Dynamical Tidal Response of a Rotating Neutron Star

PL & Eric Poisson, Phys Rev D 92, 124041 (2015)

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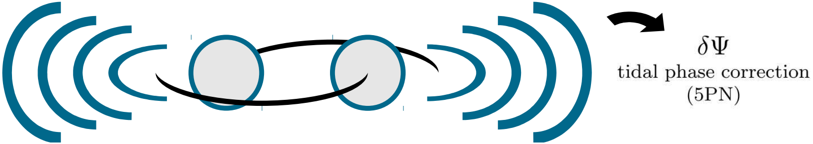
Tides in Compact Binaries

Tidal deformations impact the phase of the gravitational waves produced by the inspiral of compact bodies in a binary system.



- A body's tidal deformability is measured by its Love numbers, which encode dependence on internal structure

The Love numbers are potentially measurable with LIGO, and are useful for...

- Probing the NS EoS [*Flanagan & Hinderer 0709.1915*]
- I-Love-Q relations [*Yagi & Yunes 1302.4499*]

Relativistic Tides

A relativistic theory of tidal deformations has been developed to treat tides in compact binaries.

[*Damour & Nagar 0906.0096, Binnington & Poisson 0906.1366*]

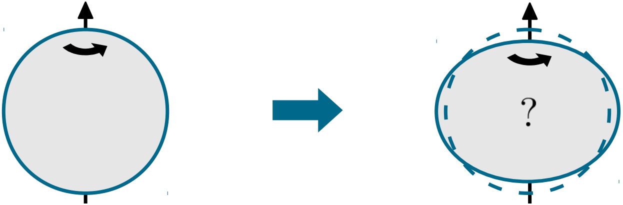
- In GR, there are two types of tidal fields
  - The gravitoelectric field $\mathcal{E}_{ab}$ raises mass multipoles
  - The gravitomagnetic field $\mathcal{B}_{ab}$ induces current multipoles

We need to incorporate spin in this framework, for astrophysical relevance. [*Pani* et al. *1503.07365, PL & Poisson 1503.07366*]

- Non-linearity of the EFE produces coupling between the body's angular momentum and the tidal field
- The coupling is analytically tractable at $\mathcal{O}(1)$ in the spin

Tides on a Spinning NS

Consider a rigidly rotating NS subject to a stationary gravitomagnetic tidal field $\mathcal{B}_{ab}$.



How is the NS affected by the applied tide?

Tides on a Spinning NS

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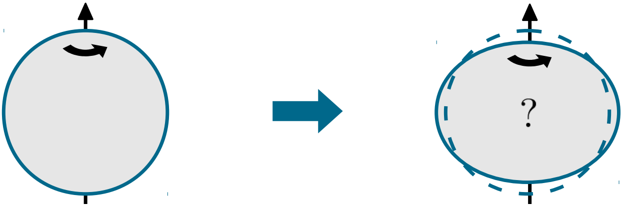


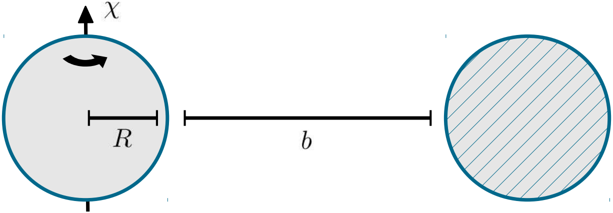
The NS responds dynamically to the tidal field.

- $\mathcal{B}_{ab}$ induces time-dependent internal fluid motions
- Interior metric variables also acquire a time dependence
- The dynamical response varies on the timescale of the NS rotation period

Nonetheless, the external metric remains perfectly stationary.

Stationary Tides

During the inspiral stage, a slowly rotating NS and its binary companion are well-separated.



$b \gg R$ implies...

- Small tides $\delta R/R \ll 1$ – work to $\mathcal{O}(1)$ in tides
- $T_{\rm int} \sim T_{\rm rot} \ll T_{\rm orb}$ – stationary tides

$\chi \ll 1$ implies...

- Slow rotation – work to $\mathcal{O}(1)$ in spin
- Linearized Kerr background – universal exterior geometry

Gravitomagnetic Tides with Spin

Generic stationary gravitomagnetic tidal moment $\mathcal{B}_{ab}$ sourced by distant matter.

Tidal environment →

Gravitomagnetic Tides with Spin

$$\underset{(\ell=2)}{\mathcal{B}_{ab}} \quad \times \quad \underset{(\ell=1)}{\chi^a}$$

$$\Downarrow$$

$$(\ell=1), \quad (\ell=2), \quad (\ell=3)$$

External Solution

- Construct metric ansatz with all possible spin-coupled tidal moments
- EFE determine metric functions up to integration constants (Love numbers)
- Solution is stationary

Tidal environment →

Gravitomagnetic Tides with Spin

Internal Solution

- Specify (barotropic) EoS and impose vorticity conservation
- Solve the EFE-Euler system for metric and fluid variables
- Solution is time-dependent

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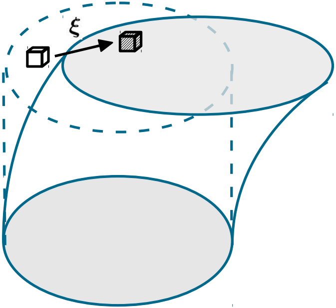
Interior and exterior metrics still match across surface!

Surface

Tidal environment →

Circulation Theorem

For a barotropic fluid, the circulation theorem says vorticity $\omega_{\alpha\beta} = \nabla_{[\alpha}\left(hu_{\beta]}\right)$ is conserved along the fluid worldines.



- The tidal perturbation displaces fluid elements by $\boldsymbol{\xi}$, and perturbs their velocity $\boldsymbol{u}$ by $\delta\boldsymbol{u}$

For a perturbation switched on adiabatically, the circulation theorem says

$$\Delta\omega_{\alpha\beta} = 0 \ .$$

Conservation of Vorticity

Let's look at the consequences of $\Delta\omega_{\alpha\beta} = 0$ for the angular components of the fluid variables.

At $\mathcal{O}(0)$ in the spin,

- $\omega_{\alpha\beta} = 0$
- $\Delta\omega_{\alpha\beta} = 0 \;\Rightarrow\; \xi_A \propto t$
- $\Rightarrow\; \delta u_A$ stationary
- Irrotational fluid motions are established

At $\mathcal{O}(1)$ in the spin,

- $\omega_{\alpha\beta} \neq 0$
- $\Delta\omega_{\alpha\beta} = 0 \;\Rightarrow\; \delta u_A \propto \xi_A$
- $\Rightarrow\; \delta u_A \propto t$
- Dynamical fluid motions are established

The EFE pass on the linear time dependence to other metric and fluid variables.

Driven Harmonic Oscillator

Consider an analogy with a driven harmonic oscillator.

- The displacement $\boldsymbol{\xi}$ satisfies an inhomogeneous DE
- We suppose the driving force $\boldsymbol{F}$ is stationary

$$\boldsymbol{\xi}(t, \mathbf{x}) = \sum_\lambda a_\lambda(t)\mathbf{z}_\lambda(\mathbf{x}) \ , \qquad f_\lambda = \int \mathbf{F} \cdot \mathbf{z}_\lambda \ d^3x$$

Mode equation: $$\ddot{a}_\lambda + {\omega_\lambda}^2 a_\lambda = f_\lambda$$

Solutions: $$a_\lambda = {\omega_\lambda}^{-2} f_\lambda \quad \text{for} \ \ \omega_\lambda \neq 0$$

$$a_\lambda = \frac{1}{2} f_\lambda t^2 \quad \text{for} \ \ \omega_\lambda = 0$$

Zero-frequency modes give rise to displacements $\boldsymbol{\xi} \propto t^2$, which imply velocities $\boldsymbol{u} \propto t$.

Zero-Frequency Modes

We propose that zero-frequency modes in NSs are driving the dynamical tidal response.

- What are these zero-frequency modes?

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- What are these zero-frequency modes?

In non-rotating stars, there exist zero-frequency modes called r-modes and g-modes.

- At $\mathcal{O}(0)$ in the spin,
  - The overlap integral between $\mathcal{B}_{ab}$ and the r-modes or g-modes is zero
- At $\mathcal{O}(1)$ in the spin,
  - The overlap integral is non-zero!

The dynamical response here may be related to the rotational modes of relativistic stars.

[*Andersson gr-qc/9706075, Lockitch* et al. *gr-qc/0008019*]

Timescale for Onset of Instability

The linear growth of the fluid velocity is a dynamical instability in our $\mathcal{O}(1)$ perturbative expansion in tides and spin.

- How long does it for the perturbation theory to break down, i.e. for $\delta u_A = u_A$?

$$T = 0.25 \left(\frac{12\ \mathrm{km}}{R}\right) \left(\frac{1.4\ M_\odot}{M'}\right)^{3/2} \left(\frac{b}{50\ \mathrm{km}}\right)^{7/2} \mathrm{s}$$

- Compare with...

$$T_{\mathrm{dynamical}} = 6 \times 10^{-4} \left(\frac{1.4\ M_\odot}{M}\right)^{1/2} \left(\frac{R}{12\ \mathrm{km}}\right)^{3/2} \mathrm{s}$$

$$T_{\mathrm{viscous}} = 9 \times 10^{7} \left(\frac{1.4\ M_\odot}{M}\right) \left(\frac{T}{10^9\ \mathrm{K}}\right)^{2} \left(\frac{R}{12\ \mathrm{km}}\right)^{5} \mathrm{s}$$

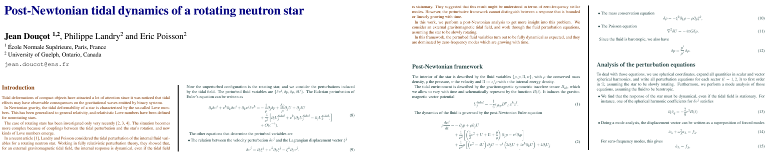
[*Friedman & Stergioulas 2013*]

Summary

When subjected to a stationary gravitomagnetic tidal field, a slowly rotating NS responds dynamically: internal fluid and metric variables acquire a linear time dependence.

- The dynamical response follows from conservation of vorticity at $\mathcal{O}(1)$ in the spin
- It is driven by the zero-frequency modes of the fluid

A post-Newtonian analysis confirms this explanation, and provides further insight into the instability.



Post-Newtonian tidal dynamics of a rotating neutron star

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Introduction

Tidal deformations of compact objects have attracted a lot of attention since it was noticed that tidal effects may have observable consequences on the gravitational waves emitted by binary systems.

In Newtonian gravity, the tidal deformability of a star is characterized by the so-called Love numbers. This has been generalized to general relativity, and relativistic Love numbers have been defined for nonrotating stars.

The case of rotating stars has been investigated only very recently [2, 3, 4]. The situation becomes more complex because of couplings between the tidal perturbation and the star's rotation, and new kinds of Love numbers emerge.

In a recent article [1], Landry and Poisson considered the tidal perturbation of the internal fluid variables for a rotating neutron star. Working in fully relativistic perturbation theory, they showed that, for an external gravitomagnetic tidal field, the internal response is dynamical, even if the tidal field is stationary. They suggested that this result might be understood in terms of zero-frequency stellar modes. However, the perturbative framework cannot distinguish between a response that is bounded or linearly growing with time.

In this work, we perform a post-Newtonian analysis to get more insight into this problem. We consider an external gravitomagnetic tidal field, and work through the fluid perturbation equations, assuming the star to be slowly rotating.

In this framework, the perturbed fluid variables turn out to be fully dynamical as expected, and they are dominated by zero-frequency modes which are growing with time.

Post-Newtonian framework

The interior of the star is described by the fluid variables $\{\rho, p, \Pi, \boldsymbol{v}\}$, with ρ the conserved mass density, p the pressure, $\boldsymbol{v}$ the velocity and $\Pi := \epsilon/\rho$ with ϵ the internal energy density.

The tidal environment is described by the gravitomagnetic symmetric tracefree tensor B_{ab}, which we allow to vary with time and schematically represent by the function $B(t)$. It induces the gravitomagnetic vector potential

$$U_j^{tidal} = -\frac{1}{6}\epsilon_{jkp}B^p{}_l\, x^k x^l. \quad (1)$$

The dynamics of the fluid is governed by the post-Newtonian Euler equation

$$\begin{aligned}\rho\frac{dv^j}{dt} = &-\partial_j p + \rho\partial_j U \\ &+ \frac{1}{c^2}\left[\left(\frac{1}{2}v^2 + U + \Pi + \frac{p}{\rho}\right)\partial_j p - v^j\partial_t p\right] \\ &+ \frac{1}{c^2}\rho\left[\left(v^2 - 4U\right)\partial_j U - v^j\left(3\partial_t U + 4v^k\partial_k U\right) + 4\partial_t U_j\right.\end{aligned} \quad (2)$$

Now the unperturbed configuration is the rotating star, and we consider the perturbations induced by the tidal field. The perturbed fluid variables are $\{\delta v^j, \delta p, \delta\rho, \delta U\}$. The Eulerian perturbation of Euler's equation can be written as

$$\begin{aligned}\partial_t\delta v^j + v^k\partial_k\delta v^j + \partial_k v^j\delta v^k = &-\frac{1}{\rho}\partial_j\delta p + \frac{\delta\rho}{\rho}\partial_j U + \partial_j\delta U \\ &+ \frac{4}{c^2}\left[\partial_t U_j^{tidal} + v^k(\partial_k U_j^{tidal} - \partial_j U_k^{tidal})\right] \\ &+ O(c^{-4}).\end{aligned} \quad (8)$$

The other equations that determine the perturbed variables are

- The relation between the velocity perturbation δv^j and the Lagrangian displacement vector ξ^j

$$\delta v^j = \partial_t\xi^j + v^k\partial_k\xi^j - \xi^k\partial_k v^j. \quad (9)$$

- The mass conservation equation

$$\delta\rho = -\xi^k\partial_k\rho - \rho\partial_k\xi^k, \quad (10)$$

- The Poisson equation

$$\nabla^2\delta U = -4\pi G\delta\rho. \quad (11)$$

Since the fluid is barotropic, we also have

$$\delta p = \frac{p'}{\rho'}\delta\rho. \quad (12)$$

Analysis of the perturbation equations

To deal with those equations, we use spherical coordinates, expand all quantities in scalar and vector spherical harmonics, and write all perturbation equations for each sector ($l = 1, 2, 3$) to first order in Ω, assuming the star to be slowly rotating. Furthermore, we perform a mode analysis of those equations, assuming the fluid to be barotropic.

- We find that the response of the star must be dynamical, even if the tidal field is stationary. For instance, one of the spherical harmonic coefficients for δv^j satisfies

$$\partial_t\hat{v}_q = -\frac{2}{9}r^3 B(t) \quad (13)$$

- Doing a mode analysis, the displacement vector can be written as a superposition of forced modes

$$\ddot{a}_\lambda + \omega_\lambda^2 a_\lambda = f_\lambda. \quad (14)$$

For zero-frequency modes, this gives

$$\ddot{a}_\lambda = f_\lambda. \quad (15)$$