac{r^2 c^2}{16\pi G}\left[-\frac{1}{2}b'^2 + \frac{1}{2}(a - rb')^2\right] + \mathcal{L}_{MOND}$

  $$\mathcal{L}_{MOND} = \frac{r^2 c^2}{16\pi G}\left[\frac{1}{2}b'^2 - \frac{c^2 b'^3}{6a_0}\right] = \frac{r^2 a_0^2}{16\pi G}\left[\frac{1}{2}\left(\frac{c^2 b'}{a_0}\right)^2 - \frac{1}{6}\left(\frac{c^2 b'}{a_0}\right)^3\right]$$

- Curvatures have an <u>even</u> number of derivatives & $\frac{1}{r}$ factors

  E.g. $R = -b'' - \frac{2b'}{r} + \frac{2a'}{r} + \frac{2a}{r^2}$ ➔ $\mathcal{L}_{MOND}$ cannot be local

- Nonlocal construction for $b(r)$ using $R_{00} = \frac{1}{2r}(rb)''$
  - $\square \equiv \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\right) \rightarrow \frac{1}{r}\partial_r^2 r$ ➔ $\frac{1}{\square}R_{00} \rightarrow \frac{1}{2}b$
- Achieving an invariant form
  - $X[g] \equiv \frac{1}{\square}1$ grows with time ➔ $u^\mu[g] \equiv \frac{g^{\mu\nu}\partial_\nu X}{\sqrt{-g^{\alpha\beta}\partial_\alpha X \partial_\beta X}}$
  - $Ƶ[g] \equiv \frac{4c^4}{a_0^2} g^{\alpha\beta}\left[\partial_\alpha \frac{1}{\square}\left(R_{\mu\nu}u^\mu u^\nu\right)\right]\left[\partial_\beta \frac{1}{\square}\left(R_{\rho\sigma}u^\rho u^\sigma\right)\right]$

$$f(Ƶ) \text{ in } \mathcal{L}_{MOND} = \frac{a_0^2}{16\pi G} f(Ƶ[g])\sqrt{-g}$$

- Gravitational bound systems have $Ƶ > 0$
  - Small $Ƶ$ ➔ $f(Ƶ) = \frac{1}{2}Ƶ - \frac{1}{6}Ƶ^{3/2} + O(Ƶ^2)$ gives TF & Lensing
  - Large $Ƶ$ ➔ $f(Ƶ) \rightarrow 0$ preserves solar system
  - E.g. $f(Ƶ) = \frac{1}{2}Ƶ \,\mathrm{Exp}\left[-\frac{1}{3}\sqrt{Ƶ}\right]$ works
- Cosmology has $Ƶ < 0$ ➔ get ΛCDM without DM
  - Small $-Ƶ$ ➔ $f(Ƶ) \sim Ƶ$ (still working on this)
  - Large $-Ƶ$ ➔ $f(Ƶ) = -\frac{\Omega_{DM}}{33}\sqrt{\frac{6c\quad{}_0}{a_0}}\left(\frac{-Ƶ}{\Omega_{rad}}\right)^{3/4}$
    - $-Ƶ \sim (1+z)^4$ ➔ good chance to make up for no DM
    - Not by hand, an inevitable result of invariant extension
  - Then check CMB & growth of structures

Conclusions

- Nonlocal, metric-based realization of MOND
  - $\mathcal{L} = \frac{c^4}{16\pi G}(R - 2\Lambda)\sqrt{-g} \; + \; \frac{a_0^2}{16\pi G} f(\text{Ƶ}[g])\sqrt{-g}$
  - View nonlocality as vacuum polarization of inflationary gravitons
- Full causal & conserved field equations derived for any $f(\text{Ƶ})$
  - See arXiv:1405.0393 (Eqn. 17 generally, Eqn. 40 for cosmology)
- Choose function $f(\text{Ƶ})$ to
  - Reproduce Tully-Fisher and lensing (small $\text{Ƶ} > 0$)
  - Preserve solar system (large $\text{Ƶ} > 0$)
  - Reproduce ΛCDM expansion history without DM ($\text{Ƶ} < 0$, out soon)
- Now test model with
  - Evolving systems (cluster cores, Bullet Cluster)
  - CMB & growth of structure