

PARAMETRIZED POST-NEWTONIAN PLUS YUKAWA (PPNY) APPROXIMATION OF NONMINIMALLY COUPLED GRAVITY

RICCARDO MARCH

IAC-CNR and INFN-LNF, Rome, Italy

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LIST OF AUTHORS

- Riccardo March (IAC-CNR and INFN-LNF, Rome, Italy)
- Jorge Paramos (Univ. Porto, Portugal)
- Orfeu Bertolami (Univ. Porto, Portugal)
- Simone Dell'Agnello (INFN-LNF, Frascati, Italy)

Work part of the **Moon-LIGHT-2 experiment** , INFN-LNF
Frascati, Italy

INTRODUCTION

- **Nonminimally coupled (NMC) gravity** is a modification of General Relativity that has been applied to astrophysical and cosmological problems as a possible alternative to the standard scenario of dark matter and dark energy.
- The nonrelativistic limit of NMC gravity consists of the Newtonian potential plus a **Yukawa perturbation**.
- A **parametrized post-Newtonian plus Yukawa (PPNY)** approximation of NMC gravity is devised (Naf and Jetzer, 2010, for $f(R)$ gravity).
- The Yukawa potential can be **long ranged with small strength**.
- Measurement of perihelion precession of Mercury orbit, resulting from **MESSENGER data**, can be converted into constraints on NMC gravity parameters.

NONMINIMALLY COUPLED GRAVITY

- The **action functional** of NMC gravity is (Bertolami et al. 2007):

$$\int \left[f^1(R)/2 + (1 + f^2(R)) \mathcal{L}_m \right] \sqrt{-g} d^4x,$$

- R is the **spacetime curvature**, g is the metric determinant, $\mathcal{L}_m = -\rho c^2$ is the Lagrangian density of matter, ρ is **mass density**.

- General Relativity** (GR) is recovered by taking

$$f^1(R) = 2\kappa R, \quad f^2(R) = 0, \quad \kappa = \frac{c^4}{16\pi G}, \quad G = \text{Newton's constant}$$

- $f^2(R) = 0$ **corresponds to** $f(R)$ **gravity theory**.
- $f^2(R)$ yields the **NMC** between geometry and matter.

METRIC AND ENERGY-MOMENTUM TENSOR

- The **metric tensor** is of the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$
 $\eta_{\mu\nu}$ is the Minkowski tensor; **$1/c$ expansion** of $g_{\mu\nu}$ (as in PPN):

$$g_{00} = -1 + h_{00}^{(2)} + h_{00}^{(4)} + O(1/c^6), \quad g_{0i} = h_{0i}^{(3)} + O(1/c^5),$$

$$g_{ij} = \delta_{ij} + h_{ij}^{(2)} + O(1/c^4).$$

- Components of **energy momentum tensor** $T_{\mu\nu}$ (as in PPN):

$$T_{00} = \rho c^2 \left(1 + \frac{v^2}{c^2} + \frac{\Pi}{c^2} - h_{00}^{(2)} \right) + O(1/c^2), \quad T_{0i} = -\rho c v_i + O(1/c),$$

$$T_{ij} = \rho v_i v_j + p \delta_{ij} + O(1/c^2),$$

- where matter is considered as a perfect fluid with **density ρ** , **velocity \mathbf{v}** , **pressure p** , and **specific energy density Π** .

ASSUMPTIONS ON FUNCTIONS OF CURVATURE

- We assume the functions $f^1(R)$ and $f^2(R)$ to be **analytic at $R=0$** . Hence f^1 and f^2 admit the Taylor expansions:

$$f^1(R) = 2\kappa \sum_{i=1}^{\infty} a_i R^i, \quad a_1 = 1, \quad \kappa = \frac{c^4}{16\pi G}$$

$$f^2(R) = \sum_{j=1}^{\infty} q_j R^j$$

- If $a_i = 0 \forall i > 1$ and $q_j = 0 \forall j$, **the action of GR is recovered.**
- The coefficients a_2, a_3, q_1, q_2 (**parameters of the NMC model**) will be used to compute the metric at the required order.
- Negative powers in f^2 would be required in order to model dark matter/dark energy (not considered here).

FIELD EQUATIONS OF NMC GRAVITY

- The first variation of the action functional with respect to the metric yields the field equations:

$$\left(f_R^1 + 2f_R^2 \mathcal{L}_m\right) R_{\mu\nu} - \frac{1}{2} f^1 g_{\mu\nu} = \nabla_{\mu\nu} \left(f_R^1 + 2f_R^2 \mathcal{L}_m\right) + (1 + f^2) T_{\mu\nu}$$

$$\nabla_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} g^{\sigma\eta} \nabla_\sigma \nabla_\eta$$

- $f_R^i = d f^i / dR$, $\mathcal{L}_m = -\rho c^2$ is the **Lagrangian density of matter**, ρ is **mass density**, $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the energy-momentum tensor, ∇_μ is the covariant derivative.
- The field equations are solved by a perturbative method.

NONRELATIVISTIC LIMIT

- At order $O(1/c^2)$ we obtain the **equations of Yukawa type**:

$$\nabla^2 R^{(2)} - \frac{1}{6a_2} R^{(2)} = -\frac{4\pi G}{3c^2 a_2} (\rho - 6q_1 \nabla^2 \rho),$$

$$\nabla^2 \left(h_{00}^{(2)} - 2a_2 R^{(2)} + 16 \frac{\pi G}{c^2} q_1 \rho \right) = -\frac{8\pi G}{c^2} \rho,$$

- $R^{(2)}$ is curvature at order $O(1/c^2)$, a_2, q_1 are **NMC parameters**.
- the solution for the 0-0 component of the metric at order $O(1/c^2)$ is the Newtonian potential **plus a Yukawa potential**:

$$h_{00}^{(2)} = 2 \frac{U}{c^2} + \left(1 - \frac{q_1}{a_2} \right) \frac{2}{3c^2} Y, \quad Y = G \int \rho(t, \vec{x}') \frac{e^{-|x - \vec{x}'|/\lambda}}{|x - \vec{x}'|} d^3 x'$$

YUKAWA POTENTIAL

- $h_{00}^{(2)}$ is the Newtonian potential **plus a Yukawa potential**:

$$h_{00}^{(2)} = 2 \frac{U}{c^2} + (1 - \theta) \frac{2}{3c^2} Y, \quad Y = G \int \rho(t, \vec{x}') \frac{e^{-|x - \vec{x}'|/\lambda}}{|x - \vec{x}'|} d^3x'$$

- The **range** λ of the Yukawa potential is

$$\lambda = \sqrt{6a_2}$$

- The **strength** α of the Yukawa potential is

$$\alpha = \frac{1}{3}(1 - \theta), \quad \theta = \frac{q_1}{a_2}, \quad q_1 = a_2 \Rightarrow \alpha = 0.$$

Long range (astronomical) effects are possible if $q_1 \cong a_2$.

PPNY APPROXIMATION

- The *i-j* components of field equations at order $O(1/c^2)$ are:

$$\nabla^2 \left(\frac{1}{2} h_{ij}^{(2)} - 2a_2 \delta_{ij} R^{(2)} + \frac{16\pi G}{c^2} q_1 \rho \delta_{ij} \right) + \frac{1}{2} \delta_{ij} R^{(2)} + 2a_2 R_{,ij}^{(2)} = \frac{c^2}{\kappa} q_1 \rho_{,ij}$$

- $R^{(2)}$ is curvature at order $O(1/c^2)$, a_2, q_1 are NMC parameters.
- Diagonal solution after gauge transformation:

$$h_{ij}^{(2)} = \left[2 \frac{U}{c^2} - (1 - \theta) \frac{2}{3c^2} Y \right] \delta_{ij}, \quad Y = G \int \rho(t, \vec{x}') \frac{e^{-|x - \vec{x}'|/\lambda}}{|x - \vec{x}'|} d^3 x'$$

- $\theta = \frac{q_1}{a_2}$, $q_1 \cong a_2 \Rightarrow \gamma \cong 1$ while in the case $f^2(R)=0$, $\gamma=1/2$.

0-0 COMPONENT AT ORDER $O(1/c^4)$

- The expression of $h_{00}^{(4)}$ is long and the leading term is:

$$h_{00}^{(4)} = h_{00}^{(4)-GR} + \frac{8\pi G^2}{c^4} \theta \left(-2q_1 + \frac{a_3 q_1}{a_2^2} - \frac{4}{3} \frac{q_2}{a_2} \right) X(\rho^2) + \dots$$

$$X(\rho^2) = \int \rho^2(t, \vec{x}') \frac{e^{-|\vec{x}-\vec{x}'|/\lambda}}{|\vec{x}-\vec{x}'|} d^3 x', \quad \theta = \frac{q_1}{a_2}$$

- a_2, a_3, q_1, q_2 are **NMC parameters**; $h_{00}^{(4)-GR}$ is the GR term;

dots ... denote the sum of **further potentials**; some are of type

$$\psi_{i,k}(t, \vec{x}) = G^2 \int \frac{\rho(t, \vec{y}) \rho(t, \vec{z}) (\vec{x} - \vec{y}) \cdot (\vec{y} - \vec{z})}{|\vec{x} - \vec{y}|^i |\vec{y} - \vec{z}|^k} e^{-(|\vec{x}-\vec{y}|+|\vec{y}-\vec{z}|)/\lambda} d^3 y d^3 z$$

$$i, k \in \{2, 3\}$$

METRIC AROUND A STATIC, SPHERICAL BODY

- **Uniform density** is assumed (M_S, R_S : mass, radius of the body):

$$g_{00} = -1 + \frac{2}{c^2} [U(r) + \alpha Y(r)] - \frac{2}{r} \left(\frac{GM_S}{c^2} \right)^2 F(r), \quad g_{0i=0},$$

$$g_{ij} = \left\{ 1 + \frac{2}{c^2} [U(r) - \alpha Y(r)] \right\} \delta_{ij}$$

- $U(r)$ is the Newtonian potential, **$Y(r)$ is the Yukawa potential**; $F(r)$ is a further potential depending on exponential functions.

The leading term in the **Yukawa strength α** for $\lambda \gg R_S$ is

$$\alpha = \frac{1}{3} (1 - \theta) + \frac{GM_S}{c^2 R_S} \theta \left[\theta \left(\frac{\mu}{2} - 1 \right) - \frac{2}{3} \nu \right] \left(\frac{\lambda}{R_S} \right)^2 + \dots$$

$$\theta = \frac{q_1}{a_2}, \quad \mu = \frac{a_3}{a_2^2}, \quad \nu = \frac{q_2}{a_2^2}, \quad \lambda = \sqrt{6a_2}$$

DEVIATION FROM GEODESICS

- The energy-momentum tensor **is not covariantly conserved**:

$$\nabla_{\mu} T^{\mu\nu} = \frac{f_R^2}{1+f_2} (g^{\mu\nu} \mathcal{L}_m - T^{\mu\nu}) \nabla_{\mu} R \neq 0 \quad \text{if} \quad f_2 \neq 0,$$

- Consequently, the trajectories **deviate from geodesics**:

$$\frac{d^2 x^{\alpha}}{ds^2} + \Gamma_{\mu\nu}^{\alpha} \frac{dx^{\mu}}{ds} \frac{dx^{\nu}}{ds} = - \frac{f_R^2}{1+f_2} g^{\alpha\beta} \partial_{\beta} R.$$

Moreover, geodesics are different with respect to GR.

- Computation of the orbit of a planet around the Sun when

$$\lambda \gg L,$$

L = semilatus rectum of the unperturbed orbit.

PERIHELION PRECESSION

- Leading term in formula for perihelion precession of a planet:

$$\delta\phi_P = \frac{6\pi GM_S}{Lc^2} + (1-\theta)^2 \frac{\pi}{3} \left(\frac{L}{\lambda}\right)^2 e^{-L/\lambda} \\ + (1-\theta) \frac{\pi GM_S}{3Lc^2} \theta \left[3\theta \left(\frac{\mu}{2} - 1\right) - 2\nu \right] \left(1 - \frac{L}{\lambda}\right) \left(\frac{L}{R_S}\right)^3 + \dots$$

$$\theta = \frac{q_1}{a_2}, \mu = \frac{a_3}{a_2^2}, \nu = \frac{q_2}{a_2^2}, \lambda = \sqrt{6a_2}, \text{ } M_S, R_S, L = \text{mass, radius, semilatus rectum}$$

- The first row contains the **GR precession + Yukawa precession**;
the second row contains the **NMC relativistic correction**;
dots ... correspond to contribution from further potentials.
Constraints on a_2, a_3, q_1, q_2 from observation of Mercury orbit.

CONSTRAINTS FROM MERCURY OBSERVATION

- Prediction of perihelion precession assuming a PPN metric:

$$\delta\phi_P^{PPN} = \left[\frac{2(1+\gamma)-\beta}{3} + 3 \times 10^3 J_2 \right] \frac{6\pi GM_S}{Lc^2}$$

γ, β are PPN parameters, J_2 is the quadrupole moment of the Sun

- Cassini bound on γ and **bound on β** from fits to planetary data including **data from MESSENGER** (Fienga et al., 2011) yield

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}, \quad \beta - 1 = (-4.1 \pm 7.8) \times 10^{-5}$$

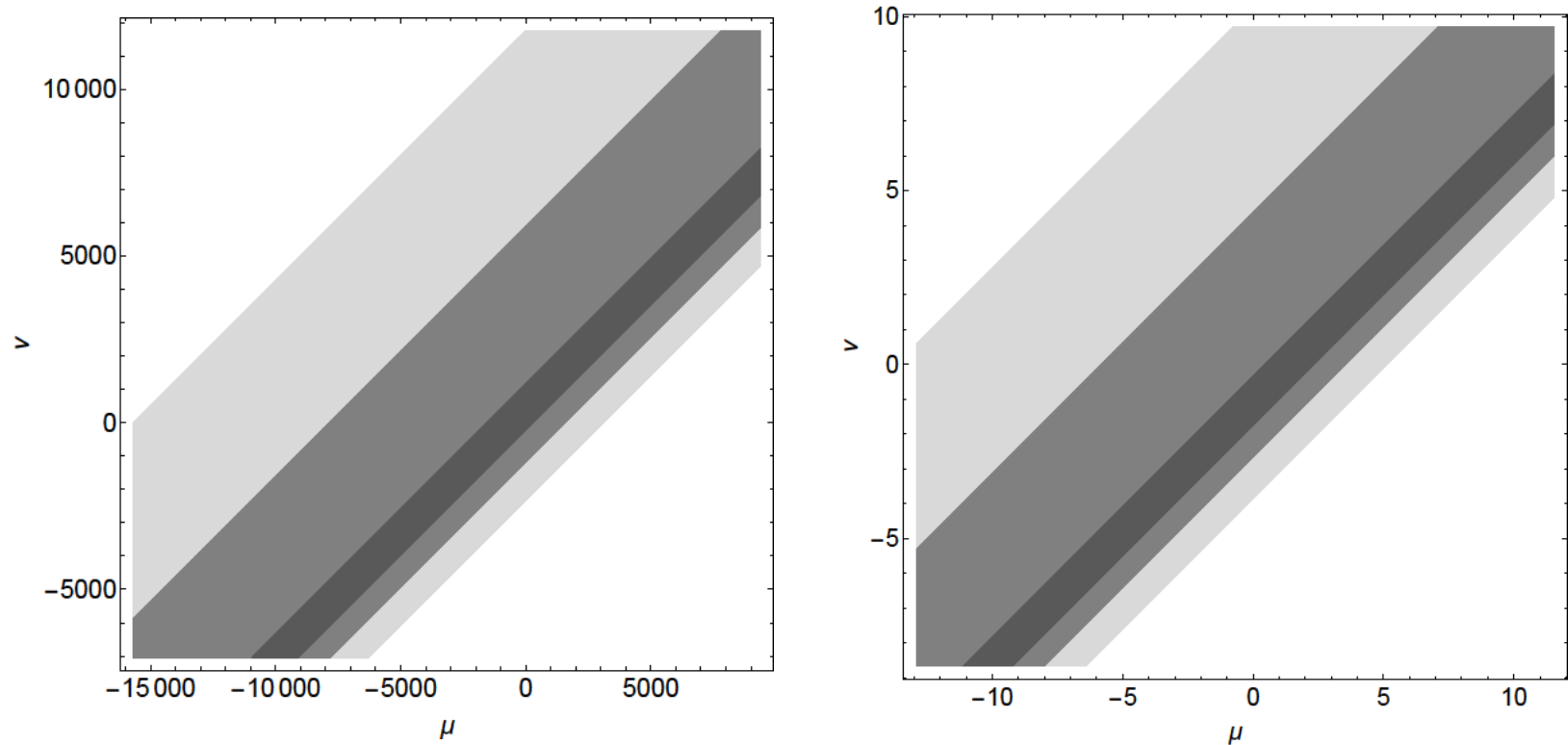
- We assume that the additional perihelion precession due to **NMC deviations from GR** is given by

$$-5.87537 \times 10^{-4} < \delta\phi_P - 42.98'' < 2.96635 \times 10^{-3}$$

Formula for $\delta\phi_P$ then yields **exclusion plots for NMC parameters**:

$$\theta = \frac{q_1}{a_2}, \quad \mu = \frac{a_3}{a_2^2}, \quad \nu = \frac{q_2}{a_2^2}, \quad \frac{L}{\lambda}$$

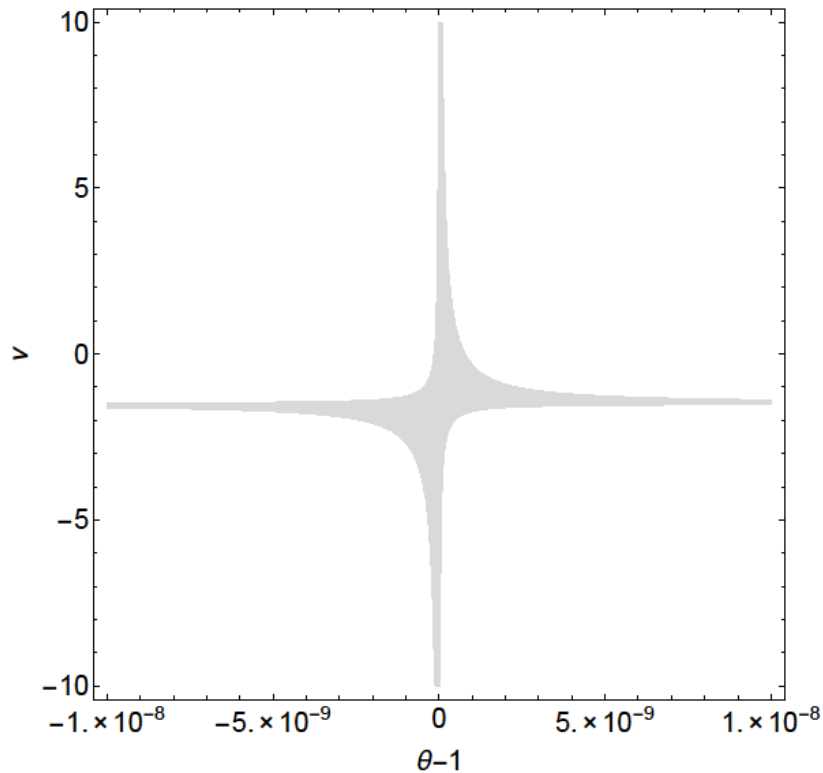
EXCLUSION PLOTS IN THE PLANE (μ, ν)



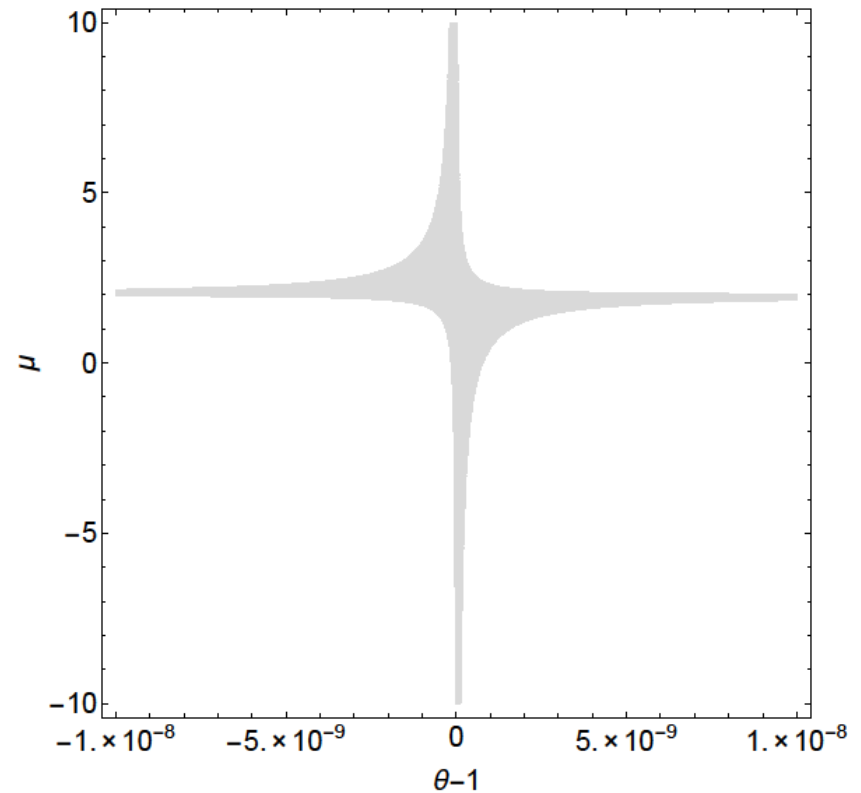
Left: $\lambda=50L$, $\theta-1=\{10^{-13}, 2 \times 10^{-13}, 10^{-12}\}$ (light, medium, dark grey)

Right: $\lambda=50L$, $\theta-1=\{10^{-10}, 2 \times 10^{-10}, 10^{-9}\}$ (light, medium, dark grey)

EXCLUSION PLOTS IN PLANES (θ, ν) AND (θ, μ)

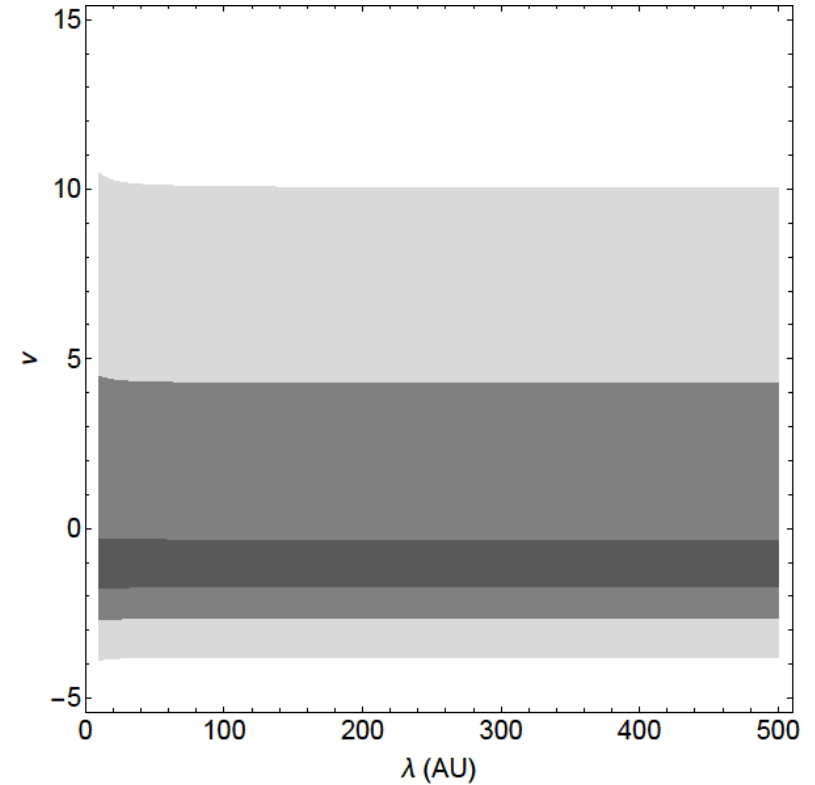
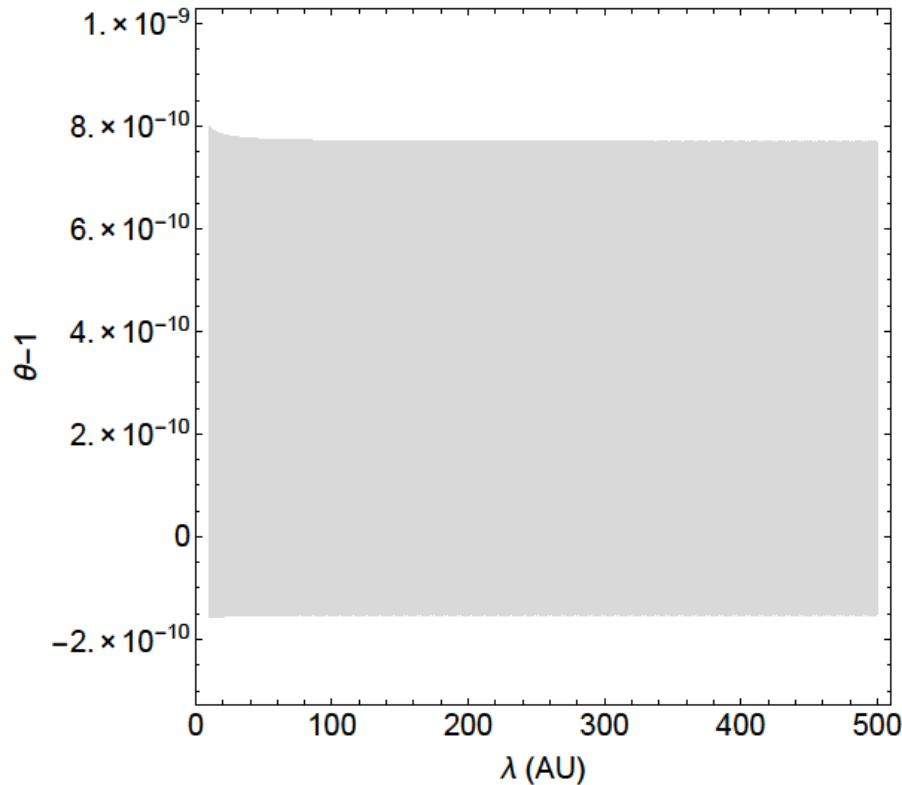


Left: $\lambda=50L, \mu=0$



Right: $\lambda=50L, \nu=0$

EXCLUSION PLOTS IN PLANES (λ, θ) AND (λ, ν)



Left: $\mu = \nu = 0$

Right: $\mu = 0$, $\theta-1 = \{10^{-10}, 2 \times 10^{-10}, 10^{-9}\}$ (light, medium, dark grey)

CONCLUSIONS

- The nonrelativistic limit of NMC gravity consists of the Newtonian potential plus a Yukawa perturbation.
- A parametrized post-Newtonian plus Yukawa (PPNY) approximation of NMC gravity has been devised.
- The Yukawa potential can be long ranged with small strength, and the NMC avoids problems like $\gamma=1/2$.
- Observations at the Solar System scale can constrain the parameters of the model.
- The results obtained might be relevant for distinguishing between GR, $f(R)$ and nonminimally coupled theories from the analysis of detailed observations data in the future.