

Causal Dynamical Triangulations: a progress report

GR21, New York City,
11 Jul 2016

Renate Loll

Institute for Mathematics, Astrophysics
& Particle Physics, Radboud University

Quantum Gravity, back to basics

Causal Dynamical Triangulations (CDT) is an attempt to bring back quantum gravity into the fold of ordinary quantum field theory, without appealing to some grand unified dynamical principle.

Analogous to QCD on the lattice, CDT uses a lattice regularization to define a theory of quantum gravity nonperturbatively. However, the lattices are dynamical and no lattice/background is distinguished.

As expected, the theory has divergences in the continuum limit as the UV regulator is removed. They must be renormalized appropriately.

A possible scenario to render the theory *nonperturbatively* well-defined and renormalizable is that of “asymptotic safety”.

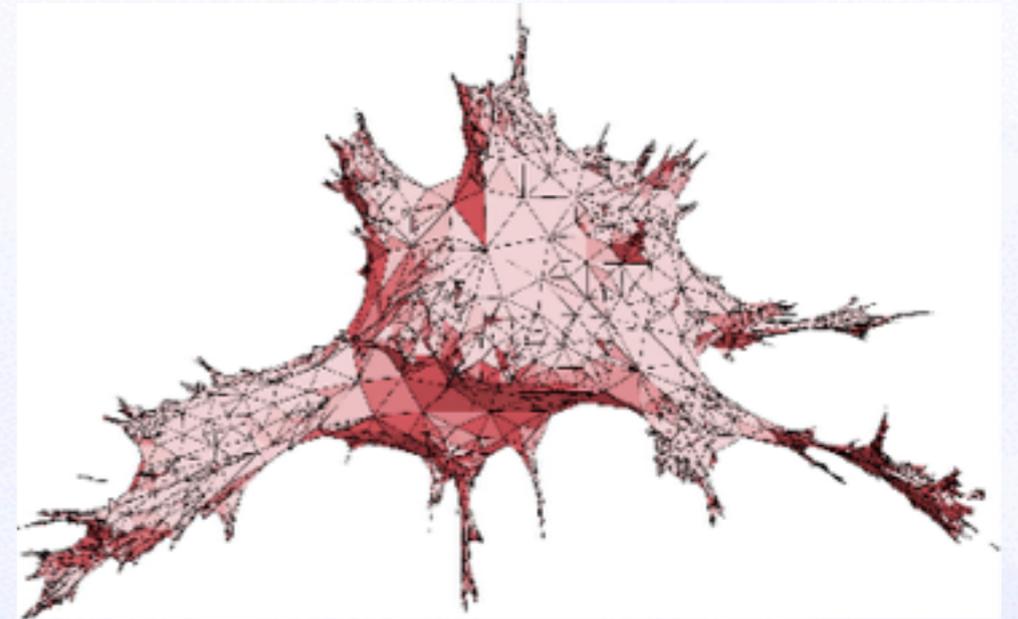
➔ c.f. Reuter’s plenary talk on Wednesday

Quantitative results so far are in a highly quantum fluctuating regime, far away from (semi-)classicality, apart from a few global observables.

The Story of (Causal) Dynamical Triangulations

This approach to quantum gravity (1998) grew out of a confluence of ideas:

- the primacy of *pure geometry* in the sense of Einstein's rods and clocks (measuring distances, not metrics $g_{\mu\nu}$);
- using *powerful numerical methods* to describe such geometry far away from a flat-space, perturbative regime;
- subsequently, the realization that the imposition of a local *causal structure* on path integral histories appears to be necessary to obtain a good classical limit in four dimensions (DT \rightarrow CDT)



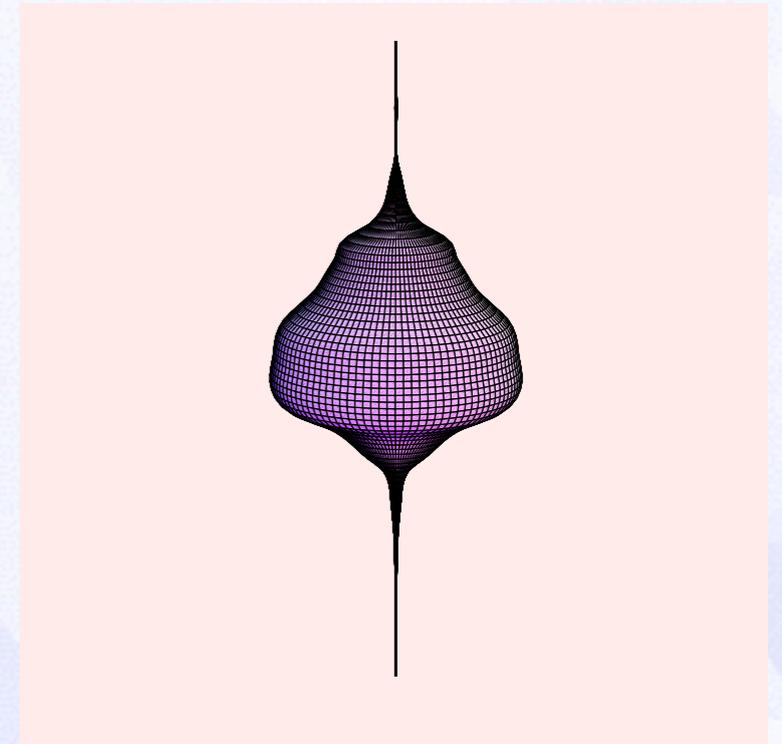
A typical path integral history (glued from triangles in 2d quantum gravity)

(J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, “Nonperturbative Quantum Gravity”, Physics Report 519 (2012) 127 [arXiv: 1203.3591])

The amazing richness of nonperturbative dynamics, uncovered by (C)DT

CDT depends on a minimalist set of ingredients and few free parameters (2), and is conceptually very simple.

However, its nonperturbative dynamics isn't. Extracting it via suitable observables isn't either.



We know little about the quantum dynamics of higher-dimensional geometry “an sich” (after Wick rotation given by statistical partition functions). — How many theories? With what universal properties?

nonperturbative = action vs. measure, energy vs. entropy

classical GR “intuition” not a good guide (e.g. dimensional reduction)

Focus of today's short CDT review talk

- lightning introduction to CDT
- “effective transfer matrix” for CDT
- new quantitative results on the phase structure
- applicability of ‘standard’ renormalization group methods
- Wilson loop observables

Results I will not talk about:

- emergence of semiclassical de Sitter space from “quantum foam”
- scale-dependent dimensionality (Planckian 2 \rightarrow classical 4)
- CDT quantum gravity in lower dimensions (2&3)
- Locally Causal Dynamical Triangulations (enforcing causal structure without global lattice “time”)
- CDT with toroidal spatial slices



Coumbe's talk in this session



Cooperman's poster contribution

Quantum Gravity from CDT★

is a *nonperturbative* implementation of the gravitational path integral,

$$Z(G_N, \Lambda) = \int_{g \in \mathcal{G}} \mathcal{D}g e^{iS_{G_N, \Lambda}^{\text{EH}}[g]}$$

Newton's constant → G_N
 cosmological constant → Λ
 spacetimes $g \in \mathcal{G}$
 Einstein-Hilbert action → $S_{G_N, \Lambda}^{\text{EH}}[g]$

much in the spirit of lattice quantum field theory, but based on *dynamical* triangular lattices, reflecting the dynamical nature of spacetime geometry:

$$Z(G_N, \Lambda) := \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a, N}}} \frac{1}{C(T)} e^{iS_{G_N, \Lambda}^{\text{Regge}}[T]}$$

UV cutoff → a
 # building blocks → N
 inequiv. triangul.s → $T \in \mathcal{G}_{a, N}$
 $C(T)$ → $|\text{Aut}(T)|$

This describes “pure gravity”; inclusion of matter fields is straightforward.

★ recent contributors: J. Ambjørn, D. Benedetti, T. Budd, J. Cooperman, D. Coumbe, B. Durhuus, J. Gizbert-Studnicki, L. Glaser, A. Görlich, J. Henson, A. Ipsen, T. Jonsson, J. Jurkiewicz, N. Klitgaard, A. Kreienbuehl, J. Laiho, T. Sotiriou, Y. Watabiki, S. Weinfurtner, J. Wheeler ...

Quantum Gravity from CDT★

is a *nonperturbative* implementation of the gravitational path integral,

$$Z(G_N, \Lambda) = \int_{g \in \mathcal{G}} \mathcal{D}g e^{iS_{G_N, \Lambda}^{\text{EH}}[g]}$$

Newton's constant $\rightarrow G_N$, cosmological constant $\rightarrow \Lambda$, spacetimes $g \in \mathcal{G}$, Einstein-Hilbert action $\rightarrow S_{G_N, \Lambda}^{\text{EH}}[g]$

much in the spirit of lattice quantum field theory, but based on *dynamical* triangular lattices, reflecting the dynamical nature of spacetime geometry:

$$Z(G_N, \Lambda) := \lim_{\substack{a \rightarrow 0 \\ N \rightarrow \infty}} \sum_{\substack{\text{inequiv.} \\ \text{triangul.s} \\ T \in \mathcal{G}_{a, N}}} \frac{1}{C(T)} e^{iS_{G_N, \Lambda}^{\text{Regge}}[T]}$$

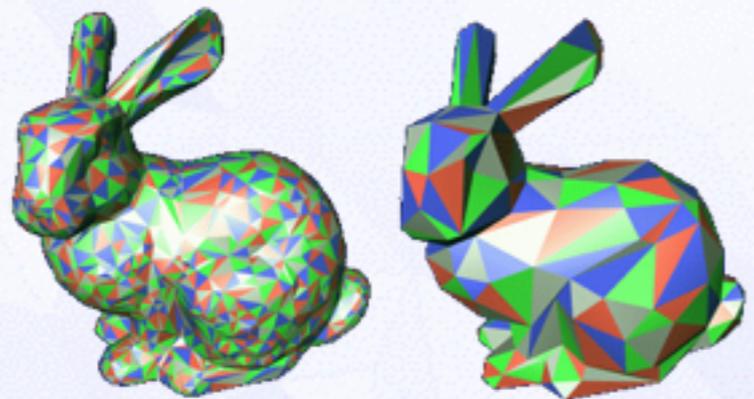
UV cutoff $\rightarrow a$, # building blocks $\rightarrow N$, $C(T) \rightarrow |\text{Aut}(T)|$

This describes “pure gravity”; inclusion of matter fields is straightforward.

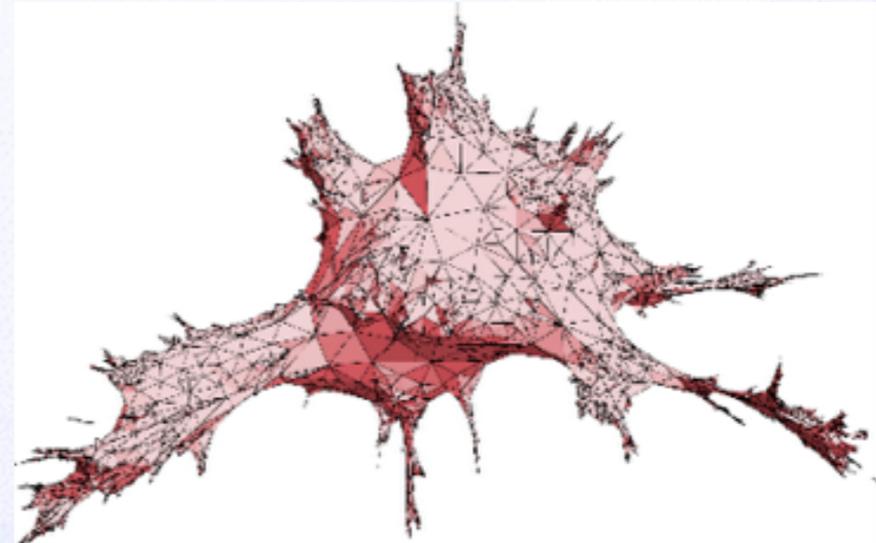
★ recent contributors: [J. Ambjørn](#), [D. Benedetti](#), [T. Budd](#), [J. Cooperman](#), [D. Coumbe](#), [B. Durhuus](#), [J. Gizbert-Studnicki](#), [L. Glaser](#), [A. Görlich](#), [J. Henson](#), [A. Ipsen](#), [T. Jonsson](#), [J. Jurkiewicz](#), [N. Klitgaard](#), [A. Kreienbuehl](#), [J. Laiho](#), [T. Sotiriou](#), [Y. Watabiki](#), [S. Weinfurtner](#), [J. Wheeler](#) ...

Key ingredients of the CDT approach:

- representing curved spacetimes by piecewise flat triangulations makes the path integral well defined at an intermediate ("regularized") stage

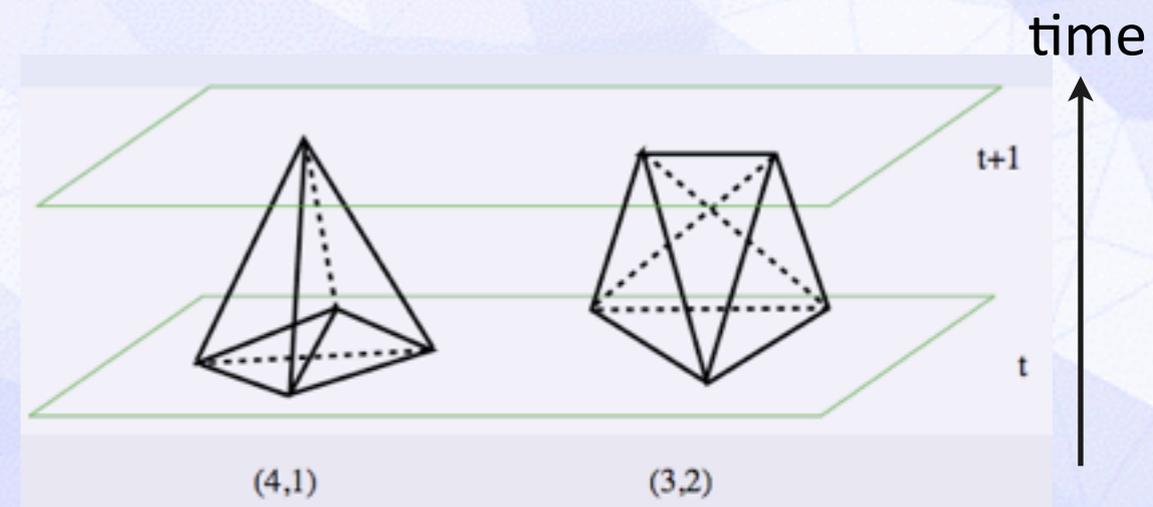


approximating a given *classical* curved surface through triangulation



Quantum Theory: approximating the space of *all* curved geometries by a space of triangulations

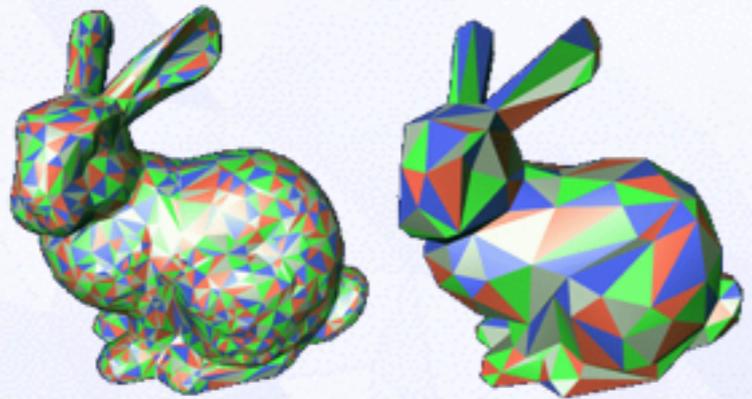
- crucial to obtain a semiclassical limit: spacetimes must have causal structure
- crucial in $d = 4$: nonperturbative comput. tools (Monte Carlo simulations) to extract quantitative results



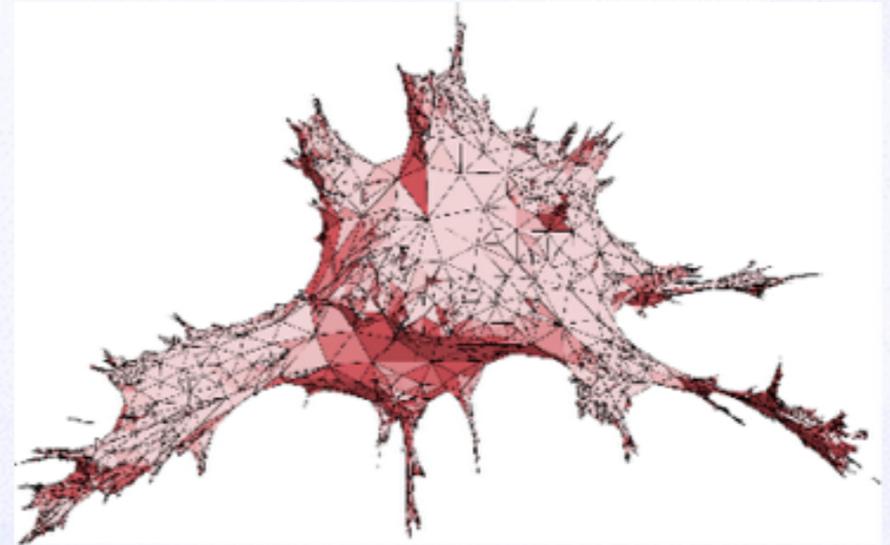
simplicial 4d building blocks of CDT

Key ingredients of the CDT approach:

- representing curved spacetimes by piecewise flat triangulations makes the path integral well defined at an intermediate ("regularized") stage

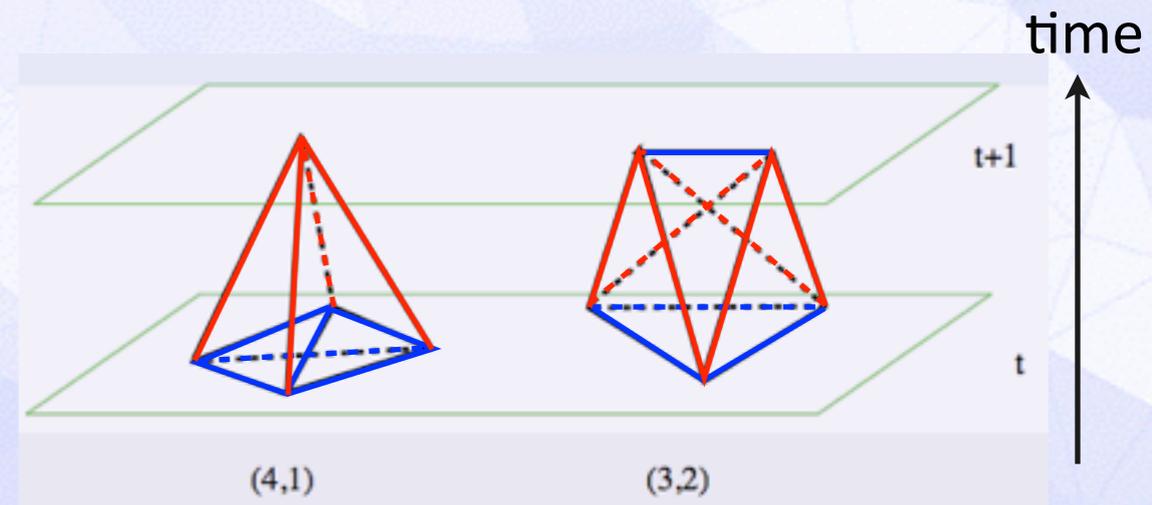


approximating a given *classical* curved surface through triangulation



Quantum Theory: approximating the space of *all* curved geometries by a space of triangulations

- crucial to obtain a semiclassical limit: spacetimes must have causal structure
- crucial in $d = 4$: nonperturbative comput. tools (Monte Carlo simulations) to extract quantitative results



- blue — spacelike edge, squared length a^2
- red — timelike edge, squared length $-\alpha a^2$, $\alpha > 0$

What makes CDT Quantum Gravity unique?

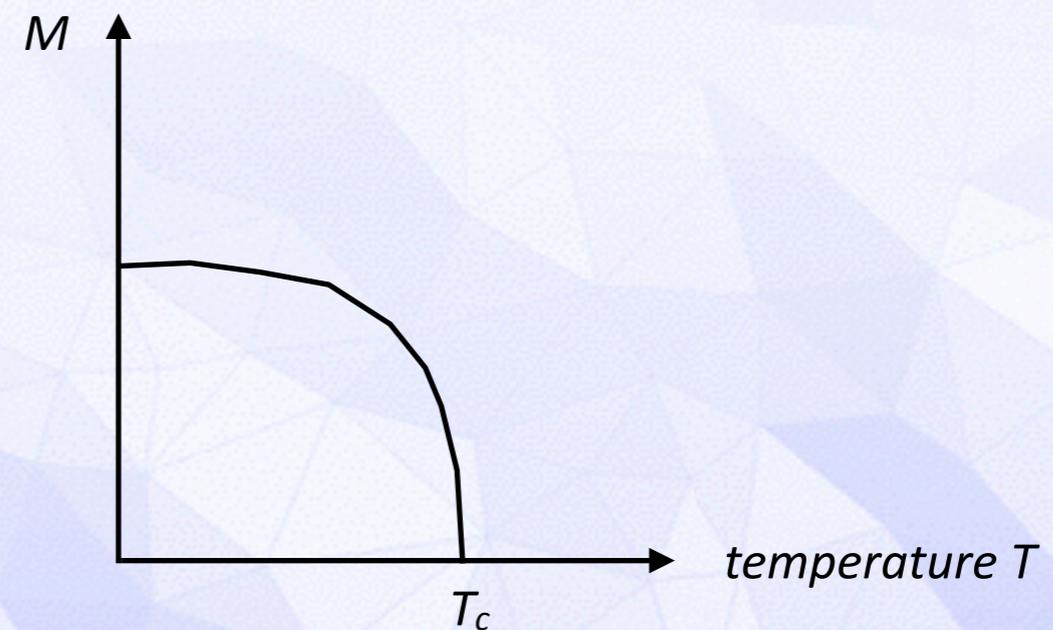
Imagine you wanted to do a nonperturbative path integral (PI) ...

- usual problem: cannot evaluate complex PI and there is no Wick rotation - do Euclidean QG instead, i.e. $\int Dg \exp(-S^{\text{eu}})$?
 - ☑ CDT has a well-defined analytic continuation; “Wick-rotated” Lorentzian PI is *not* equivalent to the Euclidean PI
- usual problem: there are redundancies because of diffeomorphism or other gauge symmetries, leading to unwanted divergences
 - ☑ CDT has no residual gauge symmetries, works with geometries
- frequent problem: PI highly divergent, no unique renormalization
 - ☑ number of configurations in CDT exponentially bounded
- frequent problem: cannot do any computations, cannot evaluate PI
 - ☑ CDT amenable to MC simulations; quantitative results, falsifiable!
- usual problem: why should PI lead to a unitary theory?
 - ☑ CDT reflection-positive w.r.t. discrete “proper time”, hence unitary!

The phase structure of CDT

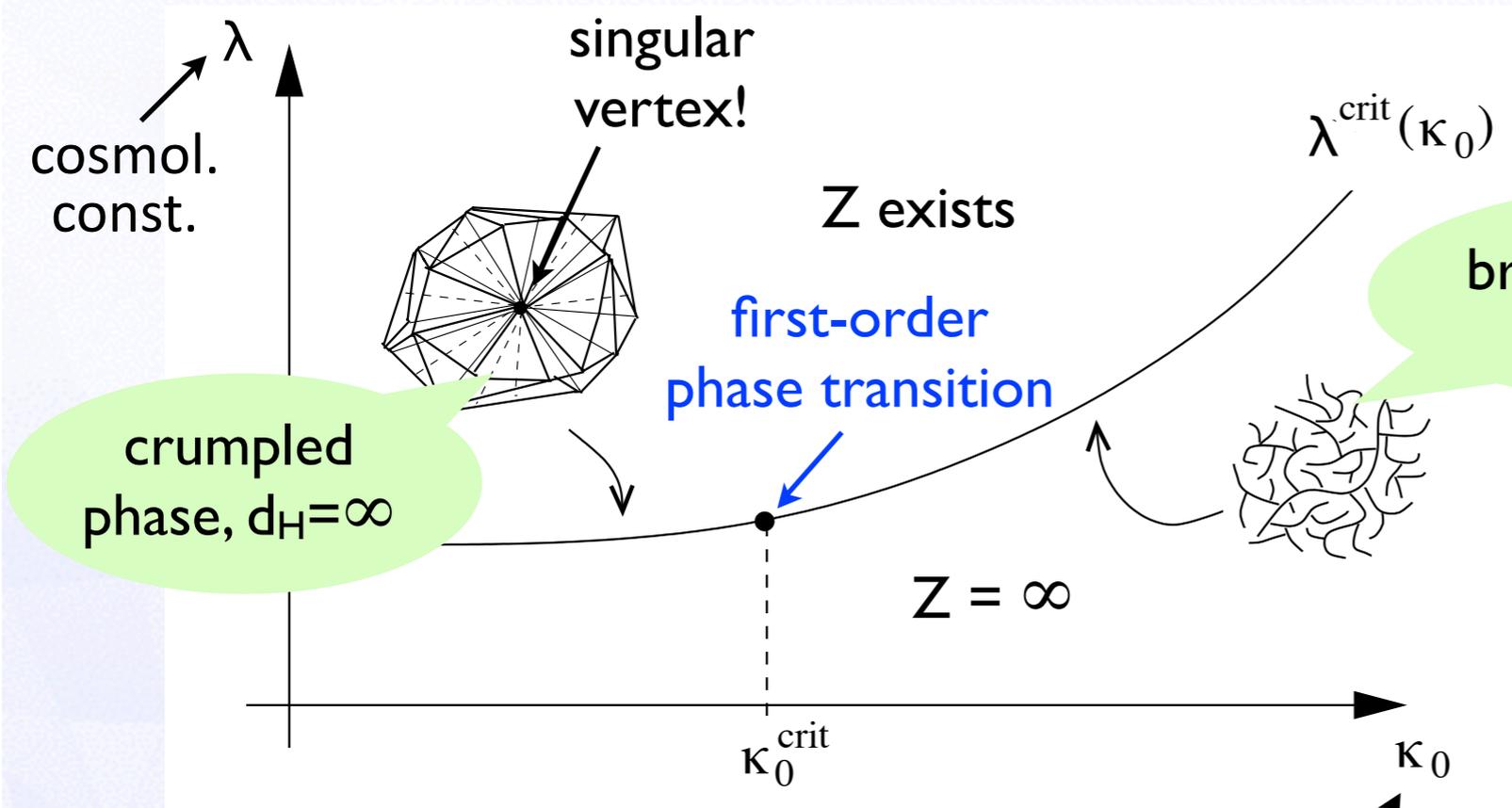
The phase space of CDT is spanned by the bare coupling constants appearing in the Wick-rotated weight factors $\exp(-S^{EH})$. Different phases, separated by phase transitions, display qualitatively different behaviour.

For a dynamical system of intrinsic geometry like CDT, what are good order parameters, characterizing the phases and the phase transitions between them?



Example: ferromagnetic (magnetic moments lined up) to paramagnetic transition in a magnet, with magnetization M as order parameter

DT phase diagram: nonperturbative surprises



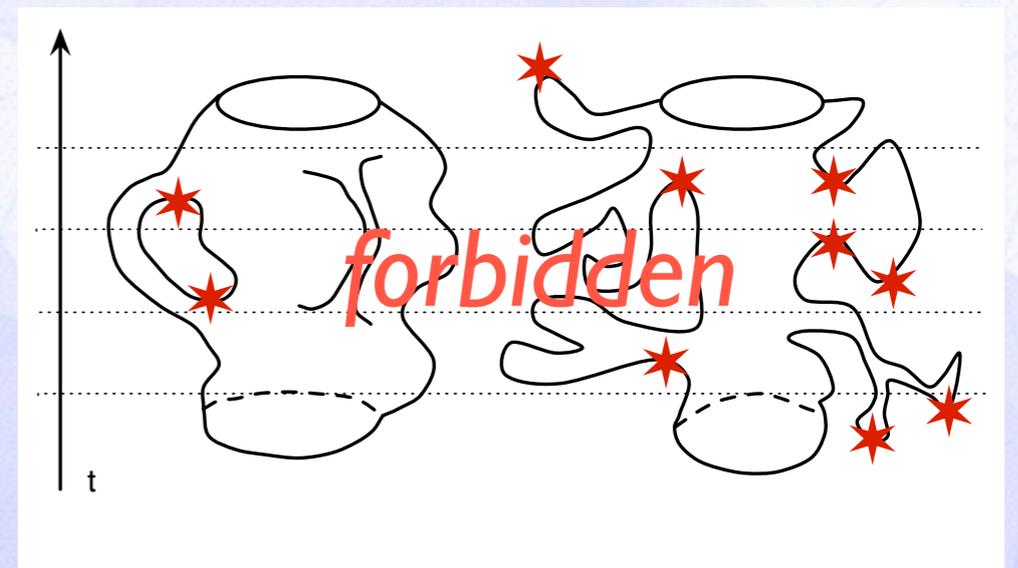
branched-polymer phase, $d_H=2$

Degenerate behaviour appears to be generic.

phase diagram DT

$\kappa_0 \sim 1/G_N^{\text{bare}}$

CDT: imposing a well-behaved causal structure on path integral configurations (suppressing spatial topology change) modifies this just enough to allow for interesting continuum limits.



N.B.: singular branching points ★

Phase diagram of CDT quantum gravity in 4D

The CDT gravitational action is *simple*:

$$S_{\text{eu}}^{\text{Regge}} = -\kappa_0 N_2 + N_4 (c\kappa_0 + \lambda) + \Delta (2N_4^{(4,1)} + N_4^{(3,2)})$$

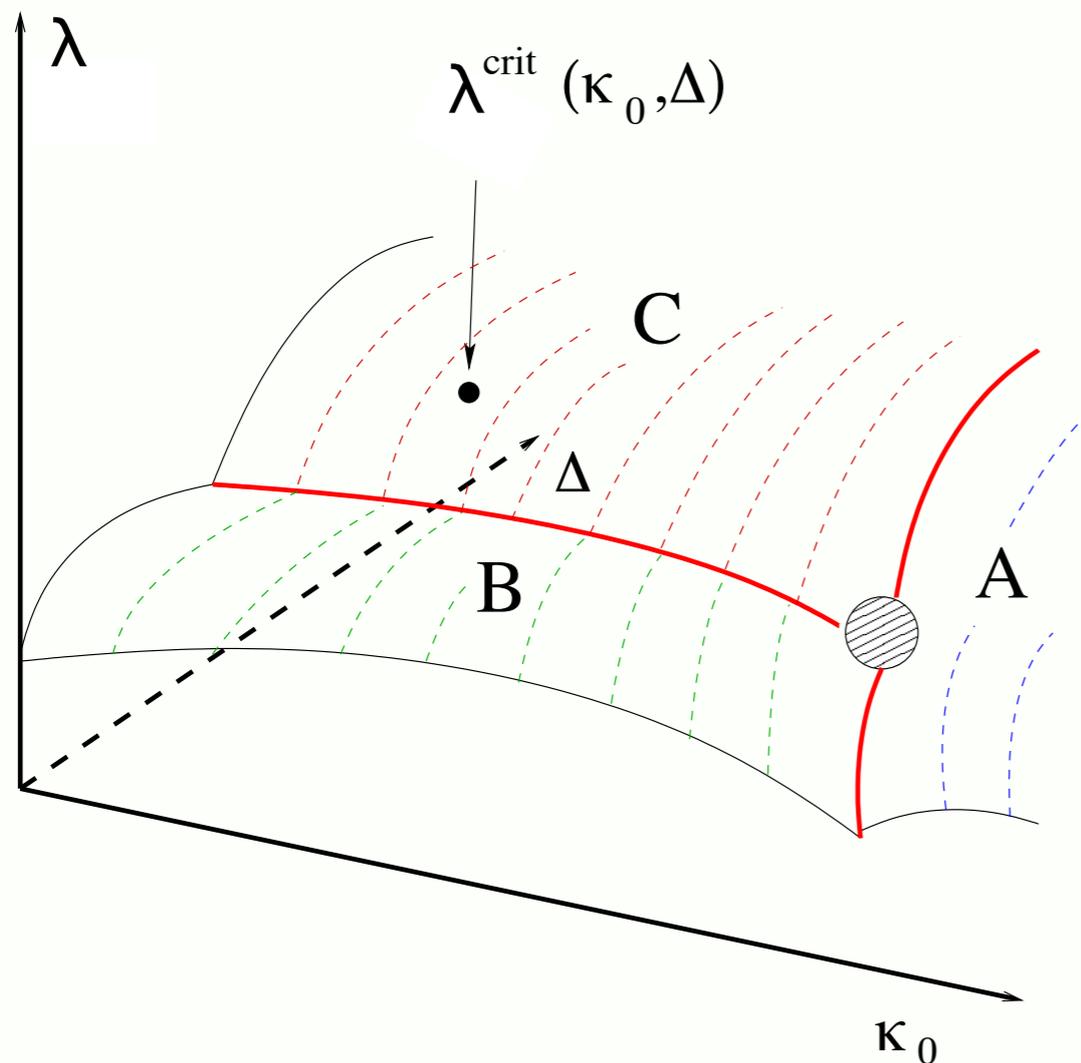
$\lambda \sim$ cosmological constant

$\kappa_0 \sim 1/G_N$ inverse Newton's constant

$\Delta \sim$ relative time/space scaling $\Delta(\alpha)$

$c \sim$ numerical constant, >0

$N_i \sim$ # of triangular building blocks of dimension i



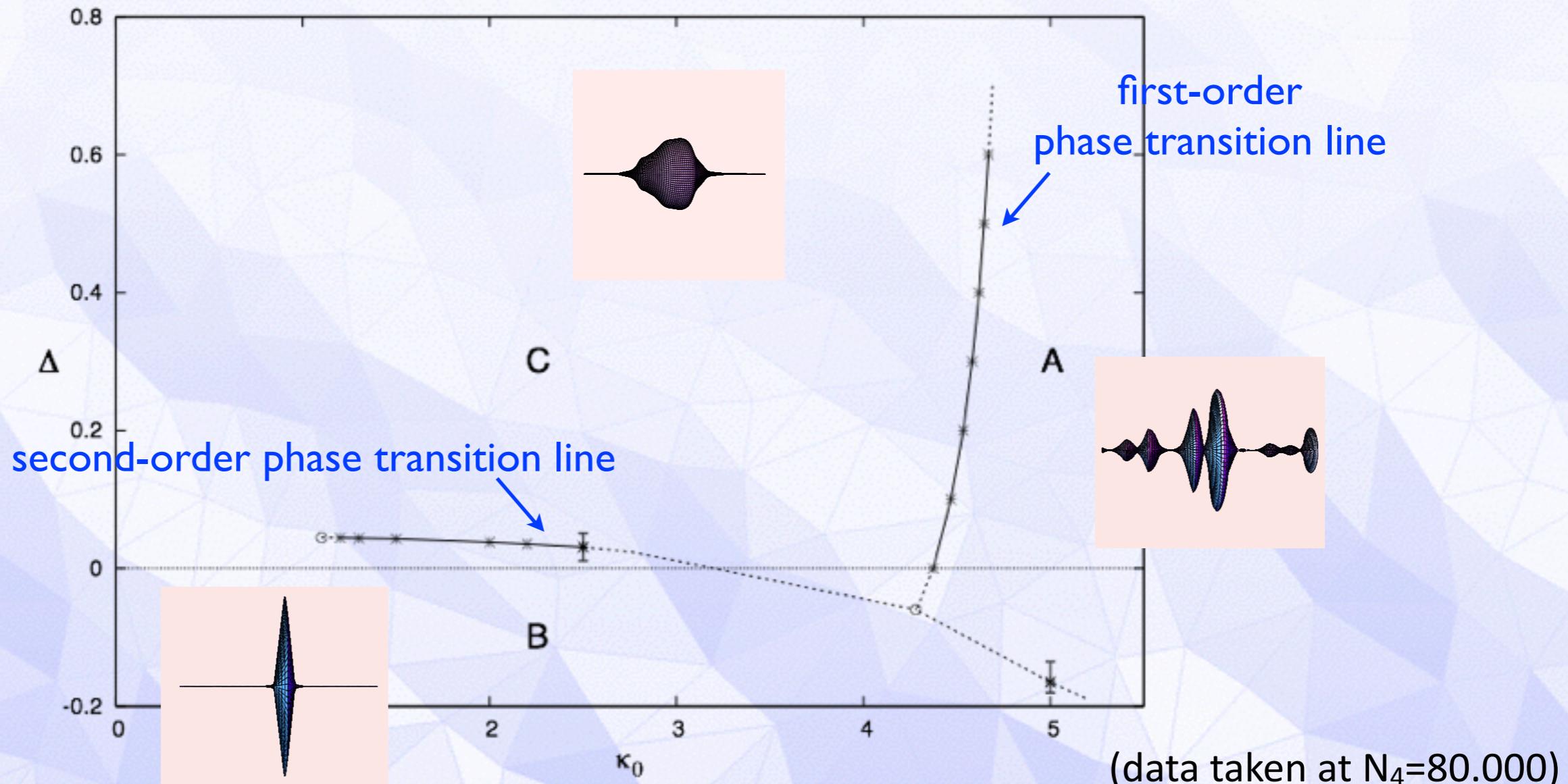
The partition function is defined for $\lambda > \lambda^{\text{crit}}(\kappa_0, \Delta)$;
 approaching the critical surface from above = taking infinite-volume limit.
 red lines \sim phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;

J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413)

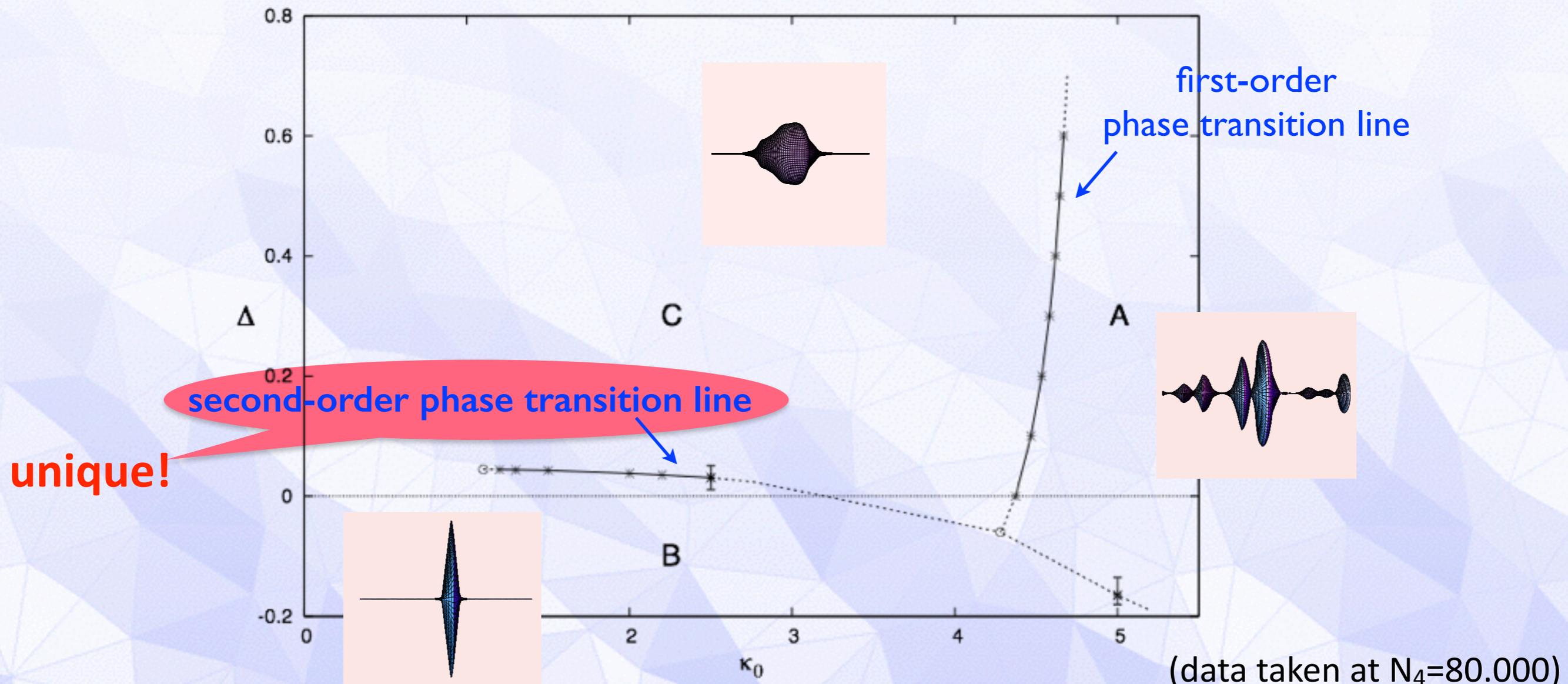
Phase diagram of CDT quantum gravity I

Unlike DT, CDT exhibits a phase of extended geometry with Hausdorff dimension 4. On the hypersurface $\lambda = \lambda^{\text{crit}}$, the “volume profile” $\langle V_3(t) \rangle$ of the dynamically generated quantum universe characterizes the phase. Only “phase C” has a large-scale limit compatible with GR.



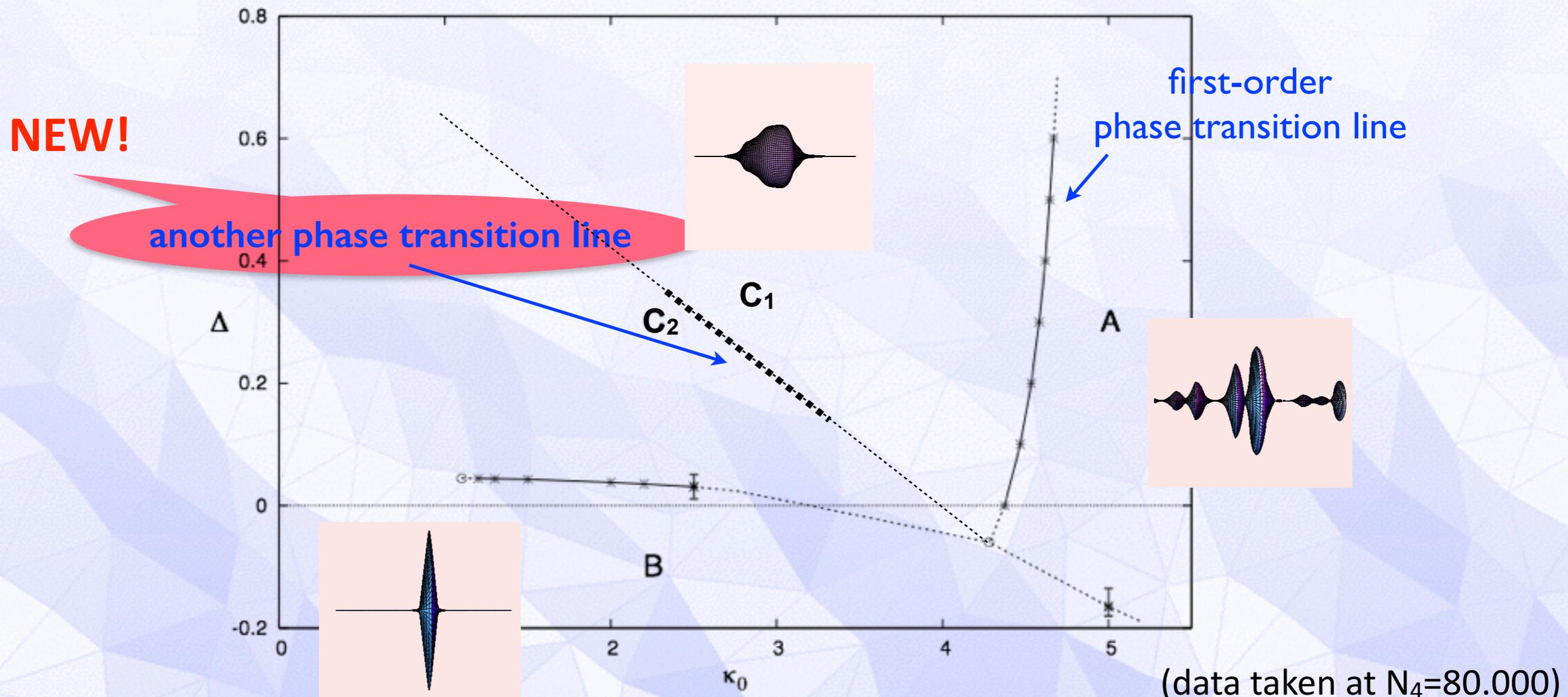
Phase diagram of CDT quantum gravity I

Unlike DT, CDT exhibits a phase of extended geometry with Hausdorff dimension 4. On the hypersurface $\lambda = \lambda^{\text{crit}}$, the “volume profile” $\langle V_3(t) \rangle$ of the dynamically generated quantum universe characterizes the phase. Only “phase C” has a large-scale limit compatible with GR.



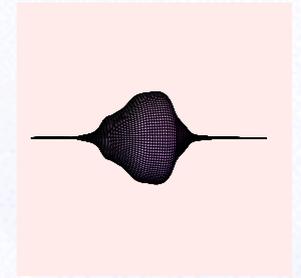
Phase diagram of CDT quantum gravity II

Recent simulations, using a small time extension of just two time steps have revealed that there is yet another transition line, dividing phase C into C_1 and C_2 (also called “phase D” or “bifurcation phase”)!



The effective transfer matrix for CDT

- instead of simulating the “entire universe” (in phase C), with $\Delta t=40\sim 80a$, consider just a single time step $\Delta t=a$



- the (matrix elements of the) associated full transfer matrix are

$$\langle T^{(3)}(t+a) | \mathbf{M} | T^{(3)}(t) \rangle = \sum_{T: T^{(3)}(t) \rightarrow T^{(3)}(t+a)} \frac{1}{C(T)} e^{-S^{\text{Regge}}[T]}$$

- remarkably, it has been shown that a much simpler object, the reduced or “effective” transfer matrix M , which only keeps track of the three-volume V_3 at fixed t , suffices to reconstruct all previous results on the average volume profile and its quantum fluctuations

$$\langle n | M | m \rangle \propto e^{-L_{\text{eff}}[n,m]}, \quad n = V_3(t+a), \quad m = V_3(t)$$

- analyzing the effective Lagrangian L_{eff} in phases B and C led to the discovery of the new phase transition

How do the phases C_1 and C_2 differ?

In phase C_1 (the old “de Sitter” phase) one finds to good precision

$$\langle n | M_{C_1} | m \rangle = e^{-\frac{1}{\Gamma} \left[\frac{(n-m)^2}{(n+m)} + \mu(n+m)^{1/3} - \lambda(n+m) \right]}$$

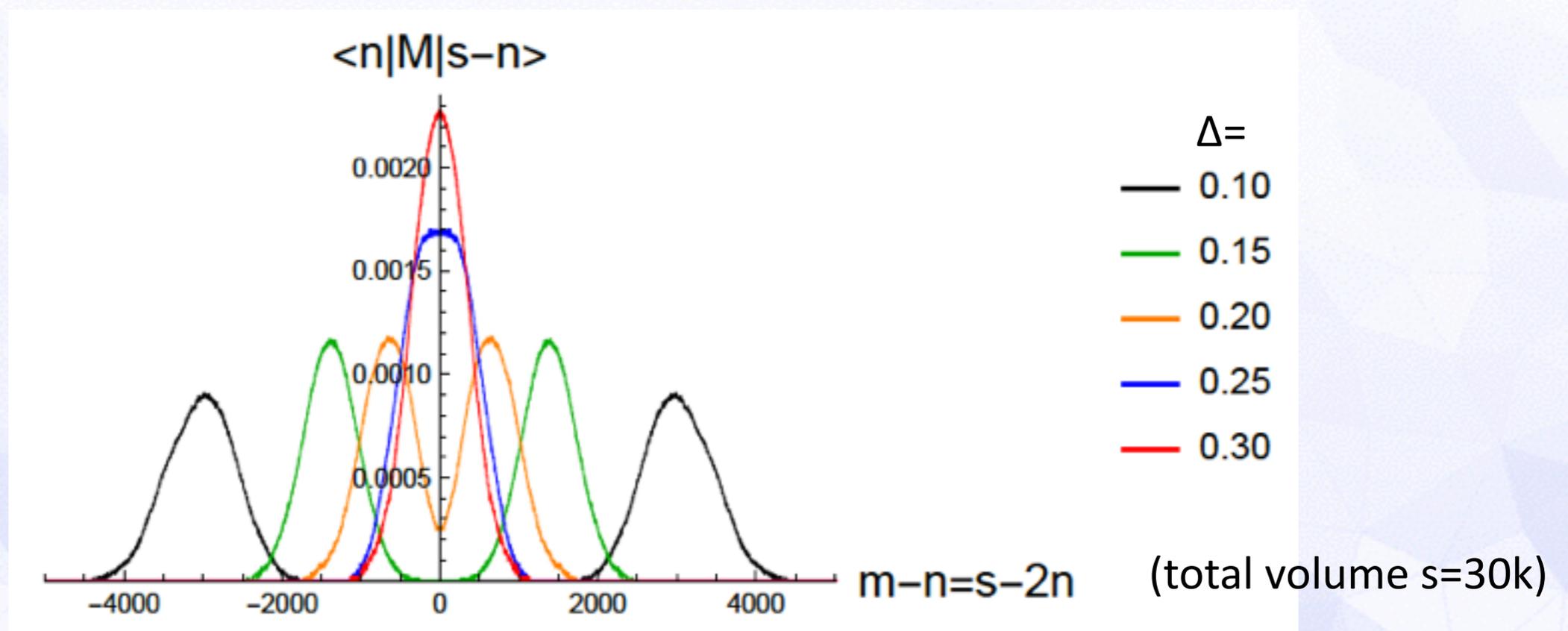
for parameters Γ, μ, λ . This can be directly compared to the minisuper-space action à la Hartle/Hawking generating de Sitter space, namely,

$$S_{\text{mini}} = \frac{1}{24\pi G_N} \int dt \sqrt{g_{tt}} \left(\frac{g^{tt} \dot{V}_3^2(t)}{V_3(t)} + k_2 V_3^{1/3}(t) - \lambda V_3(t) \right)$$

Instead, in the new phase C_2 and in phase B one finds a double-peak structure as function of the difference of the neighbouring 3-volumes:

$$\langle n | M_{C_2} | m \rangle = \left(e^{-\frac{1}{\Gamma} \frac{(n-m-c[n+m])^2}{(n+m)}} + e^{-\frac{1}{\Gamma} \frac{(n-m+c[n+m])^2}{(n+m)}} \right) e^{-\frac{1}{\Gamma} [\mu(n+m)^{1/3} - \lambda(n+m)]}$$

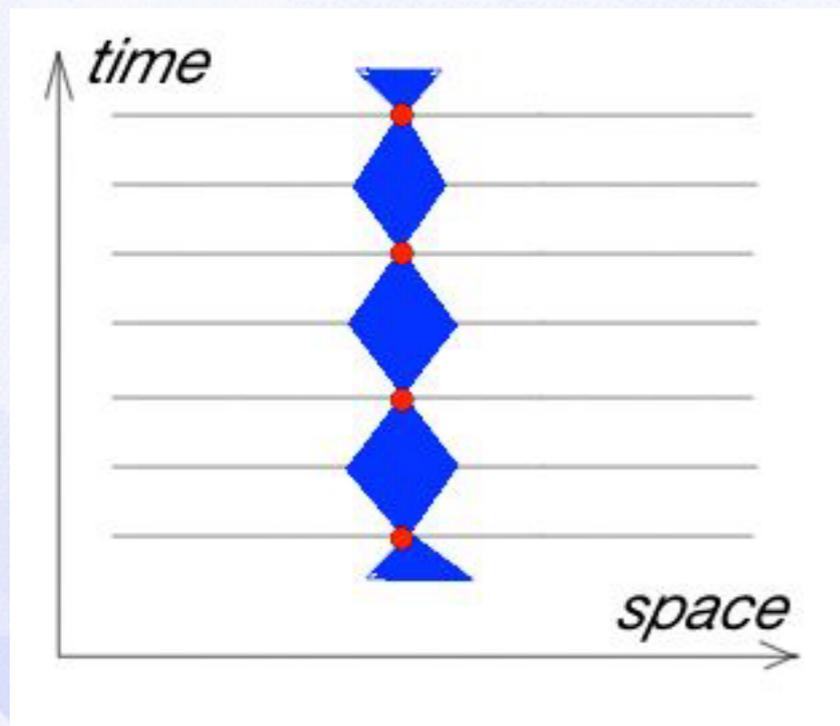
The measured matrix elements of the CDT effective transfer matrix: as the coupling Δ is lowered from 0.3 to 0.1 ($\kappa_0=2.2$), at the new C_1 - C_2 phase transition the single peak changes to a double peak.



- Why do neighbouring slices prefer unequal volumes in phase C_2 ?
- Why was this phase transition not noticed before?
- What is a good order parameter to study the transition?
- What is the order of the transition?

The answers to these questions are all related

In phase C_2 a new geometric substructure appears, namely, a vertex v of very high order $O(v)$ on every second slice of integer time, reminiscent of what happened in the crumpled phase of DT. Crossing into phase B, only a pair of such vertices remains, with a “pancake” in between.



(schematic)

- Slices with high-order vertex have lower V_3 , leading to a modulation of the standard volume profiles.
- A good order parameter appears to be $|\max[O(v(t+1))]-\max[O(v(t))]|$.
- Tentatively, this is related to a breaking of homogeneity and isotropy of geometry.

Potentially, a new candidate for defining continuum gravity!

J. Ambjørn, D. Coumbe, J. Gizbert-Studnicki, J. Jurkiewicz, JHEP 1508 (2015) 033; D.Coumbe, J. Gizbert-Studnicki, J. Jurkiewicz, arXiv:1510.086; J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, N.Klitgaard, RL, to appear

Making contact with continuum physics

As emphasized, the triangulated lattice structure of CDT is part of a regularization, which to a large degree is arbitrary. For example, one might just as well have used square-shaped building blocks.

When talking about specific numerical results in (C)DT - pertaining to physics at the Planck scale or at larger macroscopic scales - one means results obtained in a scaling limit of infinitely many building blocks (using finite-size scaling); only then do they stand a chance of being universal and not just lattice artefacts. Of course, universality is a property that needs to be demonstrated.

This way of constructing a nonperturbative theory raises ...

Some important questions

- ▶ Is there a continuum limit where physical observables become independent of the UV cut-off and of regularisation “artefacts”?
- ▶ Do standard lattice renormalization methods apply?
- ▶ Can we confirm the presence of an ultraviolet fixed point as signalled in FRG studies of the asymptotic safety scenario?
- ▶ Does QG exist as a nontrivial QFT when the UV regulator is removed?
- ▶ What *is* the UV theory/completion?

These are relevant physical questions, but highly nontrivial in nonperturbative quantum gravity, where there is no a priori background metric or measuring grid, and “geometry” and “length” are generated *dynamically*. A “naïve” correlator $G(x,y)$ and associated correlation length are not well defined.

Some important questions

- ▶ Is there a continuum limit where physical observables become independent of the UV cut-off and of regularisation “artefacts”? ✓
- ▶ Do standard lattice renormalization methods apply? ✓
- ▶ Can we confirm the presence of an ultraviolet fixed point as signalled in FRG studies of the asymptotic safety scenario?
- ▶ Does QG exist as a nontrivial QFT when the UV regulator is removed?
- ▶ What *is* the UV theory/completion?

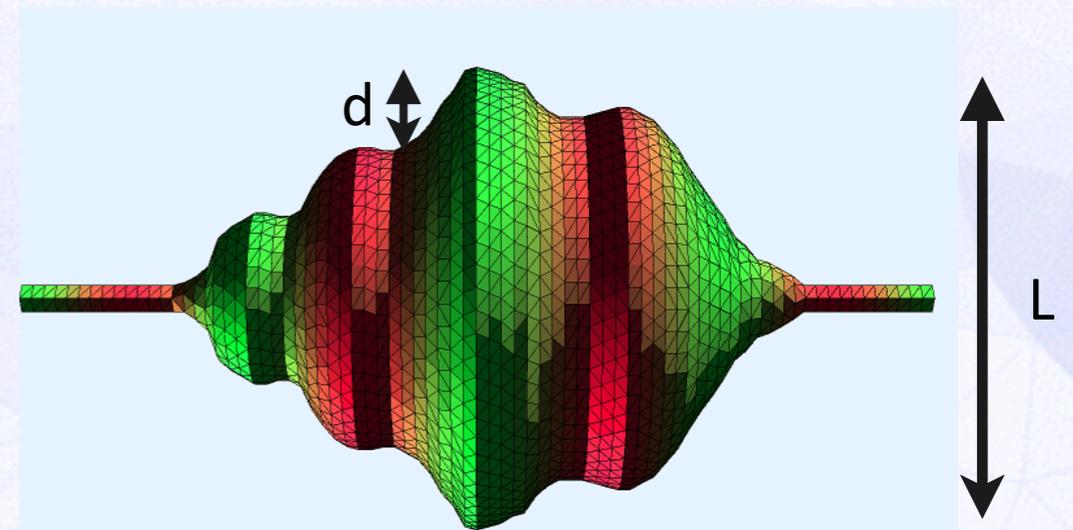
These are relevant physical questions, but highly nontrivial in nonperturbative quantum gravity, where there is no a priori background metric or measuring grid, and “geometry” and “length” are generated *dynamically*. A “naïve” correlator $G(x,y)$ and associated correlation length are not well defined.

Standard renormalization *can* be applied!

Having located lines of second-order transition points, we want to investigate the scaling behaviour of the theory in their vicinity.

We are interested in renormalization group (RG) flows probing ever shorter distances. Since there is no correlation length immediately available, we let the linear lattice size $N_4^{1/4} \rightarrow \infty$ while keeping physics constant.

Idea: use the length scales associated with the dynamically generated de Sitter universe in CDT to define physical “yardsticks”.



Under simplifying assumptions this has enabled us to perform a first explicit study of such RG flows near the B-C₂ transition. (J. Ambjørn, A. Görlich, J. Jurkiewicz, A. Kreienbühl, RL, CQG 31 (2014) 1650)

An analogous study needs to be done near the C₁-C₂ phase transition.

Observables, observables, observables ...

The RG study highlights the need for more *observables*.

In nonperturbative quantum gravity, observables must be invariantly defined, without reference to coordinates or any background (unless obtained dynamically). Standard QFT observables can sometimes be adapted to be meaningful in the functional integral over geometry.

Example: a two-point function $G_2(x,y)$ is not a good observable, since we cannot fix specific points x and y in the path integral, but

$$G_2(r) = \int \mathcal{D}[g_{\mu\nu}] e^{-S[g_{\mu\nu}]} \int dx dy \sqrt{g(x)g(y)} G_2(x,y) \delta(r - d_{g_{\mu\nu}}(x,y)) \text{ is.}$$

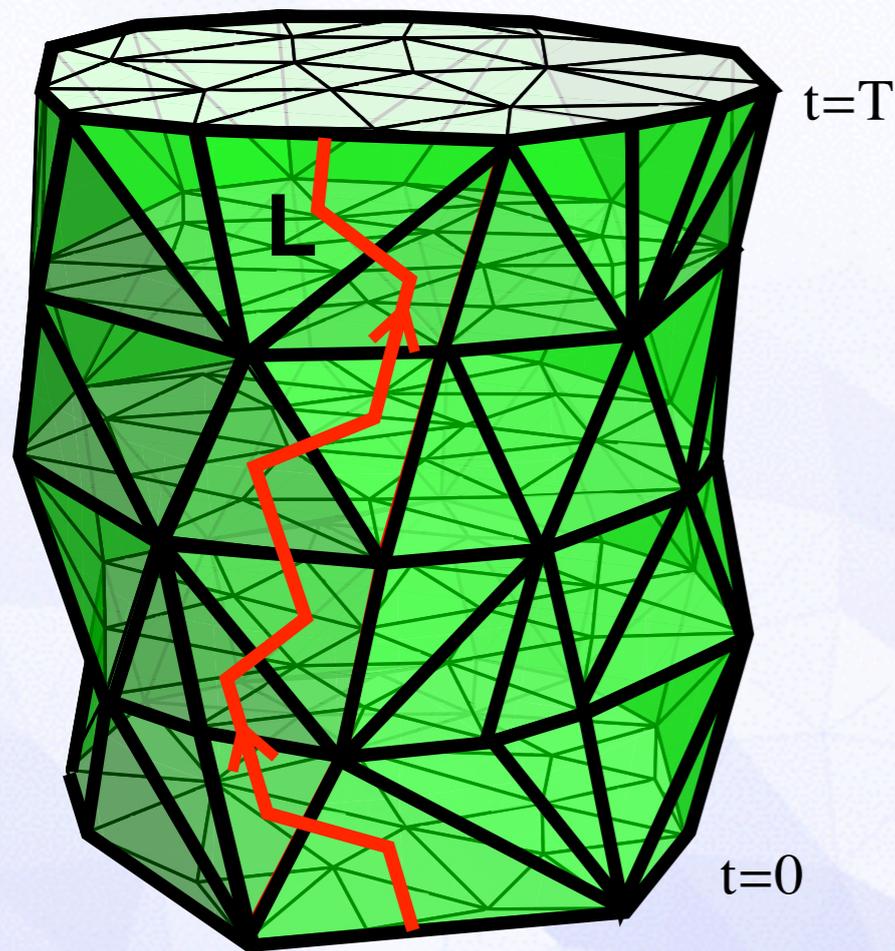
geodesic distance

Idea: can we define observables involving gravitational Wilson loops

$$W_\gamma(\Gamma) = \text{Tr} \mathcal{P} \exp \oint_\gamma \Gamma \leftarrow \text{Levi-Civita connection}$$

(referring to entire curves γ , not just points) in a similar way, to obtain curvature information about the underlying quantum spacetime?

Defining a Wilson loop observable in CDT



One lets the loop γ coincide with the world line L of a particle moving forward in time. The loops wind once around the compactified time direction of the triangulated spacetimes, which have topology $S^1 \times S^3$, as usual.

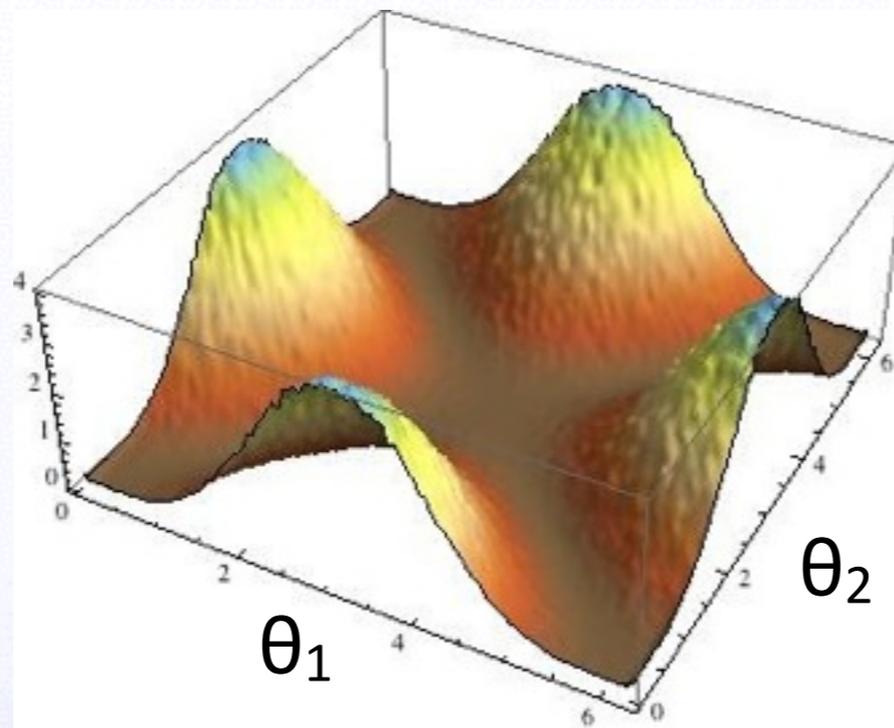
Correspondingly, one adds to the pure-gravity action a term for a free massive point particle

$$S^{\text{P.P.}} = m \int dl \quad \rightarrow \quad S_{\text{CDT}}^{\text{P.P.}} = m_0 N_L$$

where N_L = number of four-simplices along L .

In Monte Carlo simulations for the combined gravity-particle system it is straightforward to compute Wilson lines and extract two coordinate-independent trace invariants, angles θ_1, θ_2 labelling $SO(4)$ -conjugacy classes.

J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, Phys. Rev. D92 (2015) 2, 024013



The measured distribution of the invariant angles θ_i shown here is in almost perfect agreement with the theoretical result one obtains from assuming a uniform distribution of the holonomy matrices over the group manifold $SO(4)$,

$$P(\theta_1, \theta_2) = \frac{1}{\pi^2} \sin^2 \left(\frac{\theta_1 + \theta_2}{2} \right) \sin^2 \left(\frac{\theta_1 - \theta_2}{2} \right).$$

Despite being a coordinate-free approach, holonomies and Wilson loops are straightforward to define and implement in CDT, without significant discretization effects. The large loops considered here have not uncovered interesting information about (averaged) curvature yet, but other more local curvature observables are being developed.

Summary and conclusions

Despite its very simple set-up and having just two tunable couplings, CDT has an amazingly rich phase structure, with at least one, possibly two lines of second-order phase transitions.

CDT quantum gravity enjoys a number of nice features that enable it to reach where other nonperturbative approaches do not, and obtain some highly nontrivial outcomes. The recent results I focused on today underline the promise of this candidate theory of QG.

To conclude, good old quantum field theory, without exotic ingredients and adapted to the case of dynamical geometry, may provide the answer to quantum gravity after all!

Work is in progress on identifying and measuring more observables, to complete the theory further and eventually predict observable effects.



Causal Dynamical Triangulations: a progress report

GR21, New York City,
11 Jul 2016

Thank you!