Modeling the Dispersion & Polarization of Gravitational Waves to Test GR

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Motivation: Generation vs. Propagation

Correcting GR: $T^{\mu\nu} \to \bar{h}^{\mu\nu}_{\mathsf{GR}}$ process has some almost-GR solution,

$$\left(\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} + \epsilon_{\mathsf{gen}} M^{\alpha\beta}{}_{\mu\nu}\right) \bar{h}^{\mathsf{GR}}_{\alpha\beta}$$

with correction of order ϵ_{gen} (GR limit: $\epsilon_{\mathsf{gen}} = 0$) for some tensor $M^{\alpha\beta}{}_{\mu\nu}$.

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- Most non-GR tests look at leading order propagation: $k_\mu k^\mu = 0$.
- Approach: extend almost-GR corrections to the propagation of GWs. Corrections $\mathcal{O}(\epsilon_{\text{prop}})$ governs correction.
- Effects possibly dominating over ϵ_{gen}, i.e., $\epsilon_{\text{prop}} > \epsilon_{\text{gen}}$.

Dispersion & Polarization in GR and non-GR

Previous works have modified *dispersion*,

- Massive graviton, [Will, 1998]
- Massive graviton & Lorentz violating (ppE), [Mirshekari, Yunes, Will, 2012]
- Lorentz violating (SME), [Kostelecký, Tasson, 2015] & [Kostelecký, Mewes, 2016].

GW150914+GW151226: $\lambda_g > 10^{13}\,[\mathrm{km}]$ and $m_g < 1.2 \times 10^{-22}\,[\mathrm{eV}/c^2]$.
[LSC+Virgo, PRL 116, 221101 (2016); LSC+Virgo, arXiv:1606.04856]

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Previous works have modified *polarization*,

- Lorentz violating [Kostelecký, Mewes, 2016],
- Parity violating [Yunes, O'Shaughnessy, et al, 2010],
- Bigravity [Narikawa, Ueno, et al, 2015].

Generalizing the Dispersion

Inspired by electromagnetic dispersion,

$$-\omega^2 + |\vec{k}|^2 = G_0 + \hat{n}_j G_1^j + \hat{n}_i \hat{n}_j G_2^{ij} + \hat{n}_i \hat{n}_j \hat{n}_k G_3^{ijk} + \cdots$$

$$\begin{aligned}
G_0(\omega) &= a + \omega b + \omega^2 c + \cdots \\
G_1^j(\omega) &= \left(a^j + \omega b^j + \omega^2 c^j + \cdots\right) |\vec{k}| \\
G_2^{ij}(\omega) &= \left(a^{ij} + \omega b^{ij} + \omega^2 c^{ij} + \cdots\right) |\vec{k}|^2 \\
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- Expansions in frequency $\omega = 2\pi f$ and $\vec{k} = |\vec{k}|\hat{n}$ projections.
- $a, b, c, \cdots \in \mathbb{C}: \quad h(f) = A(f) \exp[i\Psi(f)]$,
  - Dissipation: $A_{\mathrm{GR}}(f) + \delta A_{\mathrm{nGR}}(f)$,
  - Non-dissipation: $\Psi_{\mathrm{GR}}(f) + \delta \Psi_{\mathrm{nGR}}(f)$.

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Sample of specific theories:

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Modified waveform (non-dissipative)

Wave packets emitted at δt_e intervals.

$$\delta t_a = \delta t_e(1+z) + \int \frac{dt}{a(t)} \left(\delta_\omega(t;\omega_a) - \delta_\omega(t;\omega_a')\right)$$

- Small dimensionless perturbations δ_ω (nGR $\ll$ GR).
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$$\delta\Psi_{\text{nGR}} = \int_{f_c}^{f} \int_{t_e}^{t_a} dt d\tilde{f} \frac{2\pi}{a(t)} \left(\delta_f(t;\tilde{f}) - \delta_f(t;f_c)\right)$$

$f = \omega/2\pi$ and coalescence frequency f_c (for binaries).

Applies to any waveform

Modified waveform (non-dissipative)

$$-\omega^2 + |\vec{k}|^2 = -(m_g^2 + \hat{n}\cdot\vec{v}) \Rightarrow \delta\Psi_{\text{nGR}}(f) = -\left(\frac{\hat{n}\cdot\vec{v}}{4\pi} + \frac{\pi}{\lambda_g^2}\right)\frac{D}{(1+z)}\frac{1}{f}.$$

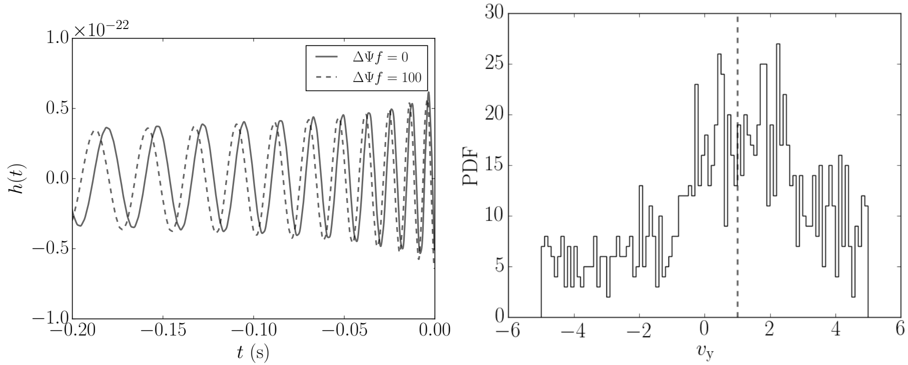

Figure: Left: Illustration of modified phase. Right: Sample of v_y component ($x \times y = z$ with $x \equiv$ vernal equinox & $z \equiv$ north pole. Done with aLIGO noise and IMR PhenomPv2 of no spin. Known source location for mock data.

Generalizing the Polarization

Motivated from position dependence: $a, b, c, \cdots \to a(x), b(x), c(x), \cdots$

- Linear (Crystal-like) and circular (Faraday-type) polarization,

$$\tilde{h}_{L,R}^{\text{emission}} = \tilde{h}_{+}^{\text{emission}} \pm i\tilde{h}_{\times}^{\text{emission}}, \quad \tilde{h}_{L,R}^{\text{arrival}} = \left(\tilde{h}_{+}^{\text{emission}} \pm i\tilde{h}_{\times}^{\text{emission}}\right) e^{ik_{L,R}(\omega) \cdot D}$$

- Mode excitation and mode mixing (analogy: neutrino oscillation):

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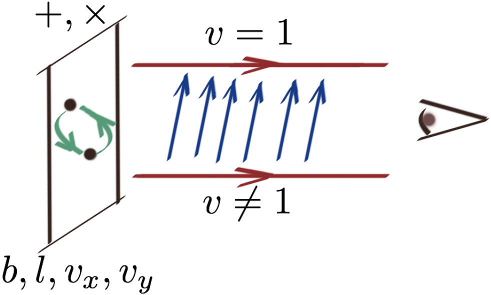


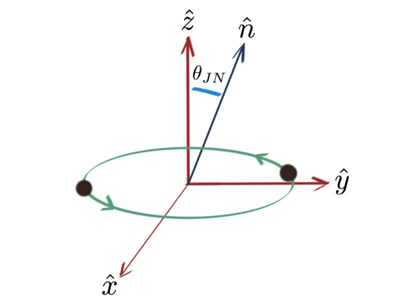
Sources

Compact binaries

Address spin precession, [Chatziioannou, Klein, et al, 2016]. Accurate (θ_{JN}, D).

Continuous waves

- Ensemble N CWs. [Isi, Weinstein, et al, 2015]
- 2+ pulsar comparison. Need to know inclination from E&M.



$$\tilde{h}_+ = \frac{1}{2}(1 + \cos^2\theta_{JN})A(f)e^{-i\Phi(f)}$$
$$\tilde{h}_\times = i\cos\theta_{JN}A(f)e^{-i\Phi(f)}$$

Closing Remarks

$$-\omega^2 + |\vec{k}|^2 = G_0(\omega) + \hat{n}_j G_1^j(\omega) + \hat{n}_i \hat{n}_j G_2^{ij}(\omega) + \cdots$$

1. Map to specific theories.
2. Position dependence:

$$a, b, c, \cdots \to a(x), b(x), c(x), \cdots$$

3. Initial bounds from GW events [Yunes, Yagi, Pretorius, PRD '16] and GW source counts [Calabrese, Battaglia, Spergel CQG '16].
4. Identify degeneracies. Determine best priors on the extra parameters. Determine best way to combine multiple sources.
5. Short-author methods paper with suggestions on future directions (soon). Prepare for O1+O2+O3 analysis with full-author report.

Acknowledgements

- Caltech LIGO-TAPIR Testing GR Group (Max, Leo, Yanbei).
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