UNIVERSITY OF TRENTO - Italy

TIFPA
Trento Institute for
Fundamental Physics
and Applications

Short gamma-ray bursts from binary neutron star mergers forming long-lived neutron stars

RICCARDO CIOLFI

Ciolfi R. & Siegel D.M. (2015a) ApJ Letters 798, L36
Ciolfi R. & Siegel D.M. (2015b), in "*Swift: 10 Years of Discovery*", PoS(SWIFT 10)108
Siegel D.M. & Ciolfi R. (2016a), ApJ 819, 14
Siegel D.M. & Ciolfi R. (2016b), ApJ 819, 15
Ciolfi R. (2016), submitted to ApJ Letters, ArXiv:1606.01743

21st International Conference
on General Relativity
and Gravitation
Columbia University, New York

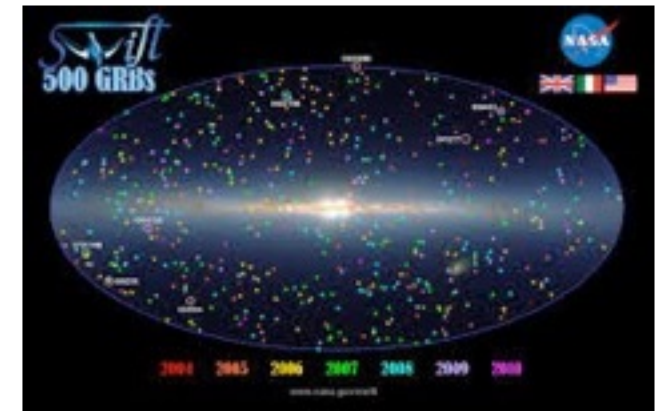
13th July 2016

COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK

Short Gamma-Ray Bursts

GAMMA-RAY BURSTS

- **bright gamma-ray flashes** with extragalactic origin, **huge luminosities**!
- followed by **afterglows** in **X-ray**, **optical** and **radio band**
- divided into **long** and **short** (prompt burst **longer**/ **shorter than 2 s**)

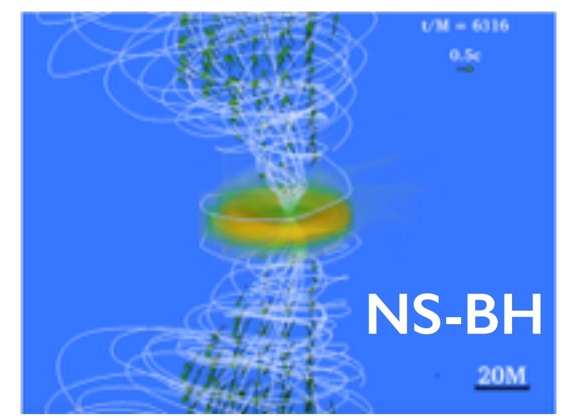


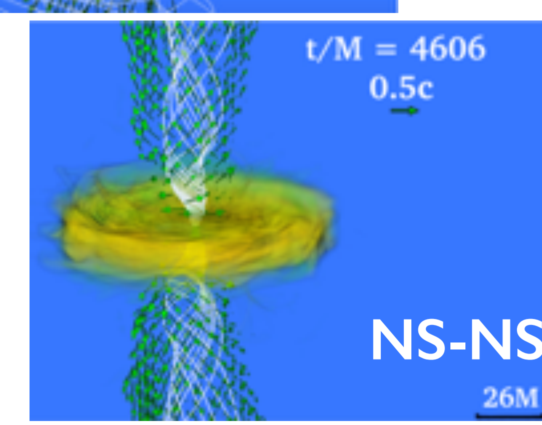
"standard" model of SGRBs

central engine is a **black hole** surrounded by a massive **accretion torus** ⟶ end result of a **BNS or NS-BH binary merger**

Paczynski 1986, Eichler et al. 1989, Narayan et al. 1992, Barthelmy et al. 2005, Fox et al. 2005, Gehrels et al. 2005, …

Paschalidis et al. 2015





Ruiz et al. 2016

X-ray afterglows of SGRBs

- SWIFT revealed that most SGRBs are accompanied by long-duration $(\sim 10^2 - 10^5\ \mathrm{s})$ and high-luminosity $(10^{46} - 10^{51}\ \mathrm{erg/s})$ X-ray afterglows
- total energy can be higher than the SGRB itself
- hardly produced by BH-torus system - they suggest ongoing energy injection from a long-lived NS

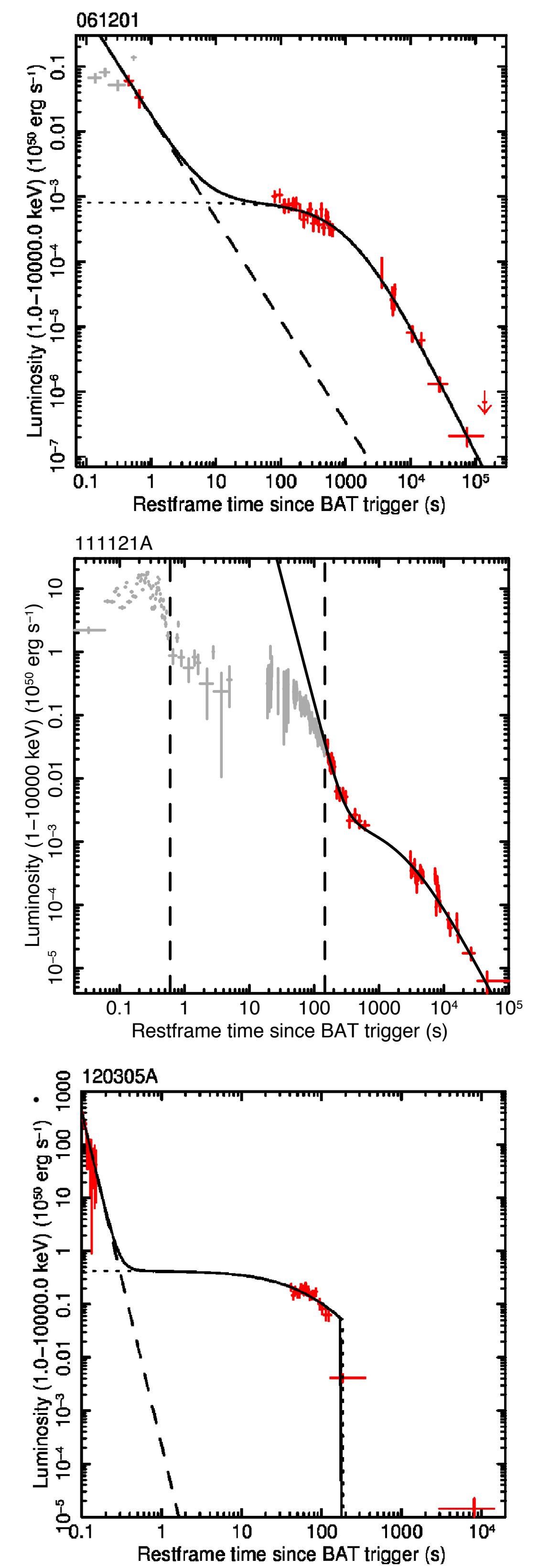
MAGNETAR MODEL

Zhang & Meszaros 2001
Metzger et al. 2008

X-ray emission $\longrightarrow$ spindown of a uniformly rotating NS with a strong surface magnetic field $\gtrsim 10^{14} - 10^{15}\,\mathrm{G}$

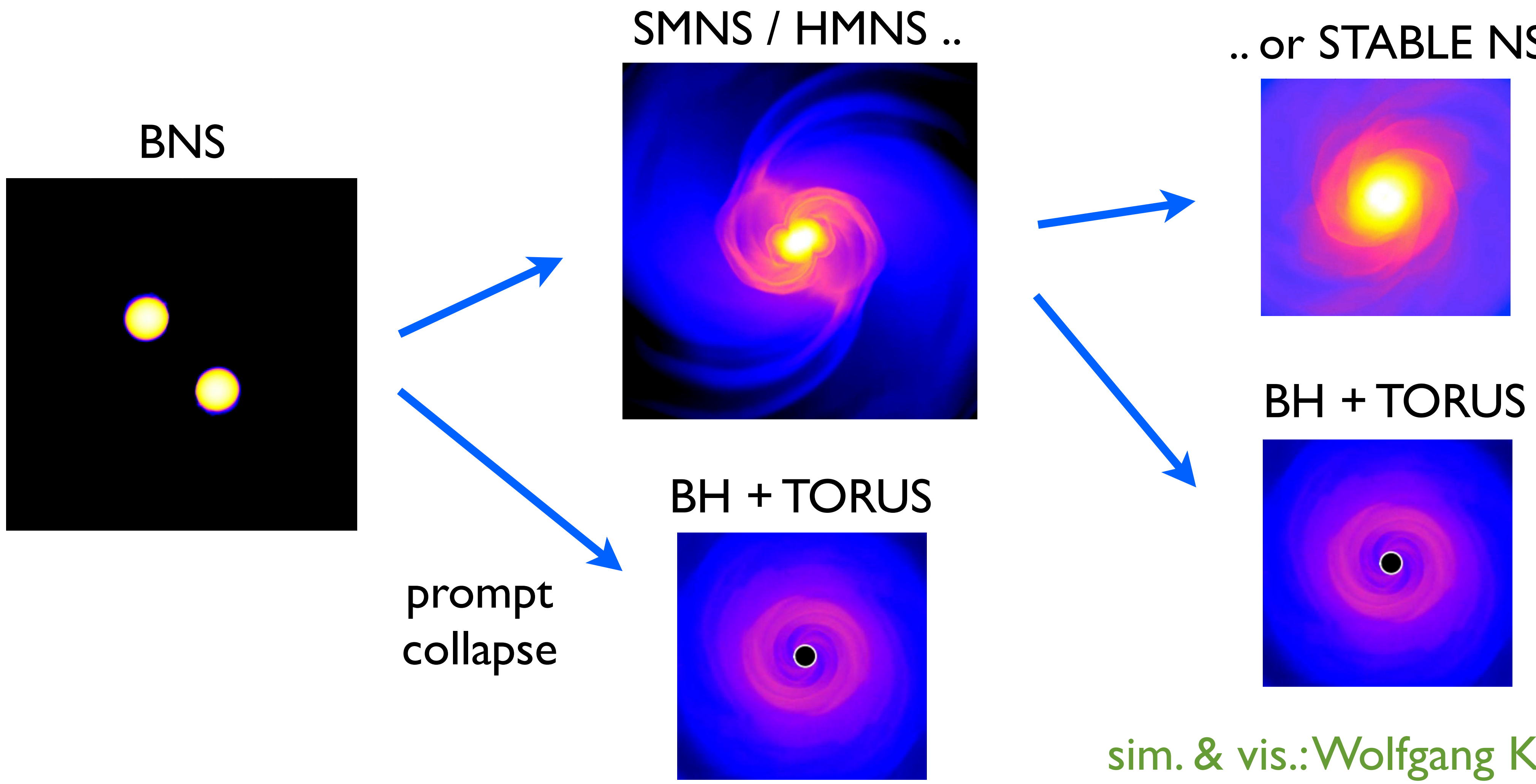
dipole spindown

$$L_{\rm sd}(t) \sim B^2 R^6 \Omega_0^4 \left(1 + \frac{t}{t_{\rm sd}}\right)^{-2}$$



Gompertz et al. 2013
Rowlinson et al. 2013

Product of BNS mergers

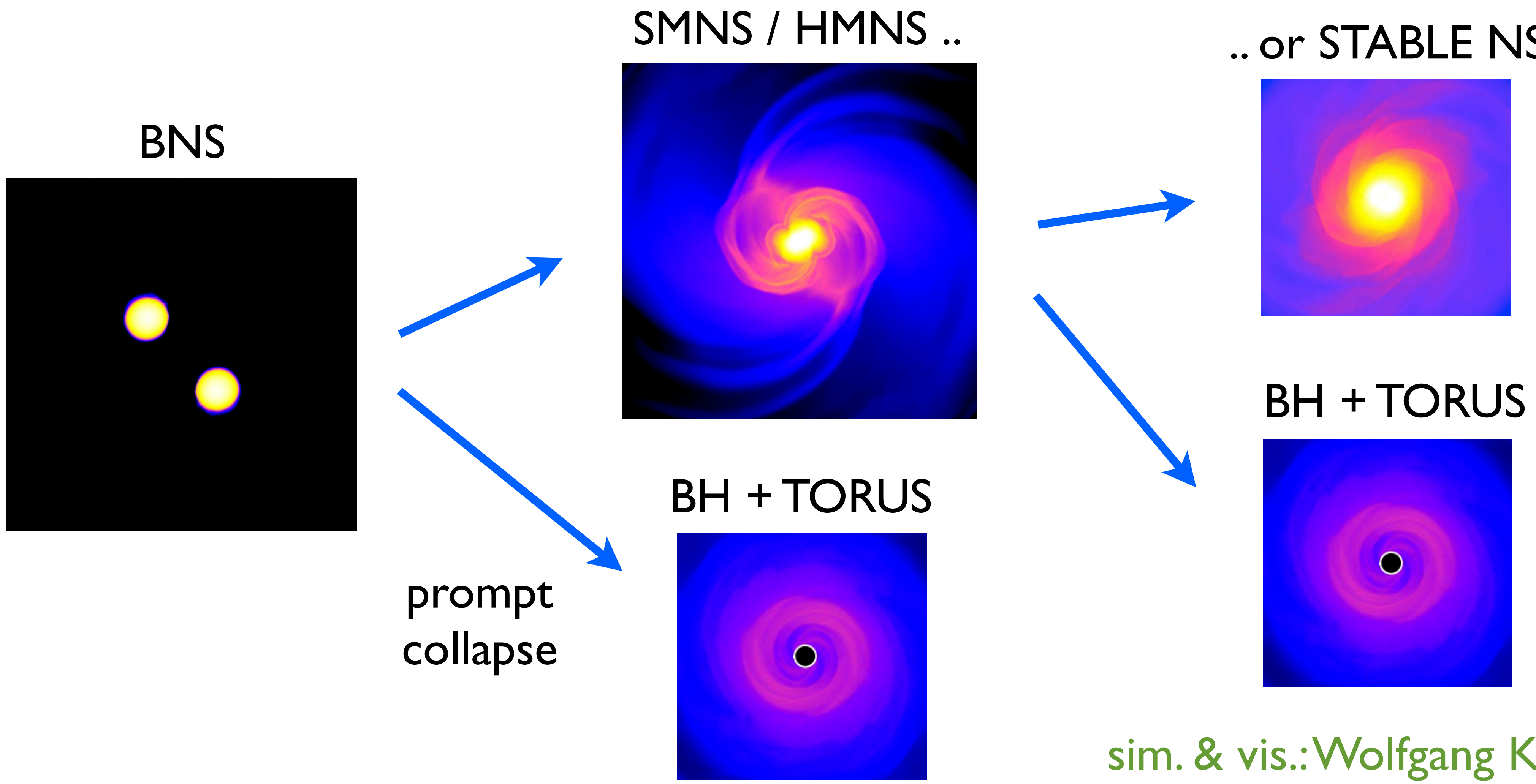


sim. & vis.: Wolfgang Kastaun

LONG-LIVED NS IS A VERY LIKELY OUTCOME OF THE MERGER

- observation of $\sim 2\ \mathrm{M}_\odot$ NSs — Demorest et al. 2010, Antoniadis et al. 2013
- progenitor masses peak around $1.3 - 1.4\ \mathrm{M}_\odot$ ➔ BMP mass likely $< 2.5\ \mathrm{M}_\odot$ — Belczynski et al. 2008
- stable NS obtained in GR BNS merger simulations — Giacomazzo & Perna 2013

Product of BNS mergers



sim. & vis.: Wolfgang Kastaun

PROBLEM OF THE LONG-LIVED NS MODEL :

strong baryon pollution can choke the formation of a relativistic jet

→ HARD TO EXPLAIN THE SGRB PROMPT EMISSION

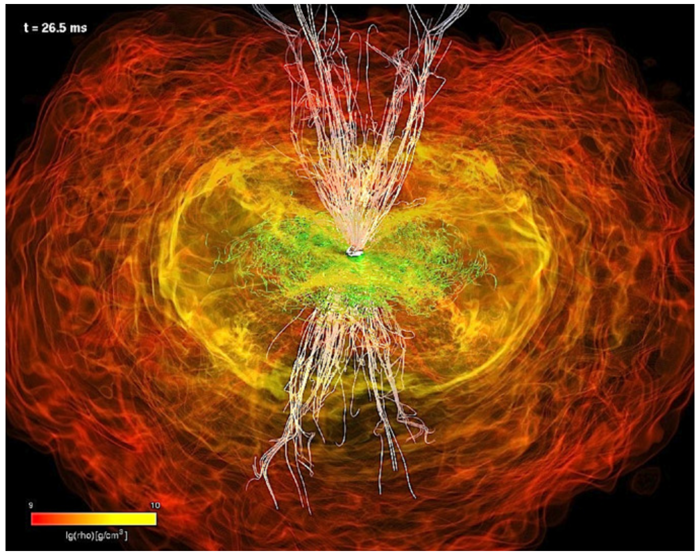
e.g., Dessart et al. 2009, Hotokezaka et al. 2013, Siegel et al. 2014

The SGRB dichotomy

- Numerical relativity picture: prompt BH-torus formation

  → can explain prompt SGRB emission ✓

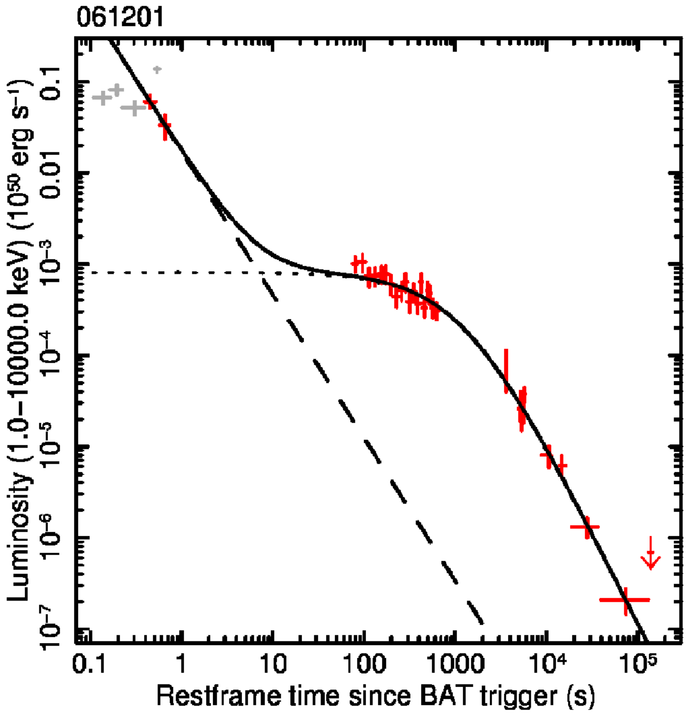
  → cannot explain X-ray afterglows ✗



- Observational picture: magnetar model

  → cannot explain prompt SGRB emission ✗

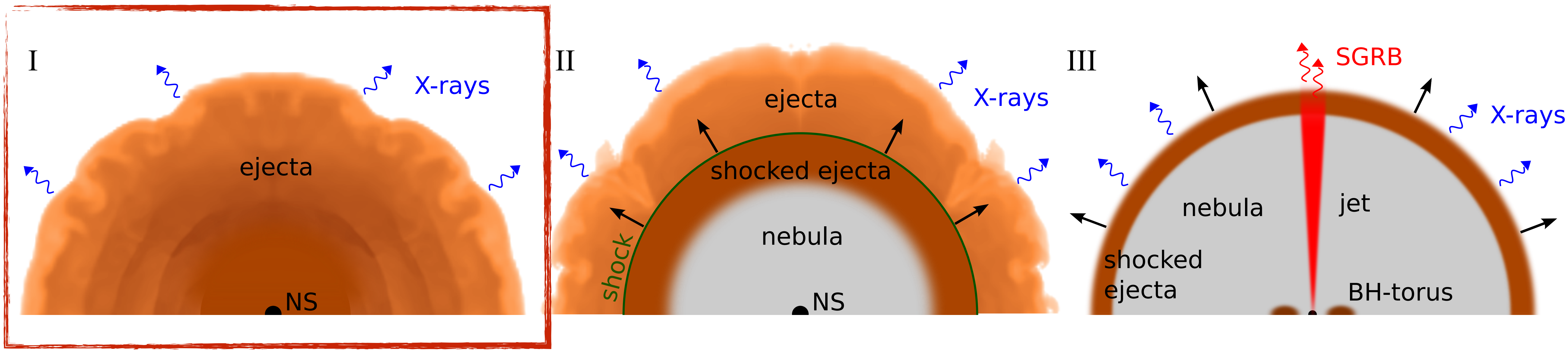
  → can explain X-ray afterglows ✓



..possible alternative solutions?

"Time-reversal" phenomenology

Ciolfi & Siegel 2015a, ApJ Letters 798, L36



(I) The differentially rotating, supramassive NS (SMNS) ejects a baryon-loaded and highly isotropic wind

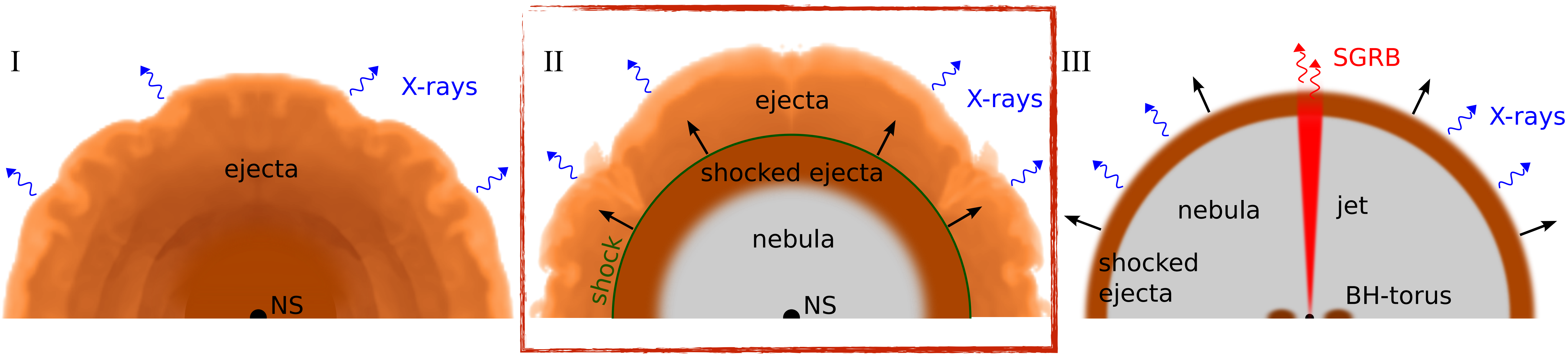
Siegel et al. 2014

(see also Siegel & Ciolfi 2015)

Dessart et al. 2009

"Time-reversal" phenomenology

Ciolfi & Siegel 2015a, ApJ Letters 798, L36

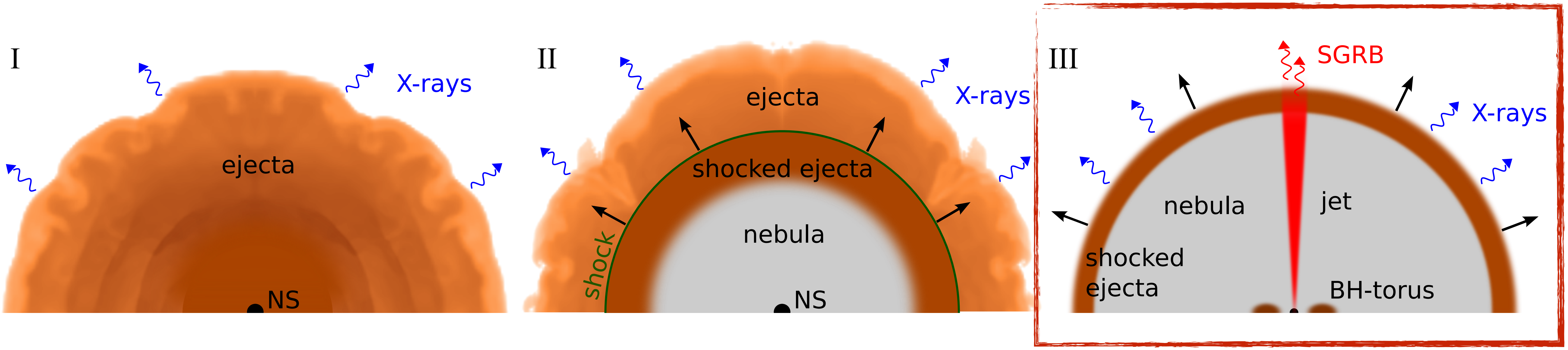


(I) The differentially rotating, supramassive NS (SMNS) ejects a baryon-loaded and highly isotropic wind

(II) The cooled-down and uniformly rotating NS emits spin-down radiation inflating a photon-pair nebula that drives a shock through the ejecta

“Time-reversal” phenomenology

Ciolfi & Siegel 2015a, ApJ Letters 798, L36



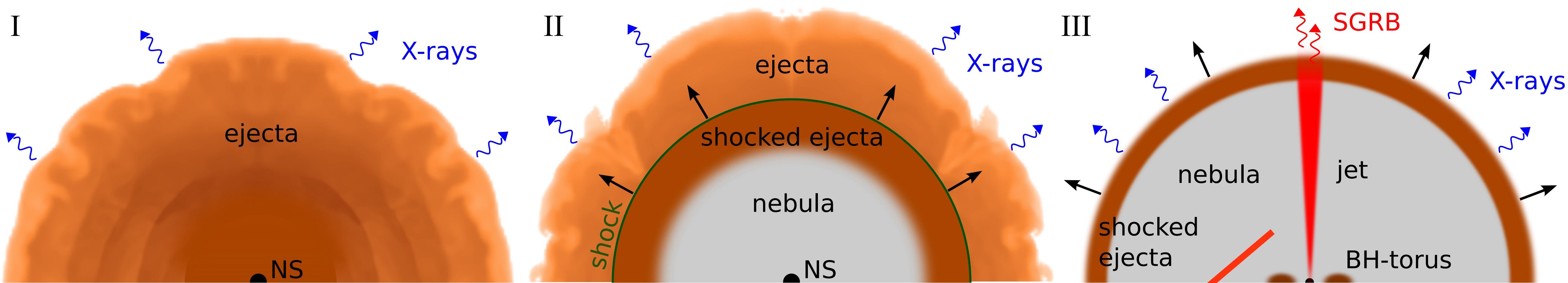
(I) The differentially rotating, supramassive NS (SMNS) ejects a baryon-loaded and highly isotropic wind

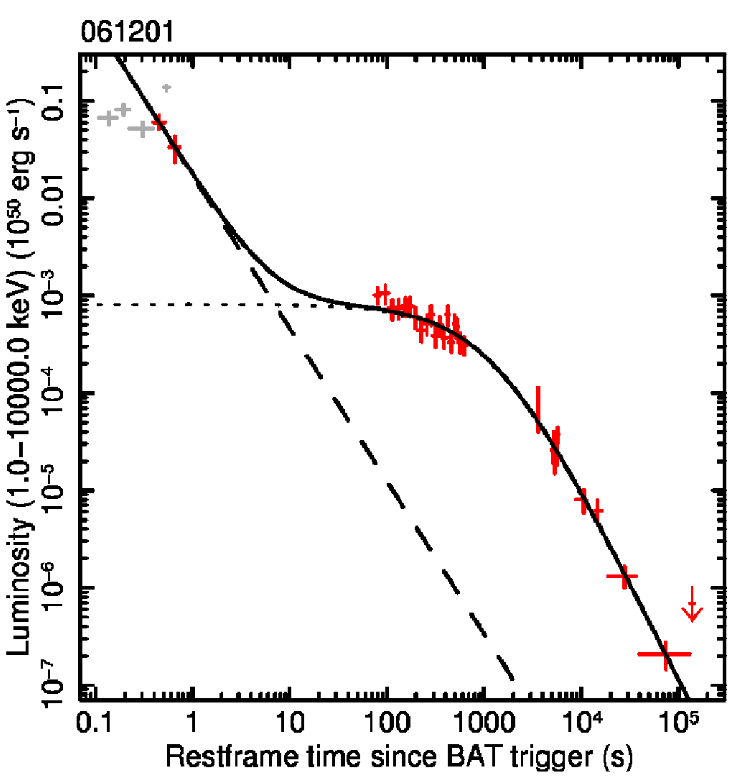
(II) The cooled-down and uniformly rotating NS emits spin-down radiation inflating a photon-pair nebula that drives a shock through the ejecta

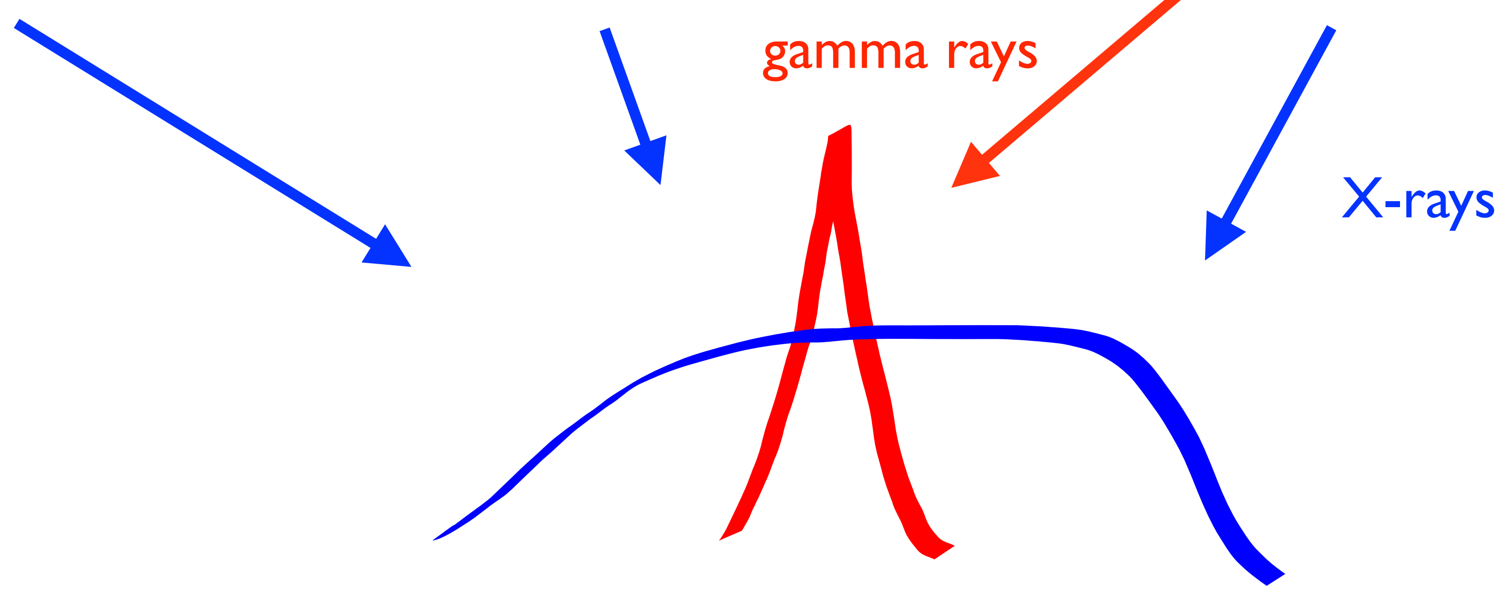
(III) The NS collapses to a black hole (BH), a relativistic jet drills through the nebula and the ejecta shell and produces the prompt SGRB, while spin-down emission diffuses outwards on a much longer timescale, producing the X-ray afterglow

(but see Margalit et al. 2015)

Electromagnetic emission







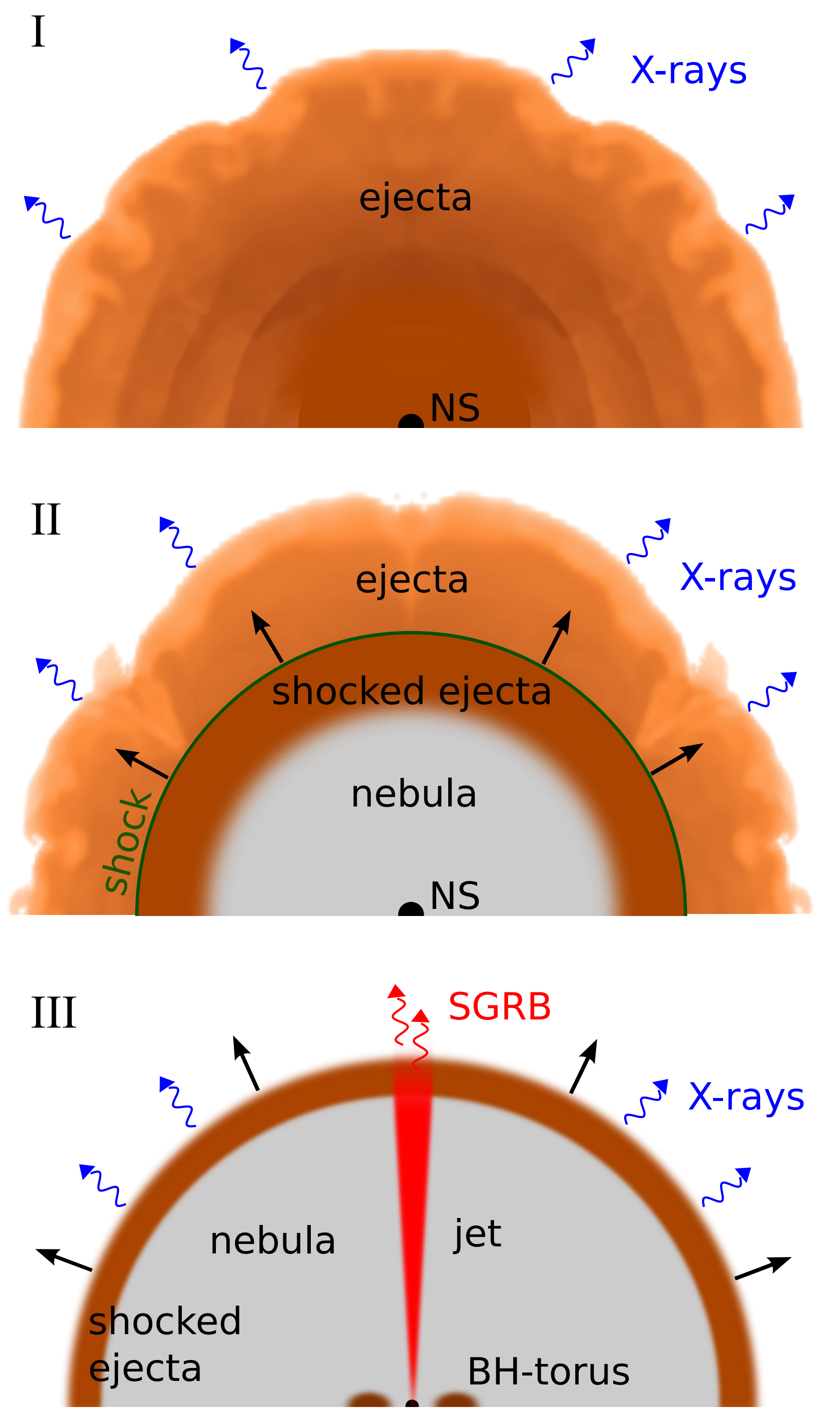
The spin-down emission is given off before but (in part) observed after the prompt SGRB radiation

TR scenario: evidence

Ciolfi & Siegel 2015a, ApJ Letters 798, L36

- proposed new scenario to solve SGRB-X-ray afterglow dichotomy ⟶ "time-reversal" scenario
- delay times can explain observed X-ray afterglow durations

  ⟶ attractive alternative to current models



Evidence:

- potential observation of X-ray plateau with SGRB in between

  ⟶ indication of time reversal

- potential observation of an orphan event without SGRB

  ⟶ isotropy of afterglow

TR scenario: implications

Ciolfi & Siegel 2015a, ApJ Letters 798, L36

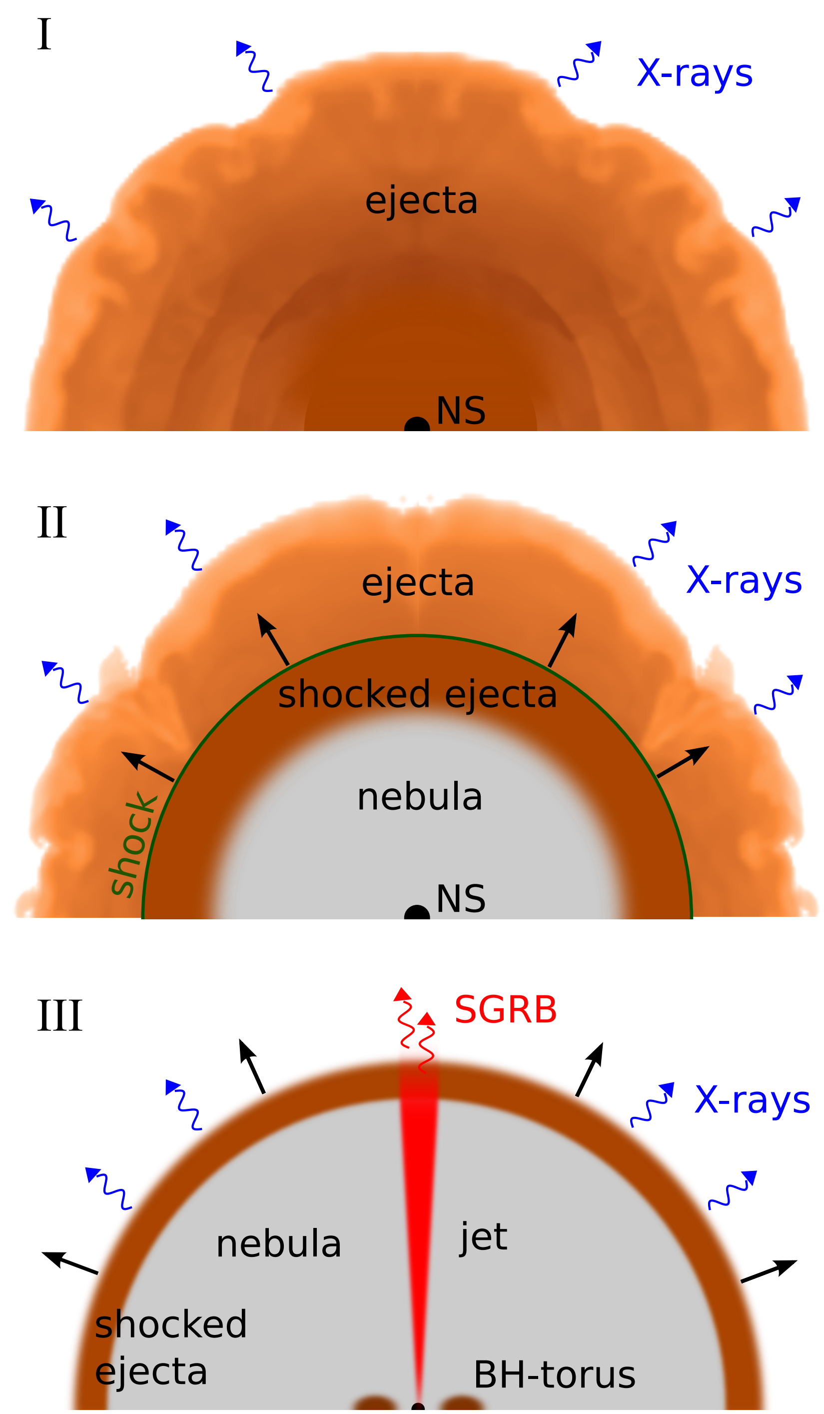
Implications:

- SGRBs with X-ray afterglows (majority of observed events) originate from BNS mergers ⟶ no BH-NS progenitors

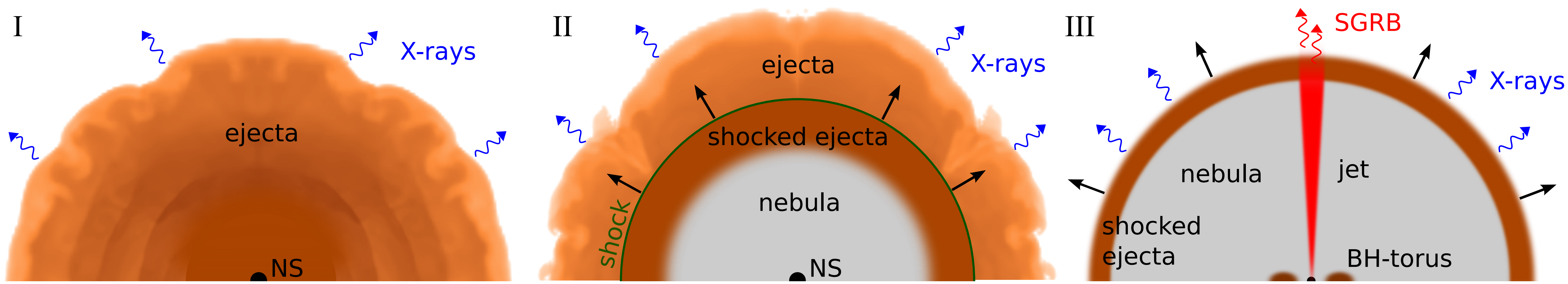
- SMNS constraints on EOS in combination with a mass estimate

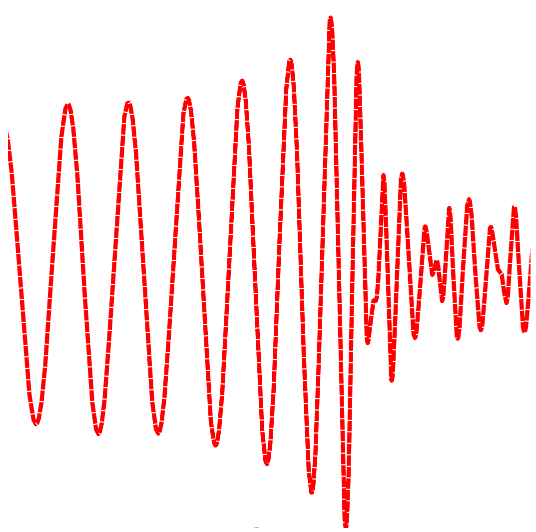
  Ciolfi & Siegel 2015b

- peak amplitude of GW emission separated from SGRB by lifetime of the NS



GW and EM observations

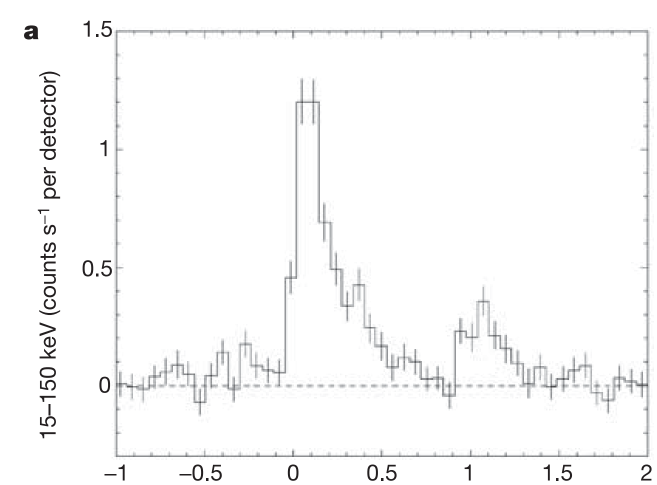




peak amplitude GW signal

$$t_{\rm coll} \sim t_{\rm sd} \sim 10^3\ {\rm s}$$

lifetime of the NS



EM SGRB signal

- GW observations ideal trigger for EM observations
- peak amplitude of GW emission separated from SGRB by lifetime of the NS

$\longrightarrow$ very precise measurement of the NS lifetime!

EM emission from the long-lived NS remnant

Siegel & Ciolfi 2016a, 2016b

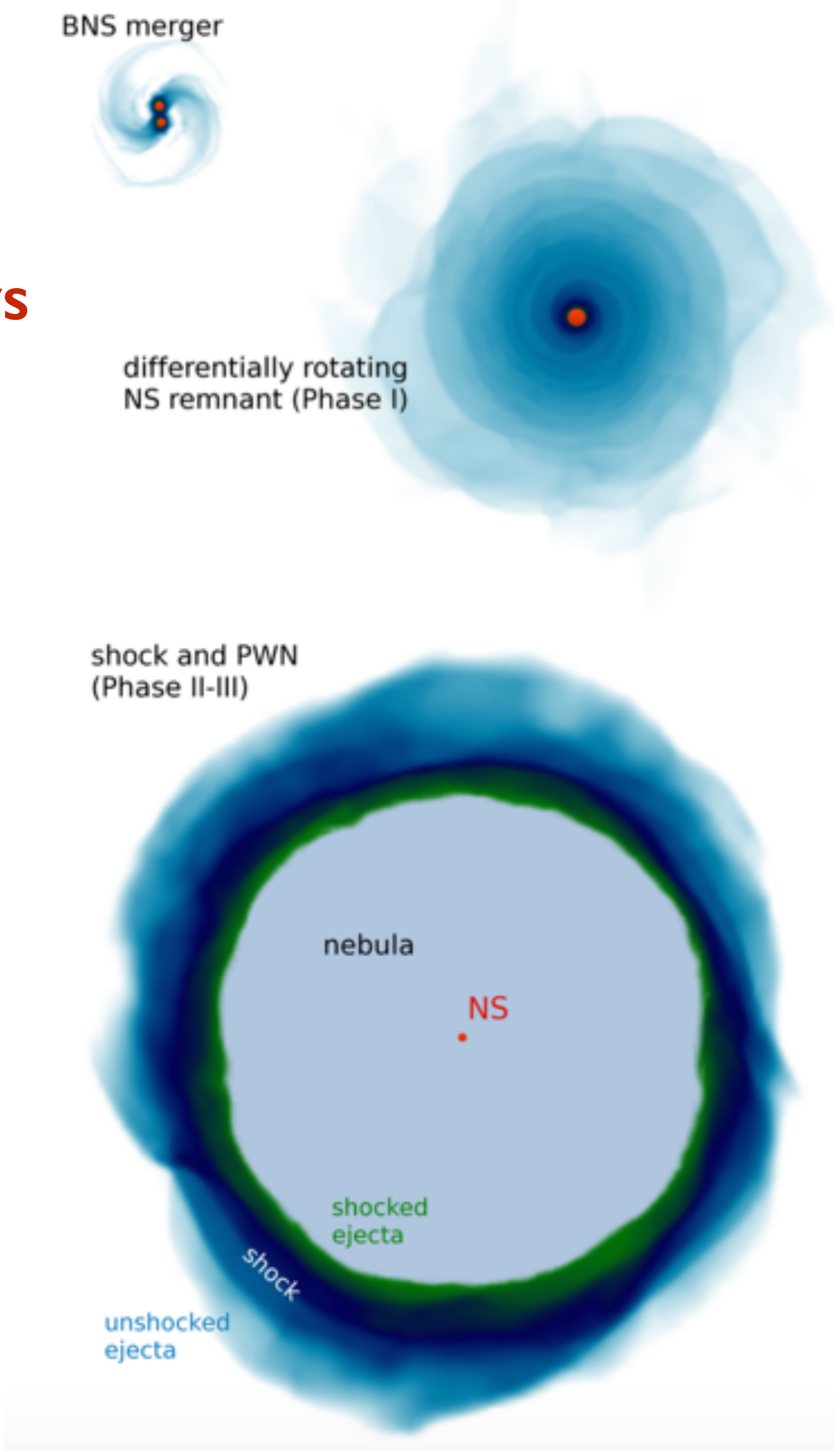
- detail model **evolving the post-merger system** (NS remnant and surrounding environment) **on large scales and up to ~days**

  → far beyond the reach of numerical relativity simulations

- takes into account the relevant **radiative processes** (thomson, compton, synchrotron, pair production/annihilation, ..) and provides **lightcurves and spectra**

- first/only **self-consistent** model of its kind

- the model **can also account for the collapse** of the NS to a BH at any time during the evolution



EM emission from the long-lived NS remnant

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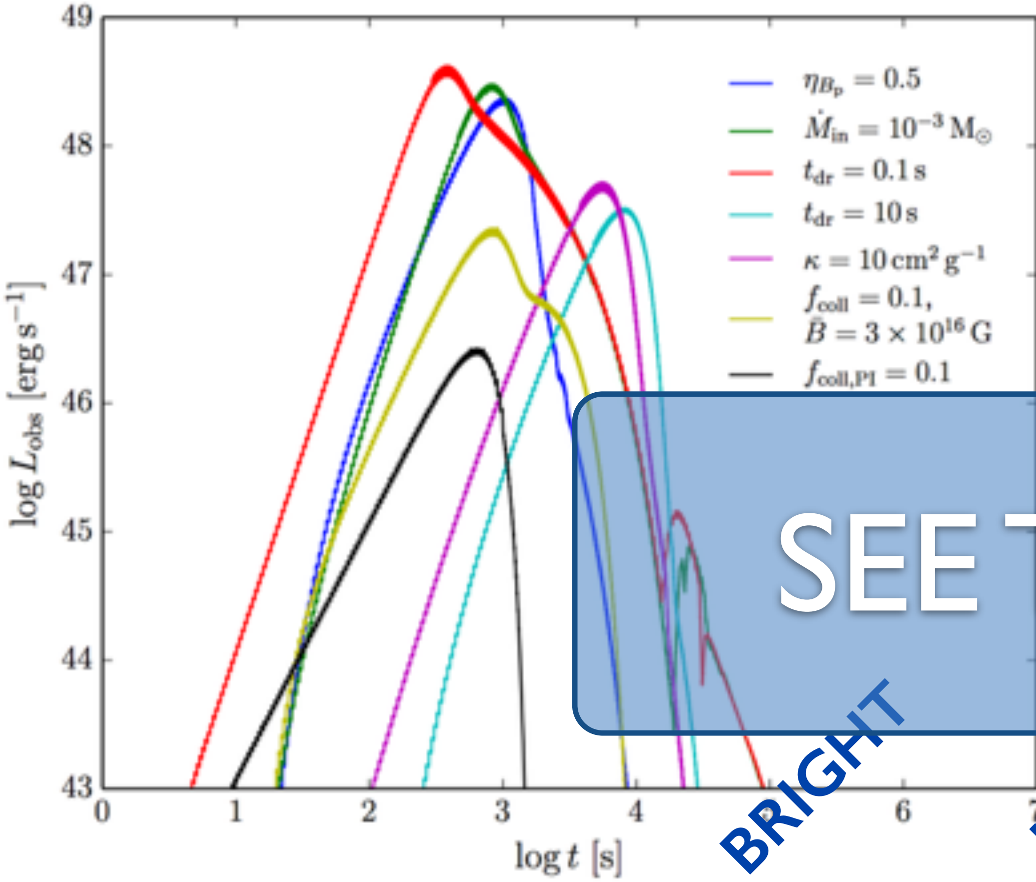


- signal peaks at 10^2-10^4 s (similar range for duration), with ~10-100 s delayed onset
- luminosities 10^{46}-10^{49} erg/s

SEE TALK BY SIEGEL

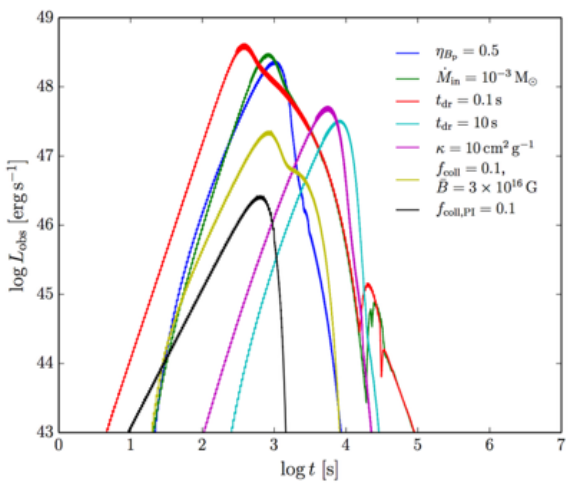
	BRIGHT	ISOTROPIC	LONG-LASTING	HIGH OCCURRENCE	DISTINGUISH BNS AND NS-BH
SGRB	✓	✗	✗	✗	✗
KILONOVA	✓ ✗	✓	✓	✓	✗
NS SPIN-DOWN	✓	✓	✓	✓	✓

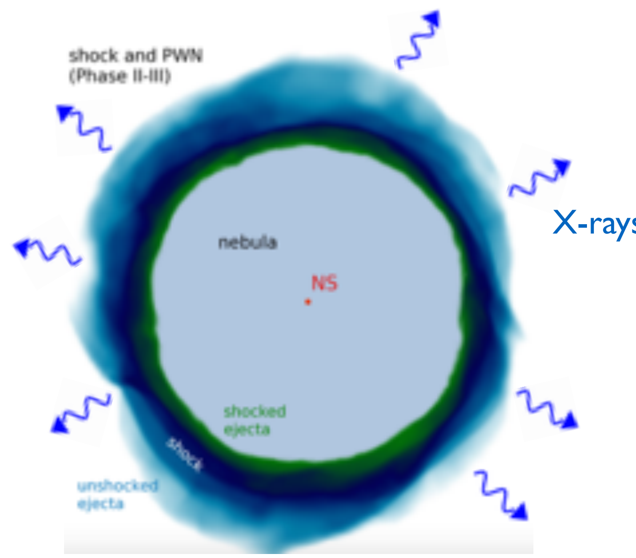
very promising EM counterpart!

X-ray flashes powered by NS spindown

Ciolfi 2016 (ArXiv:1606.01743)

- are XRFs really a subclass of long GRBs?

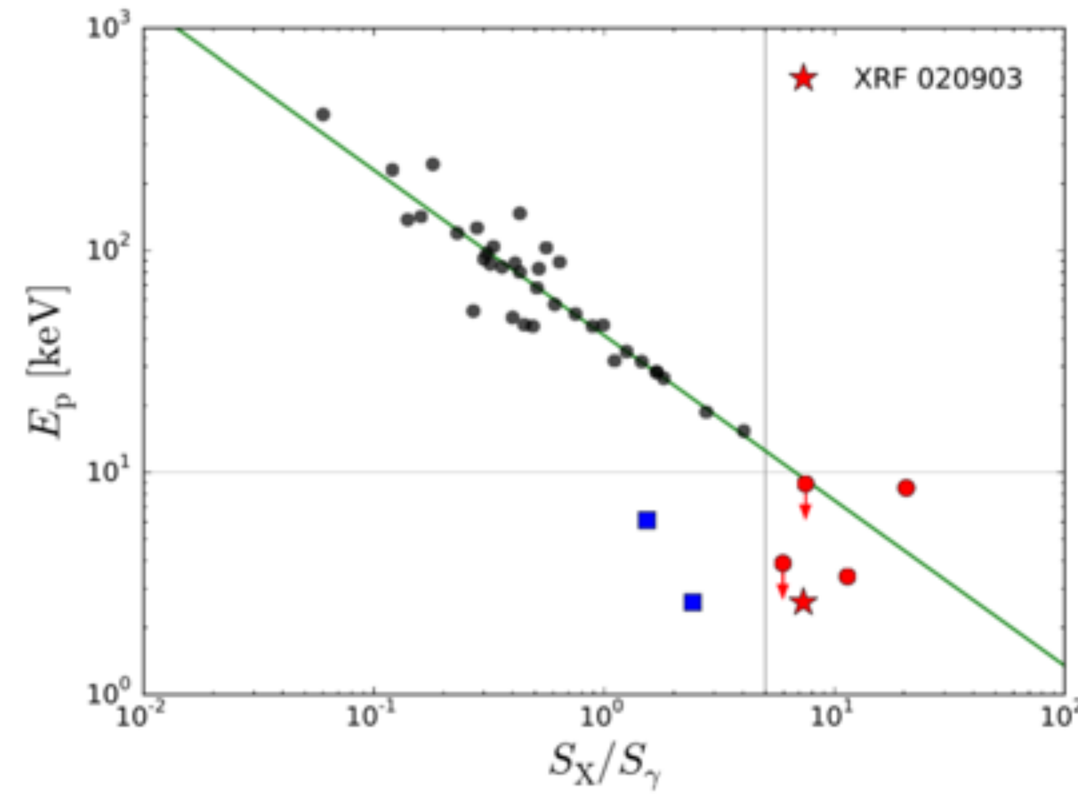


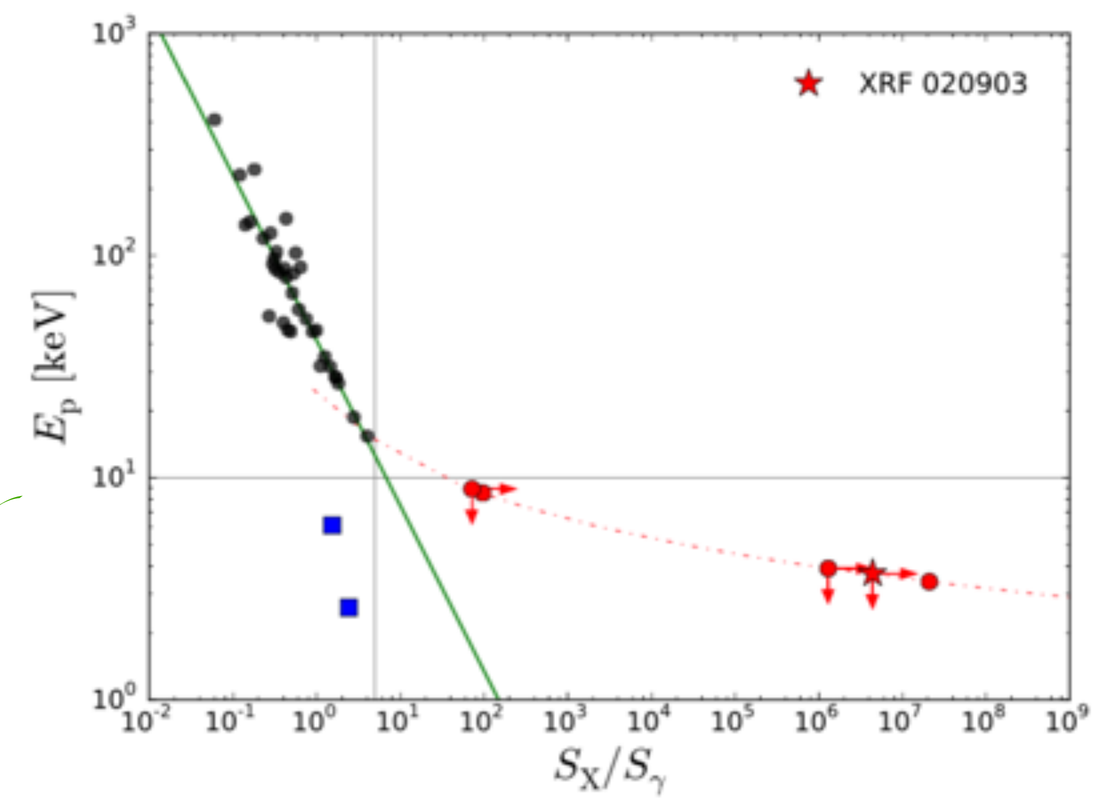


- spindown-powered X-ray emission from long-lived NSs matches the high-energy emission of soft XRFs (those emitting no gamma-rays)

spectral properties
- lack of gamma-rays
- mainly thermal
- black body T and its evolution

lightcurve properties
- luminosity
- duration





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INFN Istituto Nazionale di Fisica Nucleare

TIFPA Trento Institute for Fundamental Physics and Applications

WORKSHOP

SHORT GAMMA-RAY BURSTS
FROM OBSERVATION TO NUMERICAL SIMULATIONS

University of Trento, 8-9 September 2016
www.unitn.it/event/sgrb2016

INVITED SPEAKERS

Maria Grazia Bernardini (U. Montpellier)
Marica Branchesi (U. Urbino)
Pablo Cerda-Duran (U.Valencia)
Paolo D'avanzo (INAF Brera)
Bruno Giacomazzo (U.Trento)
Wolfgang Kastaun (U.Trento)
Kenta Kiuchi (U. Kyoto)
Paul O'Brien (U. Leicester)
Tsvi Piran (Hebrew U. Jerusalem)
Stephan Rosswog (U. Stockholm)
Ruben Salvaterra (INAF IASF)
Daniel Siegel (Columbia)
Giulia Stratta (U. Urbino)

ORGANIZERS

Riccardo Ciolfi (U.Trento)
Giancarlo Ghirlanda (INAF Brera)

Summary

- **long-lived NS** is a **very likely outcome of BNS mergers** and its EM emission can be extremely valuable to understand NS properties and SGRBs

EM counterpart point of view

see talk by Siegel on Thursday, C2 session

- X-ray signal powered by the spin-down of the long-lived NS remnant represents and **ideal counterpart to the GW signal of BNS mergers**

SGRB point of view

- long-lasting X-ray afterglows challenge the standard BH-torus picture and baryon pollution challenges the alternative magnetar model
- the time-reversal scenario offers a possible way out
- at the same time it gives a better match for the X-ray afterglow light curves

References

Ciolfi R. & Siegel D.M. (2015a) ApJ Letters 798, L36
Short gamma-ray bursts in the 'time-reversal' scenario

Ciolfi R. & Siegel D.M. (2015b) in "*Swift: 10 Years of Discovery*", PoS(SWIFT 10)108
Short gamma-ray bursts from binary neutron star mergers: the time-reversal scenario

Siegel D.M., Ciolfi R., Rezzolla L. (2014) ApJ Letters 785, L6
Magnetically driven winds from differentially rotating neutron stars and X-ray afterglows of short gamma-ray bursts

Siegel D.M. & Ciolfi R. (2015) in "*Swift: 10 Years of Discovery*", PoS(SWIFT 10)169
Magnetically-induced outflows from binary neutron star merger remnants

Siegel D.M. & Ciolfi R. (2016a) ApJ 189, 14
Electromagnetic emission from long-lived binary neutron star merger remnants I: formulation of the problem

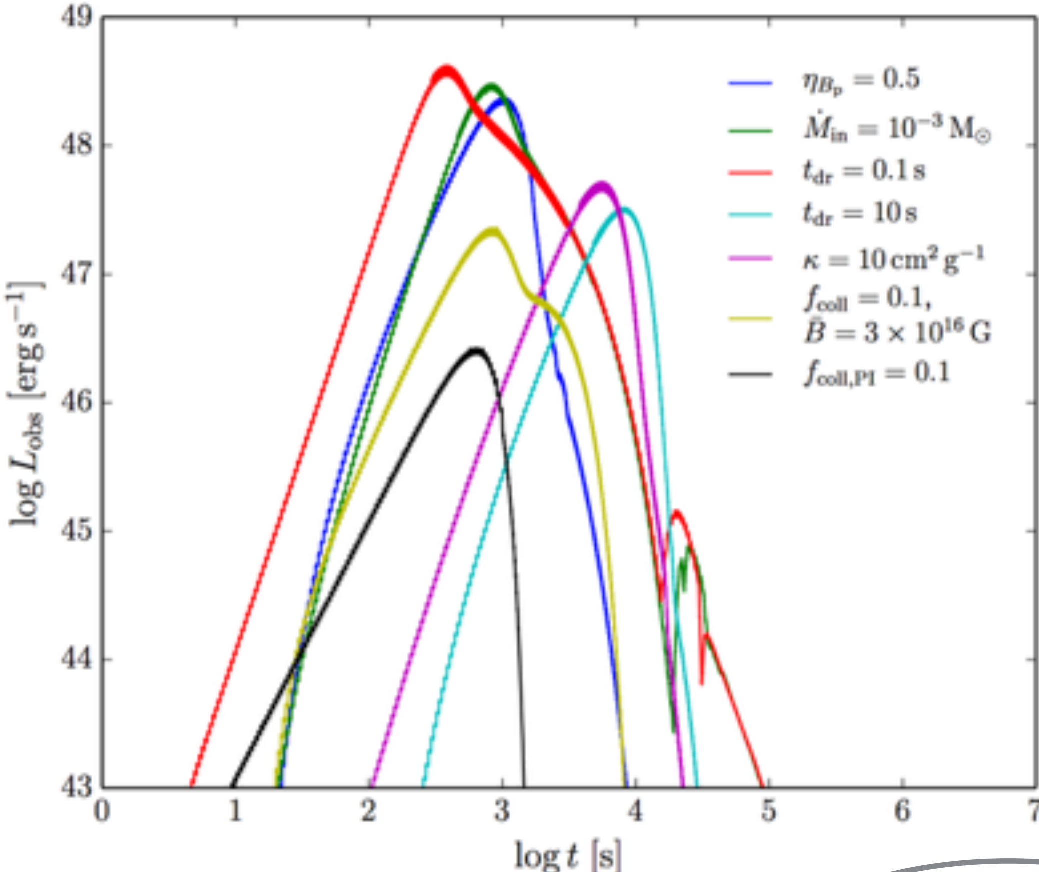
Siegel D.M. & Ciolfi R. (2016b) ApJ 189, 15
Electromagnetic emission from long-lived binary neutron star merger remnants II: light curves and spectra

Ciolfi R. (2016) submitted to ApJ Letters, ArXiv:1606.01743
X-ray flashes powered by the spindown of long-lived neutron stars

BACKUP SLIDES

EM emission from the long-lived NS remnant

Siegel & Ciolfi 2016a, 2016b

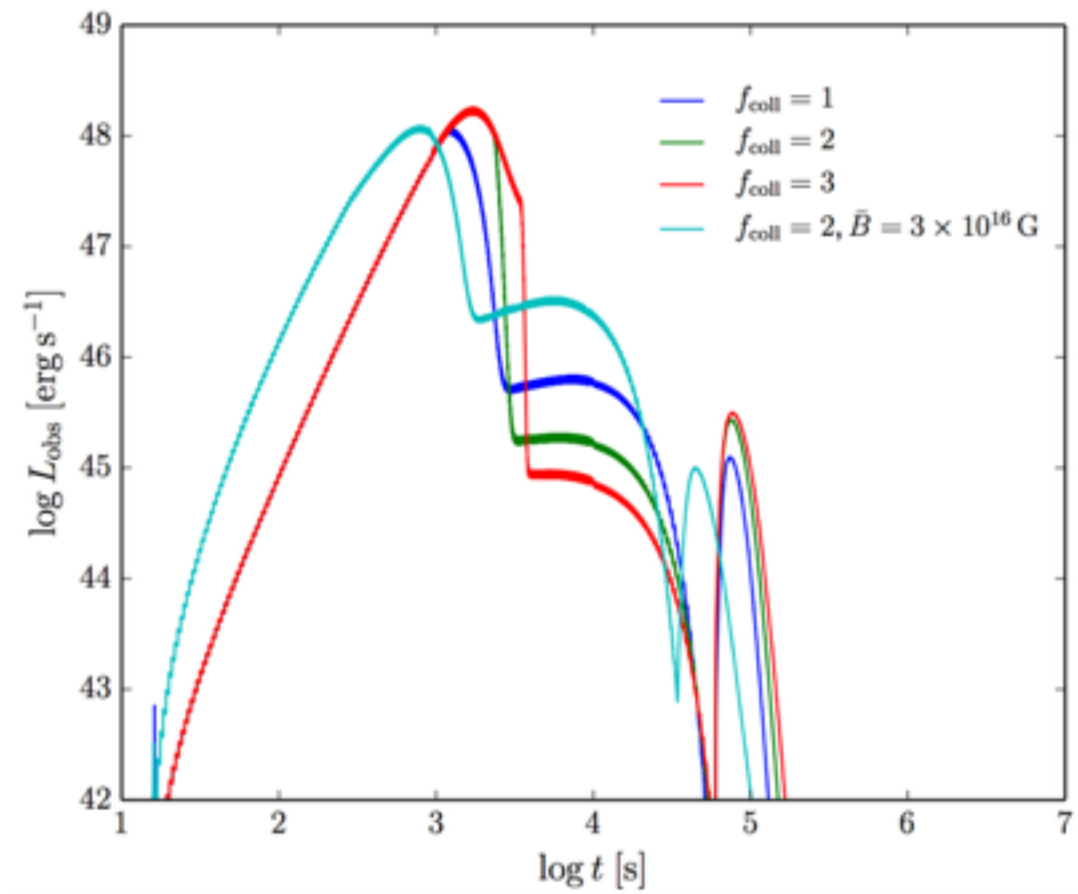


- signal peaks at 10^2-10^4 s (similar range for duration), with ~10-100 s delayed onset
- luminosities 10^{46}-10^{49} erg/s
- mostly in the soft X-rays

Luminosity in soft X-ray band (0.3-10 KeV)

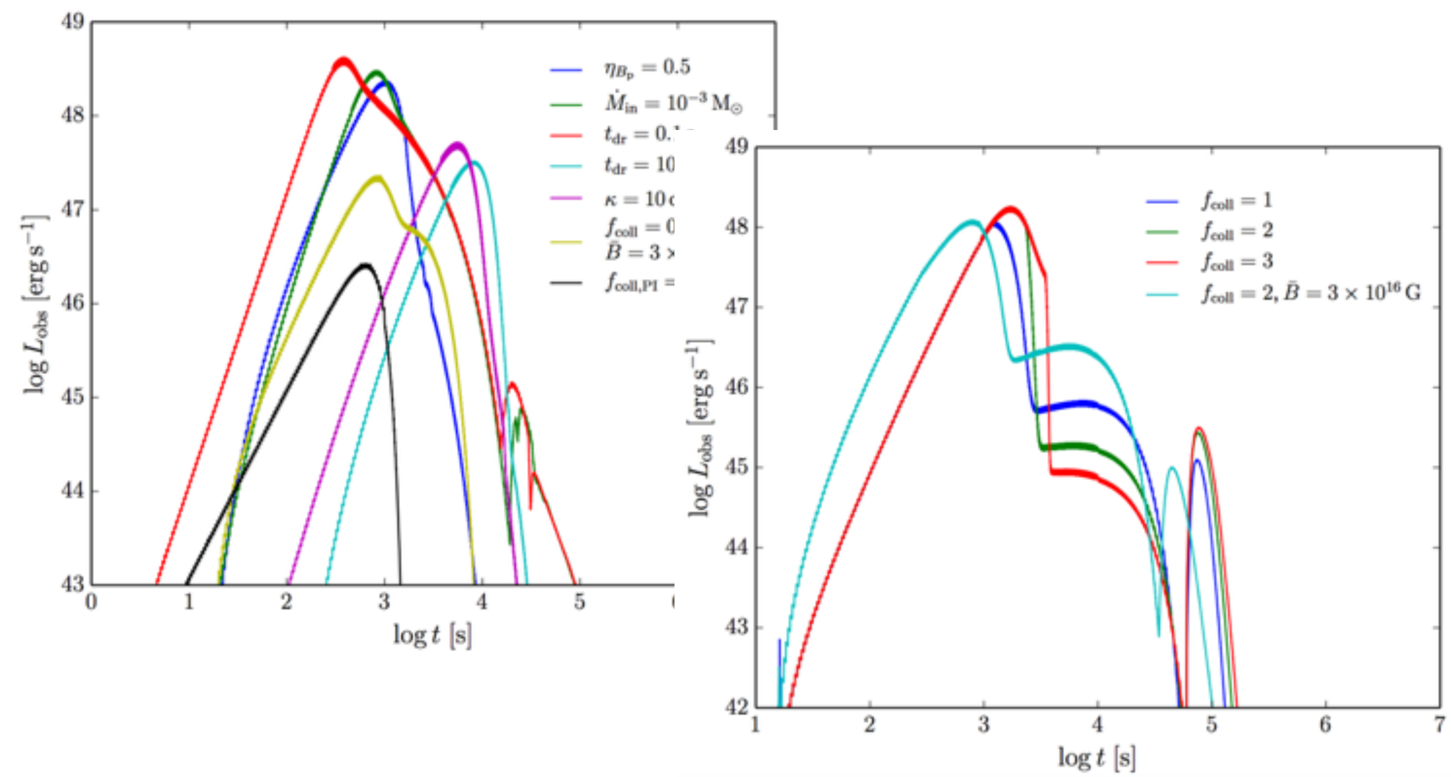
non-collapsing models

collapsing models



Comparison with SGRB afterglows

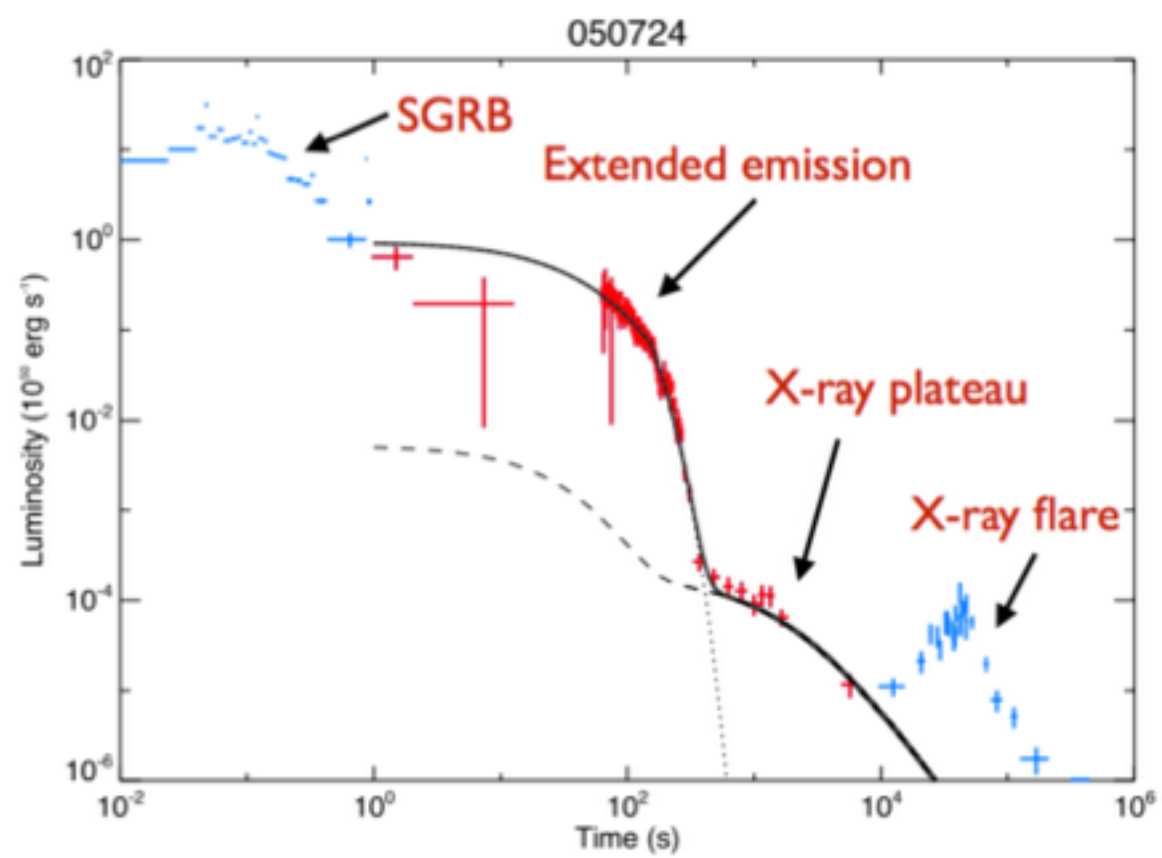
broad characteristics
in good agreement



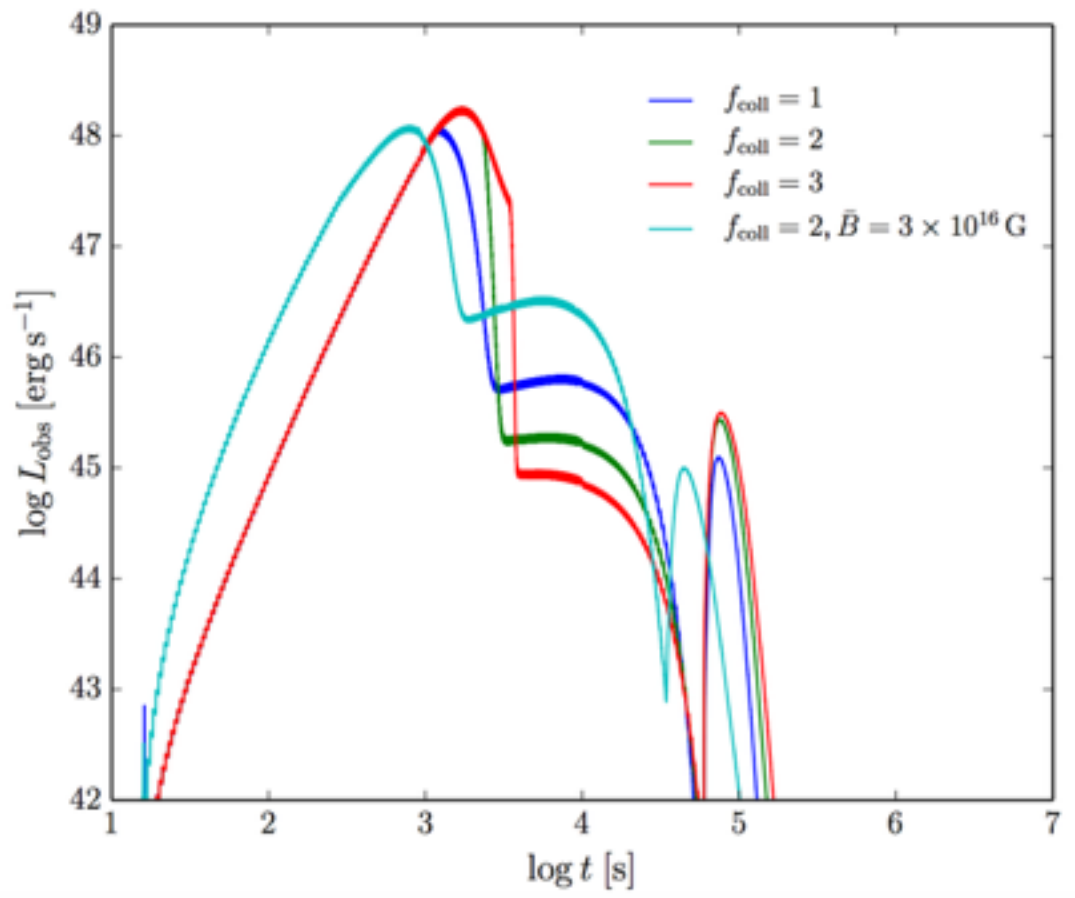
- signals cover the right band (soft X-rays)
- very nice match with range of durations and luminosities

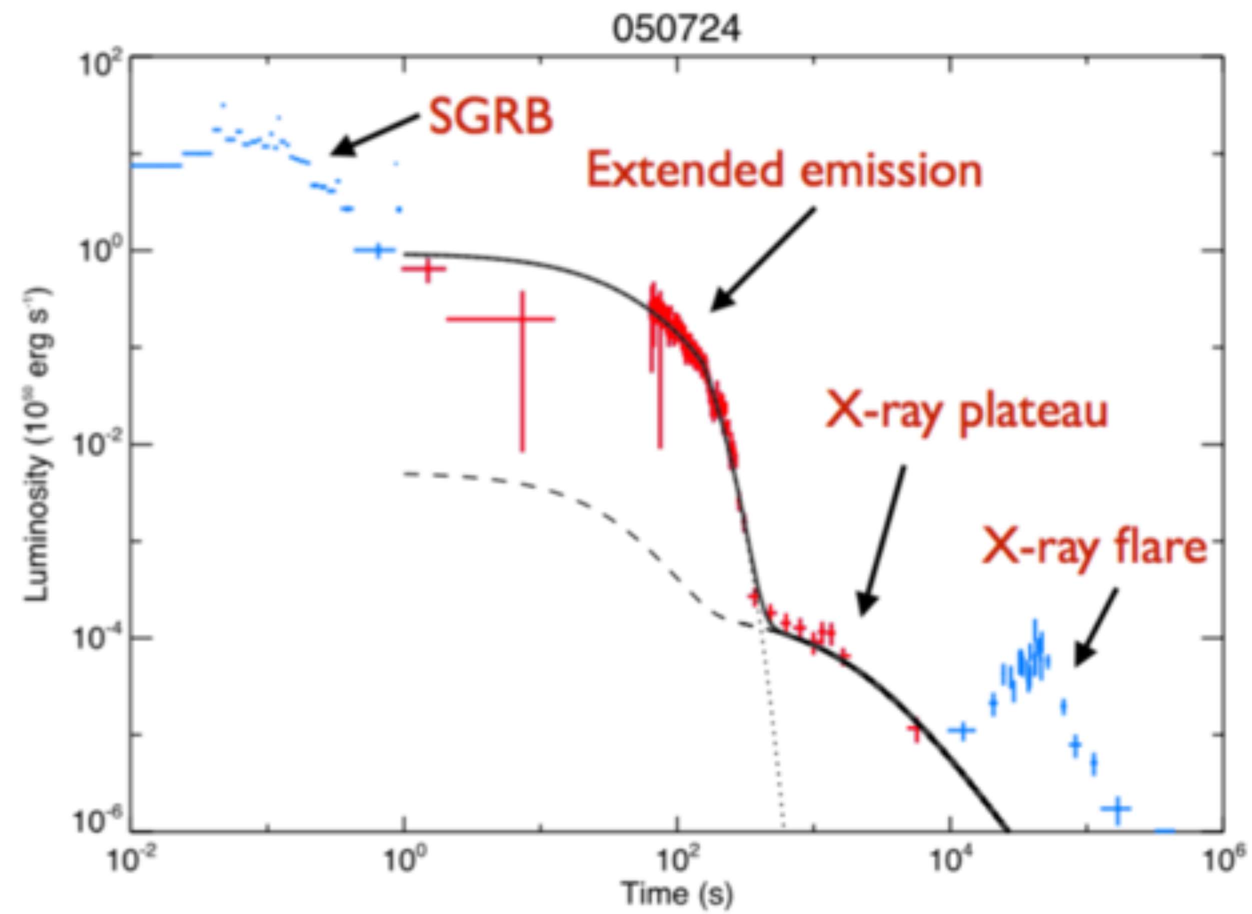
but with a closer look..

- no observations of early rising
- second plateau explained only for collapsing models
- flares explained as transition to optically thin ejecta?



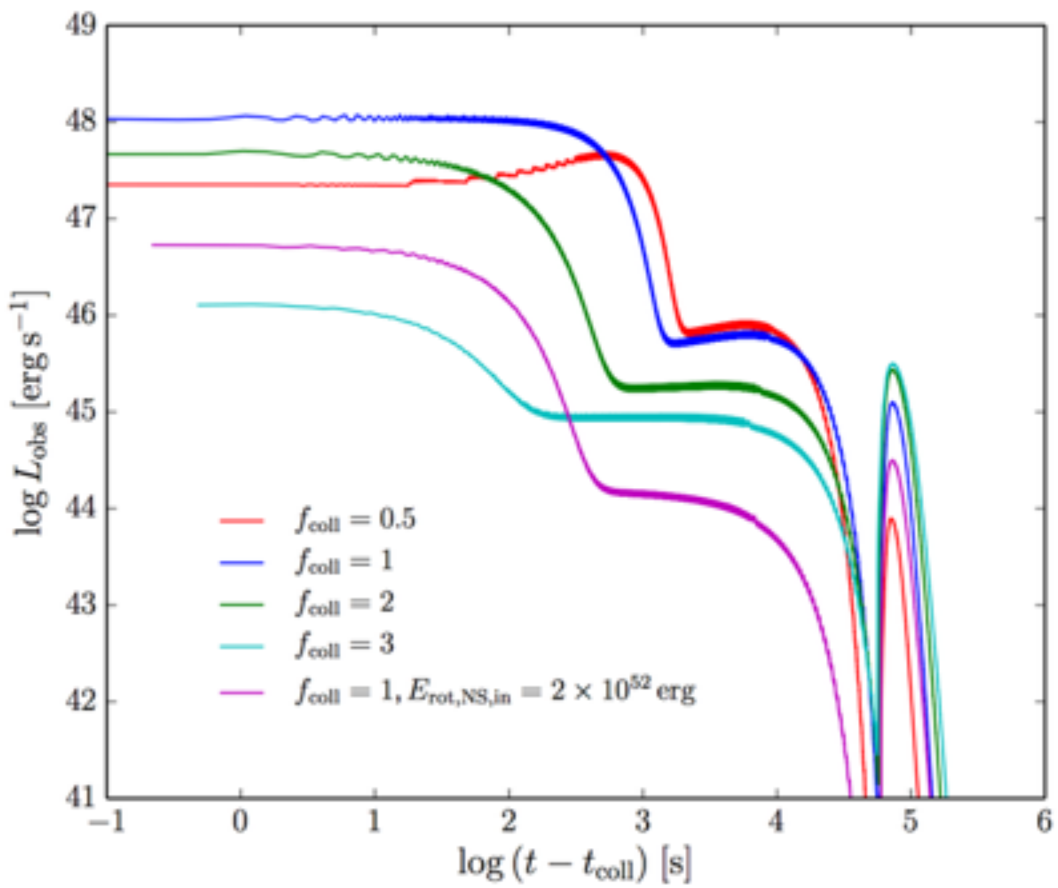
TR scenario - implications

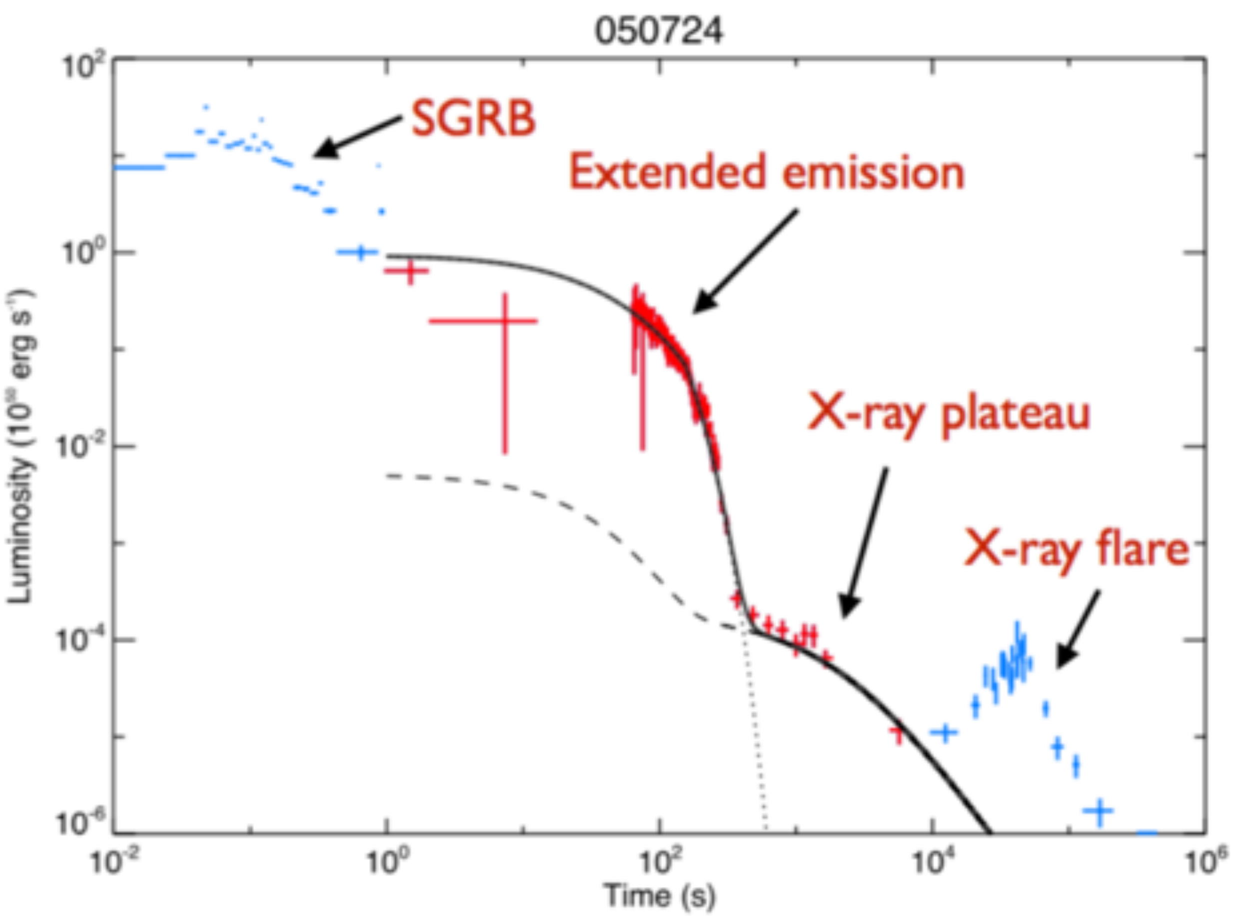




afterglows as seen by the observer assuming **SGRB** (trigger) **at merger**

TR scenario - implications

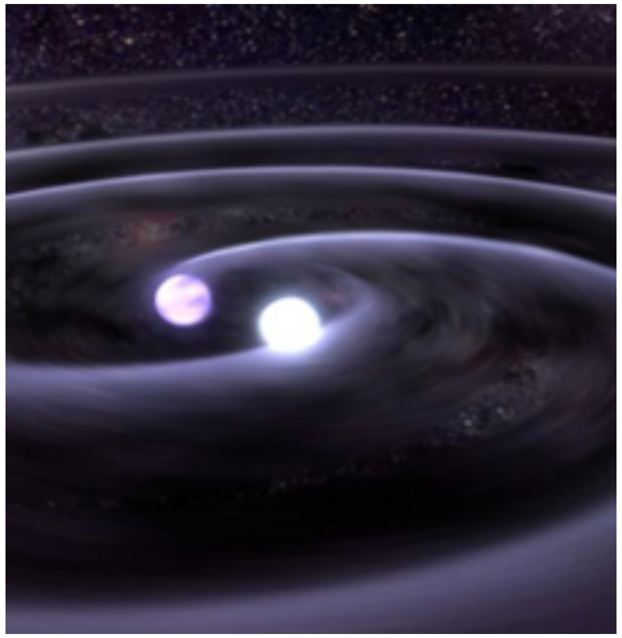




afterglows as seen by the observer assuming **SGRB** (trigger) **at collapse**

improved match assuming the time-reversal scenario!

BNS mergers and EM counterparts



binary neutron star (BNS) mergers and
neutron star-black hole (NS-BH) binary mergers

most promising gravitational wave sources
for advanced LIGO and Virgo

	detection rate	best expectation
BNS	~(0.4-400)/yr	~40/yr
NS-BH	~(0.2-300)/yr	~10/yr

Abadie et al. 2010

rewards of a combined
GW-EM detection:

- observed EM signals would incredibly enhance the chances of GW detection
- EM follow-up observations of a detected GW source is the ultimate way to unravel the nature of the system, by providing crucial and complementary information
- joint GW-EM signals can confirm the astrophysical origin of short gamma-ray bursts

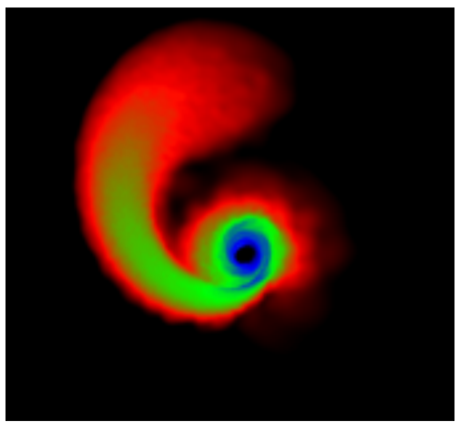
SGRBs as EM counterparts

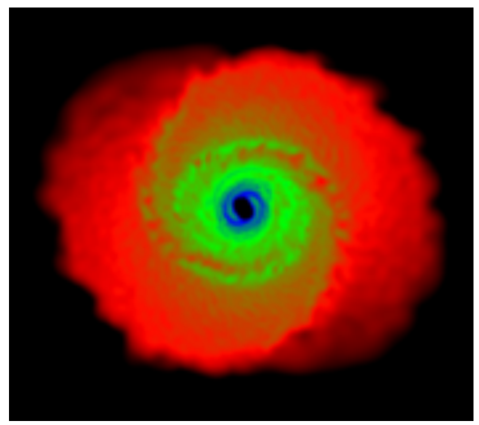
	BRIGHT	ISOTROPIC	LONG-LASTING	HIGH OCCURRENCE	DISTINGUISH BNS AND NS-BH
SGRB	✓	✗	✗	✗	✗

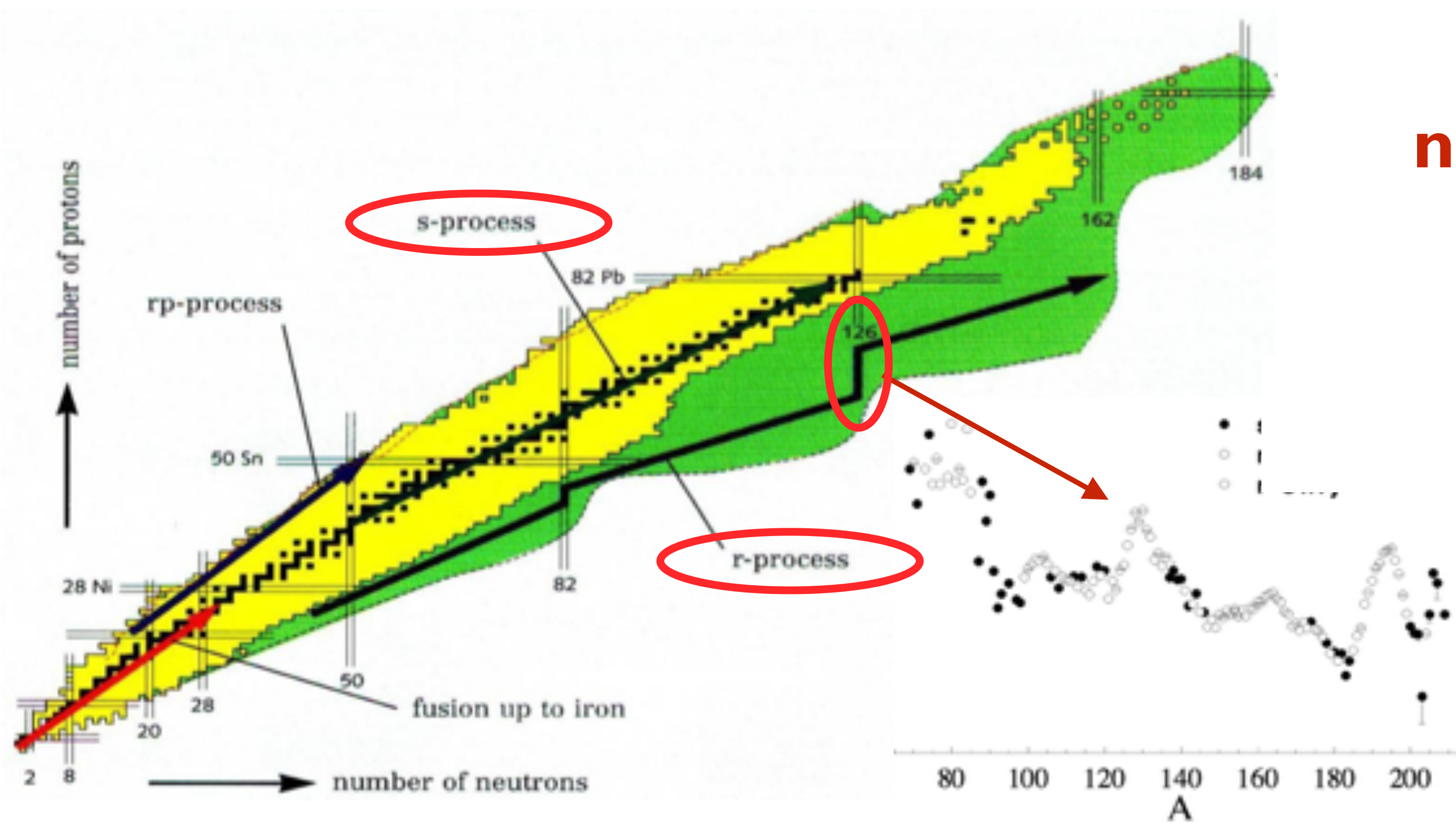
not very promising as EM counterpart

Merger ejecta and r-process nucleosynthesis

ejecta in BNS and NS-BH mergers







curtesy of A. Arcones

heavy element abundances

r-process

capture rate much faster than decay
more than one neutron capture at a time
requires very special conditions:

- High T (T > 10^9 K)
- High neutron density ($n_n > 10^{22}$ cm^{-3})

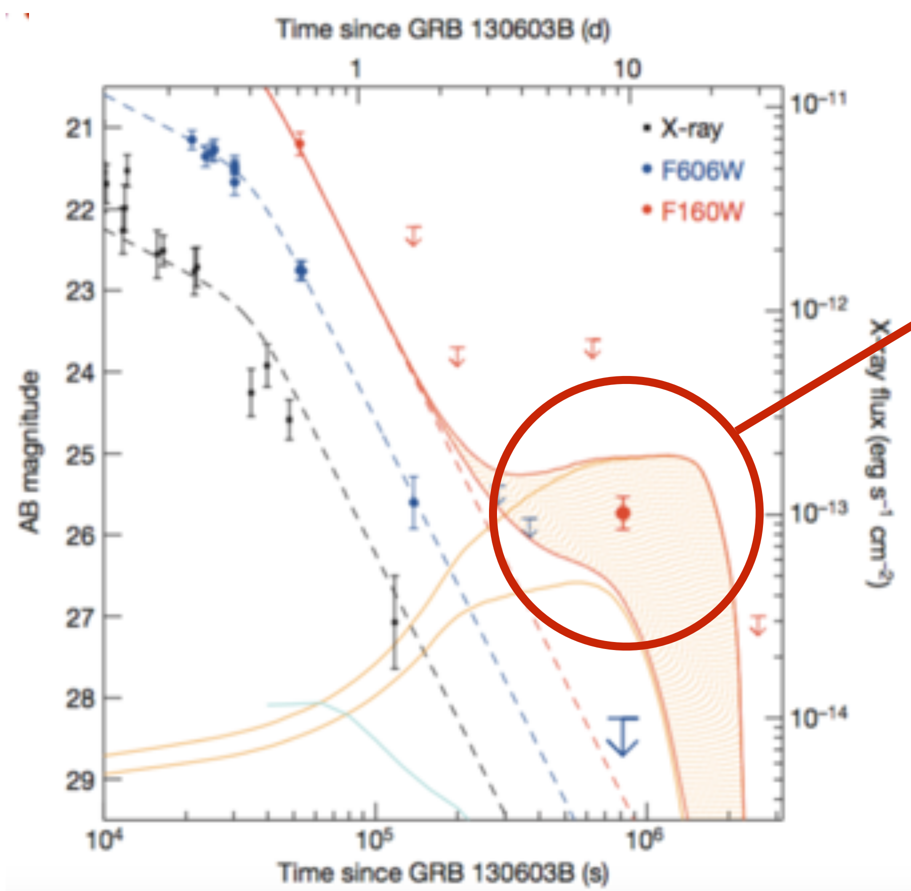
nucleosynthesis of heavy nuclei

initially unstable

radioactive decay on timescales of >days

optical signal!

Kilonovae as EM counterparts



optical rebrightening in GRB 130603B tentatively interpreted as a kilonova

connection

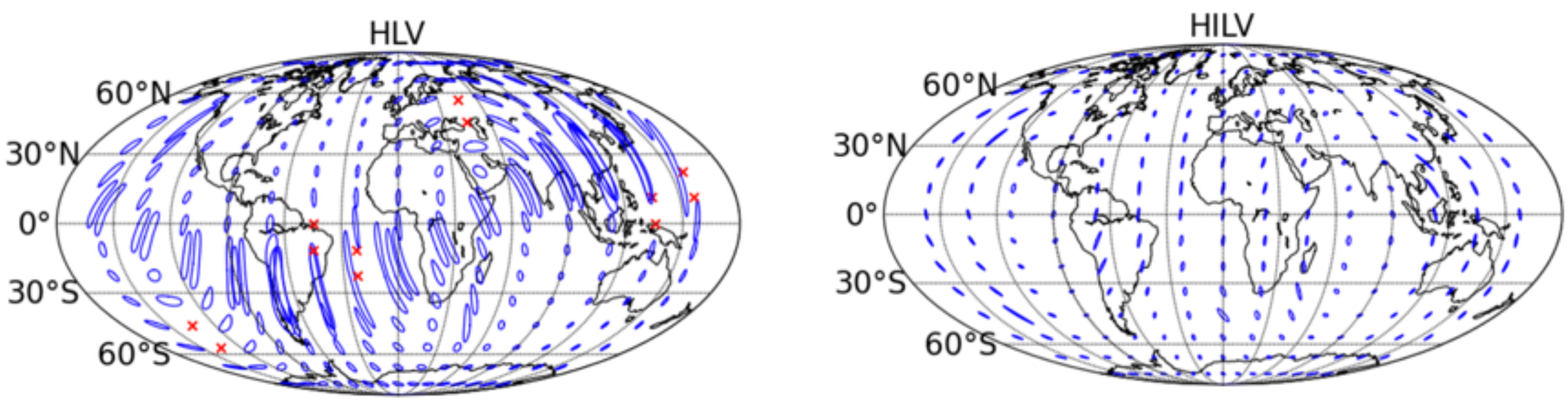
SGRB ⟷ BNS or NS-BH mergers

Tanvir et al. 2013, Berger et al. 2013

	BRIGHT	ISOTROPIC	LONG-LASTING	HIGH OCCURRENCE	DISTINGUISH BNS AND NS-BH
SGRB	✓	✗	✗	✗	✗
KILONOVA	✓ ✗	✓	✓	✓	✗

nice EM counterpart!

GW detector network



3 detectors

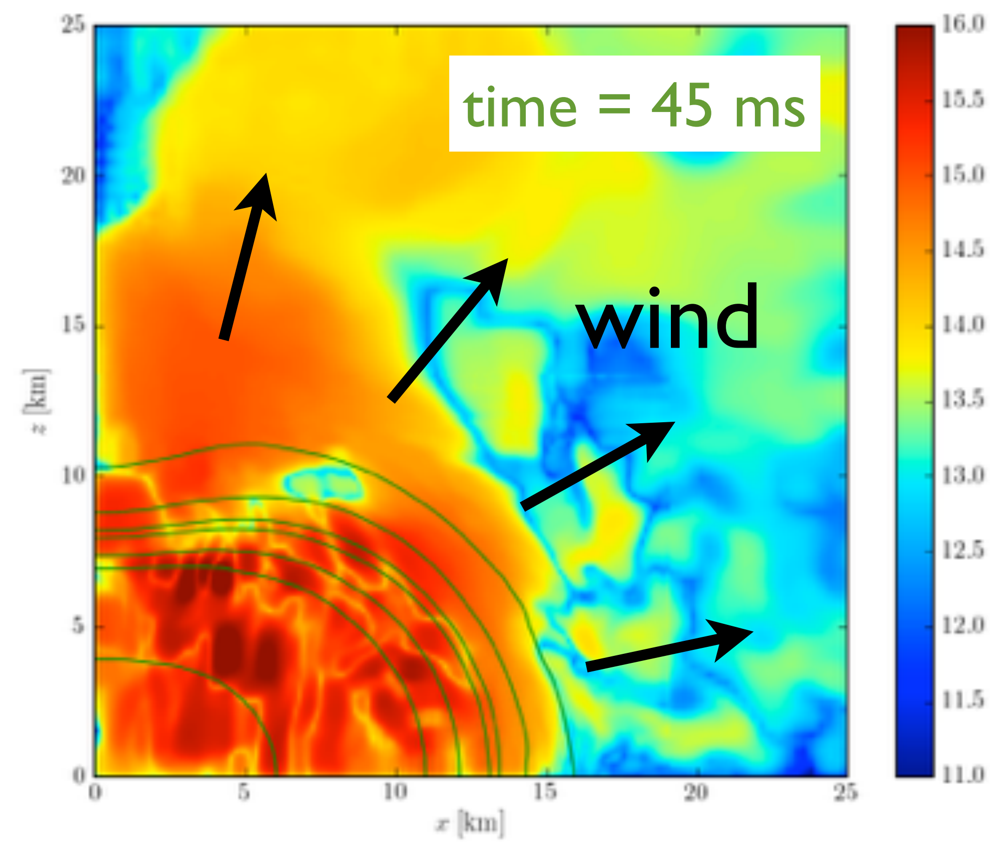
4 detectors

sky localization (90% confidence level)

Baryon-loaded wind

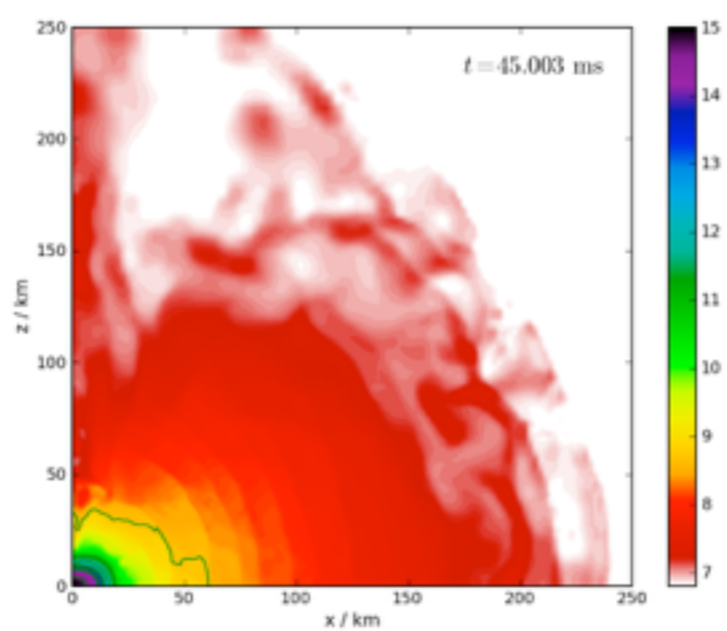
- rest-mass density of the wind $\rho \sim 10^8 \ \mathrm{g/cm^3}$
- ejection speed $v \lesssim 0.1 \ \mathrm{c}$
- mass loss rate $\dot{M} \sim 10^{-3} \ \mathrm{M_\odot/s}$
- mostly isotropic!

rand

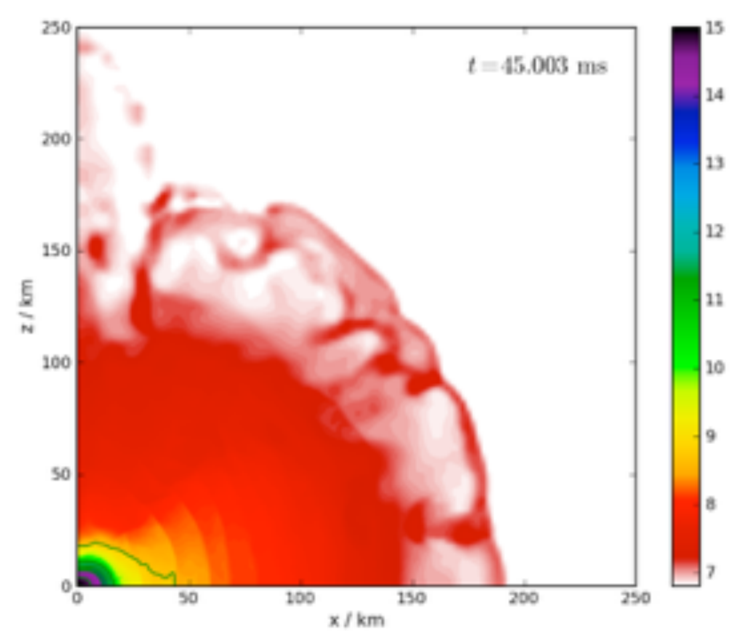


rest-mass density evolution ↓

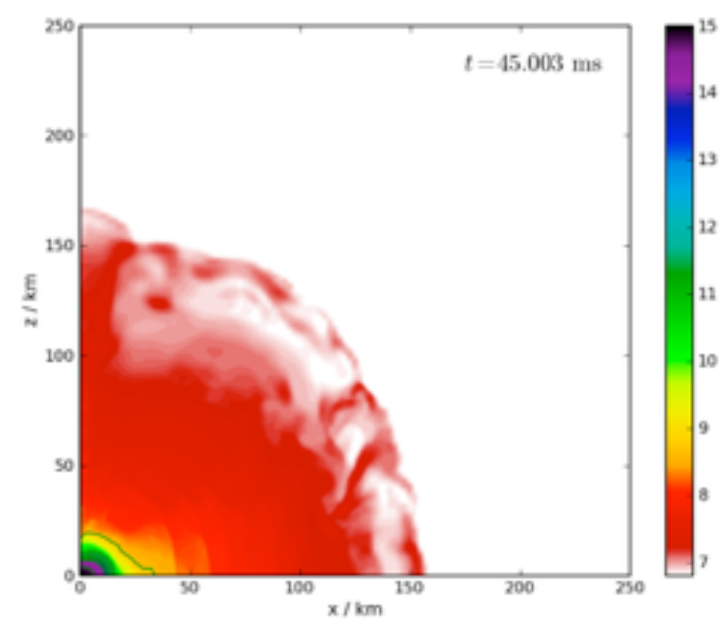
dipole 60



dipole 6

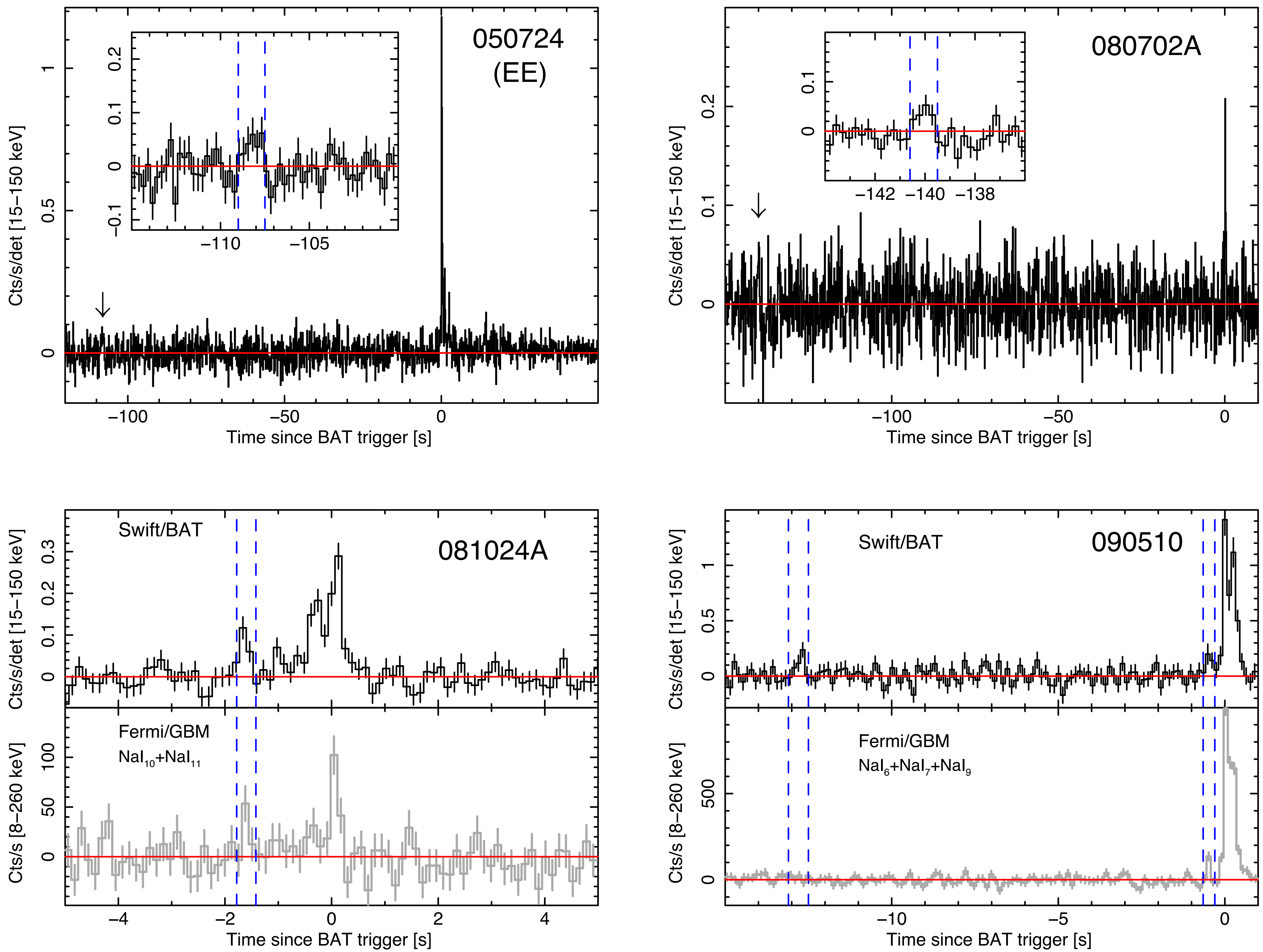


rand



SGRB precursors

Troja et al. 2010

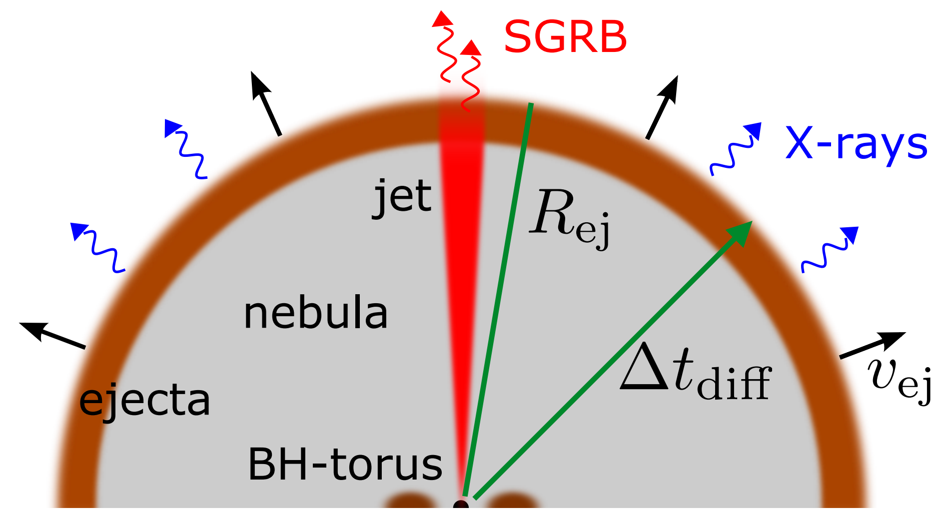


Timing argument

The scenario cannot hold unless the maximum delay is at least as large as the observed afterglow duration

- from observations: $t_{\rm coll} \gtrsim t_{\rm sd}$
- typically: $t_{\rm sd} \gg t_{\rm dr} + \Delta t_{\rm shock}$

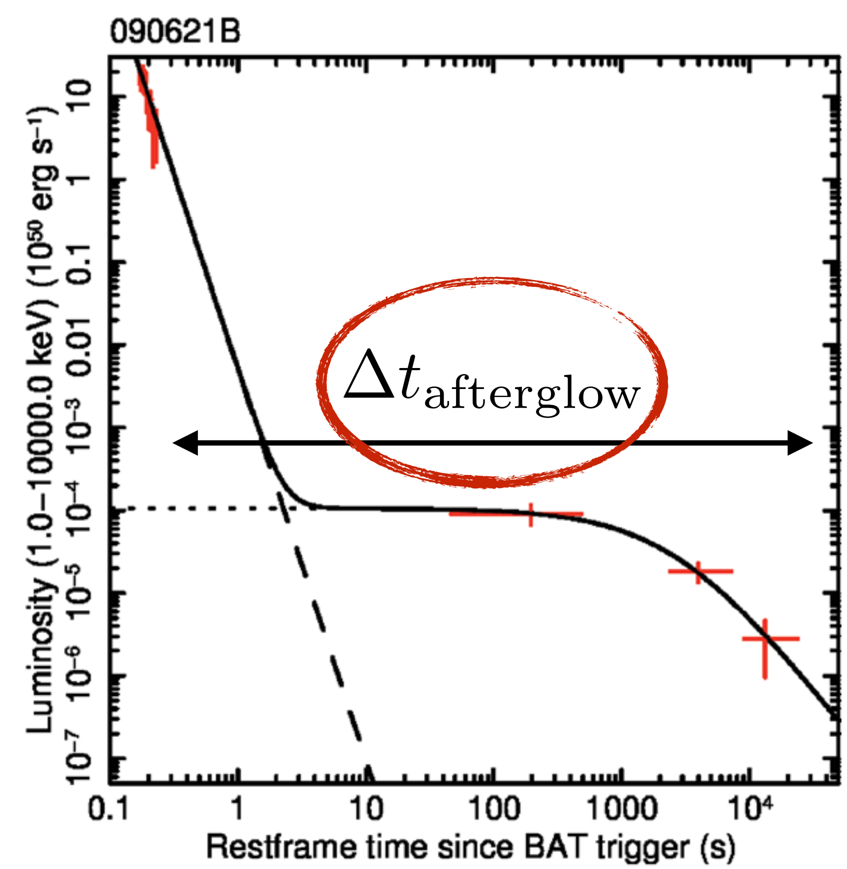
  $\longrightarrow$ at $t_{\rm coll}$ ejecta matter is swept up into thin shell



- delay for a photon emitted just before collapse ("last spin-down photon"):

$$t_{\rm NS}^{\rm delay} = \Delta t_{\rm diff} - \frac{R_{\rm ej}(t_{\rm coll} + \Delta t_{\rm diff})}{c}$$

- Delay of jet is negligible (very low densities at $t_{\rm coll}$)



Timing criterion

$$t_{\rm NS}^{\rm delay} \simeq t_{\rm NS}^{\rm delay} - t_{\rm jet}^{\rm delay} \gtrsim \Delta t_{\rm afterglow}$$

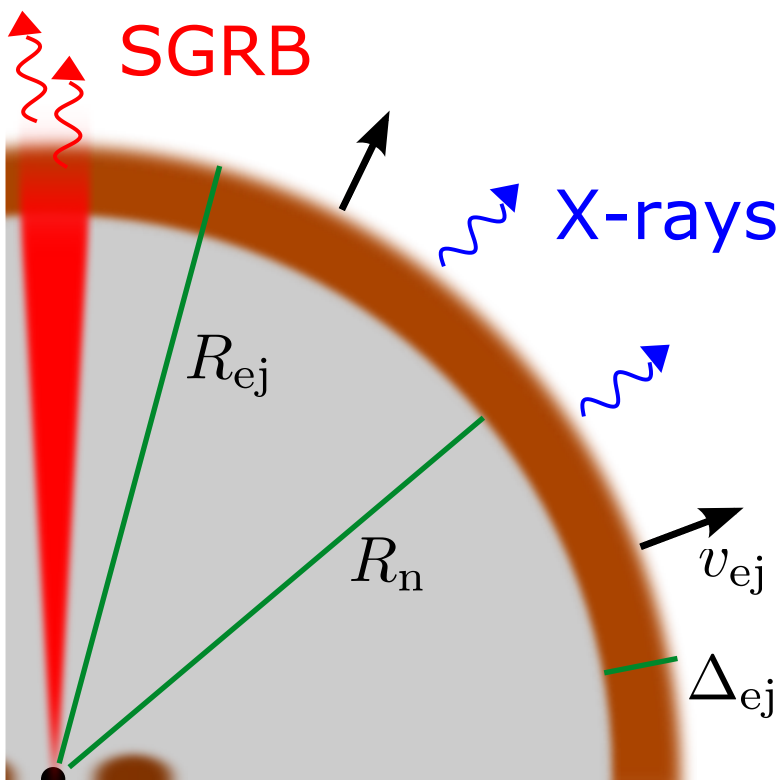
Diffusion timescales

- for static ejecta:

$$t_{\rm diff}^{\rm ej,\ stat}(t) = \frac{\Delta_{\rm ej}}{c}(1+\kappa\rho_{\rm ej}(t)\Delta_{\rm ej}) \propto t^{-2}$$

- for static nebula:

$$t_{\rm diff}^{\rm n,stat}(t) = \frac{R_{\rm n}(t)}{c}\left(1+\sqrt{\frac{4Y\sigma_{\rm T}L_{\rm sd}(t)}{\pi R_{\rm n}(t)m_{\rm e}c^3}}\right) \propto t^{-1/2}$$



$$t_{\rm NS}^{\rm delay} \lesssim t_{\rm diff}^{\rm ej,stat}(t_{\rm coll}) + t_{\rm diff}^{\rm n,stat}(t_{\rm coll}) - R_{\rm ej}(t_{\rm coll})/c$$

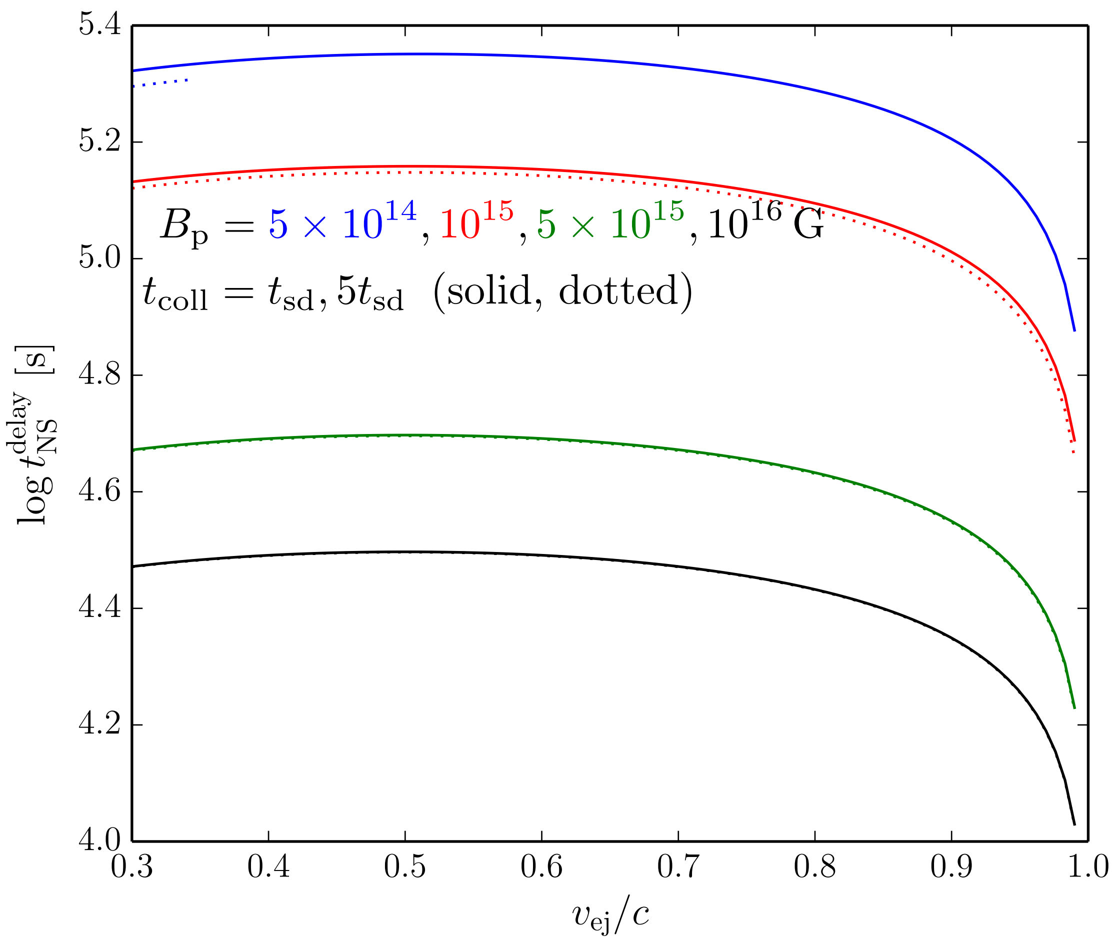
$$t_{\rm NS}^{\rm delay} \gtrsim t_{\rm diff}^{\rm ej,stat}(t^*_{\rm coll}) + t_{\rm diff}^{\rm n,stat}(t^*_{\rm coll}) - R_{\rm ej}(t^*_{\rm coll})/c$$

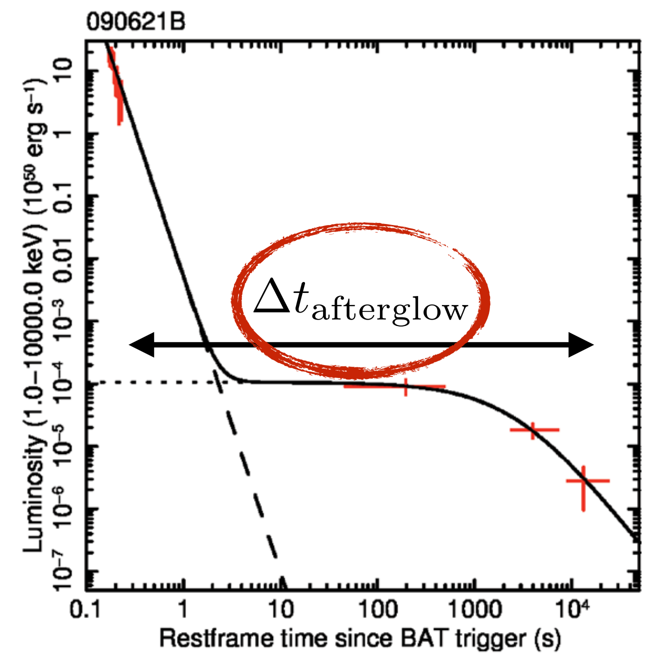
solve iteratively for $t^*_{\rm coll}$:

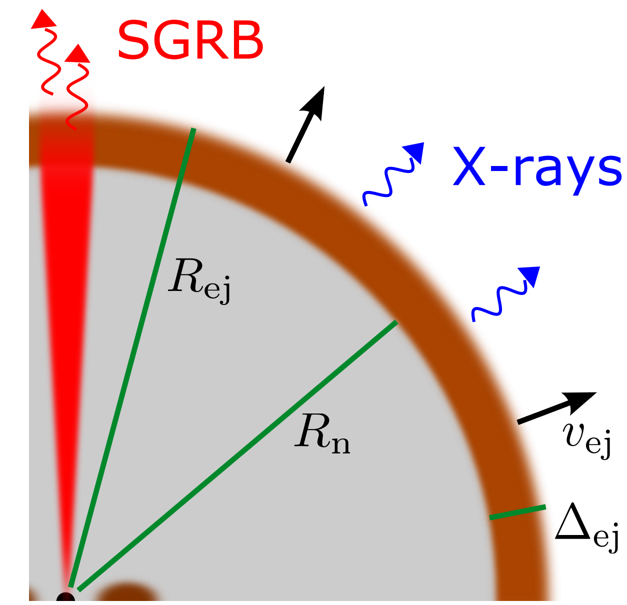
$$t^*_{\rm coll} = t_{\rm coll} + t_{\rm diff}^{\rm ej,\ stat}(t^*_{\rm coll}) + t_{\rm diff}^{\rm n,stat}(t^*_{\rm coll})$$

⟶ use lower limit to check the timing criterion

Results on delay estimation







parameter ranges:

$$B_p \sim 10^{14} - 10^{16}\,\mathrm{G}$$
$$P_{in} \sim 0.5 - 5\,\mathrm{ms}$$
$$t_{dr} \sim 0.1 - 10\,\mathrm{s}$$
$$\dot{M} \sim 10^{-4} - 10^{-2} M_\odot\,\mathrm{s}^{-1}$$
$$\Delta t_{shock} \sim 0 - 100\, t_{dr}$$
$$v_{ej}^0 \sim 0.01 - 0.1\, c$$

- for parameter ranges considered:

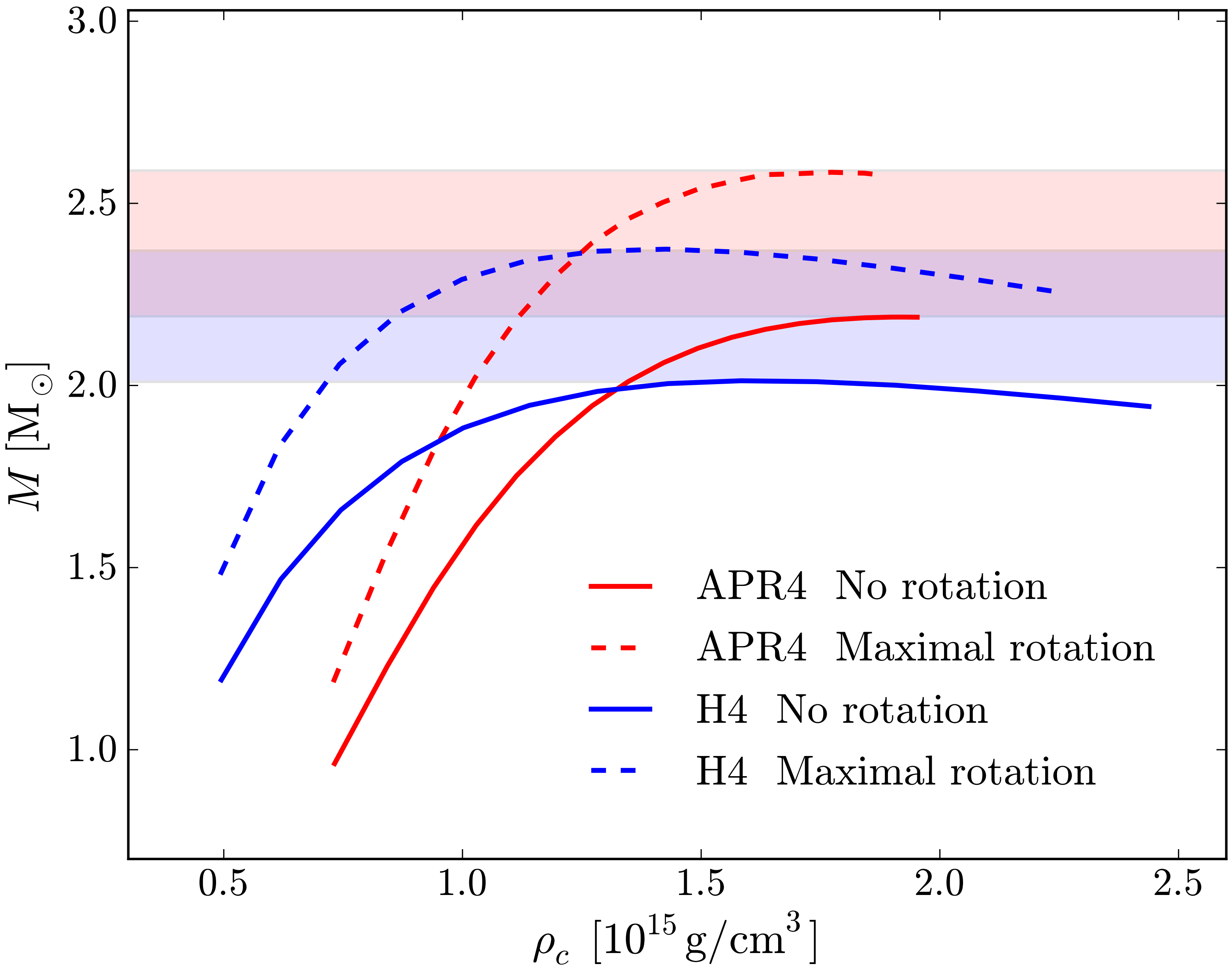
$$t_{NS}^{delay} > 3\times10^4\,\mathrm{s}$$
$$t_{NS}^{delay} \gtrsim 10^5\,\mathrm{s} \quad (B_p \lesssim 10^{15}\,\mathrm{G})$$

generally: $t_{NS}^{delay} \gtrsim \Delta t_{afterglow}$

→ "time-reversal" scenario compatible with observations

EOS constraint for a SMNS

Ciolfi & Siegel 2015b



most likely progenitor mass combination

$1.3 - 1.4\ M_\odot$

Belczynski et al. 2008

↓

$M \sim 0.9(M_1 + M_2 - 0.1)$

↓

$M \sim 2.34\ M_\odot$

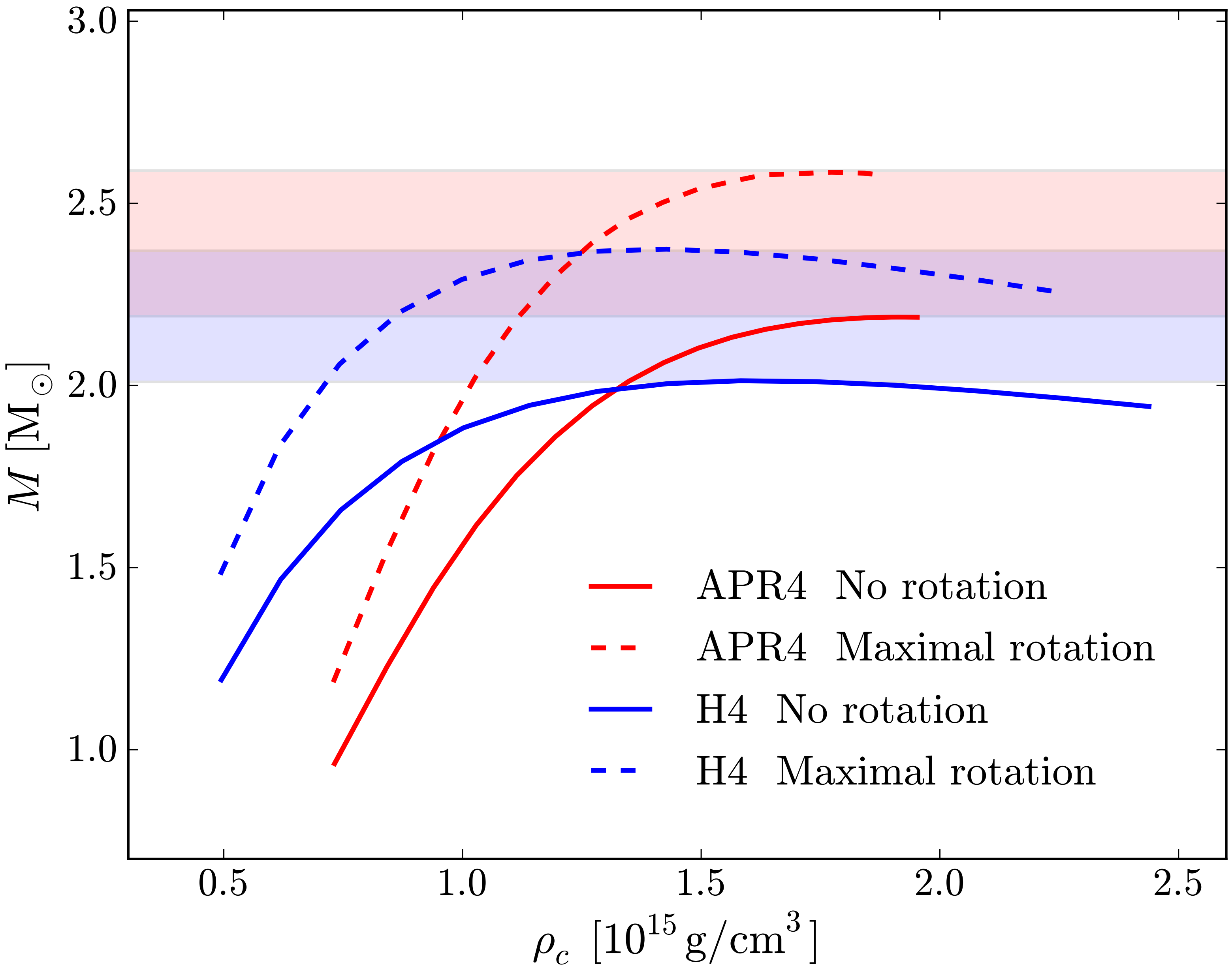
$(\ M_b \sim 2.87\ M_\odot\ )$

APR4 ✓

H4 ✓

EOS constraint for a SMNS

Ciolfi & Siegel 2015b



progenitor
mass combination

$$1.4 - 1.4\ \mathrm{M}_\odot$$

↓

$$M \sim 0.9(M_1 + M_2 - 0.1)$$

↓

$$M \sim 2.43\ \mathrm{M}_\odot$$

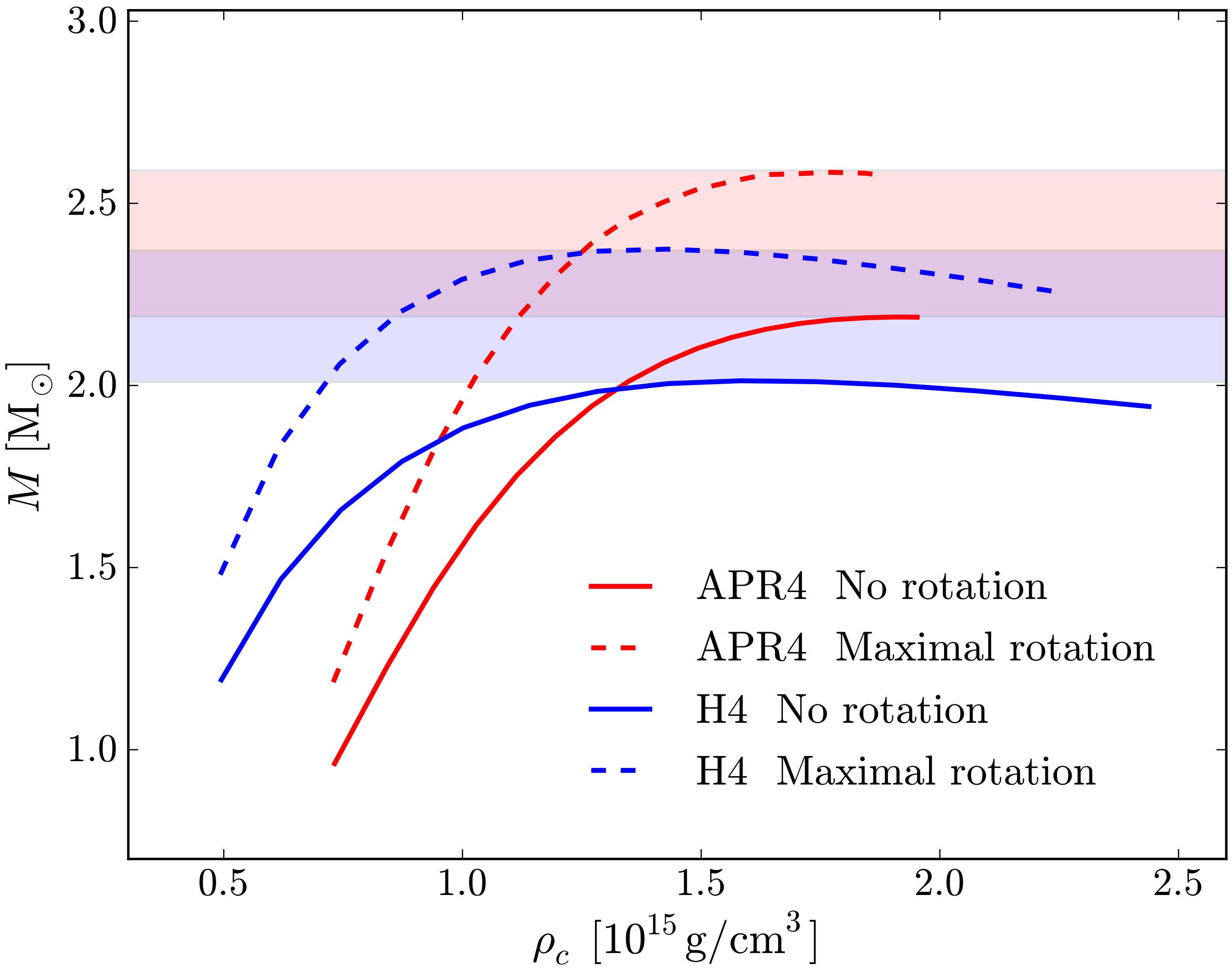
$$(\ M_b \sim 2.98\ \mathrm{M}_\odot\ )$$

APR4 ✓

H4 ✗

EOS constraint for a SMNS

Ciolfi & Siegel 2015b



progenitor mass combination

$$1.5 - 1.5\ \mathrm{M}_\odot$$

↓

$$M \sim 0.9(M_1 + M_2 - 0.1)$$

↓

$$M \sim 2.61\ \mathrm{M}_\odot$$

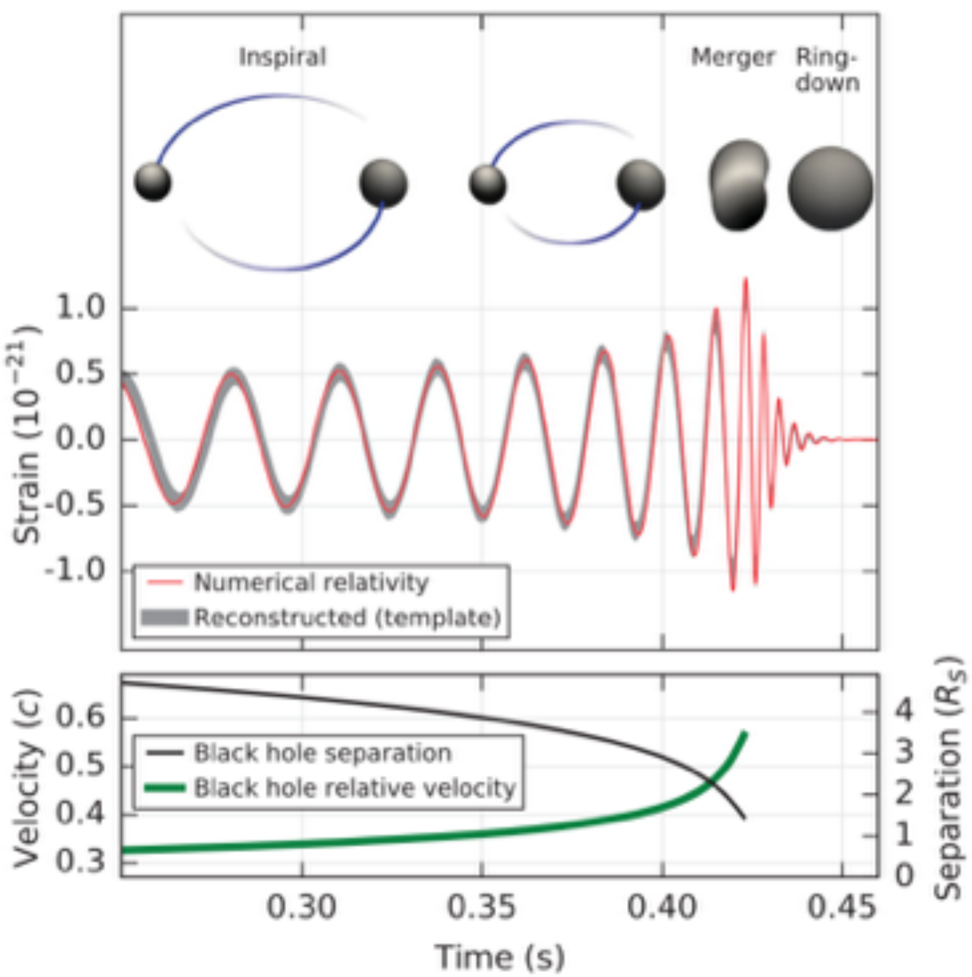
$$(\ M_b \sim 3.2\ \mathrm{M}_\odot\ )$$

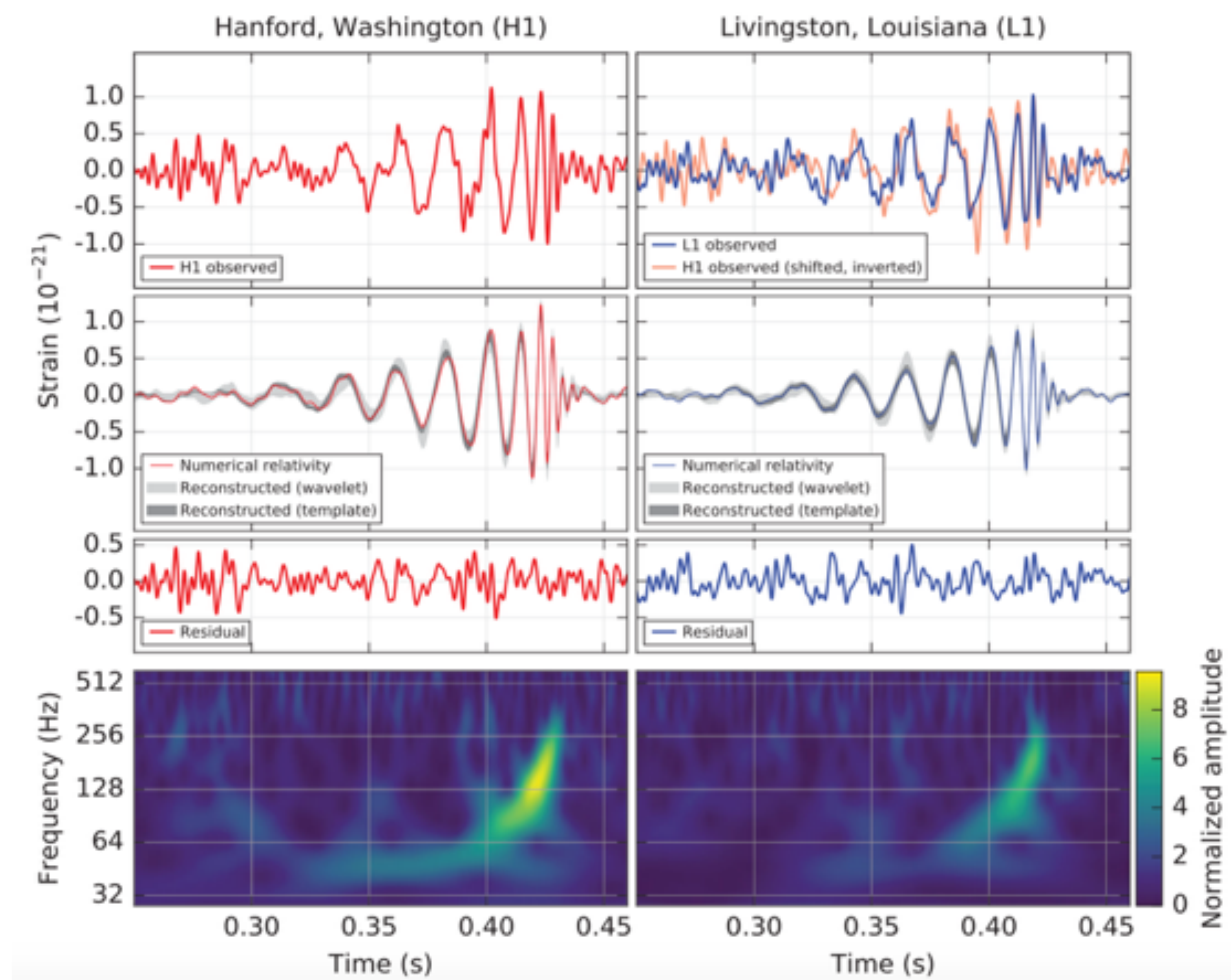
APR4 ✗

H4 ✗

FIRST GRAVITATIONAL WAVE DETECTION

LIGO Scientific Collaboration & Virgo Collaboration, PRL 116, 061102 (2016)

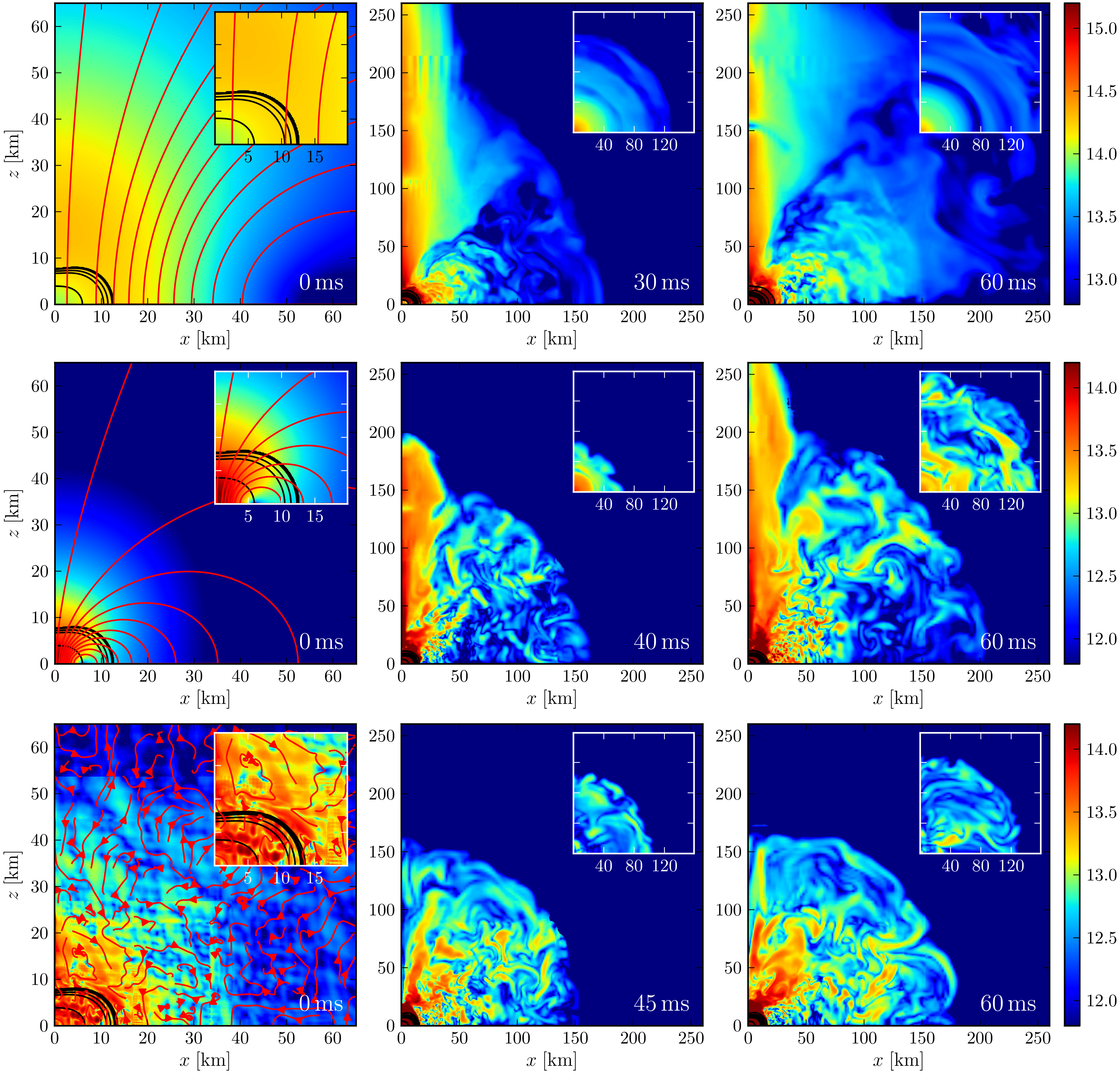




a black hole binary merger!

Phenomenology - phase I

Siegel et al. 2014
Siegel & Ciolfi 2015a



60 ms evolution
for 3 geometries

dipole 60
dipole 6
random

differential rotation powers **baryon-loaded** and **magnetized** outflow

for all MF geometries the outflow has an **isotropic** component

collimation depends strongly on MF geometry

Phenomenology - phase II-III

- uniformly rotating NS emits spin-down radiation and inflates a photon-pair nebula

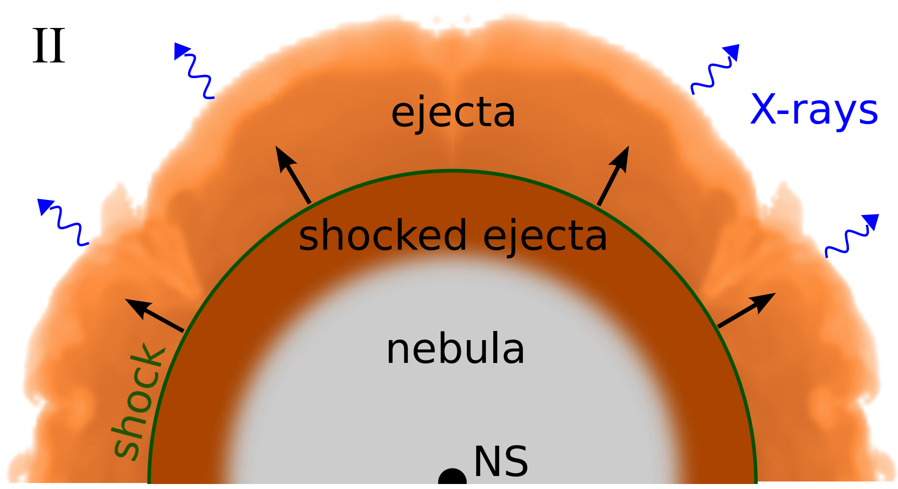
$$L_{\rm sd} \simeq 1.5\times10^{49} B_{\rm p,15}^2 R_6^3 P_{\rm in,-3}^{-4} (1+t/t_{\rm sd})^{-2}\ {\rm erg\ s^{-1}}$$

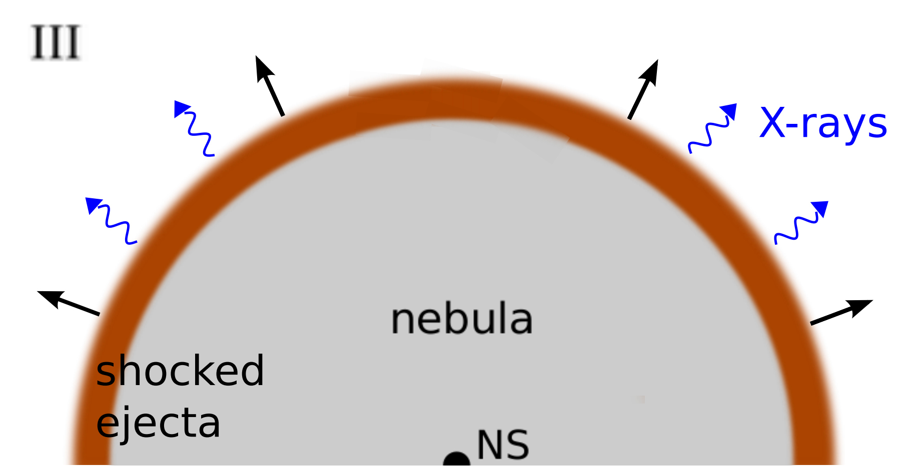
$$t_{\rm sd} \simeq 2.7\times10^3 B_{\rm p,15}^{-2} R_6^{-3} P_{\rm in,-3}^{2}\ {\rm s}$$

- high photon pressure drives a strong shock through the ejecta, sweeps up material into a thin shell

- nebula energy rapidly heats up and accelerates the ejecta shell (up to mildly relativistic speeds)

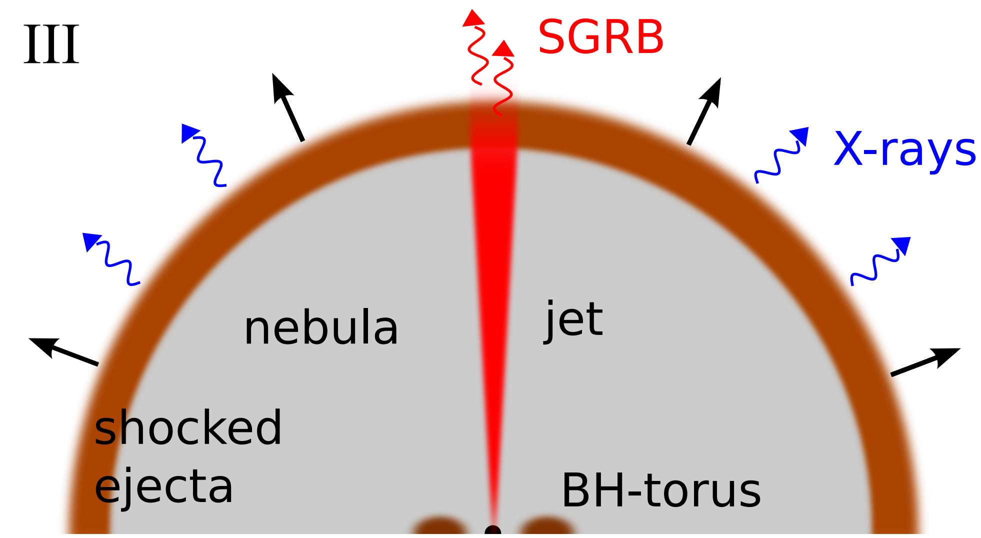
Metzger & Piro 2014





“Time-reversal” scenario

Ciolfi & Siegel 2015a, ApJ Letters 798, L36



- at $t_{\rm coll} \sim t_{\rm sd}$ the NS collapses to a BH-torus system

  ⟶ transient jet is formed in $\lesssim 0.01 - 1\,{\rm s}$ drills through the ejecta and generates the SGRB

- nebula and ejecta represent an optically thick environment

  ⟶ large fraction of spin-down energy is still trapped and diffuses outwards on much longer timescale

  ⟶ spin-down energy acquires substantial delay before emerging and producing the X-rays

EM emission from the long-lived NS remnant

Siegel & Ciolfi 2016a, 2016b

isotropy → 1D model

set of coupled ODEs for the evolution

I

$$\frac{\mathrm{d}R_{\rm ej}}{\mathrm{d}t} = v_{\rm w}(R_{\rm ej}(t), t)$$
$$\frac{\mathrm{d}E_{\rm th}}{\mathrm{d}t} = L_{\rm EM}(t) + \frac{\mathrm{d}E_{\rm th,NS}}{\mathrm{d}t} - L_{\rm rad}(t)$$

II

$$\frac{\mathrm{d}R_{\rm ej}}{\mathrm{d}t} = v_{\rm w}(R_{\rm ej}(t), t)$$
$$\frac{\mathrm{d}R_{\rm sh}}{\mathrm{d}t} = v_{\rm sh}(t)$$
$$\frac{\mathrm{d}R_{\rm n}}{\mathrm{d}t} = \frac{\mathrm{d}R_{\rm sh}}{\mathrm{d}t} - \frac{\mathrm{d}\Delta_{\rm sh}}{\mathrm{d}t}$$
$$\frac{\mathrm{d}E_{\rm th,sh}}{\mathrm{d}t} = \frac{\mathrm{d}E_{\rm sh}}{\mathrm{d}t} + \frac{\mathrm{d}E_{\rm th,vol}}{\mathrm{d}t} + \frac{\mathrm{d}E_{\rm PWN}}{\mathrm{d}t} - L_{\rm rad,in}(t)$$
$$\frac{\mathrm{d}E_{\rm th,ush}}{\mathrm{d}t} = -\frac{\mathrm{d}E_{\rm th,vol}}{\mathrm{d}t} - L_{\rm rad}(t)$$
$$\frac{\mathrm{d}E_{\rm th}}{\mathrm{d}t} = \frac{\mathrm{d}E_{\rm th,sh}}{\mathrm{d}t} + \frac{\mathrm{d}E_{\rm th,ush}}{\mathrm{d}t}$$
$$\frac{\mathrm{d}E_{\rm nth}}{\mathrm{d}t} = -\frac{E_{\rm nth}}{R_{\rm n}}\frac{\mathrm{d}R_{\rm n}}{\mathrm{d}t} - \frac{\mathrm{d}E_{\rm PWN}}{\mathrm{d}t} + L_{\rm rad,in}(t) + \eta_{\rm TS}[L_{\rm sd}(t) + L_{\rm rad,pul}(t)]$$
$$\frac{\mathrm{d}E_{B}}{\mathrm{d}t} = \eta_{B_{\rm n}}[L_{\rm sd}(t) + L_{\rm rad,pul}(t)]$$

III

$$\frac{\mathrm{d}v_{\rm ej}}{\mathrm{d}t} = a_{\rm ej}(t)$$
$$\frac{\mathrm{d}R_{\rm ej}}{\mathrm{d}t} = v_{\rm ej}(t) + \frac{1}{2}a_{\rm ej}(t)\mathrm{d}t$$
$$\frac{\mathrm{d}R_{\rm n}}{\mathrm{d}t} = \frac{\mathrm{d}R_{\rm ej}}{\mathrm{d}t}$$
$$\frac{\mathrm{d}E_{\rm th}}{\mathrm{d}t} = [1 - f_{\rm ej}(t)]\frac{\mathrm{d}E_{\rm PWN}}{\mathrm{d}t} - L_{\rm rad}(t) - L_{\rm rad,in}(t)$$
$$\frac{\mathrm{d}E_{B}}{\mathrm{d}t} = \eta_{B_{\rm n}}[L_{\rm sd}(t) + L_{\rm rad,pul}(t)]$$

+

balance equation for photons and particles

↓

$$0 = Q(\gamma) + P(\gamma) + \dot{N}_{\rm C,syn}(\gamma)$$

$$0 = \dot{n}_0 + \dot{n}_{\rm A} + \dot{n}_{\rm C}^{\rm NT} + \dot{n}_{\rm C}^{\rm T} + \dot{n}_{\rm syn} - \frac{c}{R_{\rm n}}n(\Delta\tau_{\rm C}^{\rm NT} + \Delta\tau_{\gamma\gamma}) - \dot{n}_{\rm esc}$$